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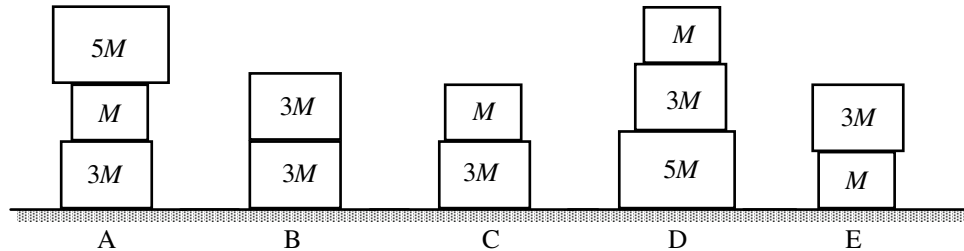
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ANSWERING RANKING TASKS

NTEx-RT1: STACKED BLOCKS—MASS OF STACK

Shown below are stacks of various blocks. All masses are given in the diagram in terms of M , the mass of the smallest block.



Rank the total mass of each stack.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ Least

OR, The magnitude of the total mass of each stack is the same but not zero. _____

OR, The magnitude of the total mass of each stack is zero. _____

OR, The ranking for the total mass of each stack cannot be determined. _____

Explain your reasoning.

Example answer formats

Stacks A and D have a total mass of $9M$, C and E have a mass of $4M$, and B has a mass of $6M$ or the ranking is $A = D > B > C = E$. Thus the ranking task answer should be expressed either as

Greatest 1 AD 2 _____ 3 B 4 CE 5 _____ Least

or

Greatest 1 A 2 D 3 B 4 C 5 E Least

Note the order of equals is not important but it is easier if people are encouraged to use alphabetical order when possible.

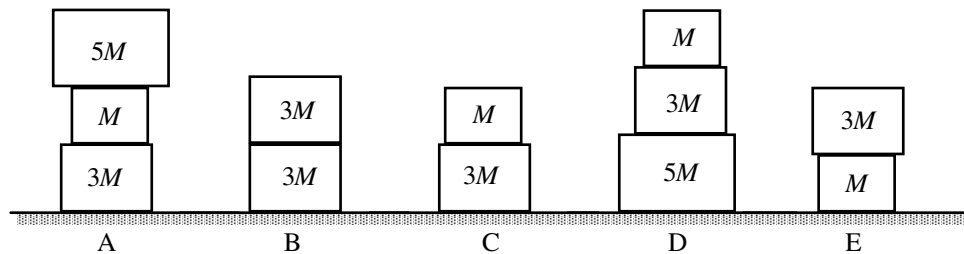
Other alternative formats are $AD > B > CE$ or $A = D > B > C = E$

An alternative but not preferred format is

Greatest 1 AD 2 B 3 CE 4 _____ 5 _____ Least

NTPRACTICE-RT2: STACKED BLOCKS—NUMBER OF BLOCKS

Shown below are stacks of various blocks. All masses are given in the diagram in terms of M , the mass of the smallest block.



Rank the total number of blocks in each stack.

Greatest 1 A 2 D 3 B 4 C 5 E Least

OR, The total number of is the same blocks for each stack. _____

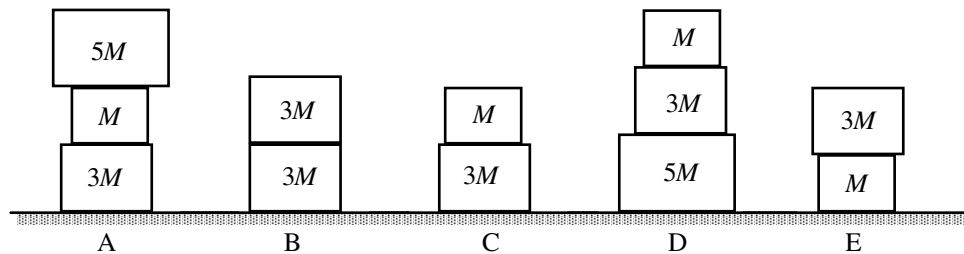
OR, The ranking for the total number of blocks in each stack cannot be determined. _____

Explain your reasoning.

There are 3 blocks in cases A and D, and two in all other cases. So a correct ranking is $A = D > B = C = E$.

NTPRACTICE-RT3: STACKED BLOCKS—AVERAGE MASS

Shown below are stacks of various blocks. All masses are given in the diagram in terms of M , the mass of the smallest block.



Rank the average mass in each stack.

Greatest 1 A 2 B 3 D 4 C 5 E Least

OR, The average mass is the same for each stack. _____

OR, The ranking for the average mass in these stacks cannot be determined. _____

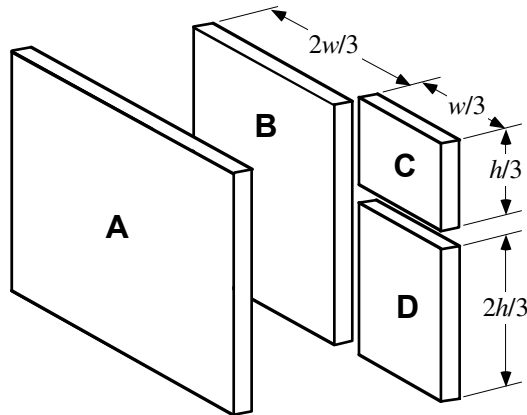
Explain your reasoning.

Adding the mass of the individual boxes in each stack we find $9M$ for cases A and D each with three boxes, $6M$ for case B with two boxes, and $4M$ for cases C and E each with two boxes. The average mass in each stack is the total mass of each stack divided by the number of boxes in each stack giving an average of $3M$ for stacks A, B, and D. The average mass is $2M$ for stacks C and E. Thus the ranking is $A = B = D > C = E$.

NT1 PRELIMINARIES

NT1A-RT1: CUTTING UP A BLOCK—DENSITY

A block of material (labeled A in the diagram) with a width w , height h , and thickness t , has a mass of M_0 distributed uniformly throughout its volume. The block is then cut into three pieces, B, C, and D, as shown.



Rank the density of the original block A, piece B, piece C, and piece D.

Greatest 1 _____ 2 _____ 3 _____ 4 _____ Least

OR, The density is the same for all these pieces. _____

OR, The ranking for the densities cannot be determined. _____

Please explain your reasoning.

Answer: The density is the same for all four pieces.

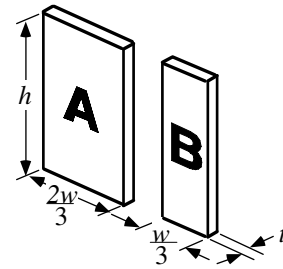
Since the mass is uniformly distributed, a piece with half the volume will also have half the mass but the density, the ratio of mass to volume, remains the same.

NT1A-WWT2: CUTTING UP A BLOCK—DENSITY

A block of material with a width w , height h , and thickness t , has a mass of M_o distributed uniformly throughout its volume. The block is then cut into two pieces, A and B, as shown. A student makes the following statement:

“The density is calculated by dividing the total mass by the volume. Since the volume is in the denominator, a large volume will give a small density. Therefore the block with the smallest volume, block B, will have the largest density.”

What, if anything, is wrong with the above statement? If something is wrong, explain the error and how to correct it. If the statement is correct, explain why.



Answer: The statement is incorrect.

The smaller block has $\frac{1}{3}$ the volume of the original block but also $\frac{1}{3}$ the mass, so the ratio of mass to volume is the same as the original block.

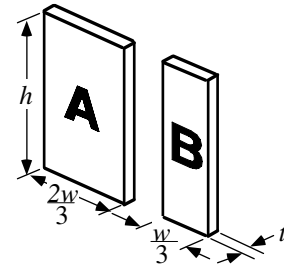
NT1A-CCT3: BREAKING UP A BLOCK—DENSITY

A block of material with a width w , height h , and thickness t , has a mass of M_0 distributed uniformly throughout its volume. The block is then broken into two pieces, A and B, as shown. Three students make the following statements:

Andy: “The density is the mass divided by the volume, and the volume of B is smaller. Since the mass is uniform and the volume is in the denominator, the density is larger for B.”

Badu: “The density of piece A is larger than the density of piece B since A is larger, thus it has more mass.”

Coen: “They both have the same density. It’s still the same material.”



Which, if any, of these three students do you agree with?

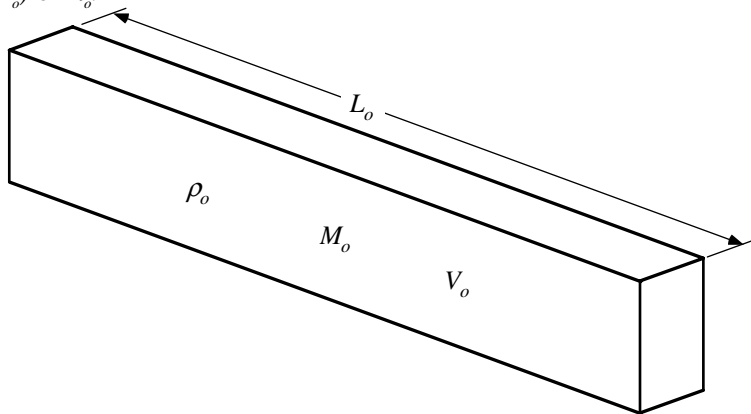
Andy _____ Badu _____ Coen _____ None of them _____

Please explain your reasoning.

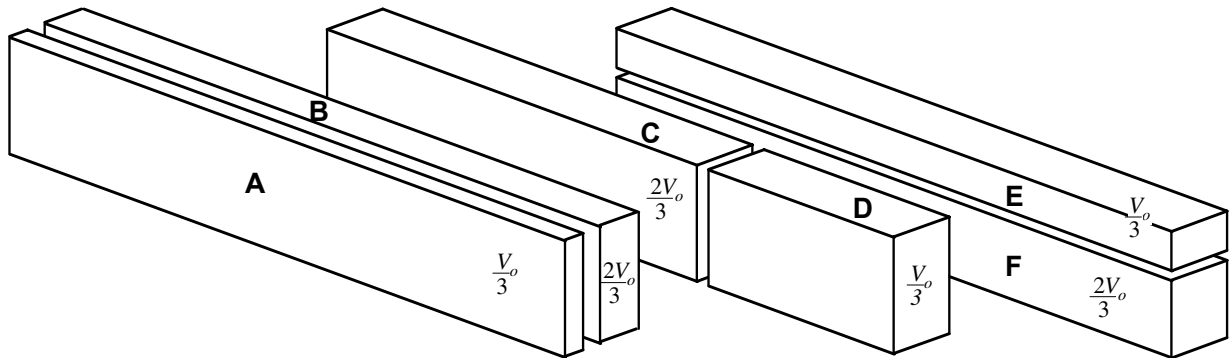
Answer: Coen is correct. Density is the ratio of mass to volume. The larger piece has twice the mass but also twice the volume of the smaller piece. Therefore the ratio of mass to volume is the same for both and the same as the density of the unbroken block.

NT1A-QRT4: SLICING UP A BLOCK—MASS & DENSITY

The block of material shown below has a length L_o and a volume V_o . An overall mass of M_o is spread uniformly throughout the volume of the block to give a density ρ_o and a linear density (in the direction of the measured length L_o) of λ_o .



Three possible ways to slice the block into unequal pieces are shown below. In each case, the larger piece has a volume $2V_o/3$ and the smaller piece has a volume $V_o/3$.

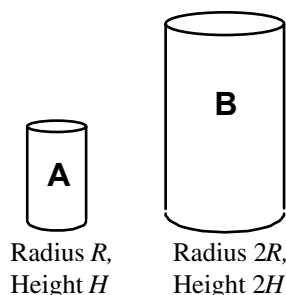


Fill in the table below with the quantities indicated for the pieces of the block labeled A – F in terms of the variables M_o , λ_o , and ρ_o .

	Mass	Mass per unit length	Mass per unit volume
Original block	M_o	λ_o	ρ_o
Piece A	$\frac{1}{3}M_o$	$\frac{1}{3}\lambda_o$	ρ_o
Piece B	$\frac{2}{3}M_o$	$\frac{2}{3}\lambda_o$	ρ_o
Piece C	$\frac{2}{3}M_o$	λ_o	ρ_o
Piece D	$\frac{1}{3}M_o$	λ_o	ρ_o
Piece E	$\frac{1}{3}M_o$	$\frac{1}{3}\lambda_o$	ρ_o
Piece F	$\frac{2}{3}M_o$	$\frac{2}{3}\lambda_o$	ρ_o

NT1A-QRT5: CYLINDRICAL RODS WITH SAME MASS—VOLUME, AREA, AND DENSITY

Two cylindrical rods are shown. Rod A has a height H and a radius R and rod B has a height $2H$ and a radius $2R$. Both rods have the same total mass. Rod A has a density ρ_A and volume V_A .



- (a) What is the volume of rod B in terms of the volume of rod A?
(Your answer should look like $V_B = n V_A$, where n is some number.)

Please explain.

The volume of Rod B = 8 times the volume of Rod A since the volume is proportional to the product of the area (which is proportional to the square of the radius) and the height of the cylinder, and the radius and the height have doubled.

- (b) What is the surface area of rod B in terms of the surface area of rod A? (Your answer should look like $SA_B = n SA_A$, where n is some number.)

Please explain.

$$SA_B = 4 SA_A,$$

The surface of each rod is made up of the circles at the top and bottom and the 'side wall.' The top and bottom of the rod are circles, and the area of a circle is proportional to the square of the radius. Since the radius is twice as big for Rod B as for Rod A, the area of the top and bottom is 4 times as big for Rod B. The 'side wall' of the cylinder has an area equal to the circumference times the height, and the circumference of a circle is proportional to the radius. Since both height and radius have doubled, the area of the side wall has also quadrupled. The overall surface area is 4 times as large for Rod B as for Rod A.

- (c) What is the density of Rod B in terms of the density of Rod A? (Your answer should look like $\rho_B = n \rho_A$, where n is some number.)

Please explain.

$$\rho_A/8$$

Since B has 8 times the volume of A with the same total mass, its density will be one-eighth that of A.

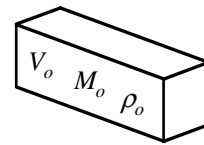
- (d) What is the mass per unit length of Rod B to the mass per unit length of Rod A? (Your answer should look like $\lambda_B = n \lambda_A$, where λ is the mass per unit length and n is some number.)

Please explain.

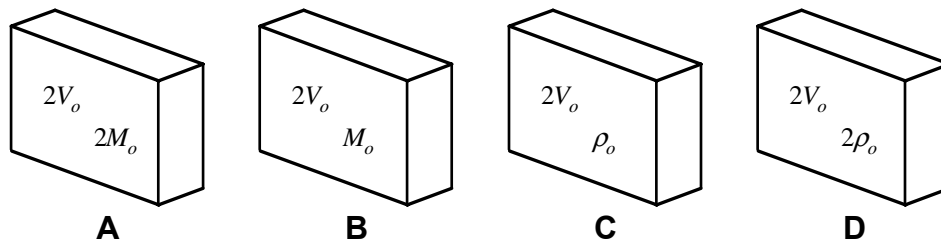
Rod B has 8 times the mass of A and twice the length so the mass per unit length of Rod B is 4 times the density of A.

NT1A-BCT6: FOUR BLOCKS—MASS AND DENSITY

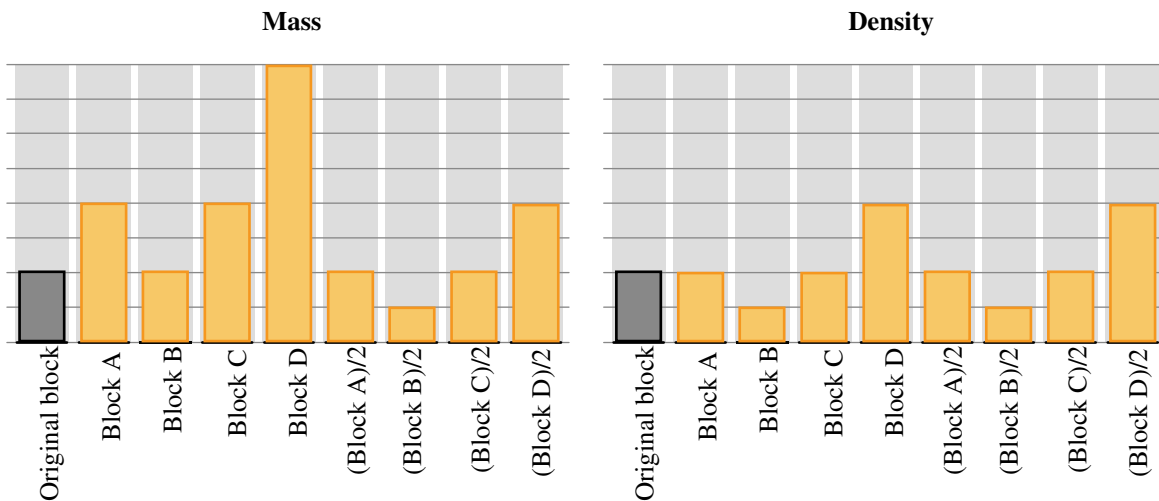
The block of material shown to the right has a volume V_o . An overall mass M_o is spread evenly throughout the volume of the block so that the block has a uniform density ρ_o .



For each block shown below, the volume is given as well as *either* the mass or the density of the block.



Construct two bar charts for the mass and density for the four blocks labeled A – D and for the pieces of the blocks if they were cut in half labeled A/2 – D/2. The mass and density for the original block is shown to set the scale of the chart.

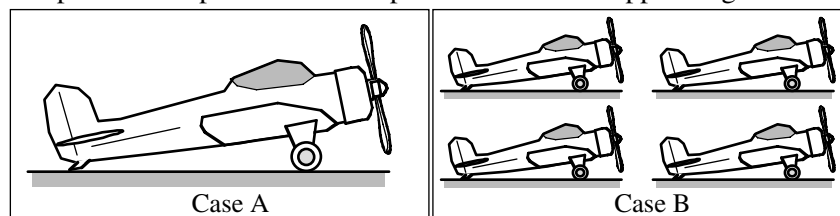


Please explain.

Answer The mass of blocks A and B are given. The mass of block C must be $2M_o$, since it has the same density as the original block and twice the volume. The mass of block D must be 4 times the mass of the original block, since it has twice the volume and twice the density and mass is volume times density. The mass of one-half of a block will be half the mass of the full block. The density of one-half of a block will be the same as the density of a full block, since the volume is halved as well as the mass, and the density is the ratio of these quantities.

NT1B-CT7: SCALE MODEL PLANES—SURFACE AREA AND WEIGHT

A woodworker has made four small airplanes and one large airplane. All airplanes are exactly the same shape, and all are made from the same kind of wood. The larger plane is twice as large in every dimension as one of the smaller planes. The planes are to be painted and then shipped as gifts.



a) The amount of paint required to paint the planes is directly proportional to the surface area. **Will the amount of paint required for the single plane in Case A be greater than, less than, or equal to the total amount of paint required for all four planes in Case B?**

Please explain your reasoning.

Answer: Equal to or the same.

Since the larger plane is twice as big in every dimension, any feature of the surface of the larger plane will have double the width and double the height of the equivalent piece of surface of the smaller plane. So the area of any piece of surface of the larger plane is four times the area of the equivalent surface of the smaller plane. Thus, the total surface area of four small planes is the same as the total surface area of one large plane.

b) The shipping cost for the planes is proportional to the weight. **Will the weight of the single plane in Case A be greater than, less than, or equal to the total weight of all four planes in Case B?**

Please explain your reasoning.

Answer: Greater than.

Since the larger plane is twice as big in every dimension, any piece of the larger plane will have double the width, double the height, and double the length of the equivalent piece of the smaller plane. So the volume of any piece of the larger plane is eight times the volume of the equivalent piece of the smaller plane, and will weigh eight times as much. Thus, the weight of four small planes is only half the weight of one large plane.