3 Statistical Process Control

Answers to Questions

3-1. In attribute control charts (such as *p*-charts and *c*-charts), the measures of quality are discrete values reflecting a simple decision criterion such as good or bad. The quality measures used in variable-control charts (such as $\bar{x}$-charts and *R*-charts) are continuous variables reflecting measurements such as weight, time, or volume.

3-2. An *R*-chart reflects the process variability, whereas an $\bar{x}$-chart indicates the tendency toward a mean value; thus, the two complement each other. That is, it is assumed that process average and variability must be in control for the process to be in control. When they are used together, the control limits are computed as $\bar{\bar{x}} + A_2\bar{R}$.

3-3. A pattern test is used to determine if sample values from a process display a consistent pattern that is the result of a nonrandom cause, even though control charts may show the process to be in control.

3-4. Width is determined by the size of the *z* value used; the smaller the value of *z*, the narrower the control limits.

3-5. A *c*-chart is used when it is not possible to determine a proportion defective (for a *p*-chart), for example, when counting the number of blemishes on a sheet of material. In a *p*-chart it must be possible to distinguish between individual defective and non-defective items.

3-6. Tolerances are product-design specifications required by the customer, whereas control limits are the upper and lower bands of a control chart indicating when a process is out of control.

3-7. Management usually selects 3-sigma limits because if the process is in control they want a high probability that the sample will fall within the control limits. With wider limits management is less likely to erroneously conclude that the process is out of control when points outside the control limits are due to normal, random variations.

3-8. Process control charts could be used to monitor service time in a restaurant, bank, hospital, store, etc.

3-9. For example, in a fast food restaurant a control chart could be used to measure service times, defective menu items, out of stock menu items, customer complaints, cleanliness, and order errors, among other things.

3-10. The process capability ratio $\left(C_p\right)$ indicates if the process is capable of meeting design specifications. The process capability index indicates if the process mean is off-center and has shifted toward the upper or lower design specifications.

3-11. $$C_p = \frac{\text{tolerance range}}{\text{process range}} = \frac{.14}{.14} = 1.00$$

The tolerance range and process range are equal so the process is capable but some defects will result.

Solutions to Problems

3-1. $\bar{p} = \dfrac{453}{(30)(100)} = 0.151;$

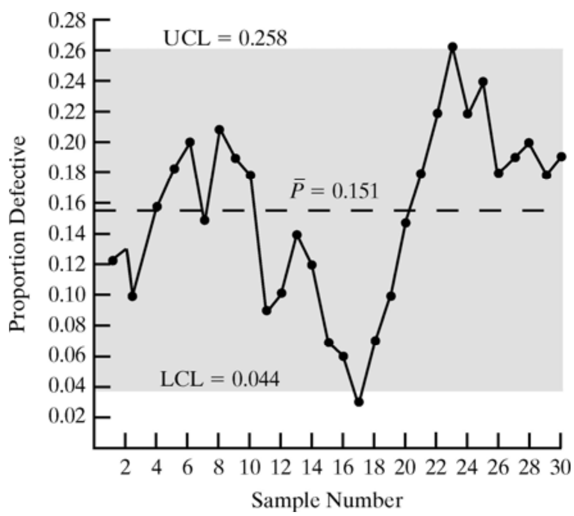
$$\sigma = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{(0.151)(0.849)}{100}} = 0.0358$$

$$\text{UCL} = \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.151 + 3\sqrt{\frac{(0.151)(0.849)}{100}} = 0.258$$

$$\text{LCL} = \bar{p} - z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

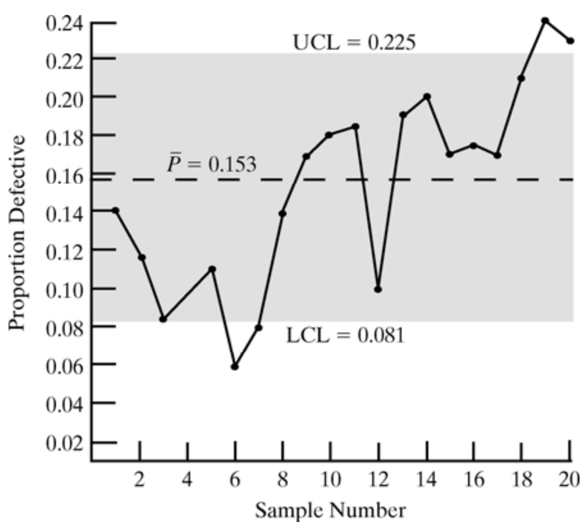
$$= 0.151 - 3\sqrt{\frac{(0.151)(0.849)}{100}} = 0.044$$



The process does not seem to be out of control, although the decreasing number of defects from sample 8 to sample 17 should probably be investigated to see why the steady improvement occurred; likewise, the steadily increasing number of defects from sample 17 to sample 25 should probably be investigated to see why the quality deteriorated.

3-2. $\bar{p} = 0.153;\ \sigma = \sqrt{\frac{\bar{p}(1-\bar{p})}{100}} = \sqrt{\frac{0.153(0.847)}{100}} = 0.036$

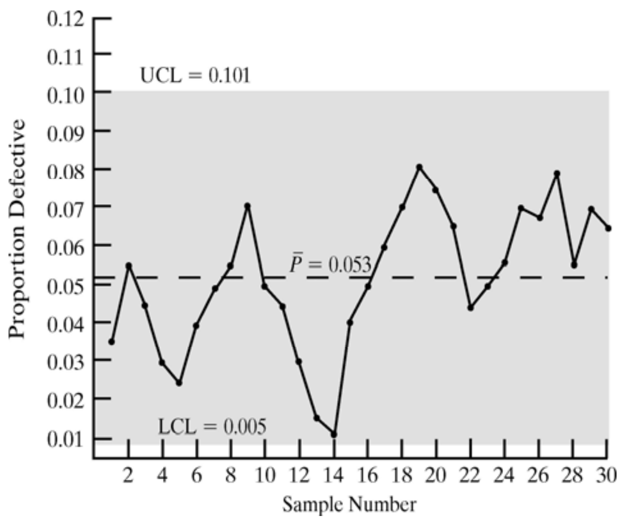
$$\begin{aligned} \text{UCL} &= \bar{p} + z\sigma \\ &= 0.153 + 2(0.036) \\ &= 0.225 \\ \text{LCL} &= \bar{p} - z\sigma \\ &= 0.153 - 2(0.036) \\ &= 0.081 \end{aligned}$$



In general, the proportion of defectives increases from sample 6 to sample 20, where it is eventually above the upper control limit. This indicates the process is moving out of control.

3-3. $\bar{p} = 0.053;\ \sigma = \sqrt{\frac{\bar{p}(1-p)}{n}} = \sqrt{\frac{0.053(0.947)}{200}} = 0.016$

$$\begin{aligned} \text{UCL} &= \bar{p} + 3\sigma \\ &= 0.053 + 3(0.016) \\ &= 0.101 \\ \text{LCL} &= \bar{p} - 3\sigma \\ &= 0.053 - 3(0.016) \\ &= 0.005 \end{aligned}$$



The process does not seem to be out of control.

3.4

$$\text{UCL} = \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$0.03 = 0.02 + z\sqrt{\frac{0.02(0.98)}{n}}$$

$$\sqrt{n} = \frac{2\sqrt{0.02(0.98)}}{0.01}$$

$$n = (28)^2$$

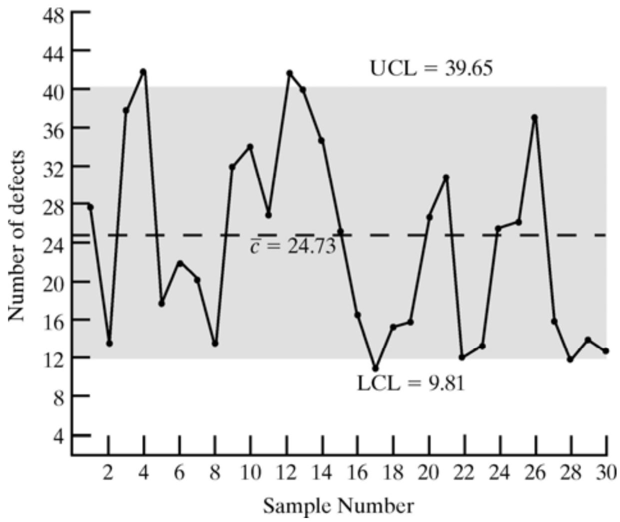
$$n = 784$$

3-5. a.

$$\bar{c} = \frac{742}{30} = 24.73$$

$$\begin{aligned} \text{UCL} &= \bar{c} + z\sqrt{\bar{c}} \\ &= 24.73 + 3\sqrt{24.73} \\ &= 39.65 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{c} - z\sqrt{\bar{c}} \\ &= 24.73 - 3\sqrt{24.73} \\ &= 9.81 \end{aligned}$$



With three points outside the control limits, the process appears to be out of control.

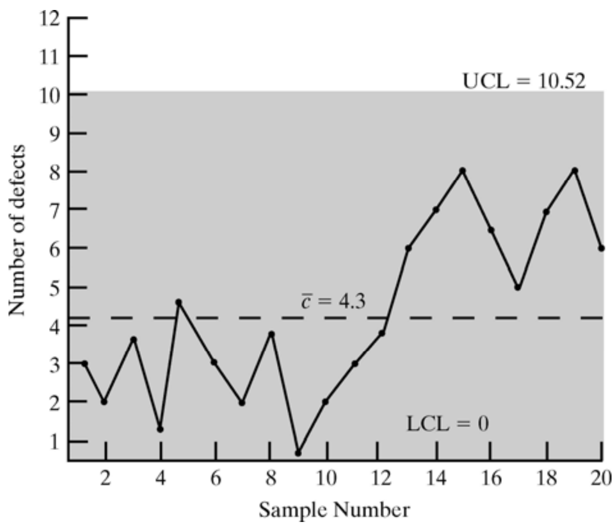
b. Nonrandom factors that might cause the process to move out of control could include (among other things) problems with the telephone order system, inexperienced operators taking orders, computer system problems, or shipping problems and delays.

3-6. a.

$$\bar{c} = \frac{86}{20} = 4.3$$

$$\begin{aligned} \text{UCL} &= \bar{c} + z\sqrt{\bar{c}} \\ &= 4.3 + 3\sqrt{4.3} \\ &= 10.52 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{c} - z\sqrt{\bar{c}} \\ &= 4.3 - 3\sqrt{4.3} \\ &= -1.92 \text{ or } 0.0 \ (\text{since the control chart cannot go below zero}) \end{aligned}$$



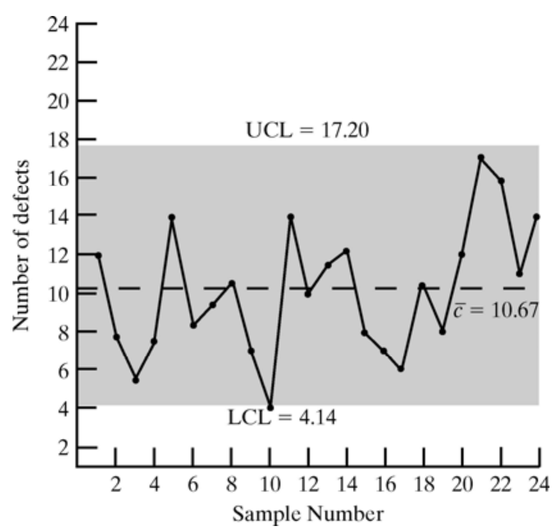
The process was not strictly out of control; however, from sample 10 to sample 20, the sample values were above the average and exhibited increasingly nonrandom behavior.

3-7.

$$\bar{c} = \frac{256}{24} = 10.67$$

$$\begin{aligned} \text{UCL} &= \bar{c} + z\sqrt{\bar{c}} \\ &= 10.67 + 2\sqrt{10.67} \\ &= 17.20 \end{aligned}$$

$$\begin{aligned} \text{LCL} &= \bar{c} - z\sqrt{\bar{c}} \\ &= 10.67 - 2\sqrt{10.67} \\ &= 4.14 \end{aligned}$$



The process does not appear to be out of control, but sample 21 is close to the UCL and the process should be investigated.

3-8. $\bar{c} = \frac{219}{30} = 7.3$

Control limits using $z = 3.00$:

$$\text{UCL} = \bar{c} + z\sqrt{\bar{c}} = 7.3 + 3\sqrt{7.3} = 7.3 + 8.11 = 15.41$$

$$\text{LCL} = \bar{c} - z\sqrt{\bar{c}} = 7.3 - 3\sqrt{7.3} = 7.3 - 8.11 \cong 0$$

All the sample observations are within the control limits suggesting that the invoice errors are in control.

3-9. $\bar{c} = \frac{255}{20} = 12.75$

$$\text{UCL} = \bar{c} + z\sqrt{\bar{c}} = 12.75 + 3\sqrt{12.75} = 23.46$$

$$\text{LCL} = \bar{c} - z\sqrt{\bar{c}} = 12.75 - 3\sqrt{12.75} = 2.04$$

All the sample observations are within the control limits suggesting that the delivery process is in control.

3-10.

Sample	**Proportion Defective**	**Sample**	**Proportion Defective**
1	.028	11	.076
2	.044	12	.048
3	.072	13	.030
4	.034	14	.024
5	.050	15	.020
6	.082	16	.032
7	.036	17	.018
8	.038	18	.042
9	.052	19	.036
10	.056	20	.024

$$\bar{p} = \frac{421}{(20)(500)} = \frac{421}{10,000} = .0421$$

$$\begin{aligned}\text{UCL} &= \bar{p} + z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ &= .0421 + 3.00\sqrt{\frac{.0421(1-.0421)}{500}} \\ &= .0421 + .027 \\ &= .069\end{aligned}$$

$$\begin{aligned}\text{LCL} &= \bar{p} - z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ &= .0421 - 3\sqrt{\frac{.0421(1-.0421)}{500}} \\ &= .0421 - .027 \\ &= .015\end{aligned}$$

Samples 3 and 11 are above the upper control limit indicating the process may be out of control.

3-11.

Sample	$\bar{x}$	R
1	2.00	2.3
2	2.08	2.6
3	2.92	2.7
4	1.78	1.9
5	2.70	3.2
6	3.50	5.0
7	2.84	2.2
8	3.26	4.6
9	2.50	1.3
10	4.14	3.5
11	2.12	3.0
12	4.38	4.0
13	2.84	3.3
14	2.70	1.1
15	3.56	5.6
16	2.96	3.1
17	3.34	6.1
18	4.16	2.4
19	3.70	2.5
20	2.72	2.9
	60.20	63.3

$$\bar{R} = \frac{\sum R}{k} = \frac{63.3}{20} = 3.17$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{20} = \frac{60.20}{20} = 3.01$$

R-chart

$$D_3 = 0,\ D_4 = 2.11,\ \text{for } n = 5$$

$$\text{UCL} = D_4\bar{R} = 2.11(3.17) = 6.69$$

$$\text{LCL} = D_3\bar{R} = 0(3.17) = 0$$

There are no R values outside the control limits, which would suggest the process is in control.

$\bar{x}$-chart

$$A_2 = 0.58$$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 3.01 + 0.58(3.17)$$
$$= 4.85$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 3.01 - 0.58(3.17)$$
$$= 1.17$$

There are no $\bar{x}$ values outside the control limits, which suggests the process is in control.

3-12. $\mu = 9$ in; $\sigma = 0.06$ in; $n = 10$

a.

$$\text{UCL} = \mu + z\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$= 9 + 3\left(\frac{0.06}{\sqrt{10}}\right)$$
$$= 9.057$$
$$\text{LCL} = \mu - z\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$= 9 - 3\left(\frac{0.06}{\sqrt{10}}\right)$$
$$= 8.943$$

b. Yes, it appears to be.

c. The control limits become narrower, but increasing the sample size will not affect the results in part b.

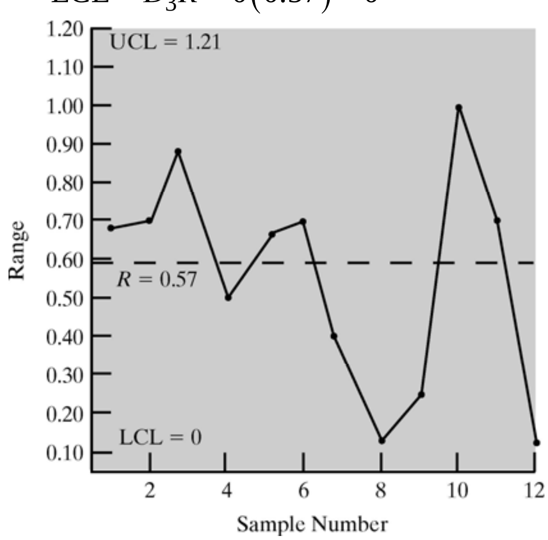
3-13.

Sample	***R***	**Sample**	***R***
1	0.67	7	0.45
2	0.69	8	0.17
3	0.93	9	0.32
4	0.52	10	0.99
5	0.64	11	0.65
6	0.71	12	0.15

a. $\bar{R} = \frac{\sum R}{k} = \frac{6.87}{12} = 0.57$

From Table 3.1 in the text, $D_3 = 0$ and $D_4 = 2.11$

$$\text{UCL} = D_4\bar{R} = 2.1(0.57) = 1.21$$
$$\text{LCL} = D_3\bar{R} = 0(0.57) = 0$$



b. The process variability is within the control limits.

3-14.

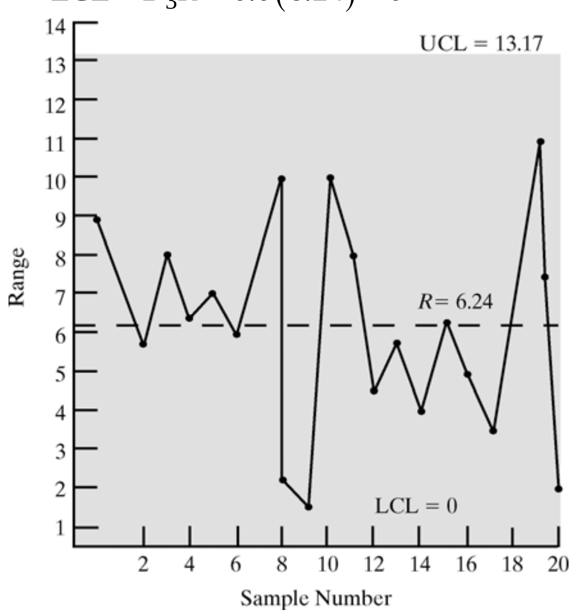
Sample	***R***	Sample	***R***
1	8.5	11	8.1
2	5.8	12	4.4
3	8.1	13	5.8
4	6.4	14	3.9
5	7.1	15	6.2
6	6.0	16	5.4
7	9.9	17	3.6
8	2.5	18	10.9
9	1.6	19	7.5
10	9.4	20	3.6

a. $\bar{R} = \frac{\sum R}{k} = \frac{12.7}{20} = 6.24$

From Table 3.1, $D_3 = 0.0$ and $D_4 = 2.11$

$\text{UCL} = D_4\bar{R} = 2.11(6.24) = 13.17$

$\text{LCL} = D_3\bar{R} = 0.0(6.24) = 0$



b. The temperature is within the control limits.

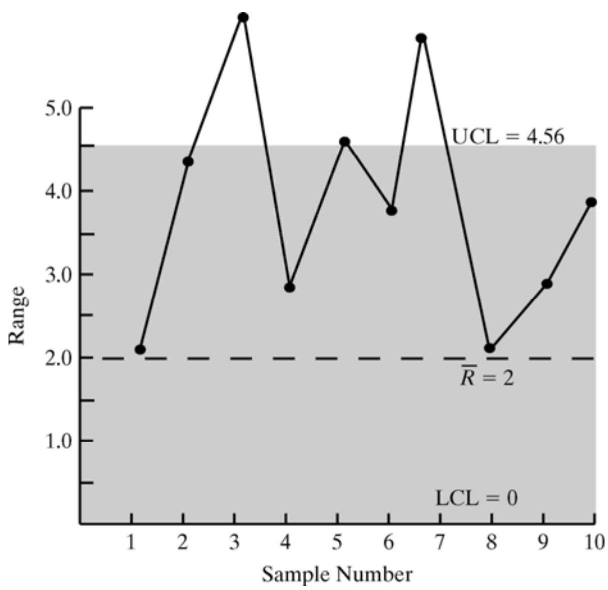
3-15. $\overline{R} = 2$

a. From Table 3.1, $D_3 = 0$ and $D_4 = 2.28$

$\text{UCL} = D_4\overline{R} = 2.28(2) = 4.56$

$\text{LCL} = D_3\overline{R} = 0(2) = 0$

Sample	***R***
1	2.1
2	4.5
3	7.0
4	3.0
5	5.0
6	4.3
7	6.5
8	2.0
9	3.2
10	4.0



The process clearly seems to be out of control. There are three of the sample points above the UCL, and all other sample values are above the center line for $\overline{R}$, indicating nonrandom variations.

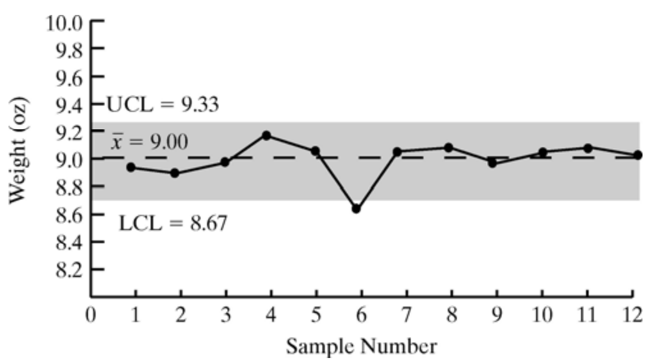
3-16.

Sample	$\bar{x}$	Sample	$\bar{x}$
1	8.89	7	9.05
2	8.88	8	9.16
3	8.99	9	8.97
4	9.19	10	9.06
5	9.04	11	9.09
6	8.71	12	9.01

$$\bar{x} = \frac{\sum x}{12} = \frac{108.04}{12} = 9.00$$

From Table 3.1, $A_2 = 0.58.$

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2\bar{R} \\ &= 9.00 + 0.58(0.57) \\ &= 9.33 \\ \text{LCL} &= \bar{\bar{x}} - A_2\bar{R} \\ &= 9 - 0.58(0.57) \\ &= 8.67 \end{aligned}$$



The process appears to be in control from both the $\bar{x}$ and R charts, although sample 6 is close to the LCL and perhaps should be investigated.

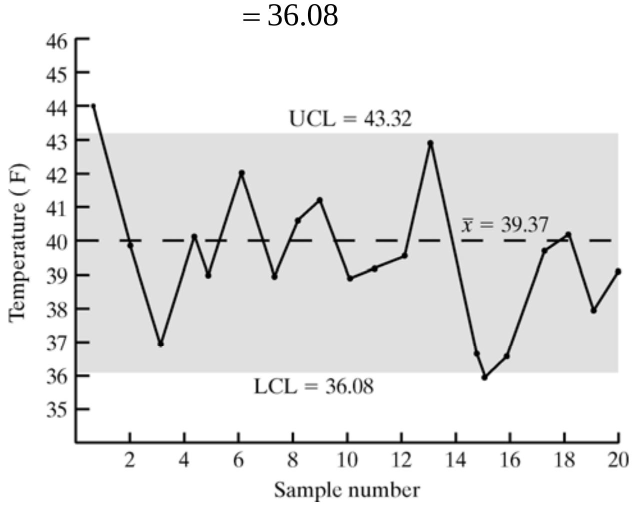
3-17.

Sample	$\bar{x}$	Sample	$\bar{x}$
1	43.9	11	39.2
2	39.7	12	39.7
3	37.2	13	42.9
4	40.4	14	37.8
5	39.0	15	36.6
6	41.8	16	37.6
7	39.4	17	39.9
8	40.7	18	40.7
9	41.6	19	38.2
10	39.0	20	39.0

$\bar{\bar{x}} = 39.7$

From Table 3.1, $A_2 = 0.58.$

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2\bar{R} \\ &= 39.7 + 0.58(6.24) \\ &= 43.32 \\ \text{LCL} &= \bar{\bar{x}} - A_2\bar{R} \\ &= 39.7 - 0.58(6.24) \\ &= 36.08 \end{aligned}$$



The process appears to be in control, with sample 1 seeming to be an aberration, however, the process should still be checked.

3-18.

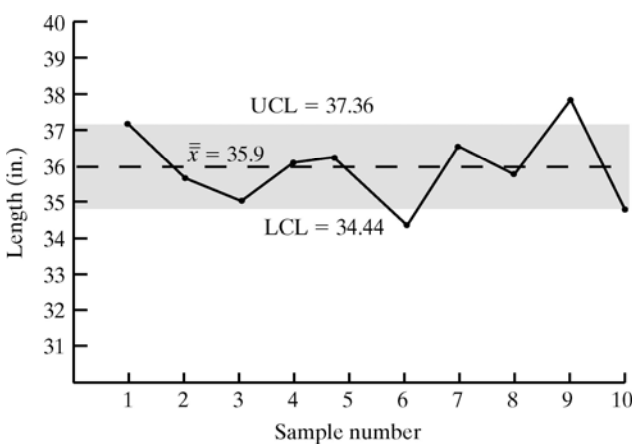
Sample	$\bar{x}$
1	37.0
2	35.8
3	35.3
4	36.1
5	36.1
6	34.3
7	36.3
8	35.6
9	37.7
10	34.5

$\bar{\bar{x}} = 35.9$

From Table 3.1, $A_2 = 0.73.$

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2\bar{R} \\ &= 35.9 + 0.73(2) \\ &= 37.36 \\ \text{LCL} &= \bar{\bar{x}} - A_2\bar{R} \\ &= 35.9 - 0.73(2) \\ &= 34.44 \end{aligned}$$

The process may be out of control (sample 6 and 9), although overall the $\bar{x}$-chart does not reflect the magnitude of the out of control situation, as does the R-chart in Problem 3-11. The process should be investigated to determine a cause for the out of control samples.



3-19.

Sample	Above/Below	Up/Down	Zone
1	B	—	B
2	B	D	B
3	B	U	C
4	A	U	B
5	A	D	C
6	B	D	A
7	A	U	C
8	A	U	B
9	B	D	C
10	A	U	C
11	A	U	C
12	A	D	C

There are no discernible nonrandom patterns.

3-20.

Sample	Above/ Below	Up/ Down	Zone	Sample	Above/ Below	Up/ Down	Zone
1	A	—	C	16	B	D	B
2	B	D	B	17	B	D	A
3	A	U	A	18	B	U	B
4	A	U	A	19	B	U	B
5	B	D	B	20	A	U	C
6	B	U	C	21	A	U	B
7	B	D	C	22	B	D	A
8	B	D	B	23	B	U	B
9	A	U	B	24	A	U	C
10	A	U	A	25	A	U	C
11	A	D	C	26	A	U	A
12	A	U	A	27	B	D	C
13	A	D	A	28	B	D	A
14	A	D	A	29	B	U	B
15	A	D	C	30	B	D	B

The zone pattern test is violated for samples 10 through 14 (2 out of 3 consecutive points in zone A but within limits).

3-21.

Sample	Above/ Below	Up/ Down	Zone	Sample	Above/ Below	Up/ Down	Zone
1	A	—	C	11	A	D	C
2	B	D	C	12	B	D	C
3	A	U	C	13	B	U	C
4	A	D	C	14	B	D	B
5	A	U	C	15	A	U	C
6	B	D	C	16	B	D	C
7	A	U	B	17	B	D	B
8	B	D	B	18	A	U	A
9	B	D	A	19	A	D	C
10	A	U	B	20	B	D	B

Samples 7 through 10 appear to violate a zone pattern test (four out of five points in zone B or beyond) so there may be a nonrandom pattern.

3-22.

Sample	Above/ Below	Up/ Down	Zone	Sample	Above/ Below	Up/ Down	Zone
1	A	—	A	11	B	U	C
2	—	D	C	12	B	U	C
3	B	D	A	13	B	U	A
4	A	U	C	14	B	D	B
5	B	D	C	15	B	D	A
6	A	U	B	16	B	U	B
7	B	D	C	17	B	U	C
8	A	U	C	18	A	U	C
9	A	U	B	19	B	D	B
10	B	D	C	20	B	U	C

There are several instances where zone pattern test rules are violated; samples 1 to 3, and sample 13 through 16. Thus, nonrandom patterns may exist.

3-23.

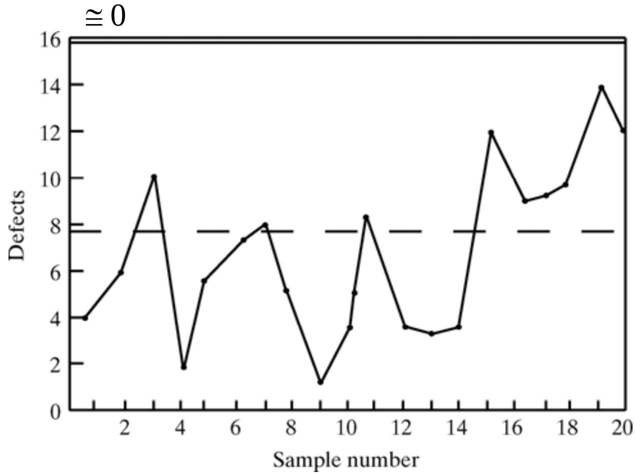
Sample	Above/ Below	Up/ Down	Sample	Above/ Below	Up/ Down
1	B	—	16	B	D
2	B	U	17	B	D
3	B	D	18	B	U
4	A	U	19	B	U
5	A	U	20	B	U
6	A	U	21	A	U
7	B	D	22	A	U
8	A	U	23	A	U
9	A	D	24	A	D
10	A	D	25	A	U
11	B	D	26	A	D
12	B	U	27	A	U
13	B	U	28	A	U
14	B	D	29	A	D
15	B	D	30	A	U

There are eight consecutive points on one side of the center line on two occasions—samples 11-20 and samples 21-30—indicating nonrandom patterns may exist.

3-24. $\bar{c} = \frac{146}{20} = 7.3$

$$\text{UCL} = \bar{c} + z\sqrt{c}$$
$$= 7.3 + (3)\sqrt{7.3}$$
$$= 7.3 + (3)(2.70)$$
$$= 15.41$$

$$\text{LCL} = \bar{c} - z\sqrt{c}$$
$$= 7.3 - (3)(2.70)$$
$$\cong 0$$



The process appears to be in control however the process may be moving toward an out-of-control situation.

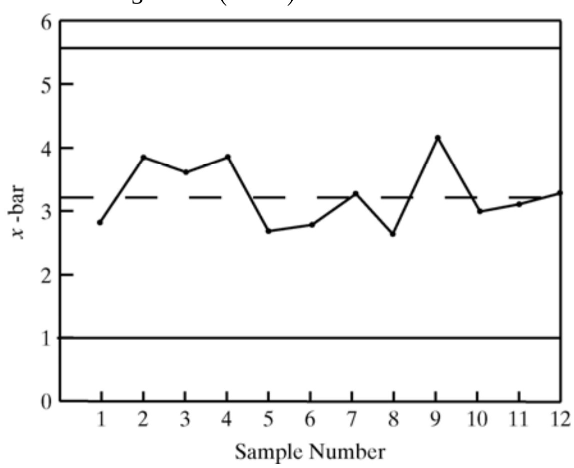
3-25. $\bar{\bar{x}} = 3.25,\ \bar{R} = 3.86,\ A_2\left(n = 5\right) = 0.58$

$$\begin{aligned}\text{UCL} &= \bar{\bar{x}} + A_2\bar{R} \\ &= 3.25 + \left(0.58\right)\left(3.86\right) \\ &= 5.48\end{aligned}$$

$$\begin{aligned}\text{LCL} &= \bar{\bar{x}} - A_2\bar{R} \\ &= 3.25 - \left(0.58\right)\left(3.86\right) \\ &= 1.01\end{aligned}$$

$$\text{UCL} = D_4\bar{R} = 2.11\left(3.86\right) = 8.14$$

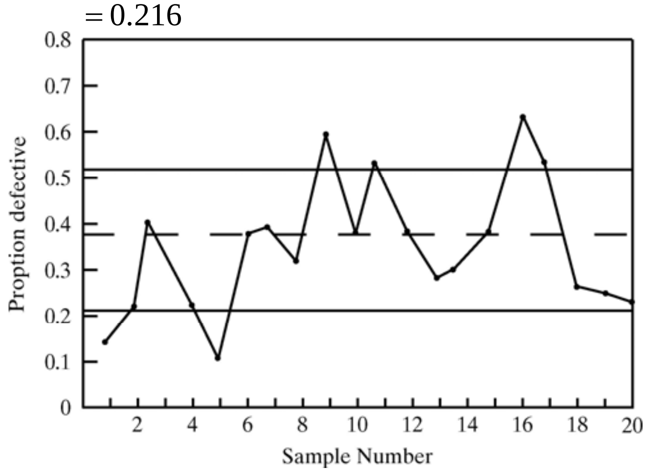
$$\text{LCL} = D_3\bar{R} = 0\left(3.86\right) = 0$$



The process appears to be in control.

3-26. $\overline{p} = \frac{718}{2000} = 0.36$

$$\begin{aligned}
\text{UCL} &= \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{100}} \\
&= 0.36 + (3)(0.048) \\
&= 0.504 \\
\text{LCL} &= \overline{p} - (3)(0.048) \\
&= 0.36 - 0.144 \\
&= 0.216
\end{aligned}$$



The process appears to be out of control.

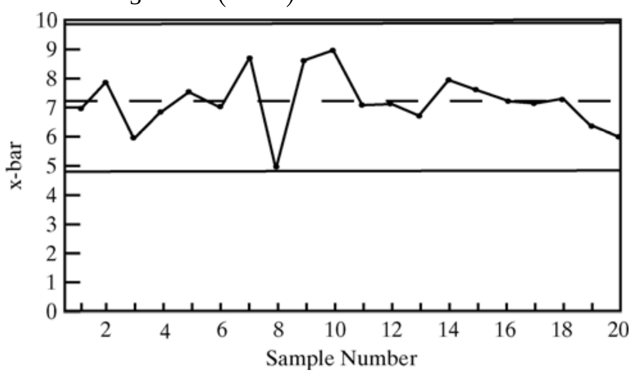
3-27.

$$\begin{aligned}
\overline{c} &= \frac{326}{20} = 16.3 \\
\text{UCL} &= \overline{c} + z\sqrt{\overline{c}} \\
&= 16.3 + (3)(4.03) \\
&= 28.412 \\
\text{LCL} &= \overline{c} - z\sqrt{\overline{c}} \\
&= 16.3 + (3)(4.03) \\
&= 4.188
\end{aligned}$$

The process is in-control.

3-28. $\bar{\bar{x}} = 7.28,\ \bar{R} = 4.25,\ A_2(n=5) = 0.58$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$= 7.28 + (0.58)(4.25) = 9.75$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$
$$= 7.28 - 2.47 = 4.81$$
$$\text{UCL} = D_4\bar{R} = 2.11(4.25) = 8.97$$
$$\text{LCL} = D_3\bar{R} = 0(4.25) = 0$$



While the process appears to be in control the mean of 7.28 appears to be significantly lower than the objective of 8 chips per cookie that management has established. Thus, the company should adjust their process to increase the number of chips and construct a new control chart.

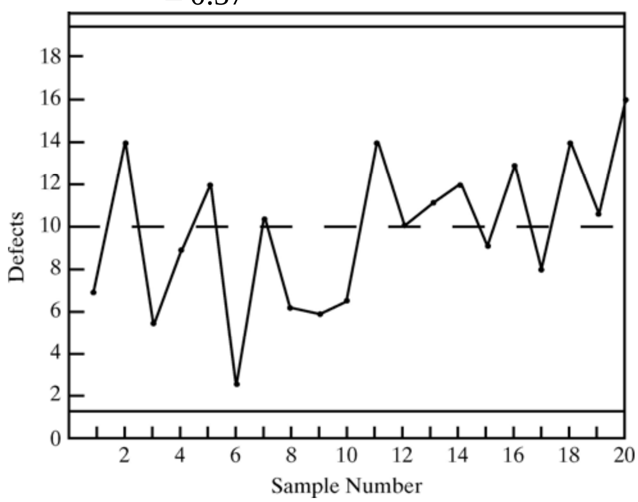
3-29.
$$\bar{p} = \frac{105}{(30)(16)} = 0.22$$
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$
$$= 0.22 + (3)(0.0756)$$
$$= 0.447$$
$$\text{LCL} = 0.22 - (3)(0.0756)$$
$$\cong 0.216$$

Although the process appears to be in control, the average proportion of “defects’’ among leaving patients, $\bar{p} = 0.22,$ seems high and the hospital should probably adopt some quality improvement measures.

3-30. $\bar{c} = \dfrac{202}{20} = 10.1$

$$\begin{aligned}
\text{UCL} &= \bar{c} + z\sqrt{\bar{c}} \\
&= 10.1 + (3)(3.18) \\
&= 19.6
\end{aligned}$$

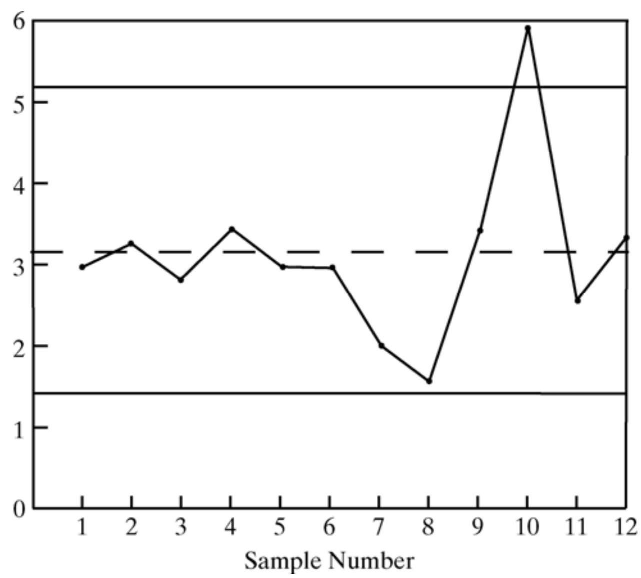
$$\begin{aligned}
\text{LCL} &= \bar{c} - z\sqrt{\bar{c}} \\
&= 10.1 - (3)(3.18) \\
&= 0.57
\end{aligned}$$



The process appears to be in control.

3-31. $\bar{\bar{x}} = 3.17,\ \bar{R} = 3.25,\ A_2(n=5) = 0.58$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$= 3.17 + (0.58)(3.25) = 5.04$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$
$$= 3.17 - 1.89 = 1.29$$
$$\text{UCL} = D_4\bar{R} = 2.11(3.25) = 6.86$$
$$\text{LCL} = D_3\bar{R} = 0(3.25) = 0$$



The process appears out of control for sample 10, however, this could be an aberration since there are no other apparent nonrandom patterns or out-of-control points. Thus, this point should probably be "thrown out'' and a new control chart developed with the remaining eleven samples.

3-32.
$$\bar{\bar{x}} = 3.17,\ \text{days}$$
$$\bar{R} = 3.25$$
$$3\sigma = A_2\bar{R} = (0.58)(3.25) = 1.89$$
$$C_p = \frac{2.00}{2(1.89)} = \frac{2.00}{3.78} = 0.53$$
$$C_{pk} = \text{minimum}\left[\frac{3.17-2.00}{1.89}, \frac{4.00-3.17}{1.89}\right]$$
$$= \text{minimum}\ (.62, .44)$$
$$= .44$$

The process is not capable of meeting the company's design specifications and defects (i.e., late deliveries) will occur.

3-33. $\bar{\bar{x}} = 7.28,\ \bar{R} = 4.25$

$$3\sigma = (0.58)(4.25) = 2.47$$

$$C_p = \frac{4.00}{2(2.47)} = \frac{4.00}{4.94} = .81$$

$$C_{pk} = \text{minimum}\left[\frac{7.28 - 6.00}{2.47}, \frac{10 - 7.28}{2.47}\right]$$

$$= \text{minimum } (.52,\ 1.10)$$

$$= .52$$

The process is not capable of meeting the design specifications and it appears that cookies will be produced with too few chips.

3-34. $\bar{\bar{x}} = 9.00$

$$\bar{R} = 0.57$$

$$3\sigma = A_2\bar{R} = (0.58)(0.575) = .33$$

$$C_p = \frac{1.00}{.66} = 1.52$$

$$C_{pk} = \text{minimum}\left[\frac{0.5}{.33}, \frac{0.5}{.33}\right]$$

$$= \text{minimum}(1.52,\ 1.52)$$

$$= 1.52$$

The process is capable of meeting design specifications.

3-35. $C_p = \frac{420}{6(55)} = \frac{420}{330} = 1.27$

$$C_{pk} = \text{minimum}\left[\frac{1,050 - 915}{3(55)}, \frac{1,335 - 1,050}{3(55)}\right]$$

$$= \text{minimum } (.82,\ 1.72)$$

$$= .82$$

While $C_p = 1.27$ indicates the process is capable, $C_{pk} = .82$ indicates the process mean has shifted toward the lower design specification, and defective (shorter lived) bulbs will be generated.

3-36. $C_p = \frac{.048}{6(.008)} = \frac{.048}{.048} = 1.00$

$$C_{pk} = \text{minimum}\left[\frac{1.281 - 1.251}{3(.008)}, \frac{1.299 - 1.281}{3(.008)}\right]$$

$$= \text{minimum}(1.25,\ 0.75)$$

$$= 0.75$$

$C_p = 1.00$ means the tolerance range and the process range are virtually the same indicating that some defective parts will occur. $C_{pk} = 0.75$ indicates that the process mean has shifted toward the upper specification indicating the process will result in some parts that are defective (too large).

3-37. The process mean would need to be shifted back toward the nominal design value of 1,125 hours. To achieve six sigma quality the process range would need to be reduced to one-half of the tolerance range. Since the tolerances are ± 210 hours, the tolerance range is 420 hours. Thus, the process range would need to be 210 hours, which are 3σ control limits of ± 105 hours. Thus, the process mean would need to be 1,125 hours with an upper control limit of 1,230 hours, and a lower control limit of 1,020 hours.

3-38. *Machine 1:*

$$C_p = \frac{.030}{.024} = 1.25$$

$$C_{pk} = \text{minimum}\left[\frac{.0995 - .082}{.012}, \frac{.112 - .0995}{.012}\right]$$

$$= \text{minimum}(1.46,\ 1.04)$$

$$= 1.04$$

Machine 1 is capable of meeting the design specifications.

Machine 2:

$$C_p = \frac{.030}{.054} = 0.56$$

$$C_{pk} = \text{minimum}\left[\frac{.1002 - .0820}{.027}, \frac{.1120 - .1022}{.027}\right]$$

$$= \text{minimum}(.67,\ .44)$$

$$= .44$$

Machine 2 is not capable of meeting the design specifications.

Machine 3:

$$C_p = \frac{.030}{.030} = 1.00$$

$$C_{pk} = \text{minimum}\left[\frac{.0951 - .0820}{.015}, \frac{.1120 - .0951}{.015}\right]$$

$$= \text{minimum}(.87,\ 1.13)$$

$$= .87$$

Machine 3 is capable of meeting the design specifications but the process center has shifted too far toward the lower design specification.

3-39.

Month	$\bar{x}$	***R***
1	7.02	5.6
2	8.16	3.7
3	8.18	4.9
4	9.18	5.4
5	10.32	4.1
6	9.54	4.1
7	6.96	3.5
8	10.38	9.0
9	8.12	8.9
10	10.26	6.9
11	9.66	9.2
12	9.10	4.3
Avg	8.91	5.8

$\bar{\bar{x}}$ = 8.91, $\bar{R}$ = 5.8, A_2 = .58, D_3 = 0, D_4 = 2.11

<u>X-bar chart</u>

UCL = 12.27
LCL = 5.54

<u>R-Chart</u>

UCL = 12.24
LCL = 0

The process is in control according to both control charts.

3-40.

$\bar{p}$ = 0.48

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.48 + 3\sqrt{\frac{0.48(1-0.48)}{150}} = 0.60$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.48 - 3\sqrt{\frac{0.48(1-0.48)}{150}} = 0.36$$

The process is not in control and does not meet the target value of 90%. It appears that improvement occurred in week 7, but the improvement was not consistent and was significantly below the target value. The hospital should reevaluate the process improvements it has implemented using quality tools and, after implementing a new program, a new control chart should be developed.

3-41.

Sample1	$\overline{x}$	***R***	Sample	$\overline{x}$	***R***
1	145.88	6.9	11	144.54	6.7
2	144.86	8.6	12	145.48	5.5
3	144.22	9.4	13	145.26	8.5
4	145.82	5.2	14	148.78	4.8
5	143.10	6.7	15	143.58	5.1
6	147.82	6.2	16	146.48	3.1
7	143.04	5.5	17	145.22	2.4
8	141.44	5.9	18	144.80	4.6
9	148.72	5.9	19	143.46	5.4
10	142.42	8.5	20	145.92	7.1

$$\overline{x} = 145.06$$
$$\overline{R} = 6.1$$
$$A_2 = 0.58$$

$\overline{x}$-chart:	$\overline{R}$-chart:
$\text{UCL} = 145.06 + 0.58(6.1)$	$\text{UCL} = D_4\overline{R} = (2.11)(6.1) = 12.87$
$= 148.60$	$\text{LCL} = D_3\overline{R} = (0)(6.1) = 0$
$\text{LCL} = 145.06 - 0.58(6.1)$	
$= 141.52$	

Sample 8 is slightly below the LCL and sample 14 is slightly above it.

$$C_p = \frac{149 - 142}{2(3.54)} = \frac{7.00}{7.08} = .99$$

$$C_{pk} = \text{minimum}\left[\frac{145.06 - 142}{3.54},\ \frac{149 - 145.06}{3.54}\right]$$

$$= \text{minimum}\ (.86,\ 1.04)$$

$$C_{pk} = .86$$

The process is not capable of meeting design specifications. Since C_p is very close to 1.00, some defective baseballs will be generated, and, $C_{pk} = 0.86$ indicates they will typically not weigh enough.

3-42. To achieve six sigma quality the process range would need to be reduced to one-half of the tolerance range. The tolerance range is 7 gms therefore the process range must be 3.5 gms which are 3σ control limits of ± 1.75 gms. Thus, the process mean would need to be 145.5 gms with an upper control limit of 147.25 and a lower control limit of 143.75.

3-43.

Sample	$\bar{x}$	***R***	**Sample**	$\bar{x}$	***R***
1	7.72	7.1	6	8.90	6.0
2	6.58	6.4	7	11.52	14.6
3	12.90	12.7	8	8.02	4.7
4	8.76	6.8	9	9.58	8.5
5	11.06	12.4	10	9.12	5.6

$\bar{\bar{x}} = 9.42, \ \bar{R} = 8.48$

$A_2 = 0.58$

$\bar{x}$-chart:	$\bar{R}$-chart:
$\bar{\bar{x}} = 9.42$	$\bar{R} = 8.48$
$\text{UCL} = 14.31$	$\text{UCL} = 17.94$
$\text{LCL} = 4.52$	$\text{LCL} = 0$

The process is in control according to both control charts.

$$C_p = \frac{12-6}{14.31-4.52} = \frac{6}{9.79} = .61$$

$$\begin{aligned} C_{pk} &= \text{minimum}\left[\frac{9.42-6}{4.92}, \ \frac{12-9.42}{4.92}\right] \\ &= \text{minimum}\ (0.70, \ 0.52) \\ &= 0.52 \end{aligned}$$

The process is not capable of meeting the design specifications and the customer waiting times will be greater than the upper specification and lower than the lower specification. Although the lower times might be considered good, it could be that customer service representatives are not devoting enough time to each customer.

3-44. (a)

Sample	$\bar{x}$	***R***
1	21.4	9
2	27.0	23
3	19.0	8
4	24.4	19
5	26.6	33
6	20.8	15
7	24.8	14
8	26.4	23
9	29.6	11
10	25.4	9
	245.4	164

$$\bar{R} = \frac{\sum R}{k} = \frac{164}{10} = 16.4$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{10} = \frac{245.4}{10} = 24.54$$

$\bar{R}$-chart

$$D_3 = 0,\ D_4 = 2.11,\ \text{for } n = 5$$

$$\text{UCL} = D_4\bar{R} = (2.11)(16.4) = 34.604$$

$$\text{LCL} = D_3\bar{R} = (0)(16.4) = 0$$

There are no R values outside the control limits, which suggests the process is in control.

$\bar{x}$-chart

$$A_2 = 0.58$$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 24.54 + (.58)(16.4) = 34.05$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 24.54 - (.58)(16.4) = 15.03$$

There are no $\bar{x}$ values outside the control limits, which suggests the process is in control.

(b)

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma} = \frac{30 - 20}{19.02} = .52$$

$$C_{pk} = \text{minumum}\left[\frac{\bar{\bar{x}} - \text{lower specification}}{3\sigma}, \frac{\text{upper specification} - \bar{\bar{x}}}{3\sigma}\right]$$

$$= \text{minimum}\left[\frac{24.54 - 20}{9.51}, \frac{30 - 24.54}{9.51}\right]$$

$$= \text{minimum}(.47,\ .57)$$

$$= .47$$

Since $C_p = .52,$ which is less than 1.0, the process range is greater than the tolerance range and the process is not capable of producing within the design specifications all the time. Since $C_{\text{pk}} = .47$ is less than 1.0, the process has moved closer to the lower design specification and will generate defects.

3-45. (a)

Sample	$\bar{x}$	R
1	4.83	3.6
2	5.38	3.3
3	5.38	1.3
4	5.75	3.3
5	5.00	2.1
6	6.40	2.9
7	6.65	2.7
8	4.57	3.5
9	6.18	4.6
10	5.32	1.2
11	4.02	4.5
12	4.57	4.2
13	4.95	3.7
14	5.05	2.1
15	4.47	3.6
	78.52	46.6

$$\bar{R} = \frac{\sum R}{k} = \frac{46.6}{15} = 3.107$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{15} = \frac{78.52}{15} = 5.23$$

$\bar{R}$-chart

$$D_3 = 0,\ D_4 = 2.00, \quad \text{for } n = 6$$

$$\text{UCL} = D_4\bar{R} = (2.00)(3.107) = 6.21$$

$$\text{LCL} = D_3\bar{R} = (0)(3.107) = 0$$

There are no R values outside the control limits, which suggest the process is in control.

$\bar{x}$-chart

$$A_2 = .48$$

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 5.23 + (.48)(3.107)$$

$$= 6.73$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 5.23 - (.48)(3.107)$$

$$= 3.74$$

There are no $\bar{x}$ values outside the control limits, which suggest the process is in control.

(b)

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma}$$

$$= \frac{6-4}{2.98} = \frac{2}{2.98}$$

$$= .67$$

$$C_{\text{pk}} = \text{minumum}\left[\frac{\bar{\bar{x}} - \text{lower specification},}{3\sigma}\ \frac{\text{upper specification} - \bar{\bar{x}}}{3\sigma}\right]$$

$$= \text{minimum}\left[\frac{5.23-4}{1.49},\ \frac{6-5.23}{1.49}\right]$$

$$= \text{minimum}(.83,\ .51)$$

$$= .51$$

Since $C_p = .51,$ which is less than 1.0, the process range is greater than the tolerance range and the process is not capable of producing within the design specifications all the time. Since $C_{\text{pk}} = .51$ is less than 1.0, the process has moved closer to the lower design and will generate defects.

3-46.

$$\bar{c} = \frac{64}{24} = 2.667$$

$$\text{UCL} = \bar{c} + \text{z}\sqrt{\bar{c}}$$
$$= 2.667 + 3\sqrt{2.667}$$
$$= 7.57$$

$$\text{LCL} = \bar{c} - \text{z}\sqrt{\bar{c}}$$
$$= 2.667 - 3\sqrt{2.667}$$
$$= 0$$

The process is only “out of control” in month 5 when there were zero falls. This month should be investigated to see if the circumstances that resulted in no falls can be repeated.

Any number of falls would seem to be poor quality, so even though this “process” is technically “in control,” the process should be improved with a six sigma goal of zero defects. While 2.667 falls per month is not a lot, it is likely too many.

3-47. (a)

Sample	$\bar{x}$	R
1	88.33	18
2	93.33	12
3	82.50	28
4	91.00	22
5	89.83	13
6	87.00	15
7	89.67	15
8	86.00	34
9	84.00	31
10	93.67	15
11	92.83	14
12	85.50	13

$$\overline{R} = \frac{\sum R}{k} = \frac{230}{12} = 19.17$$

$$\overline{\overline{x}} = \frac{\sum \overline{x}}{12} = \frac{1063.67}{12} = 88.64$$

$\overline{R}$-chart

$D_3 = 0,\ D_4 = 2.00,$ for $n = 6$

$\text{UCL} = D_4\overline{R} = (2.00)(19.17) = 38.41$

$\text{LCL} = D_3\overline{R} = (0)(19.17) = 0$

There are no R values outside the control limits, which suggests the process is in control.

$\overline{x}$-chart

$A_2 = .48$

$$\text{UCL} = \overline{\overline{x}} + A_2\overline{R} = 88.64 + (.48)(19.17) = 97.89$$

$$\text{LCL} = \overline{\overline{x}} - A_2\overline{R} = 88.64 - (.48)(19.17) = 79.38$$

There are no $\overline{x}$ values outside the control limits, which suggest the process is in control.

(b)

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma} = \frac{98-92}{(88.64-79.38)} = \frac{6}{18.71} = .32$$

$$C_{pk} = \text{minimum}\left[\frac{88.64-92}{9.27}, \frac{98-88.64}{9.27}\right] = \text{minimum}\ (-0.36, 1.01) = -0.36$$

Since $C_p = 0.32$ (<1.0), the service department is not currently capable of consistently achieving the desired customer satisfaction score. Since $C_{pk} = -0.36$ is less than 1.0 the service department will continue to generate less than desired customer scores. The service department would need to make process improvements in order to consistently achieve the desired customer satisfaction scores.

3-48. (a)

Day	$\bar{x}$	R
1	143.00	40
2	139.00	33
3	112.33	31
4	170.67	88
5	165.33	26
6	135.33	55
7	149.33	32
8	113.67	18
9	143.67	22
10	138.00	57
11	118.33	62
12	176.33	56
13	167.67	92
14	157.00	87
15	165.67	61
16	144.67	31
17	113.00	47
18	167.67	19
19	197.33	34
20	115.67	67
21	148.33	27
22	160.33	81
23	159.67	46
24	107.33	38
25	129.67	61
26	178.33	46
27	155.33	21
28	153.67	78
29	145.00	17
30	132.33	51
	4403.7	1424

$$\bar{R} = \frac{\sum R}{k} = \frac{1424}{30} = 47.47$$

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{30} = \frac{4403.7}{30} = 146.79$$

$\bar{R}$-chart

$D_3 = 0,\ D_4 = 2.574,\ \text{for } n = 3$

$\text{UCL} = D_4\bar{R} = (2.574)(47.47) = 122.18$

$\text{LCL} = D_3\bar{R} = (0)(47.47) = 0$

There are no R values outside the control limits, which suggest the process is in control.

$\overline{x}$-chart

$A_2 = 1.023$

$$\text{UCL} = \overline{\overline{x}} + A_2\overline{R} = 146.79 + (1.023)(47.47)$$
$$= 195.35$$
$$\text{LCL} = \overline{\overline{x}} - A_2\overline{R} = 146.79 - (1.023)(47.47)$$
$$= 98.23$$

There is one $\overline{x}$ value outside the control limits on day 19, which at least suggests a problem on that day should be investigated, although there is no consistent pattern of being out of control.

(b)

$$C_p = \frac{\text{upper specification limit} - \text{lower specification limit}}{6\sigma}$$
$$= \frac{135-105}{97.12} = \frac{30}{97.12}$$
$$= .31$$
$$C_{pk} = \text{minimum}\left[\frac{146.79-105}{48.56}, \frac{135-146.79}{48.56}\right]$$
$$= \text{minimum } (.86, -0.24)$$
$$= -0.24$$

Since $C_p = 0.31$ and $C_{pk} = -0.24,$ the bed turnaround time is not capable of achieving the hospital’s goal of 120 minutes. Process improvements would be necessary to achieve the desired bed turnaround times.

CASE SOLUTION 3.1: Quality Control at Rainwater Brewery

This is basically a discussion question; therefore, the student responses might vary.

The owners have stated in the case description that the chances of a batch being spoiled—and, thus, an unhealthy batch—are very unlikely. However, even a very slight risk of a contaminated batch of 1,000 bottles might be too much, given the health consequences of a spoiled batch. Testing only a small sample of even a few bottles would indicate if the batch was, in fact, bad, so a simple testing procedure such as opening and testing 5 to 10 bottles might be prudent.

Most of the quality control efforts should focus on process control procedures at the various stages of the brewing process. Obvious candidates are $\overline{x}$ and *R*-charts to monitor temperature, specific gravity, and pH during the fermentation and aging stages. Some type of process control testing of the final bottled product probably is warranted also.

Quality control methods can also be used at the beginning of the brewing process for checking materials such as bottles and caps and ingredients such as yeast, hops, and grain. Bottles and caps that are not completely clean and sterile can result in a spoiled batch, and poor-quality ingredients can obviously contribute to an “off’’ brew.

CASE SOLUTION 3.2: Quality Control at Grass, Unlimited

$$\overline{c} = \frac{249 \text{ defects}}{60 \text{ samples}} = 4.15$$

$$z = 2.00$$

$$\text{UCL} = \overline{c} + Z\sqrt{\overline{c}} = 4.15 + 2\sqrt{4.15} = 8.22$$

$$\text{LCL} = \overline{c} - Z\sqrt{\overline{c}} = 4.15 - 2\sqrt{4.15} = 0.076$$

The chart exceeds the control limits for samples 12 and 14, however, the chart appears to have been in control prior to sample 12. Although, the reasons for the out-of-control occurrences must be investigated, the chart based on samples 1 through 11 could be used, or, several observations could be collected once the process is brought back in control and used with samples 1 through 12. Thus, this chart could be implemented for continued use.

Other examples of control charts that could be used include *p*-charts for the number of errors in a sample of orders, or the number of customer complaints for a sample survey of customers. A *c*-chart could be used for the number of defects found (for cleanliness) during an inspection of facilities.

CASE SOLUTION 3.3: Improving Service Time at Dave's Burgers

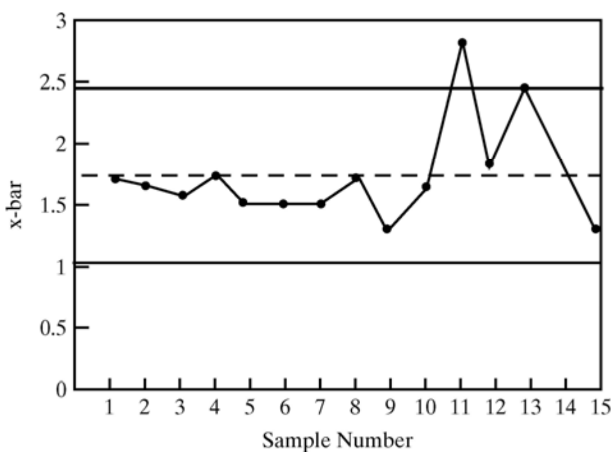
$$\bar{\bar{x}} = 1.70,\ \bar{R} = 1.40,\ A_2(n=6) = 0.48$$

$$\text{UCL} = \bar{\bar{x}} + \boldsymbol{A_2\bar{R}}$$
$$= 1.70 + (0.48)(1.40) = 2.37$$

$$\text{LCL} = \bar{\bar{x}} - \boldsymbol{A_2\bar{R}}$$
$$= 1.70 - (0.48)(1.40) = 1.03$$

$$\text{UCL} = D_4\bar{R} = 2.00(1.40) = 2.80$$

$$\text{LCL} = D_3\bar{R} = \mathbf{0}(1.40) = 0$$



The process is not in control, thus the control chart cannot be used on a continuing basis.

The out-of-control situation should be investigated and upon correction, new data should be gathered to establish a revised control chart. If that control chart is valid (i.e., in control), it may be used to monitor quality on a continuing basis.

Dave may want to chart % of customers orders completed correctly (p-chart) or the number of customer complaints (c-chart).

Answers from students, of course, may vary.