2–1. An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6$ in.

$d = 7$ in.

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$ **Ans.**

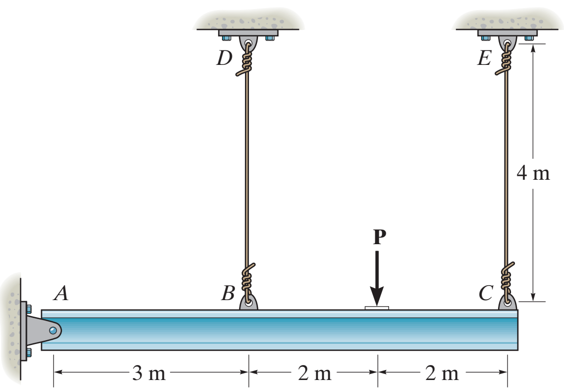
2–2. A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

$L_0 = 15$ in.

$L = \pi(5 \text{ in.})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$ **Ans.**

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



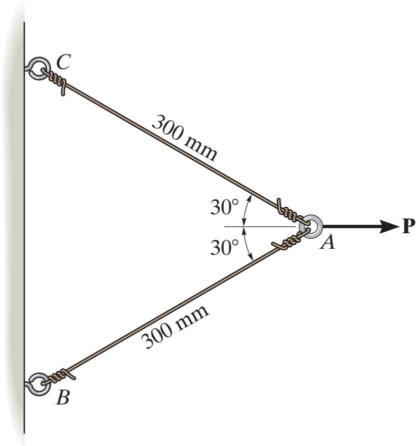
$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

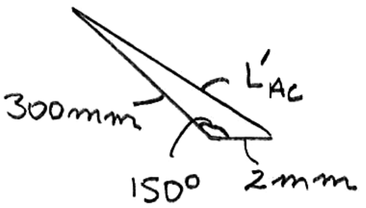
$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**

***2–4.** The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

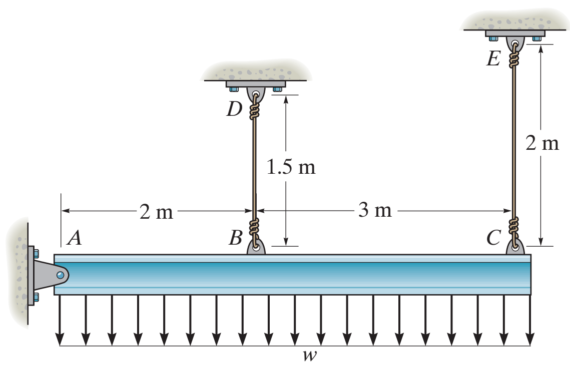


$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150°} = 301.734 \text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm}$$ **Ans.**



•2–5. The rigid beam is supported by a pin at A and wires BD and CE. If the distributed load causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



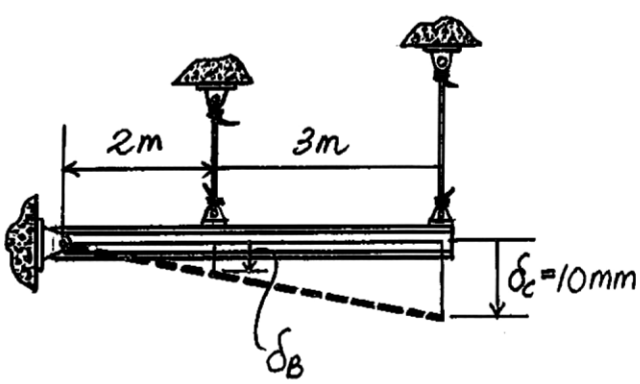
Since the vertical displacement of end C is small compared to the length of member AC, the vertical displacement δ_B of point B, can be approximated by referring to the similar triangle shown in Fig. a

$$\frac{\delta_B}{2} = \frac{10}{5}; \qquad \delta_B = 4 \text{ mm}$$

The unstretched lengths of wires BD and CE are $L_{BD} = 1500$ mm and $L_{CE} = 2000$ mm.

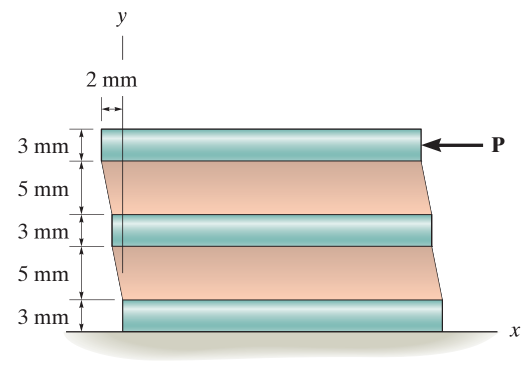
$$\left(\varepsilon_{avg}\right)_{BD} = \frac{\delta_B}{L_{BD}} = \frac{4}{1500} = 0.00267 \text{ mm/mm}$$ **Ans.**

$$\left(\varepsilon_{avg}\right)_{CE} = \frac{\delta_C}{L_{CE}} = \frac{10}{2000} = 0.005 \text{ mm/mm}$$ **Ans.**



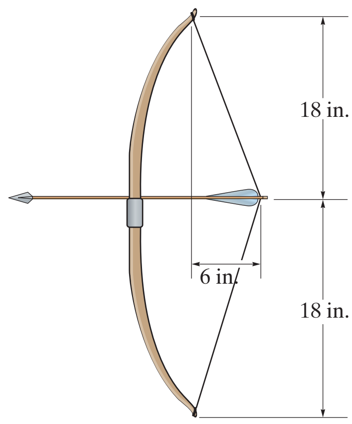
(a)

2–6. Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



$$\gamma = \tan^{-1}\left(\frac{2}{10}\right) = 11.31° = 0.197 \text{ rad}$$ **Ans.**

2–7. If the unstretched length of the bowstring is 35.5 in., determine the average normal strain in the string when it is stretched to the position shown.

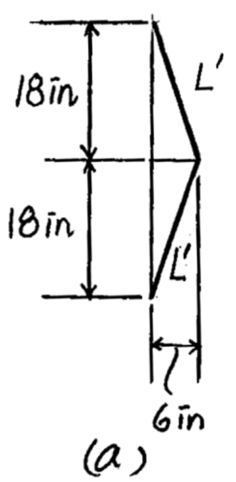


Geometry: Referring to Fig. *a*, the stretched length of the string is

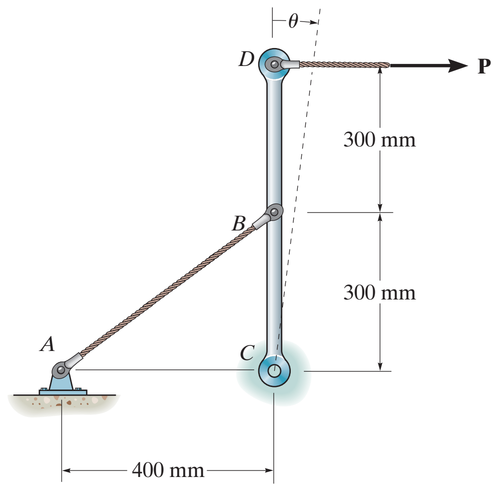
$$L = 2L' = 2\sqrt{18^2 + 6^2} = 37.947 \text{ in.}$$

Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{37.947 - 35.5}{35.5} = 0.0689 \text{ in./in.}$$ **Ans.**



***2–8.** Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



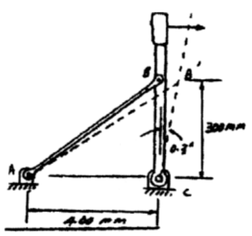
$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

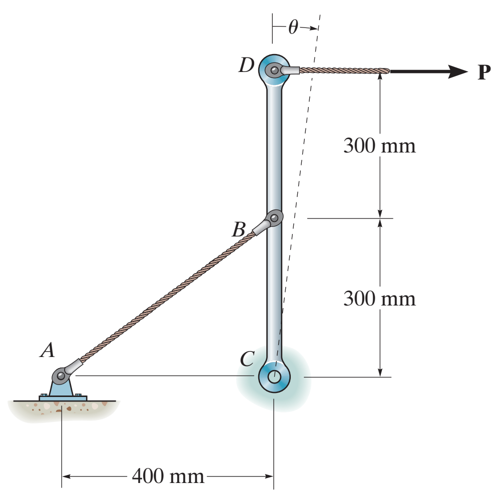
$$= 501.255 \text{ mm}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm} \qquad \textbf{Ans.}$$



•2–9. Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB} AB$$

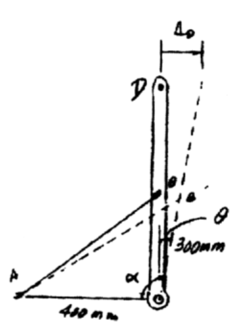
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

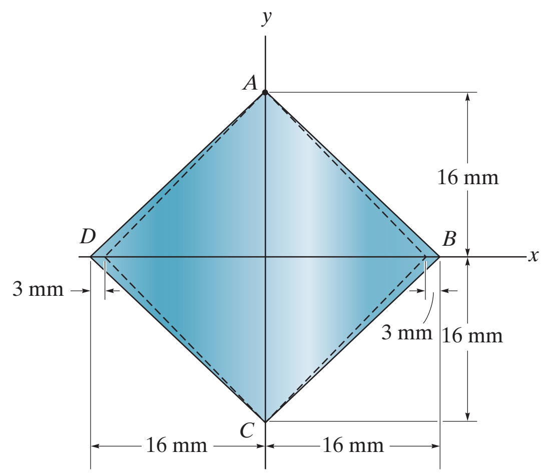
$$\alpha = 90.4185°$$

$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180°}\right)(0.4185) = 4.38 \text{ mm} \qquad \textbf{Ans.}$$



2–10. The corners B and D of the square plate are given the displacements indicated. Determine the shear strains at A and B.



Applying trigonometry to Fig. a

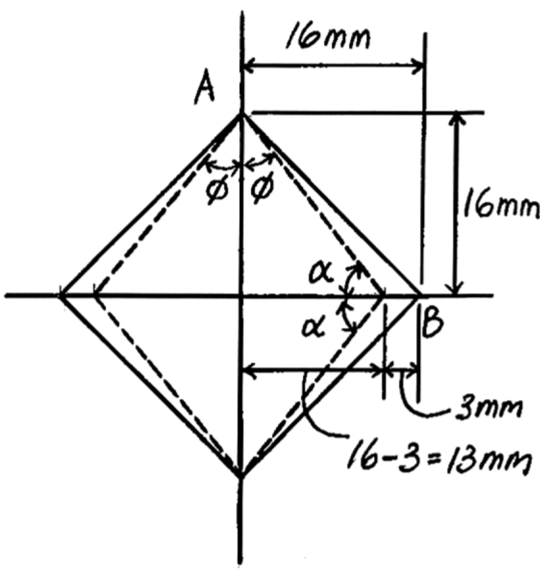
$$\phi = \tan^{-1}\left(\frac{13}{16}\right) = 39.09°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.6823 \text{ rad}$$

$$\alpha = \tan^{-1}\left(\frac{16}{13}\right) = 50.91°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.8885 \text{ rad}$$

By the definition of shear strain,

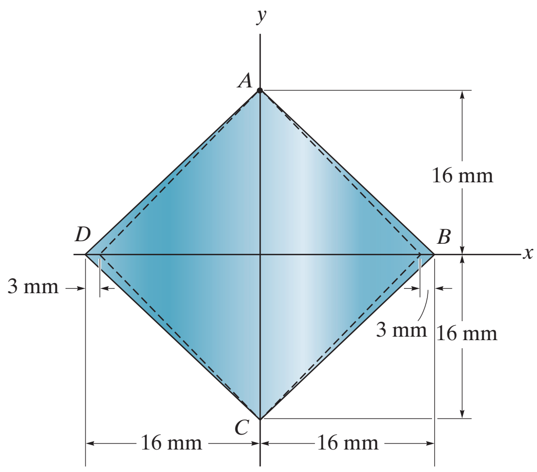
$$\left(\gamma_{xy}\right)_A = \frac{\pi}{2} - 2\phi = \frac{\pi}{2} - 2(0.6823) = 0.206 \text{ rad}$$ **Ans.**

$$\left(\gamma_{xy}\right)_B = \frac{\pi}{2} - 2\alpha = \frac{\pi}{2} - 2(0.8885) = -0.206 \text{ rad}$$ **Ans.**



(a)

2–11. The corners B and D of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonal DB.



Referring to Fig. a,

$$L_{AB} = \sqrt{16^2 + 16^2} = \sqrt{512}\text{ mm}$$

$$L_{AB'} = \sqrt{16^2 + 13^2} = \sqrt{425}\text{ mm}$$

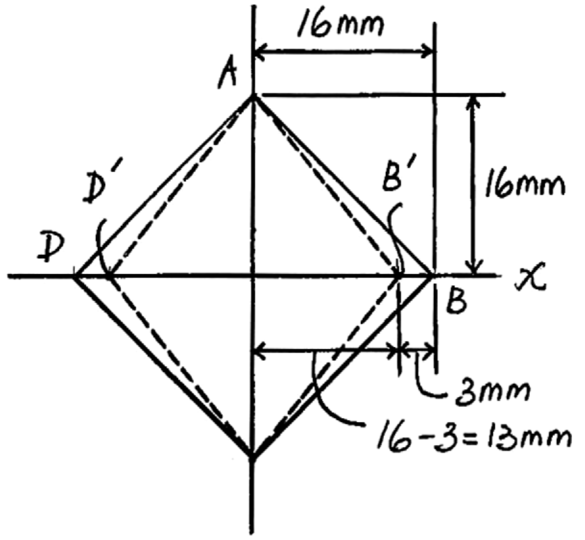
$$L_{BD} = 16 + 16 = 32\text{ mm}$$

$$L_{B'D'} = 13 + 13 = 26\text{ mm}$$

Thus,

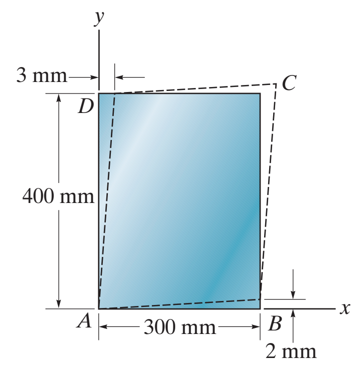
$$\left(\varepsilon_{\text{avg}}\right)_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{\sqrt{425} - \sqrt{512}}{\sqrt{512}} = -0.0889\text{ mm/mm}$$ **Ans.**

$$\left(\varepsilon_{\text{avg}}\right)_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{26 - 32}{32} = -0.1875\text{ mm/mm}$$ **Ans.**



(a)

***2–12.** The piece of rubber is originally rectangular. Determine the average shear strain γ_{xy} at A if the corners B and D are subjected to the displacements that cause the rubber to distort as shown by the dashed lines.

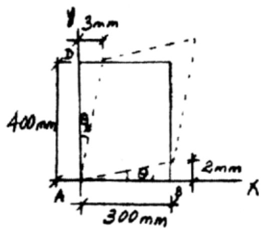


$$\theta_1 = \tan\theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

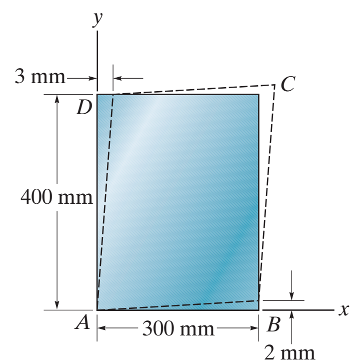
$$\theta_2 = \tan\theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

$$\gamma_{xy} = \theta_1 + \theta_2$$

$$= 0.006667 + 0.0075 = 0.0142 \text{ rad}$$ **Ans.**



•2–13. The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the diagonal DB and side AD.



$$AD' = \sqrt{(400)^2 + (3)^2} = 400.01125 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971°$$

$$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$$

$$\varphi = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966°$$

$$\alpha = 90° - 0.42971° - 0.381966° = 89.18832°$$

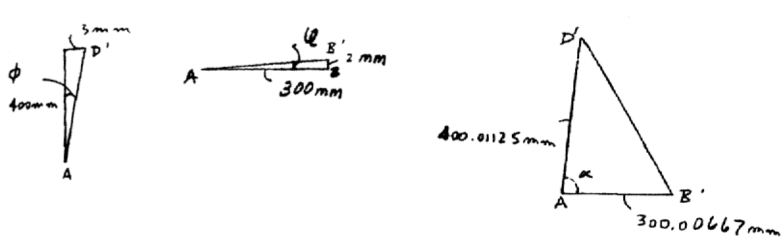
$$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832°)}$$

$$D'B' = 496.6014 \text{ mm}$$

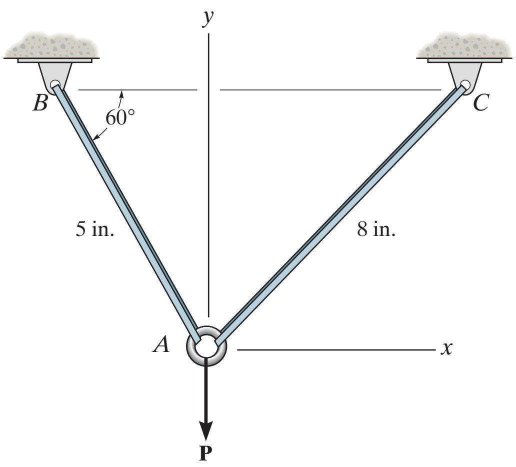
$$DB = \sqrt{(300)^2 + (400)^2} = 500 \text{ mm}$$

$$\varepsilon_{DB} = \frac{496.6014 - 500}{500} = -0.00680 \text{ mm/mm}$$ **Ans.**

$$\varepsilon_{AD} = \frac{400.01125 - 400}{400} = 0.0281(10^{-3}) \text{ mm/mm}$$ **Ans.**



2–14. Two bars are used to support a load. When unloaded, AB is 5 in. long, AC is 8 in. long, and the ring at A has coordinates $(0, 0)$. If a load $\mathbf{P}$ acts on the ring at A, the normal strain in AB becomes $\epsilon_{AB} = 0.02$ in./in., and the normal strain in AC becomes $\epsilon_{AC} = 0.035$ in./in. Determine the coordinate position of the ring due to the load.



Average Normal Strain:

$$L'_{AB} = L_{AB} + \varepsilon_{AB} L_{AB} = 5 + (0.02)(5) = 5.10 \text{ in.}$$

$$L'_{AC} = L_{AC} + \varepsilon_{AC} L_{AC} = 8 + (0.035)(8) = 8.28 \text{ in.}$$

Geometry:

$$a = \sqrt{8^2 - 4.3301^2} = 6.7268 \text{ in.}$$

$$5.10^2 = 9.2268^2 + 8.28^2 - 2(9.2268)(8.28)\cos\theta$$

$$\theta = 33.317^\circ$$

$$x' = 8.28 \cos 33.317^\circ = 6.9191 \text{ in.}$$

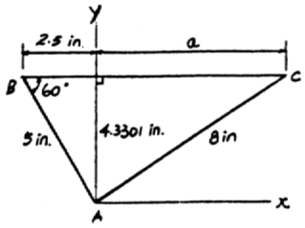
$$y' = 8.28 \sin 33.317^\circ = 4.5480 \text{ in.}$$

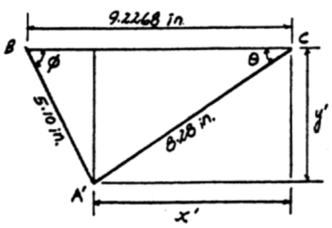
$$x = -(x' - a)$$

$$= -(6.9191 - 6.7268) = -0.192 \text{ in.}$$ **Ans.**

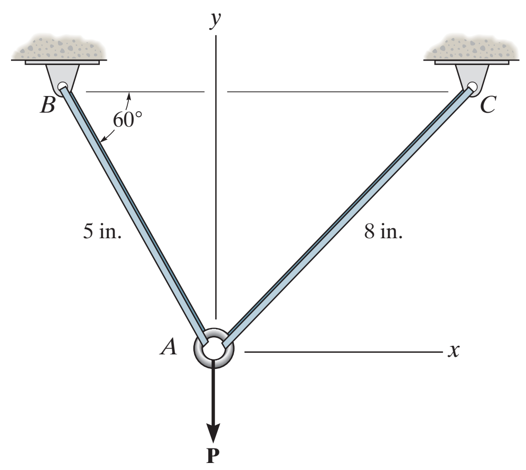
$$y = -(y' - 4.3301)$$

$$= -(4.5480 - 4.3301) = -0.218 \text{ in.}$$ **Ans.**





2–15. Two bars are used to support a load **P**. When unloaded, AB is 5 in. long, AC is 8 in. long, and the ring at A has coordinates (0, 0). If a load is applied to the ring at A, so that it moves it to the coordinate position (0.25 in., −0.73 in.), determine the normal strain in each bar.



Geometry:

$$a = \sqrt{8^2 - 4.3301^2} = 6.7268 \text{ in.}$$

$$L_{A'B} = \sqrt{(2.5 + 0.25)^2 + (4.3301 + 0.73)^2}$$

$$= 5.7591 \text{ in.}$$

$$L_{A'C} = \sqrt{(6.7268 - 0.25)^2 + (4.3301 + 0.73)^2}$$

$$= 8.2191 \text{ in.}$$

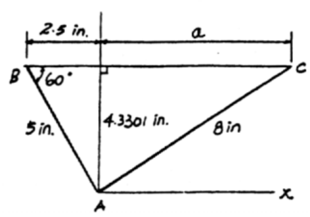
Average Normal Strain:

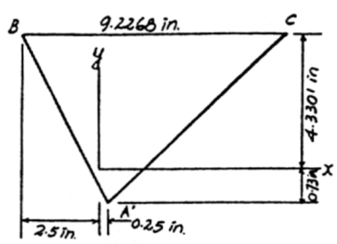
$$\varepsilon_{AB} = \frac{L_{A'B} - L_{AB}}{L_{AB}}$$

$$= \frac{5.7591 - 5}{5} = 0.152 \text{ in./in.} \qquad \textbf{Ans.}$$

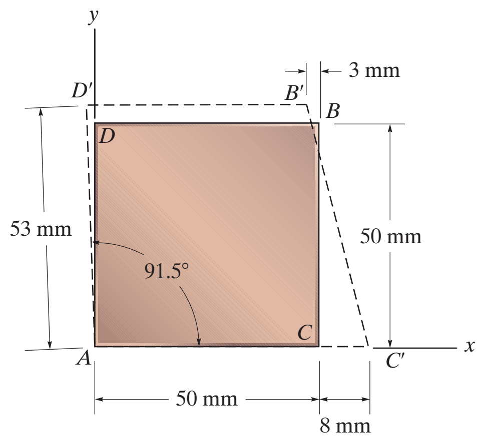
$$\varepsilon_{AC} = \frac{L_{A'C} - L_{AC}}{L_{AC}}$$

$$= \frac{8.2191 - 8}{8} = 0.0274 \text{ in./in.} \qquad \textbf{Ans.}$$





***2–16.** The square deforms into the position shown by the dashed lines. Determine the average normal strain along each diagonal, AB and CD. Side $D'B'$ remains horizontal.



Geometry:

$$AB = CD = \sqrt{50^2 + 50^2} = 70.7107\text{ mm}$$

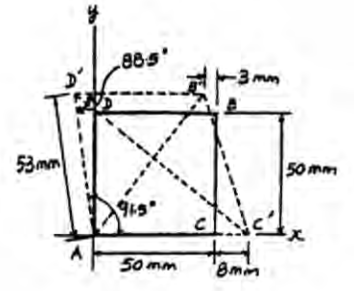
$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58)\cos 91.5°}$$

$$= 79.5860\text{ mm}$$

$$B'D' = 50 + 53\sin 1.5° - 3 = 48.3874\text{ mm}$$

$$AB' = \sqrt{53^2 + 48.3874^2 - 2(53)(48.3874)\cos 88.5°}$$

$$= 70.8243\text{ mm}$$



Average Normal Strain:

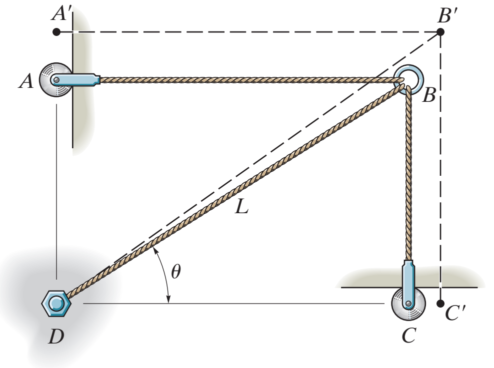
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{70.8243 - 70.7107}{70.7107} = 1.61\left(10^{-3}\right)\text{ mm/mm}$$ **Ans.**

$$\varepsilon_{CD} = \frac{C'D' - CD}{CD}$$

$$= \frac{79.5860 - 70.7107}{70.7107} = 126\left(10^{-3}\right)\text{ mm/mm}$$ **Ans.**

•2–17. The three cords are attached to the ring at B. When a force is applied to the ring it moves it to point B', such that the normal strain in AB is ϵ_{AB} and the normal strain in CB is ϵ_{CB}. Provided these strains are small, determine the normal strain in DB. Note that AB and CB remain horizontal and vertical, respectively, due to the roller guides at A and C.



Coordinates of B $(L\cos\theta, L\sin\theta)$

Coordinates of B' $(L\cos\theta + \varepsilon_{AB} L\cos\theta, L\sin\theta + \varepsilon_{CB} L\sin\theta)$

$$L_{DB'} = \sqrt{(L\cos\theta + \varepsilon_{AB} L\cos\theta)^2 + (L\sin\theta + \varepsilon_{CB} L\sin\theta)^2}$$

$$L_{DB'} = L\sqrt{\cos^2\theta(1 + 2\varepsilon_{AB} + \varepsilon_{AB}^2) + \sin^2\theta(1 + 2\varepsilon_{CB} + \varepsilon_{CB}^2)}$$

Since ε_{AB} and ε_{CB} are small,

$$L_{DB'} = L\sqrt{1 + (2\,\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)}$$

Use the binomial theorem,

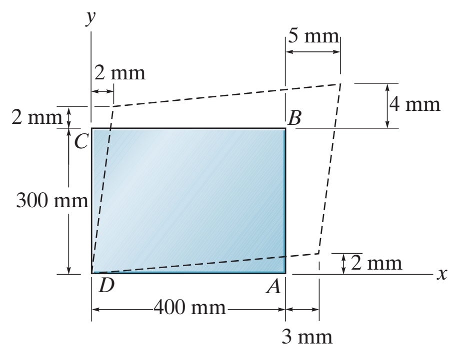
$$L_{DB'} = L\left(1 + \frac{1}{2}(2\,\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)\right)$$

$$= L\left(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta\right)$$

$$\text{Thus, } \varepsilon_{DB} = \frac{L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta) - L}{L}$$

$$\varepsilon_{DB} = \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta$$ **Ans.**

2–18. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$

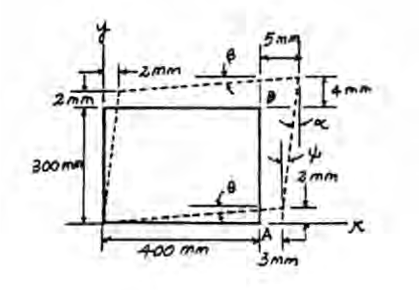
Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

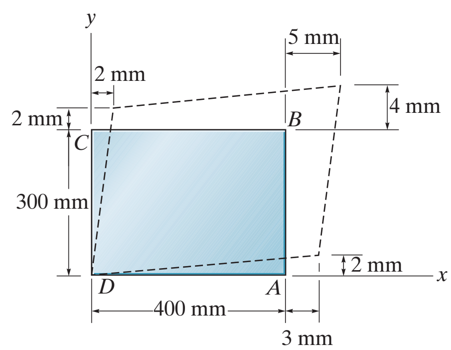
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = -(\theta + \psi)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



2–19. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$

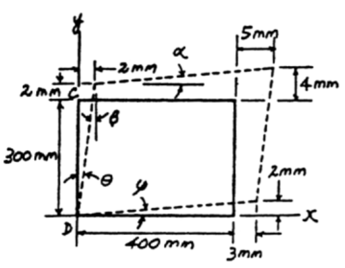
Shear Strain:

$$(\gamma_C)_{xy} = -(\alpha + \beta)$$

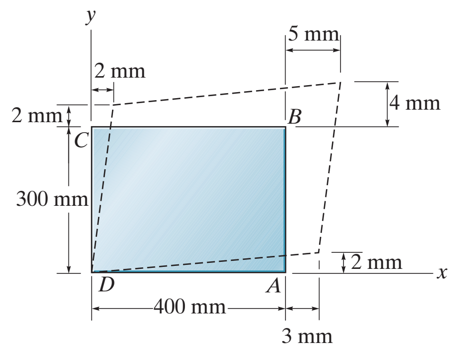
$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



***2–20.** The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals AC and DB.



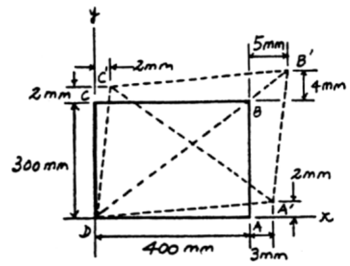
Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$

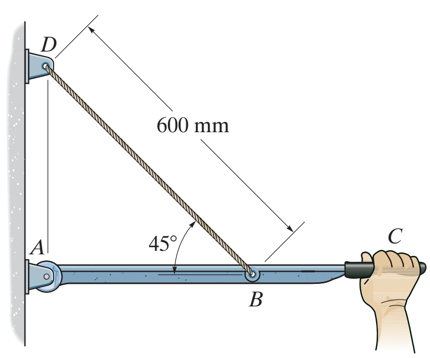
Average Normal Strain:



$$\varepsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$
$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$
$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

•2–21. The force applied to the handle of the rigid lever arm causes the arm to rotate clockwise through an angle of 3° about pin A. Determine the average normal strain developed in the wire. Originally, the wire is unstretched.

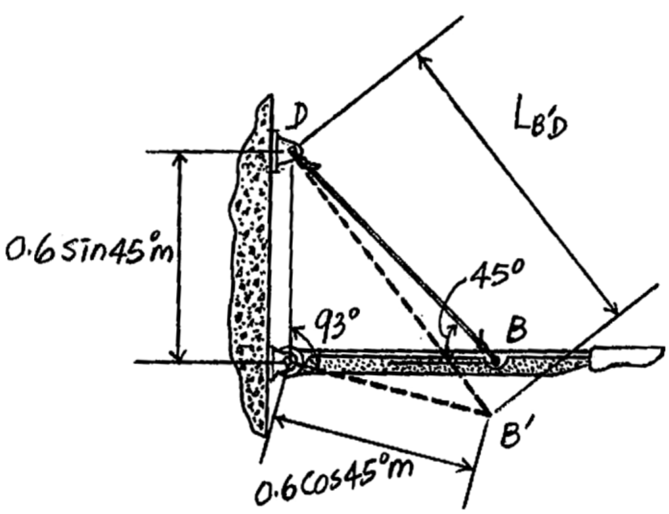


Geometry: Referring to Fig. a, the stretched length of $L_{B'D}$ can be determined using the consine law,

$$L_{B'D} = \sqrt{(0.6\cos 45°)^2 + (0.6\sin 45°)^2 - 2(0.6\cos 45°)(0.6\sin 45°)\cos 93°}$$
$$= 0.6155 \text{ m}$$

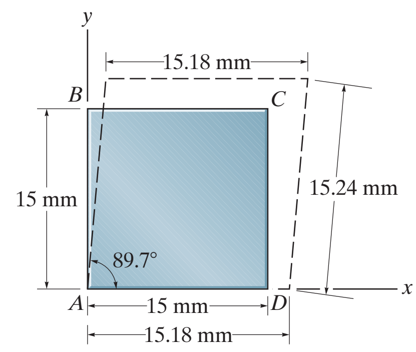
Average Normal Strain: The unstretched length of wire BD is $L_{BD} = 0.6$ m. We obtain

$$\varepsilon_{\text{avg}} = \frac{L_{B'D} - L_{BD}}{L_{BD}} = \frac{0.6155 - 0.6}{0.6} = 0.0258 \text{ m/m} \qquad \textbf{Ans.}$$



(a)

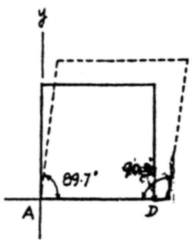
2–22. A square piece of material is deformed into the dashed position. Determine the shear strain γ_{xy} at A.



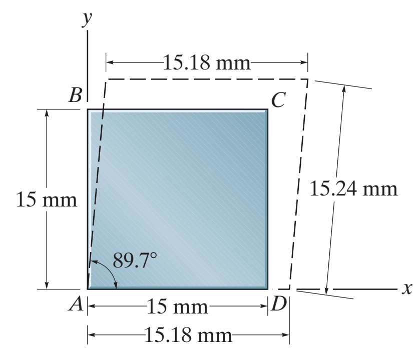
Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \left(\frac{89.7°}{180°}\right)\pi$$

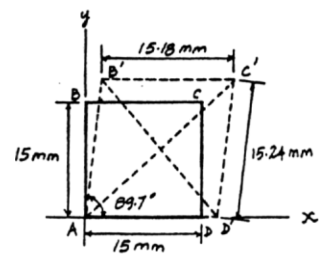
$$= 5.24\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



2–23. A square piece of material is deformed into the dashed parallelogram. Determine the average normal strain that occurs along the diagonals AC and BD.



Geometry:



$$AC = BD = \sqrt{15^2 + 15^2} = 21.2132 \text{ mm}$$

$$AC' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 90.3°}$$

$$= 21.5665 \text{ mm}$$

$$B'D' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 89.7°}$$

$$= 21.4538 \text{ mm}$$

Average Normal Strain:

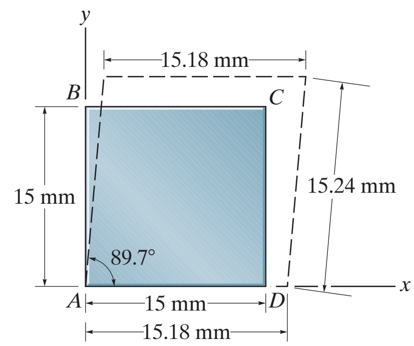
$$\varepsilon_{AC} = \frac{AC' - AC}{AC} = \frac{21.5665 - 21.2132}{21.2132}$$

$$= 0.01665 \text{ mm/mm} = 16.7\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{BD} = \frac{B'D' - BD}{BD} = \frac{21.4538 - 21.2132}{21.2132}$$

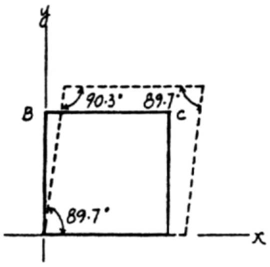
$$= 0.01134 \text{ mm/mm} = 11.3\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

*2–24. A square piece of material is deformed into the dashed position. Determine the shear strain γ_{xy} at C.

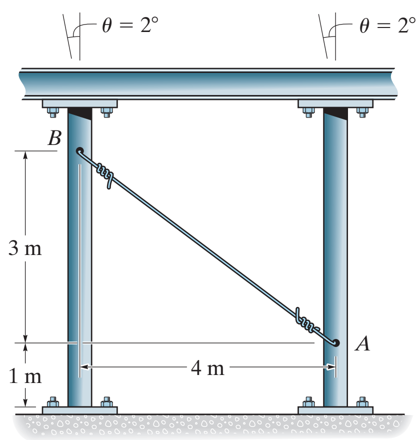


$$(\gamma_C)_{xy} = \frac{\pi}{2} - \left(\frac{89.7°}{180°}\right)\pi$$

$$= 5.24\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



•2–25. The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.



Geometry: The vertical displacement is negligible

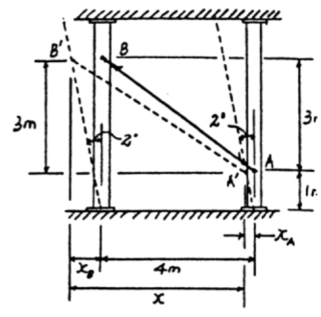
$$x_A = (1)\left(\frac{2°}{180°}\right)\pi = 0.03491 \text{ m}$$

$$x_B = (4)\left(\frac{2°}{180°}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$

$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$

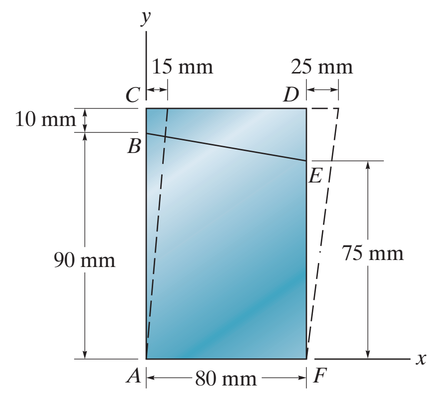


Average Normal Strain:

$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$

$$= \frac{5.08416 - 5}{5} = 16.8\left(10^{-3}\right) \text{ m/m} \qquad \textbf{Ans.}$$

2–26. The material distorts into the dashed position shown. Determine (a) the average normal strains along sides AC and CD and the shear strain γ_{xy} at F, and (b) the average normal strain along line BE.



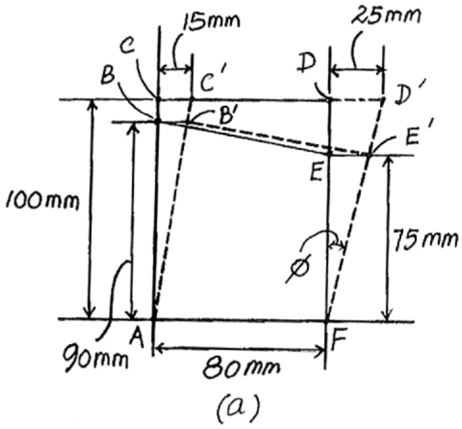
Referring to Fig. a,

$$L_{BE} = \sqrt{(90 - 75)^2 + 80^2} = \sqrt{6625}\text{ mm}$$

$$L_{AC'} = \sqrt{100^2 + 15^2} = \sqrt{10225}\text{ mm}$$

$$L_{C'D'} = 80 - 15 + 25 = 90\text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{25}{100}\right) = 14.04°\left(\frac{\pi\text{ rad}}{180°}\right) = 0.2450\text{ rad.}$$



When the plate deforms, the vertical position of point B and E do not change.

$$\frac{L_{BB'}}{90} = \frac{15}{100}; \qquad L_{BB'} = 13.5\text{ mm}$$

$$\frac{L_{EE'}}{75} = \frac{25}{100}; \qquad L_{EE'} = 18.75\text{ mm}$$

$$L_{B'E'} = \sqrt{(90 - 75)^2 + (80 - 13.5 + 18.75)^2} = \sqrt{7492.5625}\text{ mm}$$

Thus,

$$\left(\varepsilon_{\text{avg}}\right)_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{10225} - 100}{100} = 0.0112\text{ mm/mm} \qquad \textbf{Ans.}$$

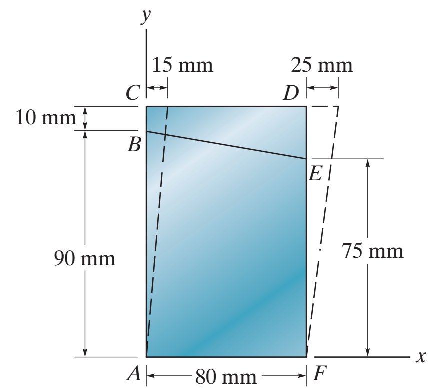
$$\left(\varepsilon_{\text{avg}}\right)_{CD} = \frac{L_{C'D'} - L_{CD}}{L_{CD}} = \frac{90-80}{80} = 0.125\text{ mm/mm} \qquad \textbf{Ans.}$$

$$\left(\varepsilon_{\text{avg}}\right)_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{7492.5625} - \sqrt{6625}}{\sqrt{6625}} = 0.0635\text{ mm/mm} \qquad \textbf{Ans.}$$

Referring to Fig. a, the angle at corner F becomes larger than 90° after the plate deforms. Thus, the shear strain is negative.

0.245 rad **Ans.**

2–27. The material distorts into the dashed position shown. Determine the average normal strain that occurs along the diagonals AD and CF.



The undeformed length of diagonals AD and CF are

$$L_{AD} = L_{CF} = \sqrt{80^2 + 100^2} = \sqrt{16400}\ \text{mm}$$

The deformed length of diagonals AD and CF are

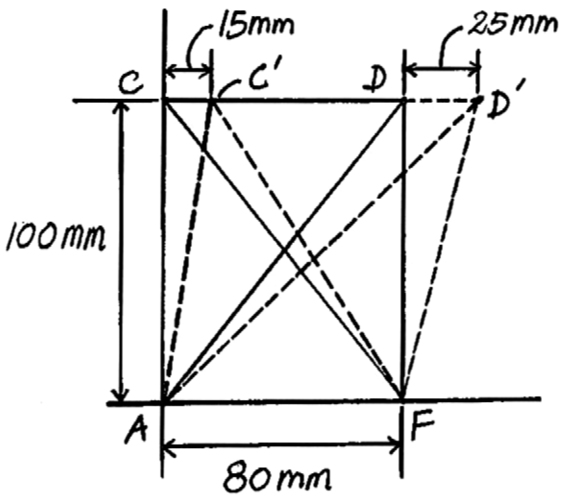
$$L_{AD'} = \sqrt{(80 + 25)^2 + 100^2} = \sqrt{21025}\ \text{mm}$$

$$L_{C'F} = \sqrt{(80 - 15)^2 + 100^2} = \sqrt{14225}\ \text{mm}$$

Thus,

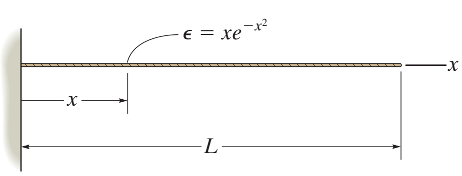
$$\left(\varepsilon_{\text{avg}}\right)_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{21025} - \sqrt{16400}}{\sqrt{16400}} = 0.132\ \text{mm/mm}$$ **Ans.**

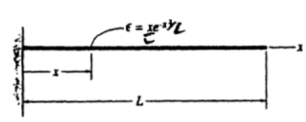
$$\left(\varepsilon_{\text{avg}}\right)_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{14225} - \sqrt{16400}}{\sqrt{16400}} = -0.0687\ \text{mm/mm}$$ **Ans.**



(a)

***2–28.** The wire is subjected to a normal strain that is defined by $\epsilon = xe^{-x^2}$, where x is in millimeters. If the wire has an initial length L, determine the increase in its length.





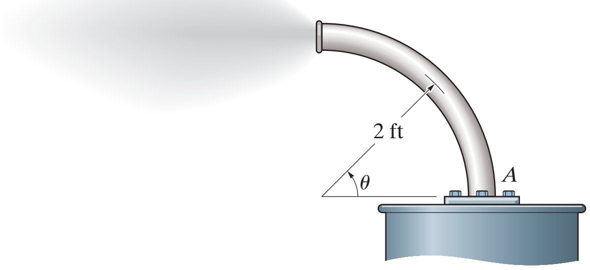
$$\delta L = \varepsilon\, dx = x\, e^{-x^2}\, dx$$

$$\Delta L = \int_0^L x\, e^{-x^2}\, dx$$

$$= -\left[\frac{1}{2} e^{-x^2}\right]\Bigg|_0^L = -\left[\frac{1}{2} e^{-L^2} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[1 - e^{-L^2}\right]$$ **Ans.**

•2–29. The curved pipe has an original radius of 2 ft. If it is heated nonuniformly, so that the normal strain along its length is $\epsilon = 0.05\cos\theta$, determine the increase in length of the pipe.



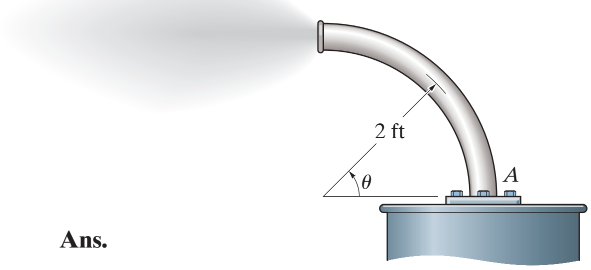
$$\varepsilon = 0.05\cos\theta$$

$$\Delta L = \int \varepsilon\, dL$$

$$= \int_0^{90^\circ} (0.05\cos\theta)(2\, d\theta)$$

$$= 0.1\int_0^{90^\circ} \cos\theta\, d\theta = \left[0.1[\sin\theta]\Big|_0^{90^\circ}\right] = 0.100\text{ ft}$$ **Ans.**

2–30. Solve Prob. 2–29 if $\epsilon = 0.08\sin\theta$.



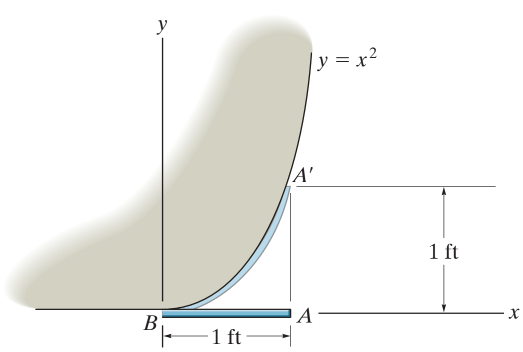
$$dL = 2\, d\theta \varepsilon = 0.08\sin\theta$$

$$\Delta L = \int \varepsilon\, dL$$

$$= \int_0^{90^\circ} (0.08\sin\theta)(2\, d\theta)$$

$$= 0.16\int_0^{90^\circ} \sin\theta\, d\theta = 0.16[-\cos\theta]\Big|_0^{90^\circ} = 0.16\text{ ft}$$ **Ans.**

2–31. The rubber band AB has an unstretched length of 1 ft. If it is fixed at B and attached to the surface at point A', determine the average normal strain in the band. The surface is defined by the function $y = (x^2)$ ft, where x is in feet.



Geometry:

$$L = \int_0^{1\text{ ft}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

However $y = x^2$ then $\dfrac{dy}{dx} = 2x$

$$L = \int_0^{1\text{ ft}} \sqrt{1 + 4x^2}\, dx$$

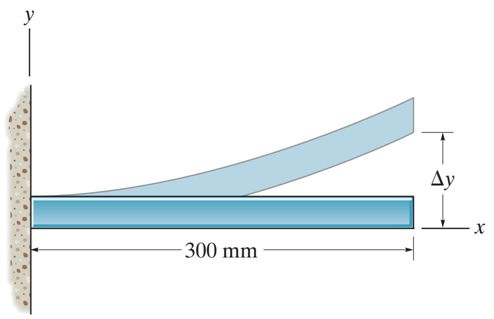
$$= \frac{1}{4}\left[2x\sqrt{1 + 4x^2} + \ln\left(2x + \sqrt{1 + 4x^2}\right)\right]\Big|_0^{1\text{ ft}}$$

$$= 1.47894\text{ ft}$$

Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{1.47894 - 1}{1} = 0.479\text{ ft/ft}$$ **Ans.**

***2–32.** The bar is originally 300 mm long when it is flat. If it is subjected to a shear strain defined by $\gamma_{xy} = 0.02x$, where x is in meters, determine the displacement Δy at the end of its bottom edge. It is distorted into the shape shown, where no elongation of the bar occurs in the x direction.



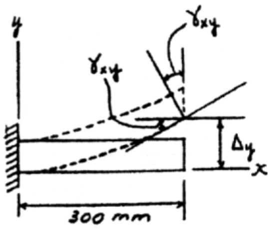
Shear Strain:

$$\frac{dy}{dx} = \tan \gamma_{xy}\,; \qquad \frac{dy}{dx} = \tan(0.02\,x)$$

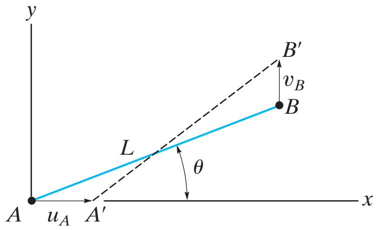
$$\int_0^{\Delta y} dy = \int_0^{300\text{ mm}} \tan(0.02\,x)dx$$

$$\Delta y = -50[\ln \cos(0.02x)]|_0^{300\text{ mm}}$$

$$= 2.03\text{ mm} \qquad \textbf{Ans.}$$



•2–33. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.



Geometry:

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^3 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\varepsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

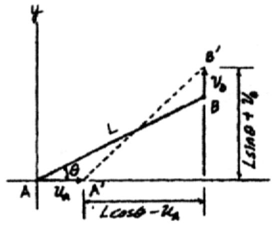
Neglecting higher terms u_A^2 and v_B^2

$$\varepsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\varepsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L} \qquad \textbf{Ans.}$$



2–34. If the normal strain is defined in reference to the final length, that is,

$$\epsilon_n' = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon_n' = \epsilon_n \epsilon_n'$.

$$\varepsilon_B = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\varepsilon_B - \varepsilon_A' = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$

$$= \frac{\Delta S'^2 - \Delta S \Delta S' - \Delta S' \Delta S + \Delta S^2}{\Delta S \Delta S'}$$

$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S' \Delta S}{\Delta S \Delta S'}$$

$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S \Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S} \right) \left(\frac{\Delta S' - \Delta S}{\Delta S'} \right)$$

$$= \varepsilon_A \varepsilon_B' \text{ (Q.E.D)}$$