Online Instructor's Solutions Manual

Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences

Arab World Edition

Ernest F. Haeussler, Jr.
The Pennsylvania State University

Richard S. Paul
The Pennsylvania State University

Richard J. Wood
Dalhousie University

Saadia Khouyibaba
American University of Sharjah

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Chapter 0

Problems 0.1

1. True; –13 is a negative integer.

2. True, because –2 and 7 are integers and $7 \neq 0$.

3. False, because –3 is not positive.

4. False, because $0 = \frac{0}{1}$.

5. False, it is fairly easy to see (although the details are not relevant for a course using this text) that $\sqrt{n}$, for n an integer, is either an integer (if n is a perfect square) or irrational (if n is not a perfect square). The perfect squares are 1, 4, 9, 16, 25, 36, ... and since 3 is not among these, $\sqrt{3}$ is not rational.

6. False, since a rational number cannot have denominator of zero. In fact, $\frac{7}{0}$ is not a number at all because we cannot divide by 0.

7. False, because $\sqrt{25} = 5$, which is a positive integer.

8. True; $\sqrt{2}$ is an irrational real number.

9. False; we cannot divide by 0.

10. False, we have $3 < \pi < 4$ so that π is at best rational and not an integer. It can be shown that π is irrational (but the details are not relevant for our purposes).

11. True

12. False, since the integer 0 is neither positive nor negative.

13. True

14. False; 2.78 and $\sqrt{5}$ are real numbers but not integers

15. True

16. True

17. False; negative integers are not natural numbers.

Problems 0.2

1. False, because 0 does not have a reciprocal.

2. True, because $\frac{7}{3} \cdot \frac{3}{7} = \frac{21}{21} = 1$.

3. False; the negative of 7 is –7 because $7 + (-7) = 0$.

4. True; $1(x \cdot y) = (1 \cdot x)(1 \cdot y)$

5. False; $-x + y = y + (-x) = y - x$.

6. True; $(x + 2)(4) = (x)(4) + (2)(4) = 4x + 8$.

7. True; $\frac{x+2}{2} = \frac{x}{2} + \frac{2}{2} = \frac{x}{2} + 1$.

8. True, because $a\left(\frac{b}{c}\right) = \frac{ab}{c}$.

9. False; $2(x \cdot y) = 2xy$ while $(2x) \cdot (2y) = (2 \cdot 2) \cdot (x \cdot y) = 4xy$.

10. True; by the associative and commutative properties, $x(4y) = (x \cdot 4)y = (4 \cdot x)y = 4xy$.

11. distributive

12. commutative

13. associative

14. definition of division and commutative property

15. commutative and distributive

16. associative

17. definition of subtraction

18. commutative

19. distributive and commutative

20. distributive

21. $2x(y - 7) = (2x)y - (2x)7 = 2xy - (7)(2x)$
$= 2xy - (7 \cdot 2)x = 2xy - 14x$

22. $(a-b)+c=[a+(-b)]+c=a+(-b+c)$
$=a+[c+(-b)]=a+(c-b)$

23. $(x+y)(2)=2(x+y)=2x+2y$

24. $x[(2y+1)+3]=x[2y+(1+3)]=x[2y+4]$
$=x(2y)+x(4)=(x\cdot 2)y+4x=(2x)y+4x$
$=2xy+4x$

25. $x(y-z+w)=x[(y-z)+w]=x(y-z)+x(w)$
$=x[y+(-z)]+xw=x(y)+x(-z)+xw$
$=xy-xz+xw$

26. $(1+a)(b+c)=1(b+c)+a(b+c)$
$=1(b)+1(c)+a(b)+a(c)=b+c+ab+ac$

27. $-2+(-4)=-6$

28. $-a+b=b-a$

29. $6+(-4)=2$

30. $7-2=5$

31. $7-(-4)=7+4=11$

32. $-5-(-13)=-5+13=8$

33. $-(-a)+(-b)=a-b$

34. $(-2)(9)=-(2\cdot 9)=-18$

35. $7(-9)=-(7\cdot 9)=-63$

36. $(-2)(-12)=2(12)=24$

37. $19(-1)=(-1)19=-(1\cdot 19)=-19$

38. $\frac{-1}{-\frac{1}{a}}=-1\left(-\frac{a}{1}\right)=a$

39. $-(-6+x)=-(-6)-x=6-x$

40. $-7(x)=-(7x)=-7x$

41. $-12(x-y)=(-12)x-(-12)(y)=-12x+12y$
(or $12y-12x$)

42. $-[-6+(-y)]=-(-6)-(-y)=6+y$

43. $-3\div(3a)=\frac{-3}{3a}=-\frac{1\cdot 3}{3\cdot a}=-\frac{1}{a}$

44. $-9\div(-27)=\frac{-9}{-27}=\frac{9}{27}=\frac{9\cdot 1}{9\cdot 3}=\frac{1}{3}$

45. $(-a)\div(-b)=\frac{-a}{-b}=\frac{a}{b}$

46. $2(-6+2)=2(-4)=-8$

47. $3[-2(3)+6(2)]=3[-6+12]=3[6]=18$

48. $(-a)(-b)(-1)=ab(-1)=-ab$

49. $(-12)(-12)=(12)(12)=144$

50. $X(1)=X$

51. $3(x-4)=3(x)-3(4)=3x-12$

52. $4(5+x)=4(5)+4(x)=20+4x$

53. $-(x-y)=-x+y=y-x$

54. $0(-x)=0$

55. $8\left(\frac{1}{11}\right)=\frac{8\cdot 1}{11}=\frac{8}{11}$

56. $\frac{5}{1}=5$

57. $\frac{14x}{21y}=\frac{2\cdot 7\cdot x}{3\cdot 7\cdot y}=\frac{2x}{3y}$

58. $\frac{2x}{-2}=\frac{2\cdot x}{-1\cdot 2}=-x$

59. $\frac{2}{3}\cdot\frac{1}{x}=\frac{2\cdot 1}{3\cdot x}=\frac{2}{3x}$

60. $\frac{a}{c}(3b)=\frac{a(3b)}{c}=\frac{3ab}{c}$

61. $(5a)\left(\frac{7}{5a}\right)=7$

62. $\frac{a}{b}\cdot\frac{1}{c}=\frac{a\cdot 1}{b\cdot c}=\frac{a}{bc}$

63. $\frac{2}{x}\cdot\frac{5}{y}=\frac{2\cdot 5}{x\cdot y}=\frac{10}{xy}$

64. $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{3+2}{6}=\frac{5}{6}$

65. $\frac{5}{12}+\frac{3}{4}=\frac{5}{12}+\frac{9}{12}=\frac{5+9}{12}=\frac{14}{12}=\frac{2\cdot 7}{2\cdot 6}=\frac{7}{6}$

66. $\frac{3}{10}-\frac{7}{15}=\frac{9}{30}-\frac{14}{30}=\frac{9-14}{30}=\frac{-5}{30}=-\frac{5\cdot 1}{5\cdot 6}=-\frac{1}{6}$

67. $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

68. $\frac{X}{\sqrt{5}}-\frac{Y}{\sqrt{5}}=\frac{X-Y}{\sqrt{5}}$

69. $\frac{3}{2}-\frac{1}{4}+\frac{1}{6}=\frac{18}{12}-\frac{3}{12}+\frac{2}{12}=\frac{18-3+2}{12}=\frac{17}{12}$

70. $\frac{2}{5}-\frac{3}{8}=\frac{16}{40}-\frac{15}{40}=\frac{16-15}{40}=\frac{1}{40}$

71. $\frac{6}{\frac{x}{y}}=6\div\frac{x}{y}=6\cdot\frac{y}{x}=\frac{6y}{x}$

72. $\frac{\frac{l}{w}}{m}=\frac{l}{w}\div\frac{m}{1}=\frac{l}{w}\cdot\frac{1}{m}=\frac{l}{wm}$

73. $\frac{\frac{-x}{y^2}}{\frac{z}{xy}}=-\frac{x}{y^2}\div\frac{z}{xy}=-\frac{x}{y^2}\cdot\frac{xy}{z}=-\frac{x^2}{yz}$

74. $\frac{0}{7}=0$

Problems 0.3

1. $(2^3)(2^2)=2^{3+2}=2^5(=32)$

2. $x^6x^9=x^{6+9}=x^{15}$

3. $a^5a^2=a^{5+2}=a^7$

4. $z^3zz^2=z^{3+1+2}=z^6$

5. $\frac{x^3x^5}{y^9y^5}=\frac{x^{3+5}}{y^{9+5}}=\frac{x^8}{y^{14}}$

6. $(x^{12})^4=x^{12\cdot 4}=x^{48}$

7. $\frac{(a^3)^7}{(b^4)^5}=\frac{a^{3\cdot 7}}{b^{4\cdot 5}}=\frac{a^{21}}{b^{20}}$

8. $\left(\frac{w}{w^3}\right)^7=\left(\frac{w}{w\cdot w^2}\right)^7=\left(\frac{1}{w^2}\right)^7=\frac{1^7}{(w^2)^7}=\frac{1}{w^{14}}$

9. $(2x^2y^3)^3=2^3(x^2)^3(y^3)^3=8x^{2\cdot 3}y^{3\cdot 3}=8x^6y^9$

10. $\left(\frac{w^2s^3}{y^2}\right)^2=\frac{(w^2s^3)^2}{(y^2)^2}=\frac{(w^2)^2(s^3)^2}{y^{2\cdot 2}}=\frac{w^{2\cdot 2}s^{3\cdot 2}}{y^4}$
$=\frac{w^4s^6}{y^4}$

11. $\frac{x^9}{x^5}=x^{9-5}=x^4$

12. $\left(\frac{2a^4}{7b^5}\right)^6=\frac{(2a^4)^6}{(7b^5)^6}$
$=\frac{2^6(a^4)^6}{7^6(b^5)^6}$
$=\frac{64a^{4\cdot 6}}{117,649b^{5\cdot 6}}$
$=\frac{64a^{24}}{117,649b^{30}}$

13. $\frac{(x^2)^5}{(x^3)^2x^4}=\frac{x^{2\cdot 5}}{x^{3\cdot 2}x^4}=\frac{x^{10}}{x^6x^4}=\frac{x^{10}}{x^{6+4}}=\frac{x^{10}}{x^{10}}=1$

14. $\sqrt{25}=5$

15. $\sqrt[4]{81}=3$

16. $\sqrt[7]{-128}=-2$

17. $\sqrt[3]{0.027}=\sqrt[3]{(0.3)^3}=0.3$

18. $\sqrt[4]{\frac{1}{16}}=\frac{\sqrt[4]{1}}{\sqrt[4]{16}}=\frac{1}{2}$

19. $\sqrt[3]{-\frac{8}{27}} = \sqrt[3]{\frac{-8}{27}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{27}} = \frac{-2}{3} = -\frac{2}{3}$

20. $(49)^{1/2} = \sqrt{49} = 7$

21. $(64)^{1/3} = \sqrt[3]{64} = 4$

22. $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

23. $(9)^{-5/2} = \frac{1}{(9)^{5/2}} = \frac{1}{\left(\sqrt{9}\right)^5} = \frac{1}{3^5} = \frac{1}{243}$

24. $(32)^{-2/5} = \frac{1}{(32)^{2/5}} = \frac{1}{\left(\sqrt[5]{32}\right)^2} = \frac{1}{(2)^2} = \frac{1}{4}$

25. $(0.09)^{-1/2} = \frac{1}{(0.09)^{1/2}} = \frac{1}{\sqrt{0.09}} = \frac{1}{0.3}$

$= \frac{1}{\frac{3}{10}} = \frac{10}{3}$

26. $\left(\frac{1}{32}\right)^{4/5} = \left(\sqrt[5]{\frac{1}{32}}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

27. $\left(-\frac{27}{64}\right)^{4/3} = \left(\sqrt[3]{-\frac{27}{64}}\right)^4 = \left(-\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$

28. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$

29. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\ \sqrt[3]{2} = 3\sqrt[3]{2}$

30. $\sqrt[3]{2x^3} = \sqrt[3]{2}\ \sqrt[3]{x^3} = x\ \sqrt[3]{2}$

31. $\sqrt{4x} = \sqrt{4}\sqrt{x} = 2\sqrt{x}$

32. $\sqrt{25y^6} = \sqrt{25} \cdot \sqrt{y^6} = 5y^3$

33. $\sqrt[4]{\frac{x}{16}} = \frac{\sqrt[4]{x}}{\sqrt[4]{16}} = \frac{\sqrt[4]{x}}{2}$

34. $\sqrt{\frac{3}{13}} = \sqrt{\frac{3}{13} \cdot \frac{13}{13}} = \sqrt{\frac{39}{13^2}} = \frac{\sqrt{39}}{\sqrt{13^2}} = \frac{\sqrt{39}}{13}$

35. $(9z^4)^{1/2} = \sqrt{9z^4} = \sqrt{3^2(z^2)^2} = \sqrt{3^2}\sqrt{(z^2)^2}$

$= 3z^2$

36. $(27x^6)^{4/3} = \left(\sqrt[3]{27x^6}\right)^4 = (3x^2)^4 = 81x^8$

37. $\left(\frac{27t^3}{8}\right)^{2/3} = \left(\left[\frac{3t}{2}\right]^3\right)^{2/3} = \left[\frac{3t}{2}\right]^2 = \frac{9t^2}{4}$

38. $\sqrt[5]{x^2y^3z^{-10}} = x^{2/5}y^{3/5}z^{-10/5} = \frac{x^{2/5}y^{3/5}}{z^2}$

39. $\frac{a^5b^{-3}}{c^2} = a^5 \cdot b^{-3} \cdot \frac{1}{c^2} = a^5 \cdot \frac{1}{b^3} \cdot \frac{1}{c^2} = \frac{a^5}{b^3c^2}$

40. $2a^{-1}b^{-3} = 2 \cdot \frac{1}{a} \cdot \frac{1}{b^3} = \frac{2}{ab^3}$

41. $x + y^{-1} = x + \frac{1}{y}$

42. $(3t)^{-2} = \frac{1}{(3t)^2} = \frac{1}{9t^2}$

43. $(3-z)^{-4} = \frac{1}{(3-z)^4}$

44. $\sqrt[5]{5x^2} = (5x^2)^{1/5} = 5^{1/5}(x^2)^{1/5} = 5^{1/5}x^{2/5}$

45. $(X^2Y^{-2})^{-2} = \left(\frac{X^2}{Y^2}\right)^{-2} = \left(\frac{Y^2}{X^2}\right)^2 = \frac{Y^4}{X^4}$

46. $\sqrt{x} - \sqrt{y} = x^{1/2} - y^{1/2}$

47. $\frac{u^{-2}v^{-6}w^3}{vw^{-5}} = \frac{w^{3-(-5)}}{u^2v^{1-(-6)}} = \frac{w^8}{u^2v^7}$

48. $\sqrt[4]{a^{-3}b^{-2}}\,a^5b^{-4} = (a^{-3}b^{-2})^{1/4}a^5b^{-4}$

$= a^{-3/4}b^{-1/2}a^5b^{-4}$

$= a^{17/4}b^{-9/2}$

$= \frac{a^{17/4}}{b^{9/2}}$

49. $x^2\sqrt[4]{xy^{-2}z^3} = x^2(xy^{-2}z^3)^{1/4} = x^2x^{1/4}y^{-2/4}z^{3/4}$
$= \dfrac{x^{9/4}z^{3/4}}{y^{1/2}}$

50. $(a+b-c)^{2/3} = \sqrt[3]{(a+b-c)^2}$

51. $(ab^2c^3)^{3/4} = \sqrt[4]{(ab^2c^3)^3} = \sqrt[4]{a^3b^6c^9}$

52. $x^{-4/5} = \dfrac{1}{x^{4/5}} = \dfrac{1}{\sqrt[5]{x^4}}$

53. $2x^{1/2} - (2y)^{1/2} = 2\sqrt{x} - \sqrt{2y}$

54. $3w^{-3/5} - (3w)^{-3/5} = \dfrac{3}{w^{3/5}} - \dfrac{1}{(3w)^{3/5}}$
$= \dfrac{3}{\sqrt[5]{w^3}} - \dfrac{1}{\sqrt[5]{(3w)^3}} = \dfrac{3}{\sqrt[5]{w^3}} - \dfrac{1}{\sqrt[5]{27w^3}}$

55. $((x^{-5})^{1/3})^{1/4} = (x^{-5/3})^{1/4}$
$= x^{-5/12}$
$= \dfrac{1}{x^{5/12}}$
$= \dfrac{1}{\sqrt[12]{x^5}}$

56. $\dfrac{6}{\sqrt{5}} = \dfrac{6}{5^{1/2}} = \dfrac{6\cdot 5^{1/2}}{5^{1/2}\cdot 5^{1/2}} = \dfrac{6\sqrt{5}}{5}$

57. $\dfrac{3}{\sqrt[4]{8}} = \dfrac{3}{8^{1/4}} = \dfrac{3\cdot 2^{1/4}}{8^{1/4}\cdot 2^{1/4}} = \dfrac{3\sqrt[4]{2}}{\sqrt[4]{16}} = \dfrac{3\sqrt[4]{2}}{2}$

58. $\dfrac{4}{\sqrt{2x}} = \dfrac{4}{(2x)^{1/2}} = \dfrac{4(2x)^{1/2}}{(2x)^{1/2}(2x)^{1/2}} = \dfrac{4\sqrt{2x}}{2x}$
$= \dfrac{2\sqrt{2x}}{x}$

59. $\dfrac{1}{\sqrt[3]{2a}} = \dfrac{1}{(2a)^{1/3}} = \dfrac{1(2a)^{2/3}}{(2a)^{1/3}(2a)^{2/3}} = \dfrac{(2a)^{2/3}}{2a}$
$= \dfrac{\sqrt[3]{4a^2}}{2a}$

60. $\dfrac{y}{\sqrt{2y}} = \dfrac{y}{(2y)^{1/2}} = \dfrac{y(2y)^{1/2}}{(2y)^{1/2}(2y)^{1/2}} = \dfrac{y\sqrt{2y}}{2y}$
$= \dfrac{\sqrt{2y}}{2}$

61. $\dfrac{2}{3\sqrt[3]{y^2}} = \dfrac{2}{3y^{2/3}} = \dfrac{2\cdot y^{1/3}}{3y^{2/3}\cdot y^{1/3}} = \dfrac{2y^{1/3}}{3y} = \dfrac{2\sqrt[3]{y}}{3y}$

62. $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

63. $\sqrt{\dfrac{18}{2}} = \sqrt{9} = 3$

64. $\dfrac{\sqrt[5]{2}}{\sqrt[4]{a^2b}} = \dfrac{\sqrt[5]{2}}{a^{2/4}b^{1/4}} = \dfrac{\sqrt[5]{2}\cdot a^{1/2}b^{3/4}}{a^{1/2}b^{1/4}\cdot a^{1/2}b^{3/4}}$
$= \dfrac{2^{1/5}a^{1/2}b^{3/4}}{ab} = \dfrac{2^{4/20}a^{10/20}b^{15/20}}{ab}$
$= \dfrac{(2^4a^{10}b^{15})^{1/20}}{ab} = \dfrac{\sqrt[20]{16a^{10}b^{15}}}{ab}$

65. $\dfrac{\sqrt{3}}{\sqrt[3]{2}} = \dfrac{\sqrt{3}}{2^{1/3}} = \dfrac{\sqrt{3}\cdot 2^{2/3}}{2^{1/3}\cdot 2^{2/3}} = \dfrac{\sqrt{3}\cdot\sqrt[3]{2^2}}{2} = \dfrac{\sqrt{3}\cdot\sqrt[3]{4}}{2}$

66. $2x^2y^{-3}x^4 = 2x^6y^{-3} = \dfrac{2x^6}{y^3}$

67. $\dfrac{3}{u^{5/2}v^{1/2}} = \dfrac{3\cdot u^{1/2}v^{1/2}}{u^{5/2}v^{1/2}\cdot u^{1/2}v^{1/2}} = \dfrac{3u^{1/2}v^{1/2}}{u^3v}$

68. $\{[(3a^3)^2]^{-5}\}^{-2} = \{[3^2a^6]^{-5}\}^{-2}$
$= \{3^{-10}a^{-30}\}^{-2}$
$= 3^{20}a^{60}$

69. $\dfrac{\sqrt{243}}{\sqrt{3}} = \sqrt{\dfrac{243}{3}} = \sqrt{81} = 9$

70. $\dfrac{3^0}{(3^{-3}x^{1/3}y^{-3})^2} = \dfrac{1}{3^{-6}x^{2/3}y^{-6}} = \dfrac{3^6y^6}{x^{2/3}}$
$= \dfrac{3^6y^6\cdot x^{1/3}}{x^{2/3}\cdot x^{1/3}} = \dfrac{(3y)^6x^{1/3}}{x}$

71. $\frac{\sqrt{s^5}}{\sqrt[3]{s^2}} = \frac{s^{5/2}}{s^{2/3}} = \frac{s^{15/6}}{s^{4/6}} = s^{11/6}$

72. $\left(\sqrt[4]{3}\right)^8 = (3^{1/4})^8 = 3^{8/4} = 3^2 = 9$

73. $\sqrt[3]{x^2yz^3}\sqrt[3]{xy^2} = \sqrt[3]{(x^2yz^3)(xy^2)} = \sqrt[3]{x^3y^3z^3}$
$= xyz$

74. $3^2(32)^{-2/5} = 3^2(2^5)^{-2/5}$
$= 3^2(2^{-2})$
$= 3^2 \cdot \frac{1}{2^2}$
$= \frac{9}{4}$

75. $\left(\sqrt[5]{a^2b}\right)^{3/5} = (a^{2/5}b^{1/5})^{3/5} = a^{6/25}b^{3/25}$
$= (a^6b^3)^{1/25}$

76. $(2x^{-1}y^2)^2 = 2^2x^{-2}y^4 = \frac{4y^4}{x^2}$

77. $\frac{3}{\sqrt[3]{y}\sqrt[4]{x}} = \frac{3}{y^{1/3}x^{1/4}} = \frac{3 \cdot y^{2/3}x^{3/4}}{y^{1/3}x^{1/4} \cdot y^{2/3}x^{3/4}}$
$= \frac{3x^{3/4}y^{2/3}}{xy}$

78. $\sqrt{x}\sqrt{x^2y^3}\sqrt{xy^2} = x^{1/2}(x^2y^3)^{1/2}(xy^2)^{1/2}$
$= x^{1/2}(xy^{3/2})(x^{1/2}y) = x^2y^{5/2}$

79. $\sqrt{75k^4} = (75k^4)^{1/2} = [(25k^4)(3)]^{1/2}$
$= [(5k^2)^2 3]^{1/2} = 5k^2 3^{1/2}$

80. $\sqrt[3]{7(49)} = \sqrt[3]{7 \cdot 7^2} = \sqrt[3]{7^3} = 7$

81. $\frac{(x^2)^3}{x^4} \div \left[\frac{x^3}{(x^3)^2}\right]^2 = \frac{x^6}{x^4} \div \frac{(x^3)^2}{(x^6)^2}$
$= x^2 \div \frac{x^6}{x^{12}} = x^2 \div x^{6-12} = x^2 \div x^{-6}$
$= x^2 \div \frac{1}{x^6} = x^2 \cdot x^6 = x^8$

82. $\sqrt{(-6)(-6)} = \sqrt{36} = 6$
Note that $\sqrt{(-6)^2} \neq -6$ since $-6 < 0$.

83. $-\frac{8s^{-2}}{2s^3} = -\frac{4}{s^3s^2} = -\frac{4}{s^5}$

84. $\left(x^3y^{-4}\sqrt{z}\right)^5 = (x^3y^{-4}z^{1/2})^5$
$= x^{15}y^{-20}z^{5/2}$
$= \frac{x^{15}z^{5/2}}{y^{20}}$

85. $(3x^3y^2 \div 2y^2z^{-3})^4 = \left(\frac{3x^3y^2}{2y^2z^{-3}}\right)^4$
$= \left(\frac{3x^3z^3}{2}\right)^4$
$= \frac{(3x^3z^3)^4}{(2)^4}$
$= \frac{3^4x^{12}z^{12}}{2^4}$
$= \frac{81x^{12}z^{12}}{16}$

86. $\text{LCD} = \sqrt[3]{x+h} \cdot \sqrt[3]{x}$
$\frac{3}{\sqrt[3]{x+h}} - \frac{3}{\sqrt[3]{x}} = \frac{3\sqrt[3]{x}}{\sqrt[3]{x+h}\sqrt[3]{x}} - \frac{3\sqrt[3]{x+h}}{\sqrt[3]{x+h}\sqrt[3]{x}}$
$= \frac{3\left(\sqrt[3]{x} - \sqrt[3]{x+h}\right)}{\sqrt[3]{x+h}\sqrt[3]{x}}$

87. $\frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

88. $\frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$

89. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}} \cdot \frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}$
$= \frac{\sqrt{2}\left(\sqrt{3}+\sqrt{6}\right)}{3-6} = \frac{\sqrt{6}+\sqrt{12}}{-3} = -\frac{\sqrt{6}+2\sqrt{3}}{3}$

90. $\frac{5}{\sqrt{6}+\sqrt{7}}\cdot\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}-\sqrt{7}}=\frac{5\left(\sqrt{6}-\sqrt{7}\right)}{6-7}$
$=\frac{5\left(\sqrt{6}-\sqrt{7}\right)}{-1}=5\left(\sqrt{7}-\sqrt{6}\right)$

91. $\frac{2\sqrt{5}}{\sqrt{3}-\sqrt{7}}\cdot\frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}$
$=\frac{2\sqrt{5}\left(\sqrt{3}+\sqrt{7}\right)}{3-7}$
$=\frac{2\left(\sqrt{15}+\sqrt{35}\right)}{-4}$
$=-\frac{\sqrt{15}+\sqrt{35}}{2}$

92. $\frac{3}{t+\sqrt{7}}\cdot\frac{t-\sqrt{7}}{t-\sqrt{7}}=\frac{3t-3\sqrt{7}}{t^2-7}$

93. $\frac{(x-3)+4}{\sqrt{x}-1}=\frac{x+1}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{(x+1)\left(\sqrt{x}+1\right)}{x-1}$

94. $\frac{5\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}-\frac{4\left(1+\sqrt{2}\right)}{\left(1-\sqrt{2}\right)\left(1+\sqrt{2}\right)}$
$=\frac{5\left(2-\sqrt{3}\right)}{4-3}-\frac{4\left(1+\sqrt{2}\right)}{1-2}$
$=\frac{5\left(2-\sqrt{3}\right)}{1}-\frac{4\left(1+\sqrt{2}\right)}{-1}$
$=5\left(2-\sqrt{3}\right)+4\left(1+\sqrt{2}\right)=4\sqrt{2}-5\sqrt{3}+14$

95. $\frac{5x^2}{4\sqrt{x}+12}=\frac{5x^2}{4\sqrt{x}+12}\cdot\frac{4\sqrt{x}-12}{4\sqrt{x}-12}$
$=\frac{20x^2\sqrt{x}-60x^2}{16x-144}=\frac{20x^2\left(\sqrt{x}-3\right)}{16(x-9)}$
$=\frac{5x^2\left(\sqrt{x}-3\right)}{4(x-9)}$

Apply It 0.4

1. If x is the width of the rectangle, then $4x$ is the length.

The perimeter P is given by

$$P = 2\times(\text{length}\times\text{width})$$
$$= 2(4x + x)$$
$$= 10x$$

The area A is given by

$$A = \text{length}\times\text{width}$$
$$= 4x\cdot x$$
$$= 4x^2$$

Problems 0.4

1. $8x - 4y + 2 + 3x + 2y - 5 = 11x - 2y - 3$

2. $4a^2 - 2ab + 3 + 5c - 3ab + 7$
$= 4a^2 - 5ab + 10 + 5c$

3. $8t^2 - 6s^2 + 4s^2 - 2t^2 + 6 = 6t^2 - 2s^2 + 6$

4. $\sqrt{x}+2\sqrt{x}+\sqrt{x}+3\sqrt{x}=7\sqrt{x}$

5. $\sqrt{a}+2\sqrt{3b}-\sqrt{c}+3\sqrt{3b}$
$=\sqrt{a}+5\sqrt{3b}-\sqrt{c}$

6. $3a + 7b - 9 - 5a - 9b - 21 = -2a - 2b - 30$

7. $7x^2 + 5xy + \sqrt{2} - 2z + 2xy - \sqrt{2}$
$= 7x^2 + 7xy - 2z$

8. $\sqrt{x}+2\sqrt{x}-\sqrt{x}-3\sqrt{x}=-\sqrt{x}$

9. $\sqrt{x}+\sqrt{2y}-\sqrt{x}-\sqrt{3z}=\sqrt{2y}-\sqrt{3z}$

10. $8z - 4w - 3w + 6z = 14z - 7w$

11. $9x + 9y - 21 - 24x + 6y - 6 = -15x + 15y - 27$

12. $4s - 5t - 2s - 5t + s + 9 = 3s - 10t + 9$

13. $5x^2 - 5y^2 + xy - 3x^2 - 8xy - 28y^2$
$= 2x^2 - 33y^2 - 7xy$

14. $2 - [3 + 4s - 12] = 2 - [4s - 9] = 2 - 4s + 9$
$= 11 - 4s$

15. $2\{3[3x^2 + 6 - 2x^2 + 10]\} = 2\{3[x^2 + 16]\}$
$= 2\{3x^2 + 48\} = 6x^2 + 96$

16. $4\{3t+15-t[1-t-1]\}=4\{3t+15-t[-t]\}$
$=4\{3t+15+t^2\}=4t^2+12t+60$

17. $-\{-6a-6b+6+10a+15b-a[2b+10]\}$
$=-\{4a+9b+6-2ab-10a\}$
$=-\{-6a+9b+6-2ab\}$
$=6a-9b-6+2ab$

18. $u^2+(5+2)u+2(5)=u^2+7u+10$

19. $x^2+(4+5)x+4(5)=x^2+9x+20$

20. $w^2+(-5+2)x+2(-5)=w^2-3w-10$

21. $x^2+(-4+7)x-28=x^2+3x-28$

22. $(2x)(5x)+[(2)(2)+(3)(5)]x+3(2)$
$=10x^2+19x+6$

23. $(t)(2t)+[(1)(7)+(-5)(2)]t+(-5)(7)$
$=2t^2-3t-35$

24. $X^2+2(X)(2Y)+(2Y)^2=X^2+4XY+4Y^2$

25. $(2x)^2-2(2x)(1)+1^2=4x^2-4x+1$

26. $7^2-2(7)(X)+X^2=49-14X+X^2$

27. $(1\cdot 2)\left(\sqrt{x}\right)^2+[(1)(5)+(-1)(2)]\sqrt{x}+(-1)(5)$
$=2x+3\sqrt{x}-5$

28. $\left(\sqrt{3x}\right)^2+2\left(\sqrt{3x}\right)(5)+(5)^2$
$=3x+10\sqrt{3x}+25$

29. $\left(\sqrt{y}\right)^2-3^2=y-9$

30. $(2s)^2-1^2=4s^2-1$

31. $(a^2)^2-(2b)^2=a^4-4b^2$

32. $x^2(x+4)-3(x+4)$
$=x^3+4x^2-3x-12$

33. $x^2(3x^2+2x-1)-4(3x^2+2x-1)$
$=3x^4+2x^3-x^2-12x^2-8x+4$
$=3x^4+2x^3-13x^2-8x+4$

34. $x(x^2+x+3)+1(x^2+x+3)$
$=x^3+x^2+3x+x^2+x+3$
$=x^3+2x^2+4x+3$

35. $t\{3(t^2-2t-8)+5[3t^2-21t]\}$
$=t\{3t^2-6t-24+15t^2-105t\}$
$=t\{18t^2-111t-24\}$
$=18t^3-111t^2-24t$

36. $[(2z)^2-1^2](4z^2+1)=[4z^2-1](4z^2+1)$
$=(4z^2)^2-1^2=16z^4-1$

37. $x(3x+2y-4)+y(3x+2y-4)+2(3x+2y-4)$
$=3x^2+2xy-4x+3xy+2y^2-4y+6x+4y-8$
$=3x^2+2y^2+5xy+2x-8$

38. $[x^2+(x+1)]^2$
$=(x^2)^2+2x^2(x+1)+(x+1)^2$
$=x^4+2x^3+2x^2+x^2+2x+1$
$=x^4+2x^3+3x^2+2x+1$

39. $(2a)^3+3(2a)^2(3)+3(2a)(3)^2+(3)^3$
$=8a^3+36a^2+54a+27$

40. $(2a)^3-3(2a)^2(3)+3(2a)(3)^2-3^3$
$=8a^3-36a^2+54a-27$

41. $(2x)^3-3(2x)^2(3)+3(2x)(3)^2-3^3$
$=8x^3-36x^2+54x-27$

42. $x^3+3x^2(2y)+3x(2y)^2+(2y)^3$
$=x^3+6x^2y+12xy^2+8y^3$

43. $\frac{z^2}{z}-\frac{18z}{z}=z-18$

44. $\frac{2x^3}{x}-\frac{7x}{x}+\frac{4}{x}=2x^2-7+\frac{4}{x}$

45. $\frac{6u^5}{3u^2}+\frac{9u^3}{3u^2}-\frac{1}{3u^2}=2u^3+3u-\frac{1}{3u^2}$

46. $\frac{3y-4-9y-5}{3y}$

$=\frac{-6y-9}{3y}$

$=\frac{-6y}{3y}-\frac{9}{3y}$

$=-2-\frac{3}{y}$

47.
$$\begin{array}{r|l} & x \\ x+5 & x^2+5x-3 \\ & \underline{x^2+5x} \\ & \qquad\quad -3 \end{array}$$

Answer: $x+\frac{-3}{x+5}$

48.
$$\begin{array}{r|l} & x-1 \\ x-4 & x^2-5x+4 \\ & \underline{x^2-4x} \\ & \quad -x+4 \\ & \quad \underline{-x+4} \\ & \qquad\quad 0 \end{array}$$

Answer: $x-1$

49.
$$\begin{array}{r|l} & 3x^2-8x+17 \\ x+2 & 3x^3-2x^2+\ \ x-3 \\ & \underline{3x^3+6x^2} \\ & \quad -8x^2+\ \ x \\ & \quad \underline{-8x^2-16x} \\ & \qquad\qquad 17x-3 \\ & \qquad\qquad \underline{17x+34} \\ & \qquad\qquad\quad -37 \end{array}$$

Answer: $3x^2-8x+17+\frac{-37}{x+2}$

50.
$$\begin{array}{r|l} & x^3-x^2+4x-4 \\ x+1 & x^4+0x^3+3x^2+0x+2 \\ & \underline{x^4+\ \ x^3} \\ & \quad -x^3+3x^2 \\ & \quad \underline{-x^3-\ \ x^2} \\ & \qquad\quad 4x^2+0x \\ & \qquad\quad \underline{4x^2+4x} \\ & \qquad\qquad -4x+2 \\ & \qquad\qquad \underline{-4x-4} \\ & \qquad\qquad\qquad 6 \end{array}$$

Answer: $x^3-x^2+4x-4+\frac{6}{x+1}$

51.
$$\begin{array}{r|l} & x^2-2x+4 \\ x+2 & x^3+0x^2+0x+0 \\ & \underline{x^3+2x^2} \\ & \quad -2x^2+0 \\ & \quad \underline{-2x^2-4x} \\ & \qquad\quad 4x+0 \\ & \qquad\quad \underline{4x+8} \\ & \qquad\qquad -8 \end{array}$$

Answer: $x^2-2x+4-\frac{8}{x+2}$

52.
$$\begin{array}{r|l} & x-2 \\ 3x+2 & 3x^2-4x+3 \\ & \underline{3x^2+2x} \\ & \quad -6x+3 \\ & \quad \underline{-6x-4} \\ & \qquad\quad 7 \end{array}$$

Answer: $x-2+\frac{7}{3x+2}$

53.
$$\begin{array}{r|l} & z+2 \\ z^2-z+1 & z^3+z^2+z \\ & \underline{z^3-z^2+z} \\ & \quad 2z^2 \\ & \quad \underline{2z^2-2z+2} \\ & \qquad\quad 2z-2 \end{array}$$

Answer: $z+2+\frac{2z-2}{z^2-z+1}$

Problems 0.5

1. $5b(x+1)$

2. $2y(3y-2)$

3. $5x(2y+z)$

4. $3x^2y(1-3xy^2)$

5. $4bc(2a^3-3ab^2d+b^3cd^2)$

6. $z^2-7^2=(z+7)(z-7)$

7. $(x+2)(x-3)$

8. $(s-4)(s-2)$

9. $(p+3)(p+1)$

10. $(5y)^2-2^2=(5y+2)(5y-2)$

11. $(x+6)(x-4)$

12. $(2t)^2-(3s)^2=(2t+3s)(2t-3s)$

13. $x^2+2(3)(x)+3^2=(x+3)^2$

14. $(t-6)(t-12)$

15. $5(x^2+5x+6)=5(x+3)(x+2)$

16. $3(t^2+4t-5)=3(t-1)(t+5)$

17. $3(x^2-1^2)=3(x+1)(x-1)$

18. $(5x+1)(x+3)$

19. $(4x+3)(x-1)$

20. $2s(6s^2+5s-4)=2s(3s+4)(2s-1)$

21. $(3z)^2+2(3z)(5)+5^2=(3z+5)^2$

22. $u^{3/5}v(u^2-4v^2)=u^{3/5}v(u+2v)(u-2v)$

23. $(2x^{3/5})^2-1^2=(2x^{3/5}+1)(2x^{3/5}-1)$

24. $2x(x^2+x-6)=2x(x+3)(x-2)$

25. $(xy)^2-2(xy)(2)+2^2=(xy-2)^2$

26. $[2(2x+1)]^2=2^2(2x+1)^2$
$=4(2x+1)^2$

27. $2x^2[2x(1-2x)]^2$
$=2x^2(2x)^2(1-2x)^2$
$=2x^2(4x^2)(1-2x)^2$
$=8x^4(1-2x)^2$

28. $x(x^2y^2-16xy+64)=x[(xy)^2-2(xy)(8)+8^2]$
$=x(xy-8)^2$

29. $x(5x+2)+2(5x+2)=(5x+2)(x+2)$

30. $x(x^2-4)+2(4-x^2)$
$=x(x^2-4)-2(x^2-4)$
$=(x^2-4)(x-2)$
$=(x+2)(x-2)(x-2)$
$=(x+2)(x-2)^2$

31. $(x+1)(x-1)+(x-2)(x+1)$
$=(x+1)[(x-1)+(x-2)]$
$=(x+1)(2x-3)$

32. $y^2(y^2+8y+16)-(y^2+8y+16)$
$=(y^2+8y+16)(y^2-1)$
$=(y+4)^2(y+1)(y-1)$

33. $tu(t^2-3)+w^2(t^2-3)$
$=(t^2-3)(tu+w^2)$
$=\left(t+\sqrt{3}\right)\left(t-\sqrt{3}\right)(tu+w^2)$

34. $b^3+4^3=(b+4)(b^2-4(b)+4^2)$
$=(b+4)(b^2-4b+16)$

35. $x^3-1^3=(x-1)[x^2+1(x)+1^2]$
$=(x-1)(x^2+x+1)$

36. $(x^3)^2-1^2=(x^3+1)(x^3-1)$
$=(x+1)(x^2-x+1)(x-1)(x^2+x+1)$

37. $3^3 + (2x)^3 = (3+2x)[3^2 - 3(2x) + (2x)^2]$
$= (3+2x)(9 - 6x + 4x^2)$

38. $(x+4)^2(x-2)[(x+4)+(x-2)]$
$= (x+4)^2(x-2)(2x+2)$
$= 2(x+4)^2(x-2)(x+1)$

39. $[P(1+r)] + [P(1+r)]r = [P(1+r)](1+r)$
$= P(1+r)^2$

40. $(3X+5I)[(X-3I)-(X+2I)] = (3X+5I)(-5I)$
$= -5I(3X+5I)$

41. $(x^2)^2 - 4^2 = (x^2+4)(x^2-4)$
$= (x^2+4)(x+2)(x-2)$

42. $(16y^2)^2 - (z^2)^2 = (16y^2+z^2)(16y^2-z^2)$
$= (16y^2+z^2)(4y+z)(4y-z)$

43. $(y^4)^2 - 1^2 = (y^4+1)(y^4-1)$
$= (y^4+1)(y^2+1)(y^2-1)$
$= (y^4+1)(y^2+1)(y+1)(y-1)$

44. $(t^2)^2 - 2^2 = (t^2+2)(t^2-2)$
$= (t^2+2)\left[t^2 - \left(\sqrt{2}\right)^2\right]$
$= (t^2+2)\left(t+\sqrt{2}\right)\left(t-\sqrt{2}\right)$

45. $(X^2+5)(X^2-1) = (X^2+5)(X+1)(X-1)$

46. $(x^2-9)(x^2-1) = (x+3)(x-3)(x+1)(x-1)$

47. $b(a^4 - 8a^2 + 16) = b(a^2-4)^2$
$= b[(a+2)(a-2)]^2$
$= b(a+2)^2(a-2)^2$

Problems 0.6

1. $\dfrac{a^2-9}{a^2-3a} = \dfrac{(a-3)(a+3)}{a(a-3)} = \dfrac{a+3}{a}$

2. $\dfrac{x^2-3x-10}{x^2-4} = \dfrac{(x+2)(x-5)}{(x+2)(x-2)} = \dfrac{x-5}{x-2}$

3. $\dfrac{x^2-9x+20}{x^2+x-20} = \dfrac{(x-5)(x-4)}{(x+5)(x-4)} = \dfrac{x-5}{x+5}$

4. $\dfrac{3x^2-27x+24}{2x^3-16x^2+14x} = \dfrac{3(x-8)(x-1)}{2x(x-7)(x-1)}$
$= \dfrac{3(x-8)}{2x(x-7)}$

5. $\dfrac{15x^2+x-2}{3x^2+20x-7} = \dfrac{(5x+2)(3x-1)}{(3x-1)(x+7)} = \dfrac{5x+2}{x+7}$

6. $\dfrac{12x^2-19x+4}{6x^2-17x+12} = \dfrac{(4x-1)(3x-4)}{(2x-3)(3x-4)} = \dfrac{4x-1}{2x-3}$

7. $\dfrac{y^2(-1)}{(y-3)(y+2)} = -\dfrac{y^2}{(y-3)(y+2)}$

8. $\dfrac{(a+b)(a-b)(a-b)^2}{2(a-b)(a+b)} = \dfrac{(a-b)^2}{2}$

9. $\dfrac{(ax-b)(c-x)}{(x-c)(ax+b)} = \dfrac{(ax-b)(-1)(x-c)}{(x-c)(ax+b)}$
$= \dfrac{(ax-b)(-1)}{ax+b}$
$= \dfrac{b-ax}{ax+b}$

10. $\dfrac{X^2}{8} \cdot \dfrac{4}{X} = \dfrac{4X^2}{8X} = \dfrac{X}{2}$

11. $\dfrac{2(x-1)}{(x-4)(x+2)} \cdot \dfrac{(x+4)(x+1)}{(x+1)(x-1)}$
$= \dfrac{2(x-1)(x+4)(x+1)}{(x-4)(x+2)(x+1)(x-1)}$
$= \dfrac{2(x+4)}{(x-4)(x+2)}$

12. $\dfrac{3x^2}{7x} \cdot \dfrac{14}{x} = \dfrac{3x}{7} \cdot \dfrac{14}{x} = \dfrac{3(14)x}{7x} = 6$

13. $\dfrac{15u}{v^3} \cdot \dfrac{v^4}{3u} = \dfrac{15uv^4}{3uv^3} = 5v$

14. $\dfrac{c+d}{c} \cdot \dfrac{2c}{c-d} = \dfrac{2c(c+d)}{c(c-d)} = \dfrac{2(c+d)}{c-d}$

15. $\frac{4x}{3} \div 2x = \frac{4x}{3} \cdot \frac{1}{2x} = \frac{4x}{6x} = \frac{2}{3}$

16. $\frac{4x}{1} \cdot \frac{2x}{3} = \frac{4x(2x)}{3} = \frac{8x^2}{3}$

17. $\frac{-9x^3}{1} \cdot \frac{3}{x} = \frac{-27x^3}{x} = -27x^2$

18. $\frac{21t^5}{t^2} \cdot \frac{1}{-7} = -\frac{21t^5}{-7t^2} = -3t^3$

19. $\frac{x-3}{1} \cdot \frac{x-4}{(x-3)(x-4)} = \frac{x-3}{1} \cdot \frac{1}{x-3} = \frac{x-3}{x-3} = 1$

20. $\frac{(x+3)^2}{x} \div (x+3) = \frac{(x+3)^2}{x} \cdot \frac{1}{x+3}$

$= \frac{(x+3)^2}{x(x+3)} = \frac{x+3}{x}$

21. $\frac{10x^3}{(x+1)(x-1)} \cdot \frac{x+1}{5x} = \frac{10x^3(x+1)}{5x(x+1)(x-1)} = \frac{2x^2}{x-1}$

22. $\frac{(x-3)(x+2)}{(x+3)(x-3)} \cdot \frac{(x+3)(x-1)}{(x+2)(x-2)}$

$= \frac{x+2}{x+3} \cdot \frac{(x+3)(x-1)}{(x+2)(x-2)}$

$= \frac{(x+2)(x+3)(x-1)}{(x+3)(x+2)(x-2)}$

$= \frac{x-1}{x-2}$

23. $\frac{(x+6)(x+2)}{(x+6)(x+3)} \cdot \frac{(x-5)(x+3)}{(x-5)(x+2)} = \frac{x+2}{x+3} \cdot \frac{x+3}{x+2}$

$= \frac{(x+2)(x+3)}{(x+3)(x+2)}$

$= 1$

24. $\frac{(x+3)^2}{4x-3} \cdot \frac{(3+4x)(3-4x)}{7(x+3)}$

$= \frac{(x+3)^2(3+4x)(3-4x)}{7(4x-3)(x+3)}$

$= \frac{(x+3)(3+4x)(-1)(4x-3)}{7(4x-3)}$

$= -\frac{(x+3)(3+4x)}{7}$

25. $\frac{(2x+3)(2x-3)}{(x+4)(x-1)} \cdot \frac{(1+x)(1-x)}{2x-3}$

$= \frac{(2x+3)(2x-3)(1+x)(1-x)}{(x+4)(x-1)(2x-3)}$

$= \frac{(2x+3)(1+x)(-1)(x-1)}{(x+4)(x-1)}$

$= -\frac{(2x+3)(1+x)}{x+4}$

26. $\frac{y(6x^2+7x-3)}{x(y-1)+5(y-1)} \cdot \frac{x(y-1)+4(y-1)}{x^2y(x+4)}$

$= \frac{y(3x-1)(2x+3)(y-1)(x+4)}{(y-1)(x+5)x^2y(x+4)}$

$= \frac{(3x-1)(2x+3)}{x^2(x+5)}$

27. $\frac{x^2+5x+6}{x+3} = \frac{(x+3)(x+2)}{x+3} = x+2$

28. $\frac{-1+x}{x-1} = \frac{x-1}{x-1} = 1$

29. LCD = $3t$

$\frac{2}{t} + \frac{1}{3t} = \frac{6}{3t} + \frac{1}{3t} = \frac{6+1}{3t} = \frac{7}{3t}$

30. LCD = X^3

$\frac{9}{X^3} - \frac{1}{X^2} = \frac{9}{X^3} - \frac{X}{X^3} = \frac{9-X}{X^3}$

31. LCD = $x^3 - 1$

$1 - \frac{x^3}{x^3-1} = \frac{x^3-1}{x^3-1} - \frac{x^3}{x^3-1}$

$= \frac{x^3-1-x^3}{x^3-1}$

$= \frac{-1}{x^3-1}$

$= \frac{1}{1-x^3}$

32. LCD = $s + 4$

$\frac{4}{s+4} + s = \frac{4}{s+4} + \frac{s(s+4)}{s+4} = \frac{4+s(s+4)}{s+4}$

$= \frac{s^2+4s+4}{s+4} = \frac{(s+2)^2}{s+4}$

33. LCD = $(3x - 1)(x + 1)$

$$\frac{1}{3x-1}+\frac{x}{x+1}=\frac{x+1}{(3x-1)(x+1)}+\frac{x(3x-1)}{(3x-1)(x+1)}$$
$$=\frac{(x+1)+x(3x-1)}{(3x-1)(x+1)}$$
$$=\frac{3x^2+1}{(x+1)(3x-1)}$$

34. LCD = $(x - 1)(x + 1)$

$$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{(x+1)(x+1)}{(x-1)(x+1)}-\frac{(x-1)(x-1)}{(x-1)(x+1)}$$
$$=\frac{(x+1)^2-(x-1)^2}{(x+1)(x-1)}$$
$$=\frac{x^2+2x+1-(x^2-2x+1)}{(x+1)(x-1)}=\frac{4x}{(x+1)(x-1)}$$

35. LCD = $(x-3)(x+1)(x+3)$

$$\frac{1}{(x-3)(x+1)}+\frac{1}{(x+3)(x-3)}=\frac{x+3}{(x-3)(x+1)(x+3)}+\frac{x+1}{(x-3)(x+1)(x+3)}$$
$$=\frac{(x+3)+(x+1)}{(x-3)(x+1)(x+3)}$$
$$=\frac{2x+4}{(x-3)(x+1)(x+3)}$$
$$=\frac{2(x+2)}{(x-3)(x+1)(x+3)}$$

36. LCD = $(x - 4)(2x + 1)(2x - 1)$

$$\frac{4}{(x-4)(2x+1)}-\frac{x}{(x-4)(2x-1)}=\frac{4(2x-1)}{(x-4)(2x+1)(2x-1)}-\frac{x(2x+1)}{(x-4)(2x+1)(2x-1)}$$
$$=\frac{4(2x-1)-x(2x+1)}{(x-4)(2x+1)(2x-1)}$$
$$=\frac{-2x^2+7x-4}{(x-4)(2x+1)(2x-1)}$$

37. LCD = $(x - 1)(x + 5)$

$$\frac{4}{x-1}-3+\frac{-3x^2}{-(x-1)(x+5)}=\frac{4(x+5)}{(x-1)(x+5)}-\frac{3(x-1)(x+5)}{(x-1)(x+5)}+\frac{3x^2}{(x-1)(x+5)}$$
$$=\frac{4x+20-3(x^2+4x-5)+3x^2}{(x-1)(x+5)}$$
$$=\frac{35-8x}{(x-1)(x+5)}$$

38. LCD = $(2x-1)(x+2)(3x-1)$

$$\frac{x+1}{(2x-1)(x+2)}-\frac{x-1}{(3x-1)(x+2)}+\frac{1}{3x-1}=\frac{(x+1)(3x-1)}{(2x-1)(x+2)(3x-1)}-\frac{(x-1)(2x-1)}{(2x-1)(x+2)(3x-1)}+\frac{(2x-1)(x+2)}{(2x-1)(x+2)(3x-1)}$$
$$=\frac{(x+1)(3x-1)-(x-1)(2x-1)+(2x-1)(x+2)}{(2x-1)(x+2)(3x-1)}$$
$$=\frac{3x^2+8x-4}{(2x-1)(x+2)(3x-1)}$$

39. $\left(1+\frac{1}{x}\right)^2=\left(\frac{x}{x}+\frac{1}{x}\right)^2=\left(\frac{x+1}{x}\right)^2=\frac{x^2+2x+1}{x^2}$

40. $\left(\frac{1}{x}+\frac{1}{y}\right)^2=\left(\frac{y}{xy}+\frac{x}{xy}\right)^2=\left(\frac{y+x}{xy}\right)^2=\frac{y^2+2xy+x^2}{x^2y^2}$

41. $\left(\frac{1}{x}-y\right)^{-1}=\left(\frac{1}{x}-\frac{xy}{x}\right)^{-1}=\left(\frac{1-xy}{x}\right)^{-1}=\frac{x}{1-xy}$

42. $\left(a+\frac{1}{b}\right)^2=\left(\frac{ab}{b}+\frac{1}{b}\right)^2=\left(\frac{ab+1}{b}\right)^2$
$$=\frac{a^2b^2+2ab+1}{b^2}$$

43. Multiplying numerator and denominator by x gives $\frac{5x+2}{3x}$.

44. Multiplying numerator and denominator by x gives $\frac{x+3}{x^2-9}=\frac{x+3}{(x+3)(x-3)}=\frac{1}{x-3}$.

45. Multiplying numerator and denominator by $2x(x+2)$ gives $\frac{3(2x)(x+2)-1(x+2)}{x(2x)(x+2)+x(2x)}=\frac{(x+2)[3(2x)-1]}{2x^2[(x+2)+1]}=\frac{(x+2)(6x-1)}{2x^2(x+3)}$.

Review Problems

1. The integers of S are $-2, 0, 2$.

2. The rational numbers of S are
$-2, -\frac{7}{3}, 0, 2, 1.\overline{34}$.

3. The irrational numbers of S are $-\sqrt{3}, \pi, \frac{e}{2}$.

4. The natural numbers of S consist of 2 only.

5. The real numbers of S are
$-2, -\frac{7}{3}, -\sqrt{3}, 0, 2, \pi, 1.\overline{34}, \frac{e}{2}$ (all except $\sqrt{-2}$).

6. $x^3-3x[2x^2-2(4-3x)+1]$
$=x^3-3x[2x^2-8+6x+1]$
$=x^3-3x[2x^2+6x-7]$
$=x^3-6x^3-18x^2+21x$
$=-5x^3-18x^2+21x$

7. $(2m-3n)^2-(3m+2n)(m-2)$
$=(4m^2-12mn+9n^2)-(3m^2-6m+2nm-4n)$
$=4m^2-12mn+9n^2-3m^2+6m-2mn+4n$
$=m^2-14mn+9n^2+6m+4n$

8. $(x^2-3x+1)(5x+1)-3x+4$

$=5x^3+x^2-15x^2-3x+5x+1-3x+4$

$=5x^3-14x^2-x+5$

9. $2x^2-\{3x[x-(2x^2-1)]-(4-3x)+1\}$

$=2x^2-\{3x[x-2x^2+1]-4+3x+1\}$

$=2x^2-\{3x^2-6x^3+3x-4+3x+1\}$

$=2x^2-\{3x^2-6x^3+6x-3\}$

$=2x^2-3x^2+6x^3-6x+3$

$=6x^3-x^2-6x+3$

10. $\dfrac{3x}{x^2+x-2}+\dfrac{4}{x+2}-\dfrac{x}{x-1}$

$=\dfrac{3x}{(x+2)(x-1)}+\dfrac{4}{x+2}-\dfrac{x}{x-1}$

$=\dfrac{3x}{(x+2)(x-1)}+\dfrac{4(x-1)}{(x+2)(x-1)}-\dfrac{x(x+2)}{(x-1)(x+2)}$

$=\dfrac{3x+4(x-1)-x(x+2)}{(x+2)(x-1)}$

$=\dfrac{3x+4x-4-x^2-2x}{(x+2)(x-1)}$

$=\dfrac{-x^2+5x-4}{(x+2)(x-1)}$

$=\dfrac{-(x-4)(x-1)}{(x+2)(x-1)}=-\dfrac{x-4}{x+2}$

11. $\dfrac{x+3}{x^2-4x+4}\div\dfrac{x^2+6x+9}{x^2-2x}$

$=\dfrac{x+3}{x^2-4x+4}\cdot\dfrac{x^2-2x}{x^2+6x+9}$

$=\dfrac{x+3}{(x-2)^2}\cdot\dfrac{x(x-2)}{(x+3)^2}$

$=\dfrac{x}{(x-2)(x+3)}$

12. $\dfrac{\frac{1}{u}-u}{\frac{2}{u}-\frac{2}{u^2}}$

$=\dfrac{u^2\left(\frac{1}{u}-u\right)}{u^2\left(\frac{2}{u}-\frac{2}{u^2}\right)}=\dfrac{u-u^3}{2u-2}$

$=\dfrac{u(1-u^2)}{2(u-1)}=\dfrac{u(1+u)(1-u)}{-2(1-u)}$

$=-\dfrac{u(1+u)}{2}$

13. $\dfrac{b^{-1}-a^{-1}}{ab^{-2}-ba^{-2}}$

$=\dfrac{\frac{1}{b}-\frac{1}{a}}{\frac{a}{b^2}-\frac{b}{a^2}}=\dfrac{a^2b^2\left(\frac{1}{b}-\frac{1}{a}\right)}{a^2b^2\left(\frac{a}{b^2}-\frac{b}{a^2}\right)}$

$=\dfrac{a^2b-ab^2}{a^3-b^3}=\dfrac{ab(a-b)}{(a-b)(a^2+ab+b^2)}$

$=\dfrac{ab}{a^2+ab+b^2}$

14. x^3-4x

$=x(x^2-4)=x(x+2)(x-2)$

15. $2x(x+3)^2(x-2)-x^2(x-2)^2(x+3)$

$=x(x+3)(x-2)\big[2(x+3)-x(x-2)\big]$

$=x(x+3)(x-2)[2x+6-x^2+2x]$

$=x(x+3)(x-2)(-x^2+4x+6)$

16. $5x^2-7x-6$

$=(5x+3)(x-2)$

17. $3x^2-5x+6$ is prime (cannot be factored)

18. $9x^2-12x+4$

$=(3x-2)(3x-2)$

$=(3x-2)^2$

19. $x^2-6x+9-y^2$

$=(x^2-6x+9)-y^2$

$=(x-3)^2-y^2$

$=(x-3+y)(x-3-y)$

20. x^2+9 is prime (cannot be factored)

21. $8x^3+512$

$=8(x^3+64)=8(x^3+4^3)$

$=8(x+4)(x^2-4x+16)$

22. $\left(\dfrac{8x^4y^{-6}}{27x^{-2}y^3}\right)^{1/3}$

$=\left(\dfrac{8x^6}{27y^9}\right)^{1/3}=\dfrac{8^{1/3}x^{6/3}}{27^{1/3}y^{9/3}}=\dfrac{2x^2}{3y^3}$

23. $\left(\dfrac{m^3n^{-4}}{4m^{-5}n^2}\right)^{-2}$

$=\left(\dfrac{m^8}{4n^6}\right)^{-2}=\left(\dfrac{4n^6}{m^8}\right)^2=\dfrac{16n^{12}}{m^{16}}$

24. $\left(\dfrac{4x^{\frac{1}{3}}}{9x^{\frac{-2}{3}}}\right)^{\frac{1}{2}}$

$=\left(\dfrac{4x^{\frac{1}{3}+\frac{2}{3}}}{9}\right)^{1/2}=\sqrt{\dfrac{4x}{9}}=\dfrac{2\sqrt{x}}{3}=\dfrac{2x^{1/2}}{3}$

25. $\dfrac{\sqrt[3]{27x^4y^{10}z^6}}{2xyz}$

$=\dfrac{\sqrt[3]{3^3x^3xy^9yz^6}}{2xyz}=\dfrac{3xy^3z^2\sqrt[3]{xy}}{2xyz}$

$=\dfrac{3y^2zx^{1/3}y^{1/3}}{2}=\dfrac{3x^{1/3}y^{7/3}z}{2}$

26. $\dfrac{\sqrt[5]{16x^4y^4}\cdot\sqrt[5]{4x^7y}}{3x^2y}$

$=\dfrac{\sqrt[5]{16x^4y^4\cdot 4x^7y}}{3x^2y}=\dfrac{\sqrt[5]{2^6x^{11}y^5}}{3x^2y}$

$=\dfrac{\sqrt[5]{2^5\cdot 2x^{10}xy^5}}{3x^2y}=\dfrac{2x^2y\sqrt[5]{2x}}{3x^2y}=\dfrac{2(2x)^{1/5}}{3}$

27. $\dfrac{1}{\sqrt{x}+\sqrt{x+2}}$

$=\dfrac{1}{\sqrt{x}+\sqrt{x+2}}\cdot\dfrac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}-\sqrt{x+2}}$

$=\dfrac{\sqrt{x}-\sqrt{x+2}}{\left(\sqrt{x}\right)^2-\left(\sqrt{x+2}\right)^2}=\dfrac{\sqrt{x}-\sqrt{x+2}}{x-(x+2)}$

$=\dfrac{\sqrt{x}-\sqrt{x+2}}{-2}=\dfrac{\sqrt{x+2}-\sqrt{x}}{2}$

28. $\dfrac{x}{\sqrt[3]{2x^4y^2}}$

$=\dfrac{x}{\sqrt[3]{2x^4y^2}}\cdot\dfrac{\sqrt[3]{2^2x^2y}}{\sqrt[3]{2^2x^2y}}$

$=\dfrac{x\sqrt[3]{2^2x^2y}}{\sqrt[3]{2^3x^6y^3}}=\dfrac{x\sqrt[3]{4x^2y}}{2x^2y}=\dfrac{\sqrt[3]{4x^2y}}{2xy}$

29. $\dfrac{\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$$= \frac{\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \cdot \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$$
$$= \frac{\sqrt{x}\left(\sqrt{x+h}-\sqrt{x}\right)}{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2} = \frac{\sqrt{x(x+h)}-x}{x+h-x}$$
$$= \frac{\sqrt{x(x+h)}-x}{h}$$

30. If x is the width of the base, then the height is $x + 3$. The area of triangle is therefore

$$A = \frac{\text{base} \times \text{height}}{2} = \frac{x(x+3)}{2} = \frac{x^2+3x}{2}\ \text{cm}^2$$

Chapter Test

1. $a[b + (c + d)] = a[(b + c) + d] = a[d + (b + c)]$
$= a[(d + b) + c]$

2. $\dfrac{-aby}{-ax} = \dfrac{-a \cdot by}{-a \cdot x} = \dfrac{by}{x}$

3. $\dfrac{0}{0}$ is not defined (we cannot divide by 0).

4. $\dfrac{(x^2)^3(x^3)^2}{(x^3)^4} = \dfrac{x^{2\cdot 3}x^{3\cdot 2}}{x^{3\cdot 4}} = \dfrac{x^6x^6}{x^{12}} = \dfrac{x^{12}}{x^{12}}$
$x^{12-12} = x^0 = 1$

5. $2\sqrt{8} - 5\sqrt{27} + \sqrt[3]{128} = 2\sqrt{4\cdot 2} - 5\sqrt{9\cdot 3} + \sqrt[3]{64\cdot 2}$
$= 2\cdot 2\sqrt{2} - 5\cdot 3\sqrt{3} + 4\sqrt[3]{2}$
$= 4\sqrt{2} - 15\sqrt{3} + 4\sqrt[3]{2}$

6. $\left(\dfrac{256}{x^{12}}\right)^{-3/4} = \left(\left[\dfrac{4}{x^3}\right]^4\right)^{-3/4} = \left[\dfrac{4}{x^3}\right]^{-3} = \dfrac{4^{-3}}{(x^3)^{-3}}$
$= \dfrac{4^{-3}}{x^{-9}} = \dfrac{x^9}{4^3} = \dfrac{x^9}{64}$

7. $\dfrac{(a^2b^{-3}c^4)^5}{(a^{-1}c^{-2})^{-3}} = \dfrac{a^{10}b^{-15}c^{20}}{a^3c^6} = \dfrac{a^7c^{14}}{b^{15}}$

8. $\dfrac{1}{\left(\frac{\sqrt{2}x^{-2}}{\sqrt{16}x^3}\right)^2} = \dfrac{1}{\frac{\left(2^{1/2}\right)^2(x^{-2})^2}{(16^{1/2})^2(x^3)^2}} = \dfrac{1}{\frac{2x^{-4}}{16x^6}} = \dfrac{1}{\frac{1}{8x^{10}}} = 8x^{10}$

9. $-2(6u^3 + 6u^2 - 2(u^2 - 5 + 2u))$
$= -2(6u^3 + 6u^2 - 2u^2 + 10 - 4u)$
$= -2(6u^3 + 4u^2 - 4u + 10)$
$= -12u^3 - 8u^2 + 8u - 20$

10. $3y(4y^3 + 2y^2 - 3y) - 2(4y^3 + 2y^2 - 3y)$
$= 12y^4 + 6y^3 - 9y^2 - 8y^3 - 4y^2 + 6y$
$= 12y^4 - 2y^3 - 13y^2 + 6y$

11.
$$\begin{array}{r|l} & 3x - \tfrac{1}{2} \\ \hline 2x+3 & 6x^2 + 8x + 1 \\ & \underline{6x^2 + 9x} \\ & \quad -x + 1 \\ & \quad \underline{-x - \tfrac{3}{2}} \\ & \qquad \tfrac{5}{2} \end{array}$$

Answer: $3x - \dfrac{1}{2} + \dfrac{\frac{5}{2}}{2x+3}$

12. $5r^2t^2(s + 2rs^2t - 3)$

13. $(a + 7)(a + 5)$

14. $(3y - 4)(3y - 2)$

15. $(a+5)^2(a+1)^2[(a+5)+(a+1)]$
$= (a+5)^2(a+1)^2(2a+6)$
$= 2(a+5)^2(a+1)^2(a+3)$

16. $2x(2x^2 - 3x - 2) = 2x(2x+1)(x-2)$

17. $\dfrac{(t+3)(t-3)t^2}{t(t+3)(t-3)^2} = \dfrac{t}{t-3}$

18. $\dfrac{x(x+2)}{3(x-4)(x-2)} \cdot \dfrac{(x-2)^2}{(x-3)(x+2)}$
$= \dfrac{x(x+2)(x-2)^2}{3(x-4)(x-2)(x-3)(x+2)}$
$= \dfrac{x(x-2)}{3(x-4)(x-3)}$

19. Multiplying numerator and denominator by $3(x+3)(x+2)$ gives

$$\frac{3(x-1)-1(3)(x+3)}{3(3)(x+3)(x+2)+(x-7)(x+3)(x+2)}$$

$$=\frac{-12}{(x+3)(x+2)[9+(x-7)]}=-\frac{12}{(x+3)(x+2)^2}.$$

20. $\text{LCD}=\sqrt{3+x}\cdot\sqrt{x}$

$$\frac{x\sqrt{x}}{\sqrt{3+x}}+\frac{2}{\sqrt{x}}=\frac{x\sqrt{x}\cdot\sqrt{x}}{\sqrt{3+x}\sqrt{x}}+\frac{2\sqrt{3+x}}{\sqrt{3+x}\sqrt{x}}$$

$$=\frac{x^2+2\sqrt{3+x}}{\sqrt{3+x}\sqrt{x}}$$

21. $$\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{5}}\cdot\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\frac{2\sqrt{3}\left(\sqrt{3}-\sqrt{5}\right)}{3-5}$$

$$=\frac{2\cdot 3-2\sqrt{15}}{-2}$$

$$=-3+\sqrt{15}$$

Chapter 1

Apply It 1.1

1. **a.** If 1 Bahraini dinar is 15.64 Egyptian pounds, then x Bahraini dinars are equivalent to $15.64x$ Egyptian pounds.

b. x Egyptian pounds equate to $\frac{x}{15.64}$ Bahraini dinars.

2. Let x be the profit realized by the company in 2009. We know that in 2010 the profit increased by 22.2%, so

$$x + 0.222x = 45$$
$$1.222x = 45$$
$$x = \frac{45}{1.222} \approx 36.825$$

Therefore, the company realized a profit of 36.825 million dinars in 2009.

Problems 1.1

1. $9x - x^2 = 0;\ 1, 0$

Set $x = 1$:

$9(1) - (1)^2 \stackrel{?}{=} 0$

$9 - 1 \stackrel{?}{=} 0$

$8 \neq 0$

Set $x = 0$:

$9(0) - (0)^2 \stackrel{?}{=} 0$

$0 - 0 \stackrel{?}{=} 0$

$0 = 0$

Thus, 0 satisfies the equation, but 1 does not.

2. $12 - 7x = -x^2;\ 4, 3$

Set $x = 4$:

$12 - 7(4) \stackrel{?}{=} -(4)^2$

$12 - 28 \stackrel{?}{=} -16$

$-16 = -16$

Set $x = 3$:

$12 - 7(3) \stackrel{?}{=} -(3)^2$

$12 - 21 \stackrel{?}{=} -9$

$-9 = -9$

Thus, 4 and 3 satisfy the equation.

3. $x^2 + x - 6 = 0;\ 2, 3$

Set $x = 2$: $(2)^2 + 2 - 6 \stackrel{?}{=} 0$

$4 + 2 - 6 \stackrel{?}{=} 0$

$0 = 0$

Set $x = 3$: $(3)^2 + 3 - 6 \stackrel{?}{=} 0$

$9 + 3 - 6 \stackrel{?}{=} 0$

$6 \neq 0$

Thus, 2 satisfies the equation, but 3 does not.

4. $x(6 + x) - 2(x + 1) - 5x = 4;\ -2, 0$

Set $x = -2$:

$(-2)(6-2) - 2(-2+1) - 5(-2) \stackrel{?}{=} 4$

$-2(4) - 2(-1) + 10 \stackrel{?}{=} 4$

$-8 + 2 + 10 \stackrel{?}{=} 4$

$4 = 4$

Set $x = 0$:

$0(6) - 2(1) - 5(0) \stackrel{?}{=} 4$

$-2 \neq 4$

Thus, –2 satisfies the equation, but 0 does not.

5. $x(x+1)^2(x+2) = 0;\ 0, -1, 2$

Set $x = 0$:

$0(1)^2(2) \stackrel{?}{=} 0$

$0 = 0$

Set $x = -1$:

$(-1)(0)^2(1) \stackrel{?}{=} 0$

$0 = 0$

Set $x = 2$:

$2(3)^2(4) \stackrel{?}{=} 0$

$72 \neq 0$

Thus, 0 and –1 satisfy the equation, but 2 does not.

6. Adding 5 to both sides; equivalence guaranteed

7. Dividing both sides by 8; equivalence guaranteed

8. Raising both sides to the fourth power; equivalence not guaranteed

9. Dividing both sides by 2; equivalence guaranteed

10. Dividing both sides by x; equivalence not guaranteed

11. Multiplying both sides by $x - 1$; equivalence not guaranteed

12. Dividing both sides by $x + 1$; equivalence not guaranteed

13. Multiplying both sides by $\frac{2x-3}{2x}$; equivalence not guaranteed

14. Adding $9 - x$ to both sides and then dividing both sides by 2; equivalence guaranteed

15. $4x = 10$
$x = \frac{10}{4} = \frac{5}{2}$

16. $0.2x = 7$
$x = \frac{7}{0.2} = 35$

17. $7y^2 = 0$
$y = 0$

18. $2x - 4x = -5$
$-2x = -5$
$x = \frac{-5}{-2} = \frac{5}{2}$

19. $-8x = 12 - 20$
$-8x = -8$
$x = \frac{-8}{-8} = 1$

20. $4 - 7x = 3$
$-7x = -1$
$x = \frac{-1}{-7} = \frac{1}{7}$

21. $5x - 3 = 9$
$5x = 12$
$x = \frac{12}{5}$

22. $\sqrt{3}x + 2 = 11$
$\sqrt{3}x = 9$
$x = \frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

23. $7x + 7 = 2(x + 1)$
$7x + 7 = 2x + 2$
$5x + 7 = 2$
$5x = -5$
$x = \frac{-5}{5} = -1$

24. $4s + 3s - 1 = 41$
$7s - 1 = 41$
$7s = 42$
$s = \frac{42}{7} = 6$

25. $5(p-7) - 2(3p-4) = 3p$
$5p - 35 - 6p + 8 = 3p$
$-p - 27 = 3p$
$-27 = 4p$
$p = -\frac{27}{4}$

26. $t = 2 - 2[2t - 3(1 - t)]$
$t = 2 - 2[2t - 3 + 3t]$
$t = 2 - 2[5t - 3]$
$t = 2 - 10t + 6$
$11t = 8$
$t = \frac{8}{11}$

27. $\frac{x}{7} = 3x + 5$
$x = 21x + 35$
$-20x = 35$
$x = -\frac{35}{20} = -\frac{7}{4}$

28. $\frac{5y}{7} - \frac{6}{7} = 2 - 4y$
$5y - 6 = 14 - 28y$
$33y = 20$
$y = \frac{20}{33}$

29. $7 + \frac{4x}{9} = \frac{x}{2}$
Multiplying both sides by $9 \cdot 2$ gives
$9 \cdot 2 \cdot 7 + 2(4x) = 9(x)$
$126 + 8x = 9x$
$x = 126$

30. $\frac{x}{3}-4=\frac{x}{5}$

Multiplying both sides by 15 gives
$5x - 60 = 3x$
$2x = 60$
$x = 30$

31. $r=\frac{4}{3}r-5$

Multiplying both sides by 3 gives
$3r = 4r - 15$
$-r = -15$
$r = 15$

32. $\frac{2x}{11}+\frac{11x}{2}=4$

Multiplying both sides by 22 gives
$4x + 121x = 88$
$125x = 88$
$x=\frac{88}{125}$

33. $y-\frac{y}{2}+\frac{y}{3}-\frac{y}{4}=\frac{y}{5}$

Multiplying both sides by 60 gives
$60y - 30y + 20y - 15y = 12y$
$35y = 12y$
$23y = 0$
$y = 0$

34. $\frac{2y-3}{4}=\frac{6y+7}{3}$

Multiplying both sides by 12 gives
$3(2y - 3) = 4(6y + 7)$
$6y - 9 = 24y + 28$
$-18y = 37$
$y=-\frac{37}{18}$

35. $\frac{t}{4}+\frac{5}{3}t=\frac{7}{2}(t-1)$

Multiplying both sides by 12 gives
$3t + 20t = 42(t - 1)$
$23t = 42t - 42$
$42 = 19t$
$t=\frac{42}{19}$

36. $t+\frac{t}{3}-\frac{t}{4}+\frac{t}{36}=10$

Multiplying both sides by 36 gives
$36t + 12t - 9t + t = 360$
$40t = 360$
$t = 9$

37. $\frac{7+2(x+1)}{3}=\frac{6x}{5}$

Multiplying both sides by 15 gives
$35 + 10(x + 1) = 18x$
$35 + 10x + 10 = 18x$
$45 = 8x$
$x=\frac{45}{8}$

38. $\frac{x+2}{3}-\frac{2-x}{6}=x-2$

Multiplying both sides by 6 gives
$2(x + 2) - (2 - x) = 6(x - 2)$
$2x + 4 - 2 + x = 6x - 12$
$3x + 2 = 6x - 12$
$2 = 3x - 12$
$14 = 3x$
$x=\frac{14}{3}$

39. $\frac{x}{5}+\frac{2(x-4)}{10}=7$

Multiplying both sides by 10 gives
$2x + 2(x - 4) = 70$
$2x + 2x - 8 = 70$
$4x = 78$
$x=\frac{78}{4}=\frac{39}{2}$

40. $\frac{9}{5}(3-x)=\frac{3}{4}(x-3)$

Multiplying both sides by 20 gives
$36(3 - x) = 15(x - 3)$
$108 - 36x = 15x - 45$
$153 = 51x$
$x = 3$

41. $\frac{2x-7}{3}+\frac{8x-9}{14}=\frac{3x-5}{21}$

Multiplying both sides by 42 gives

$$14(2x-7)+3(8x-9)=2(3x-5)$$
$$28x-98+24x-27=6x-10$$
$$52x-125=6x-10$$
$$46x=115$$
$$x=\frac{115}{46}=\frac{5}{2}$$

42. $\frac{4}{3}(5x-2)=7[x-(5x-2)]$

$$4(5x-2)=21(x-5x+2)$$
$$20x-8=-84x+42$$
$$104x=50$$
$$x=\frac{50}{104}=\frac{25}{52}$$

43. $(2x-5)^2+(3x-3)^2=13x^2-5x+7$

$$4x^2-20x+25+9x^2-18x+9=13x^2-5x+7$$
$$13x^2-38x+34=13x^2-5x+7$$
$$-33x=-27$$
$$x=\frac{-27}{-33}=\frac{9}{11}$$

44. $\frac{5}{x}=25$

$$5=25x$$
$$x=\frac{5}{25}$$
$$x=\frac{1}{5}$$

45. $\frac{4}{x-1}=2$

$$4=2(x-1)$$
$$4=2x-2$$
$$6=2x$$
$$x=3$$

46. $\frac{3x-5}{x-3}=0$

$$3x-5=0$$
$$3x=5$$
$$x=\frac{5}{3}$$

47. Multiplying both sides by $x+3$ gives $5=0$, which is false. Thus there is no solution, so the solution set is $\varnothing$.

48. $\frac{3}{5-2x}=\frac{7}{2}$

$$3(2)=7(5-2x)$$
$$6=35-14x$$
$$14x=29$$
$$x=\frac{29}{14}$$

49. $\frac{x+3}{x}=\frac{2}{5}$

$$5(x+3)=2x$$
$$5x+15=2x$$
$$3x=-15$$
$$x=-5$$

50. $\frac{q}{5q-4}=\frac{1}{3}$

$$3q=5q-4$$
$$-2q=-4$$
$$q=2$$

51. $\frac{5q}{3-q}=2$

$$5q=2(3-q)$$
$$5q=6-2q$$
$$7q=6$$
$$q=\frac{6}{7}$$

52. $\frac{2x-3}{4x-5}=6$

$$2x-3=24x-30$$
$$27=22x$$
$$x=\frac{27}{22}$$

53. $\frac{1}{p-1}=\frac{2}{p-2}$

$$p-2=2(p-1)$$
$$p-2=2p-2$$
$$p=0$$

54. $\frac{1}{x}+\frac{1}{7}=\frac{3}{7}$

$$\frac{1}{x}=\frac{3}{7}-\frac{1}{7}$$
$$\frac{1}{x}=\frac{2}{7}$$
$$x=\frac{7}{2}$$

55. $\dfrac{2}{x-1}=\dfrac{3}{x-2}$

$2(x-2)=3(x-1)$

$2x-4=3x-3$

$-x=1$

$x=-1$

56. $\dfrac{2t+1}{2t+3}=\dfrac{3t-1}{3t+4}$

$(2t+1)(3t+4)=(3t-1)(2t+3)$

$6t^2+11t+4=6t^2+7t-3$

$4t=-7$

$t=-\dfrac{7}{4}$

57. $\dfrac{x+2}{x-1}+\dfrac{x+1}{3-x}=0$

$(x+2)(3-x)+(x+1)(x-1)=0$

$3x-x^2+6-2x+x^2-1=0$

$x+5=0$

$x=-5$

58. $\dfrac{y-6}{y}-\dfrac{6}{y}=\dfrac{y+6}{y-6}$

$(y-6)^2-6(y-6)=y(y+6)$

$y^2-12y+36-6y+36=y^2+6y$

$y^2-18y+72=y^2+6y$

$72=24y$

$y=3$

59. $\dfrac{y-2}{y+2}=\dfrac{y-2}{y+3}$

$(y-2)(y+3)=(y-2)(y+2)$

$y^2+y-6=y^2-4$

$y=2$

60. $\dfrac{-5}{2x-3}=\dfrac{7}{3-2x}+\dfrac{11}{3x+5}$

$-5(3x+5)=-7(3x+5)+11(2x-3)$

$-15x-25=-21x-35+22x-33$

$-15x-25=x-68$

$-16x=-43$

$x=\dfrac{43}{16}$

61. $\dfrac{9}{x-3}=\dfrac{3x}{x-3}$

$9=3x$

$x=3$

But the given equation is not defined for $x=3$, so there is no solution. The solution set is $\varnothing$.

62. $\dfrac{x}{x+3}-\dfrac{x}{x-3}=\dfrac{3x-4}{x^2-9}$

$x(x-3)-x(x+3)=3x-4$

$x^2-3x-x^2-3x=3x-4$

$-6x=3x-4$

$-9x=-4$

$x=\dfrac{4}{9}$

63. $\sqrt{x+5}=4$

$\left(\sqrt{x+5}\right)^2=4^2$

$x+5=16$

$x=11$

64. $\sqrt{z-2}=3$

$\left(\sqrt{z-2}\right)^2=3^2$

$z-2=9$

$z=11$

65. $\sqrt{2x+3}-4=0$

$\sqrt{2x+3}=4$

$\left(\sqrt{2x+3}\right)^2=4^2$

$2x+3=16$

$2x=13$

$x=\dfrac{13}{2}$

66. $4-\sqrt{3x+1}=0$

$4=\sqrt{3x+1}$

$4^2=\left(\sqrt{3x+1}\right)^2$

$16=3x+1$

$15=3x$

$x=5$

67. $\sqrt{\dfrac{x}{2}+1}=\dfrac{2}{3}$

$\left(\sqrt{\dfrac{x}{2}+1}\right)^2=\left(\dfrac{2}{3}\right)^2$

$\dfrac{x}{2}+1=\dfrac{4}{9}$

$\frac{x}{2} = -\frac{5}{9}$

$x = 2\left(-\frac{5}{9}\right) = -\frac{10}{9}$

68. $(x+6)^{1/2} = 7$

$[(x+6)^{1/2}]^2 = 7^2$

$x + 6 = 49$

$x = 43$

69. $\sqrt{4x-6} = \sqrt{x}$

$\left(\sqrt{4x-6}\right)^2 = \left(\sqrt{x}\right)^2$

$4x - 6 = x$

$3x = 6$

$x = 2$

70. $\sqrt{x+1} = \sqrt{2x-3}$

$\left(\sqrt{x+1}\right)^2 = \left(\sqrt{2x-3}\right)^2$

$x+1 = 2x-3$

$-x = -4$

$x = 4$

71. $(x-5)^{3/4} = 27$

$[(x-5)^{3/4}]^{4/3} = 27^{4/3}$

$x-5 = 81$

$x = 86$

72. $\sqrt{y^2-9} = 9-y$

$\left(\sqrt{y^2-9}\right)^2 = (9-y)^2$

$y^2 - 9 = 81 - 18y + y^2$

$18y = 90$

$y = \frac{90}{18} = 5$

73. $\sqrt{y} + \sqrt{y+2} = 3$

$\sqrt{y+2} = 3 - \sqrt{y}$

$\left(\sqrt{y+2}\right)^2 = \left(3-\sqrt{y}\right)^2$

$y + 2 = 9 - 6\sqrt{y} + y$

$6\sqrt{y} = 7$

$\left(6\sqrt{y}\right)^2 = 7^2$

$36y = 49$

$y = \frac{49}{36}$

74. $\sqrt{x} - \sqrt{x+1} = 1$

$\sqrt{x} = \sqrt{x+1} + 1$

$\left(\sqrt{x}\right)^2 = \left(\sqrt{x+1}+1\right)^2$

$x = x + 1 + 2\sqrt{x+1} + 1$

$-2 = 2\sqrt{x+1}$

$-1 = \sqrt{x+1}$, which is impossible because $\sqrt{a} \ge 0$ for all a. Thus there is no solution. The solution set is $\varnothing$.

75. $\sqrt{a^2+2a} = 2+a$

$\left(\sqrt{a^2+2a}\right)^2 = (2+a)^2$

$a^2 + 2a = 4 + 4a + a^2$

$-2a = 4$

$a = -2$

76. $\sqrt{\frac{1}{w}} - \sqrt{\frac{2}{5w-2}} = 0$

$\sqrt{\frac{1}{w}} = \sqrt{\frac{2}{5w-2}}$

$\left(\sqrt{\frac{1}{w}}\right)^2 = \left(\sqrt{\frac{2}{5w-2}}\right)^2$

$\frac{1}{w} = \frac{2}{5w-2}$

$5w - 2 = 2w$

$3w = 2$

$w = \frac{2}{3}$

77. $I = Prt$

$r = \frac{I}{Pt}$

78. $P\left(1+\frac{p}{100}\right) - R = 0$

$P\left(1+\frac{p}{100}\right) = R$

$P = \frac{R}{1+\frac{p}{100}}$

79. $p = 8q - 1$
$p + 1 = 8q$
$q = \frac{p+1}{8}$

80. $p = 10 - 2q$
$2q = 10 - p$
$q = \frac{10-p}{2}$

81. $r = \frac{2mI}{B(n+1)}$
$\frac{r[B(n+1)]}{2m} = I$
$I = \frac{rB(n+1)}{2m}$

82. $S = \frac{R[(1+i)^n - 1]}{i}$
$Si = R[(1+i)^n - 1]$
$R = \frac{Si}{(1+i)^n - 1}$

83. $A = \frac{R[1-(1+i)^{-n}]}{i}$
$R = \frac{Ai}{1-(1+i)^{-n}}$

84. $\frac{x-a}{b-x} = \frac{x-b}{a-x}$
$(x - a)(a - x) = (x - b)(b - x)$
$(x - a)(a - x)(-1) = (x - b)(b - x)(-1)$
$(x - a)(x - a) = (x - b)(x - b)$
$x^2 - 2ax + a^2 = x^2 - 2bx + b^2$
$a^2 - b^2 = 2ax - 2bx$
$(a + b)(a - b) = 2x(a - b)$
$a + b = 2x$ (for $a \neq b$)
$\frac{a+b}{2} = x$

85. $S = P(1+r)^n$
$\frac{S}{P} = (1+r)^n$
$\left(\frac{S}{P}\right)^{1/n} = [(1+r)^n]^{1/n}$
$\left(\frac{S}{P}\right)^{1/n} = 1 + r$
$\left(\frac{S}{P}\right)^{1/n} - 1 = r$

86. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$
$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$
$\frac{1}{q} = \frac{p-f}{pf}$
$q = \frac{pf}{p-f}$

87. $r = \frac{2mI}{B(n+1)}$
$r(n+1) = \frac{2mI}{B}$
$n+1 = \frac{2mI}{rB}$
$n = \frac{2mI}{rB} - 1$

88. $P = 2l + 2w$
$660 = 2l + 2(160)$
$660 = 2l + 320$
$340 = 2l$
$l = \frac{340}{2} = 170$
The length of the rectangle is 170 m.

89. $V = \pi r^2 h$
$355 = \pi r^2 (16)$
$355 = 16\pi r^2$
$r^2 = \frac{355}{16\pi}$
$r = \sqrt{\frac{355}{16\pi}}$
The radius of the can is $\sqrt{\frac{355}{16\pi}} \approx 2.66$ centimeters.

90. $c = x - 0.0825x = 0.9175x$

91. Revenue equals cost when $450x = 380x + 3500$.
$450x = 380x + 3500$
$70x = 3500$
$x = 50$
50 toddlers need to be enrolled.

92. $V = C\left(1 - \frac{n}{N}\right)$
$2000 = 3200\left(1 - \frac{n}{8}\right)$
$2000 = 3200 - 400n$
$400n = 1200$
$n = 3$
The furniture will have a value of \$2000 after 3 years.

93. Reem's weekly salary for working h hours is $47h + 28$. She saves one quarter of this amount.
$\frac{1}{4}(47h + 28) = 550$
$47h + 28 = 2200$
$47h = 2172$
$h = \frac{2172}{47} \approx 46.2$
Reem must work approximately 46.2 hours per week.

94. $y = a(1 - by)x$
$y = ax(1 - by)$
$y = ax - abxy$
$y + abxy = ax$
$y(1 + abx) = ax$
$y = \frac{ax}{1 + abx}$

95. $y = \frac{1.4x}{1 + 0.09x}$
With $y = 10$ the equation is
$10 = \frac{1.4x}{1 + 0.09x}$
$10(1 + 0.09x) = 1.4x$
$10 + 0.9x = 1.4x$
$10 = 0.5x$
$x = 20$
The prey density should be 20.

96. Let x = the maximum number of customers.
$\frac{8}{x - 92} = \frac{10}{x - 46}$
$8(x - 46) = 10(x - 92)$
$8x - 368 = 10x - 920$
$552 = 2x$
$x = 276$
The maximum number of customers is 276.

97. $t = \frac{d}{r - c}$
$t(r - c) = d$
$tr - tc = d$
$tr - d = tc$
$c = \frac{tr - d}{t} = r - \frac{d}{t}$

98. Let x be the horizontal distance from the base of the tower to the house. By the Pythagorean theorem, $x^2 + 100^2 = (x + 2)^2$.
$x^2 + 10,000 = x^2 + 4x + 4$
$10,000 = 4x + 4$
$9996 = 4x$
$x = \frac{9996}{4} = 2499$
The distance from the base of the tower to the house is 2499 meters.

99. Let e be Maram's expenses in Nova Scotia before the HST tax. Then the HST tax is $0.15e$ and the total receipts are $e + 0.15e = 1.15e$. The percentage of the total that is HST is
$\frac{0.15e}{1.15e} = \frac{0.15}{1.15} = \frac{15}{115} = \frac{3}{23}$ or approximately 13%.

100. $s = \sqrt{30fd}$
Set $s = 45$ and (for dry concrete) $f = 0.8$.
$45 = \sqrt{30(0.8)d}$
$45 = \sqrt{24d}$
$(45)^2 = \left(\sqrt{24d}\right)^2$
$2025 = 24d$
$d = \frac{2025}{24} = \frac{675}{8} = 84\frac{3}{8} \approx 84$ m

Apply It 1.2

3. **a.** The year 2010 corresponds to $t = 20$, so we replace t by 20 in the given formula:
$N = 175.15 \times 20^2 + 3641.68 \times 20 + 26032$
$= 70060 + 72833.6 + 26032$
$= 168925.6$
Thus, in 2010, the number of female secondary-school teachers in Saudi Arabia was approximately 168,926.

b. To find the year in which the number of teachers is 252,041 we solve the equation

$175.15t^2 + 3641.68t + 26032 = 252041$
$175.15t^2 + 3641.68t - 226009 = 0$

Applying the quadratic formula with $a = 175.15$, $b = 3641.68$ and $c = -226009$ gives

$$t = \frac{-3641.68 \pm \sqrt{(3641.68)^2 - 4(175.15)(-226009)}}{2(175.15)}$$
$$= \frac{-3641.68 \pm 13099.76}{350.30} \approx -47.7,\ 27.0$$

As t is the number of years after 1990, the positive solution 27 is the reasonable one, so the number of female secondary-school teachers will be 252,041 in 2017.

Problems 1.2

1. $x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$
$x - 2 = 0$
$x = 2$

2. $(t + 1)(t + 2) = 0$
$t + 1 = 0$ or $t + 2 = 0$
$t = -1$ or $t = -2$

3. $t^2 - 6t + 8 = 0$
$(t-4)(t-2) = 0$
$t - 4 = 0$ or $t - 2 = 0$
$t = 4$ or $t = 2$

4. $(x - 2)(x + 5) = 0$
$x - 2 = 0$ or $x + 5 = 0$
$x = 2$ or $x = -5$

5. $x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x - 3 = 0$ or $x + 1 = 0$
$x = 3$ or $x = -1$

6. $(x - 4)(x + 4) = 0$
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ or $x = -4$

7. $u^2 - 13u = -36$
$u^2 - 13u + 36 = 0$
$(u-4)(u-9) = 0$
$u - 4 = 0$ or $u - 9 = 0$
$u = 4$ or $u = 9$

8. $2(z^2 + 4z + 4) = 0$
$2(z+2)^2 = 0$
$z + 2 = 0$
$z = -2$

9. $x^2 - 4 = 0$
$(x - 2)(x + 2) = 0$
$x - 2 = 0$ or $x + 2 = 0$
$x = 2$ or $x = -2$

10. $3u(u-2) = 0$
$u = 0$ or $u - 2 = 0$
$u = 0$ or $u = 2$

11. $t^2 - 5t = 0$
$t(t-5) = 0$
$t = 0$ or $t - 5 = 0$
$t = 0$ or $t = 5$

12. $x^2 + 9x + 14 = 0$
$(x + 7)(x + 2) = 0$
$x + 7 = 0$ or $x + 2 = 0$
$x = -7$ or $x = -2$

13. $9x^2 + 4 = -12x$
$9x^2 + 12x + 4 = 0$
$(3x+2)^2 = 0$
$3x + 2 = 0$
$3x = -2$
$x = -\frac{2}{3}$

14. $2z^2 + 9z - 5 = 0$
$(2z - 1)(z + 5) = 0$
$2z - 1 = 0$ or $z + 5 = 0$
$z = \frac{1}{2}$ or $z = -5$

15. $v(3v-5) = -2$
$3v^2 - 5v = -2$
$3v^2 - 5v + 2 = 0$
$(3v-2)(v-1) = 0$
$3v - 2 = 0$ or $v - 1 = 0$
$v = \frac{2}{3}$ or $v = 1$

16. $-6x^2 + x + 2 = 0$
$6x^2 - x - 2 = 0$
$(2x+1)(3x-2) = 0$
$2x+1 = 0$ or $3x-2 = 0$
$x = -\frac{1}{2}$ or $x = \frac{2}{3}$

17. $u^2 = 2u$
$u^2 - 2u = 0$
$u(u-2) = 0$
$u = 0$ or $u-2 = 0$
$u = 0$ or $u = 2$

18. $2p^2 = 3p$
$2p^2 - 3p = 0$
$p(2p-3) = 0$
$p = 0$ or $2p-3 = 0$
$p = 0$ or $p = \frac{3}{2}$

19. $r^2 + r - 12 = 0$
$(r-3)(r+4) = 0$
$r-3 = 0$ or $r+4 = 0$
$r = 3$ or $r = -4$

20. $x(x+4)(x-1) = 0$
$x = 0$ or $x+4 = 0$ or $x-1 = 0$
$x = 0$ or $x = -4$ or $x = 1$

21. $(w-3)^2(w+1)^2 = 0$
$w-3 = 0$ or $w+1 = 0$
$w = 3$ or $w = -1$

22. $s^3 - 16s = 0$
$s(s^2 - 16) = 0$
$s(s+4)(s-4) = 0$
$s = 0$ or $s+4 = 0$ or $s-4 = 0$
$s = 0$ or $s = -4$ or $s = 4$

23. $x(x^2 - 4x - 5) = 0$
$x(x-5)(x+1) = 0$
$x = 0$ or $x-5 = 0$ or $x+1 = 0$
$x = 0$ or $x = 5$ or $x = -1$

24. $6x^3 + 5x^2 - 4x = 0$
$x(6x^2 + 5x - 4) = 0$
$x(2x-1)(3x+4) = 0$
$x = 0$ or $2x-1 = 0$ or $3x+4 = 0$
$x = 0$ or $x = \frac{1}{2}$ or $x = -\frac{4}{3}$

25. $x^2 + 2x + 1 - 5x + 1 = 0$
$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$
$x-1 = 0$ or $x-2 = 0$
$x = 1$ or $x = 2$

26. $(x-3)(x^2-4) = 0$
$(x-3)(x-2)(x+2) = 0$
$x-3 = 0$ or $x-2 = 0$ or $x+2 = 0$
$x = 3$ or $x = 2$ or $x = -2$

27. $7(z-4)(z+1)(z+7) = 0$
$z-4 = 0$ or $z+1 = 0$ or $z+7 = 0$
$z = 4$ or $z = -1$ or $z = -7$

28. $p(p-3)^2 - 4(p-3)^3 = 0$
$(p-3)^2[p - 4(p-3)] = 0$
$(p-3)^2(12 - 3p) = 0$
$3(p-3)^2(4-p) = 0$
$p-3 = 0$ or $4-p = 0$
$p = 3$ or $p = 4$

29. $(x^2-1)(x^2-2) = 0$
$(x+1)(x-1)\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right) = 0$
$x+1 = 0$ or $x-1 = 0$
or $x+\sqrt{2} = 0$ or $x-\sqrt{2} = 0$
$x = -1$ or $x = 1$
or $x = -\sqrt{2}$ or $x = \sqrt{2}$

30. $x^2 - 2x - 15 = 0$
$x^2 - 2x = 15$
$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 15 + \left(\frac{-2}{2}\right)^2$
$(x-1)^2 = 16$
$x-1 = \pm\sqrt{16} = \pm 4$
$x = 1 \pm 4 = 5, -3$

31. Before completing the square, divide through by 3 to make the coefficient of x^2 equal to 1.

$$x^2 - 2x = 8$$
$$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 8 + \left(\frac{-2}{2}\right)^2$$
$$(x-1)^2 = 9$$
$$x - 1 = \pm\sqrt{9} = \pm 3$$
$$x = 1 \pm 3 = 4, -2$$

32. $x^2 + 4x = 6$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 6 + \left(\frac{4}{2}\right)^2$$
$$(x+2)^2 = 10$$
$$x + 2 = \pm\sqrt{10}$$
$$x = \sqrt{10} - 2 \quad \text{or} \quad -\sqrt{10} - 2$$

33. Before completing the square, divide through by 4 to make the coefficient of x^2 equal to 1.

$$x^2 - x - \frac{1}{4} = 0$$
$$x^2 - x = \frac{1}{4}$$
$$x^2 - x + \left(\frac{-1}{2}\right)^2 = \frac{1}{4} + \left(\frac{-1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$
$$x = \frac{1}{2} + \frac{\sqrt{2}}{2} \quad \text{or} \quad \frac{1}{2} - \frac{\sqrt{2}}{2}$$

34. $x^2 - 2x - 15 = 0$

$a = 1$, $b = -2$, $c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{4 - 4(1)(-15)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{64}}{2}$$
$$= \frac{2 \pm 8}{2}$$
$$x = \frac{2+8}{2} = 5 \quad \text{or} \quad x = \frac{2-8}{2} = -3$$

35. $x^2 + 2x - 24 = 0$

$a = 1$, $b = 2$, $c = -24$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{4 - 4(1)(-24)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{100}}{2}$$
$$= \frac{-2 \pm 10}{2}$$
$$x = \frac{-2+10}{2} = 4 \quad \text{or} \quad x = \frac{-2-10}{2} = -6$$

36. $q^2 - 5q = 0$

$a = 1$, $b = -5$, $c = 0$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{5 \pm \sqrt{25 - 4(1)(0)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{25}}{2}$$
$$= \frac{5 \pm 5}{2}$$
$$q = \frac{5+5}{2} = 5 \quad \text{or} \quad q = \frac{5-5}{2} = 0$$

37. $16x^2 - 40x + 25 = 0$

$a = 16$, $b = -40$, $c = 25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-40) \pm \sqrt{1600 - 4(16)(25)}}{2(16)}$$
$$= \frac{40 \pm 0}{32}$$
$$= \frac{5}{4}$$

38. $p^2 - 2p - 7 = 0$

$a = 1,\ b = -2,\ c = -7$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)} = \frac{2 \pm \sqrt{32}}{2} = 1 \pm 2\sqrt{2}$$

$p = 1 + 2\sqrt{2}$ or $p = 1 - 2\sqrt{2}$

39. $2 - 2x + x^2 = 0$

$x^2 - 2x + 2 = 0$

$a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2}$$

no real roots

40. $4x^2 + 5x - 2 = 0$

$a = 4,\ b = 5,\ c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(4)(-2)}}{2(4)} = \frac{-5 \pm \sqrt{57}}{8}$$

$x = \frac{-5 + \sqrt{57}}{8}$ or $x = \frac{-5 - \sqrt{57}}{8}$

41. $4 - 2n + n^2 = 0$

$n^2 - 2n + 4 = 0$

$a = 1,\ b = -2,\ c = 4$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{4 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{-12}}{2}$$

no real roots

42. $w^2 - 2w + 1 = 0$

$a = 1,\ b = -2,\ c = 1$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} = \frac{2 \pm \sqrt{0}}{2} = 1$$

43. $0.02w^2 - 0.3w = 20$

$0.02w^2 - 0.3w - 20 = 0$

$a = 0.02,\ b = -0.3,\ c = -20$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-0.3) \pm \sqrt{0.09 - 4(0.02)(-20)}}{2(0.02)} = \frac{0.3 \pm \sqrt{1.69}}{0.04} = \frac{0.3 \pm 1.3}{0.04}$$

$w = \frac{0.3 + 1.3}{0.04} = \frac{1.6}{0.04} = 40$ or

$w = \frac{0.3 - 1.3}{0.04} = \frac{-1.0}{0.04} = -25$

44. $0.01x^2 + 0.2x - 0.6 = 0$

$a = 0.01,\ b = 0.2,\ c = -0.6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.2 \pm \sqrt{0.04 - 4(0.01)(-0.6)}}{2(0.01)} = \frac{-0.2 \pm \sqrt{0.064}}{0.02} = \frac{-0.2 \pm \sqrt{(0.0064)(10)}}{0.02} = \frac{-0.2 \pm 0.08\sqrt{10}}{0.02} = -10 \pm 4\sqrt{10}$$

$x = -10 + 4\sqrt{10}$ or $x = -10 - 4\sqrt{10}$

45. $3x^2+2x=6$

$3x^2+2x-6=0$

$a=3, b=2, c=-6$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{4-4(3)(-6)}}{2(3)}$$
$$=\frac{-2\pm\sqrt{76}}{6}$$
$$=\frac{-2\pm2\sqrt{19}}{6}$$
$$=\frac{-1\pm\sqrt{19}}{3}$$
$$x=\frac{-1+\sqrt{19}}{3} \text{ or } x=\frac{-1-\sqrt{19}}{3}$$

46. $-2x^2-6x+5=0$

$a=-2, b=-6, c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6)\pm\sqrt{36-4(-2)(5)}}{2(-2)}$$
$$=\frac{6\pm\sqrt{76}}{-4}$$
$$=\frac{6\pm2\sqrt{19}}{-4}$$
$$=\frac{-3\pm\sqrt{19}}{2}$$
$$x=\frac{-3+\sqrt{19}}{2} \text{ or } x=\frac{-3-\sqrt{19}}{2}$$

47. $(x^2)^2-5(x^2)+6=0$

Let $w=x^2$. Then

$w^2-5w+6=0$

$(w-3)(w-2)=0$

$w=3, 2$

Thus $x^2=3$ or $x^2=2$, so $x=\pm\sqrt{3}, \pm\sqrt{2}$.

48. $(X^2)^2-3(X)^2-10=0$

Let $w=X^2$. Then

$w^2-3w-10=0$

$(w-5)(w+2)=0$

$w=5, -2$

Thus $X^2=5$ or $X^2=-2$, so the real solutions are $X=\pm\sqrt{5}$.

49. $3\left(\frac{1}{x}\right)^2-7\left(\frac{1}{x}\right)+2=0$

Let $w=\frac{1}{x}$. Then

$3w^2-7w+2=0$

$(3w-1)(w-2)=0$

$w=\frac{1}{3}, 2$

Thus, $x=3, \frac{1}{2}$.

50. $(x^{-1})^2-x^{-1}-6=0$

Let $w=x^{-1}$. Then

$w^2-w-6=0$

$(w-3)(w+2)=0$

$w=3, -2$

Thus, $x=-\frac{1}{2}, \frac{1}{3}$.

51. $(x^{-2})^2-9(x^{-2})+20=0$

Let $w=x^{-2}$. Then

$w^2-9w+20=0$

$(w-5)(w-4)=0$

$w=5, 4$

Thus, $\frac{1}{x^2}=5$ or $\frac{1}{x^2}=4$, so $x^2=\frac{1}{5}$ or $x^2=\frac{1}{4}$.

$x=\pm\frac{\sqrt{5}}{5}, \pm\frac{1}{2}$.

52. $\left(\frac{1}{x^2}\right)^2-9\left(\frac{1}{x^2}\right)+8=0$

Let $w=\frac{1}{x^2}$. Then

$w^2-9w+8=0$

$(w-8)(w-1)=0$

$w=8, 1$

Thus, $\frac{1}{x^2}=8$ or $\frac{1}{x^2}=1$, so $x^2=\frac{1}{8}$ or $x^2=1$.

$x=\pm\frac{\sqrt{2}}{4}, \pm1$.

53. $(X-5)^2+7(X-5)+10=0$
Let $w = X - 5$. Then
$w^2+7w+10=0$
$(w+2)(w+5)=0$
$w=-2, -5$
Thus, $X - 5 = -2$ or $X - 5 = -5$, so $X = 3, 0$.

54. $(3x+2)^2-5(3x+2)=0$
Let $w = 3x + 2$. Then
$w^2-5w=0$
$w(w-5)=0$
$w=0, 5$
Thus $3x + 2 = 0$ or $3x + 2 = 5$, so $x=-\frac{2}{3}, 1$.

55. $\left(\frac{1}{x-4}\right)^2-7\left(\frac{1}{x-4}\right)+12=0$
Let $w=\frac{1}{x-4}$. Then
$w^2-7w+12=0$
$(w-4)(w-3)=0$
$w=4, 3$
Thus, $\frac{1}{x-4}=4$ or $\frac{1}{x-4}=3$, so $x-4=\frac{1}{4}$ or $x-4=\frac{1}{3}$. $x=\frac{17}{4}, \frac{13}{3}$.

56. $x^2=\frac{x+3}{2}$
$2x^2=x+3$
$2x^2-x-3=0$
$(2x-3)(x+1)=0$
Thus, $x=\frac{3}{2}, -1$.

57. $\frac{x}{2}=\frac{7}{x}-\frac{5}{2}$
Multiplying both sides by the LCD, $2x$, gives
$x^2=14-5x$
$x^2+5x-14=0$
$(x-2)(x+7)=0$
Thus, $x = 2, -7$.

58. $\frac{3}{x-4}+\frac{x-3}{x}=2$
Multiplying both sides by the LCD, $x(x-4)$, gives
$3x+(x-3)(x-4)=2x(x-4)$
$3x+x^2-7x+12=2x^2-8x$
$x^2-4x+12=2x^2-8x$
$0=x^2-4x-12$
$0=(x-6)(x+2)$
Thus, $x = 6, -2$.

59. $\frac{1}{3x+1}+\frac{2}{x+1}=3$
Multiplying both sides by the LCD, $(3x+1)(x+1)$, gives
$x+1+2(3x+1)=3(3x+1)(x+1)$
$x+1+6x+2=9x^2+12x+3$
$0=9x^2+5x$
$0=x(9x+5)$
Thus, $x=0, -\frac{5}{9}$.

60. $\frac{3x+2}{x+1}-\frac{2x+1}{2x}=1$
Multiplying both sides by the LCD, $2x(x+1)$, gives
$2x(3x+2)-(2x+1)(x+1)=2x(x+1)$
$6x^2+4x-(2x^2+3x+1)=2x^2+2x$
$4x^2+x-1=2x^2+2x$
$2x^2-x-1=0$
$(2x+1)(x-1)=0$
Thus, $x=-\frac{1}{2}, 1$.

61. $\frac{6(w+1)}{2-w}+\frac{w}{w-1}=3$
Multiplying both sides by the LCD, $(2-w)(w-1)$, gives
$6(w+1)(w-1)+w(2-w)=3(2-w)(w-1)$
$6(w^2-1)+2w-w^2=3(-w^2+3w-2)$
$5w^2+2w-6=-3w^2+9w-6$
$8w^2-7w=0$
$w(8w-7)=0$
Thus, $w=0, \frac{7}{8}$.

62. $\frac{2}{r-2}-\frac{r+1}{r+4}=0$
Multiplying both sides by the LCD, $(r-2)(r+4)$, gives
$2(r+4)-(r-2)(r+1)=0$

$2r+8-(r^2-r-2)=0$

$-r^2+3r+10=0$

$r^2-3r-10=0$

$(r-5)(r+2)=0$

Thus, $r = 5, -2$.

63. $\frac{t-1}{t-2}+\frac{t-3}{t-4}=\frac{t-5}{t^2-6t+8}$

Multiplying both sides by the LCD, $(t-2)(t-4)$, gives

$(t-1)(t-4)+(t-3)(t-2)=t-5$

$t^2-5t+4+t^2-5t+6=t-5$

$2t^2-11t+15=0$

$(2t-5)(t-3)=0$

Thus, $x = 3, \frac{5}{2}$.

64. $\frac{2x-3}{2x+5}+\frac{2x}{3x+1}=1$

Multiplying both sides by the LCD, $(2x+5)(3x+1)$, gives

$(2x-3)(3x+1)+2x(2x+5)=(2x+5)(3x+1)$

$6x^2-7x-3+4x^2+10x=6x^2+17x+5$

$10x^2+3x-3=6x^2+17x+5$

$4x^2-14x-8=0$

$2x^2-7x-4=0$

$(2x+1)(x-4)=0$

Thus, $x=-\frac{1}{2}, 4$.

65. $\frac{2}{x+1}+\frac{3}{x}=\frac{4}{x+2}$

Multiplying both sides by the LCD, $x(x+1)(x+2)$, gives

$2x(x+2)+3(x+1)(x+2)=4x(x+1)$

$2x^2+4x+3x^2+9x+6=4x^2+4x$

$5x^2+13x+6=4x^2+4x$

$x^2+9x+6=0$

$a = 1, b = 9, c = 6$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-9\pm\sqrt{9^2-4(1)(6)}}{2(1)}$

$=\frac{-9\pm\sqrt{57}}{2}$

Thus, $x=\frac{-9+\sqrt{57}}{2}, \frac{-9-\sqrt{57}}{2}$.

66. $\frac{2}{x^2-1}-\frac{1}{x(x-1)}=\frac{2}{x^2}$

Multiplying both sides by the LCD, $x^2(x+1)(x-1)$, gives

$2x^2-x(x+1)=2(x+1)(x-1)$

$2x^2-x^2-x=2x^2-2$

$x^2-x=2x^2-2$

$0=x^2+x-2$

$0=(x+2)(x-1)$

$x = -2$ or $x = 1$

But $x = 1$ does not check. The solution is –2.

67. If $x \neq -3$, the equation is $5-\frac{3}{x}=\frac{1-x}{x}$.

Multiplying both sides by x gives

$5x-3=1-x$

$6x=4$

$x=\frac{2}{3}$

68. $\left(\sqrt{2x-3}\right)^2=(x-3)^2$

$2x-3=x^2-6x+9$

$0=x^2-8x+12$

$0=(x-6)(x-2)$

$x = 6$ or $x = 2$

Only $x = 6$ checks.

69. $\left(2\sqrt{x+1}\right)^2=(x+3)^2$

$4(x+1)=x^2+6x+9$

$0=x^2+2x+5$

$a = 1, b = 2, c = 5$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-2\pm\sqrt{4-4(1)(5)}}{2(1)}$

$=\frac{-2\pm\sqrt{-16}}{2}$

There are no real roots.

70. $(q+2)^2=\left(2\sqrt{4q-7}\right)^2$

$q^2+4q+4=16q-28$

$q^2 - 12q + 32 = 0$
$(q-4)(q-8) = 0$
Thus, $q = 4, 8$.

71. $\left(\sqrt{x}\right)^2 + 2\left(\sqrt{x}\right) - 5 = 0$

Let $w = \sqrt{x}$, then $w^2 + 2w - 5 = 0$
$a = 1, b = 2, c = -5$

$w = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \dfrac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{24}}{2}$

$= \dfrac{-2 \pm 2\sqrt{6}}{2}$

$= -1 \pm \sqrt{6}$

Since $w = \sqrt{x}$ and $-1-\sqrt{6} < 0$, $w = -1-\sqrt{6}$ does not check. Thus $w = -1+\sqrt{6}$, so

$x = \left(-1+\sqrt{6}\right)^2 = 7 - 2\sqrt{6}$.

72. $\sqrt{z+3} = \sqrt{3z} + 1$

$\left(\sqrt{z+3}\right)^2 = \left(\sqrt{3z}+1\right)^2$

$z + 3 = 3z + 2\sqrt{3z} + 1$

$-2z + 2 = 2\sqrt{3z}$

$-z + 1 = \sqrt{3z}$

$(-z+1)^2 = \left(\sqrt{3z}\right)^2$

$z^2 - 2z + 1 = 3z$

$z^2 - 5z + 1 = 0$

$a = 1, b = -5, c = 1$

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{5 \pm \sqrt{21}}{2}$

Only $z = \dfrac{5 - \sqrt{21}}{2}$ checks.

73. $\sqrt{x} - 2 = \sqrt{2x-8}$

$\left(\sqrt{x}-2\right)^2 = \left(\sqrt{2x-8}\right)^2$

$x - 4\sqrt{x} + 4 = 2x - 8$

$-4\sqrt{x} = x - 12$

$\left(-4\sqrt{x}\right)^2 = (x-12)^2$

$16x = x^2 - 24x + 144$

$0 = x^2 - 40x + 144$

$0 = (x-4)(x-36)$
$x = 4$ or $x = 36$
Only $x = 4$ checks.

74. $\sqrt{x} + 1 = \sqrt{3x+1}$

$\left(\sqrt{x}+1\right)^2 = \left(\sqrt{3x+1}\right)^2$

$x + 2\sqrt{x} + 1 = 3x + 1$

$0 = 2x - 2\sqrt{x} = 2\sqrt{x}\left(\sqrt{x}-1\right)$

$x = 0$ or $x = 1$

75. $\left(\sqrt{y-2}+2\right)^2 = \left(\sqrt{2y+3}\right)^2$

$y - 2 + 4\sqrt{y-2} + 4 = 2y + 3$

$4\sqrt{y-2} = y + 1$

$\left(4\sqrt{y-2}\right)^2 = (y+1)^2$

$16y - 32 = y^2 + 2y + 1$

$0 = y^2 - 14y + 33$

$0 = (y-11)(y-3)$
Thus, $y = 11, 3$.

76. $\left(\sqrt{x+3}+1\right)^2 = \left(3\sqrt{x}\right)^2$

$x + 3 + 2\sqrt{x+3} + 1 = 9x$

$2\sqrt{x+3} = 8x - 4$

$\sqrt{x+3} = 4x - 2$

$\left(\sqrt{x+3}\right)^2 = (4x-2)^2$

$x + 3 = 16x^2 - 16x + 4$

$0 = 16x^2 - 17x + 1$

$0 = (16x-1)(x-1)$

$x = \dfrac{1}{16}$ or $x = 1$

Only $x = 1$ checks.

77. $\left(\sqrt{\sqrt{t}+2}\right)^2 = \left(\sqrt{3t+1}\right)^2$

$\sqrt{t} + 2 = 3t + 1$

$\sqrt{t} = 3t - 1$

$$\left(\sqrt{t}\right)^2 = (3t-1)^2$$
$$t = 9t^2 - 6t + 1$$
$$0 = 9t^2 - 7t + 1$$
$a = 9$, $b = -7$, $c = 1$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(9)(1)}}{2(9)} = \frac{7 \pm \sqrt{13}}{18}$$
Only $\frac{7+\sqrt{13}}{18}$ checks.

78. $x = \frac{-(-2.7) \pm \sqrt{(-2.7)^2 - 4(0.04)(8.6)}}{2(0.04)}$

≈ 64.15 or 3.35

79. $x^2 + (0.1)x - 0.2 = 0$

$a = 1$, $b = 0.1$, $c = -0.2$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.1 \pm \sqrt{0.01 - 4(1)(-0.2)}}{2(1)} = \frac{-0.1 \pm \sqrt{0.81}}{2} = \frac{-0.1 \pm 0.9}{2}$$
$x = 0.40$ or $x = -0.50$.

80. Let l be the length of the picture, then its width is $l - 2$.

$l(l-2) = 48$

$l^2 - 2l - 48 = 0$

$(l-8)(l+6) = 0$

$l - 8 = 0$ or $l + 6 = 0$

$l = 8$ or $l = -6$

Since length cannot be negative, $l = 8$. The width of the picture is $l - 2 = 8 - 2 = 6$ cm.
The dimensions of the picture are 6 cm by 8 cm.

81. The amount that the temperature has risen over the X days is

(X degrees per day)(X days) = X^2 degrees.

$X^2 + 15 = 51$

$X^2 = 36$

$X = \pm\sqrt{36}$

$X = 6$ or $X = -6$

The temperature has been rising 6 degrees per day for 6 days.

82. $\overline{M} = \frac{Q(Q+10)}{44}$

$44\overline{M} = Q^2 + 10Q$

$0 = Q^2 + 10Q - 44\overline{M}$

From the quadratic formula with $a = 1$, $b = 10$, $c = -44\overline{M}$,
$$Q = \frac{-10 \pm \sqrt{100 - 4(1)(-44\overline{M})}}{2(1)} = \frac{-10 + 2\sqrt{25 + 44\overline{M}}}{2} = -5 \pm \sqrt{25 + 44\overline{M}}$$
Thus, $-5 + \sqrt{25 + 44\overline{M}}$ is a root.

83. $g = -200P^2 + 200P + 20$

Set $g = 60$.

$60 = -200P^2 + 200P + 20$

$200P^2 - 200P + 40 = 0$

$5P^2 - 5P + 1 = 0$

From the quadratic formula with $a = 5$, $b = -5$, $c = 1$,
$$P = \frac{5 \pm \sqrt{25 - 4(5)(1)}}{2(5)} = \frac{5 \pm \sqrt{5}}{10}$$
$P \approx 0.28$ or $P \approx 0.72$

28% and 72% of yeast gave an average weight gain of 60 grams.

84. **a.** $(2n-1)v^2 - 2nv + 1 = 0$

From the quadratic formula with $a = 2n - 1$, $b = -2n$, $c = 1$,
$$v = \frac{-(-2n) \pm \sqrt{4n^2 - 4(2n-1)(1)}}{2(2n-1)}$$
$$v = \frac{2n \pm \sqrt{4n^2 - 8n + 4}}{2(2n-1)}$$
$$v = \frac{2n \pm 2\sqrt{n^2 - 2n + 1}}{2(2n-1)} = \frac{n \pm \sqrt{(n-1)^2}}{2n-1}$$
Because of the condition that $n \ge 1$, it follows that $n - 1$ is nonnegative. Thus, $\sqrt{(n-1)^2} = n - 1$ and we have

$$v = \frac{n \pm (n-1)}{2n-1}.$$

$v = 1$ or $v = \frac{1}{2n-1}.$

b. $nv^2 - (2n+1)v + 1 = 0$

From the quadratic formula with $a = n$, $b = -(2n + 1)$, and $c = 1$,

$$v = \frac{-[-(2n+1)] \pm \sqrt{[-(2n+1)]^2 - 4(n)(1)}}{2n}$$

$$v = \frac{2n+1 \pm \sqrt{4n^2+1}}{2n}$$

Because $\sqrt{4n^2+1}$ is greater than $2n$, choosing the plus sign gives a numerator greater than $2n + 1 + 2n$, or $4n + 1$, so v is greater than $\frac{4n+1}{2n} = 2 + \frac{1}{2n}$. Thus v is greater than 2. This contradicts the restriction on v. On the other hand, because $\sqrt{4n^2+1}$ is greater than 1, choosing the minus sign gives a numerator less than $2n$, so v is less than $\frac{2n}{2n} = 1$. This meets the condition on v. Thus we choose

$$v = \frac{2n+1-\sqrt{4n^2+1}}{2n}.$$

85. a. When the object strikes the ground, h must be 0, so

$0 = 39.2t - 4.9t^2 = 4.9t(8-t)$

$t = 0$ or $t = 8$

The object will strike the ground 8 s after being thrown.

b. Setting $h = 68.2$ gives

$68.2 = 39.2t - 4.9t^2$

$4.9t^2 - 39.2t + 68.2 = 0$

$$t = \frac{39.2 \pm \sqrt{(-39.2)^2 - 4(4.9)(68.2)}}{2(4.9)}$$

$$\approx \frac{39.2 \pm 14.1}{9.8}$$

$t \approx 5.4$ s or $t \approx 2.6$ s.

86. The revenue covers the cost when $R = C$, that is, when

$-5x^2 + 600x = 10000 + 150x$

$-5x^2 + 450x - 10000 = 0$

Using the quadratic formula, we get

$$x = \frac{-450 \pm \sqrt{450^2 - 4(-5)(-10000)}}{2(-5)}$$

$$= \frac{-450 \pm 50}{-10} = 40 \text{ or } 50$$

So when 40 or 50 laptops are sold, the revenue covers the cost; since profit equals revenue minus cost, the university's profit is zero in this case. (Note that for x between 40 and 50 we have $R > C$, so revenue more than covers cost and the university makes a positive profit.)

87. a. To find household consumption expenditure in 2011, replace x by 11 in the equation $H = 0.138x^2 + 0.416x + 2.57$ to get

$H = 0.138 \times 11^2 + 0.416 \times 11 + 2.57$

$= 16.698 + 4.576 + 2.57$

$= 23.844$

For the year 2023, replace x by 23:

$H = 0.138 \times 23^2 + 0.416 \times 23 + 2.57$

$= 73.002 + 9.568 + 2.57$

$= 85.14$

Therefore, the household consumption expenditure amounts in 2011 and 2023 are \$23.844 billion and \$85.14 billion, respectively.

b. To determine when household consumption expenditure will be \$100 billion, we replace H by 100 in the equation $H = 0.138x^2 + 0.416x + 2.57$ and solve for x, as follows:

$100 = 0.138x^2 + 0.416x + 2.57$

$0.138x^2 + 0.416x - 97.43 = 0$

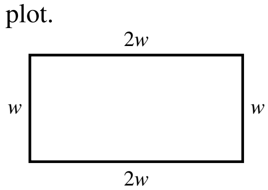
$$x = \frac{-0.416 \pm \sqrt{0.416^2 - 4(0.138)(-97.43)}}{2(0.138)}$$

$x \approx 25$ or -28

Since x is the number of years after 2000, we accept only the positive solution, 25. Hence household consumption expenditure in Qatar will be \$100 billion in 2025.

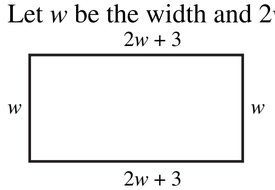
Problems 1.3

1. Let w be the width and $2w$ be the length of the plot.



Then area = 800.
$(2w)w = 800$
$2w^2 = 800$
$w^2 = 400$
$w = 20$ m
Thus the length is 40 m, so the amount of fencing needed is $2(40) + 2(20) = 120$ m.

2. Let w be the width and $2w + 3$ be the length.



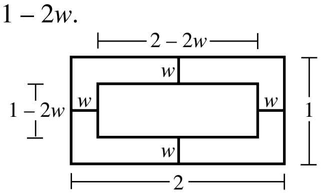
Then perimeter = 300.

$$2w + 2(2w + 3) = 300$$
$$6w + 6 = 300$$
$$6w = 294$$
$$w = 49 \text{ m}$$

Thus the length is $2(49) + 3 = 101$ m.
The dimensions are 49 m by 101 m.

3. Let n = number of cubic meters in each part. Then we have
$1n + 3n + 5n = 765$
$9n = 765$
$n = 85$
Thus he needs $1n = 1(85) = 85\ \text{m}^3$ of portland cement, $3n = 3(85) = 255\ \text{m}^3$ of sand, and $5n = 5(85) = 425\ \text{m}^3$ of crushed stone.

4. Let w = width (in km) of strip to be cut. Then the remaining forest has dimensions $2 - 2w$ by $1 - 2w$.



Considering the area of the remaining forest, we have

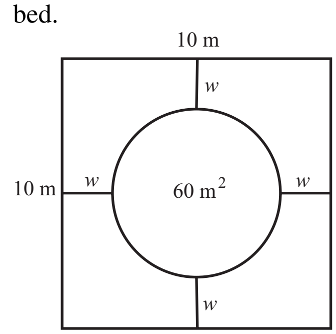
$$(2 - 2w)(1 - 2w) = \frac{3}{4}$$
$$2 - 6w + 4w^2 = \frac{3}{4}$$
$$8 - 24w + 16w^2 = 3$$
$$16w^2 - 24w + 5 = 0$$
$$(4w - 1)(4w - 5) = 0$$

Hence $w = \frac{1}{4}, \frac{5}{4}$. But $w = \frac{5}{4}$ is impossible since one dimension of original forest is 1 km. Thus the width of the strip should be $\frac{1}{4}$ km.

5. Let n = number of ounces in each part. Then we have
$2n + 1n = 16$
$3n = 16$
$n = \frac{16}{3}$
Thus the turpentine needed is
$(1)n = \frac{16}{3} = 5\frac{1}{3}$ ounces.

6. Since diameter of circular end is 140 mm, the radius is 70 mm. Area of circular end is $\pi(\text{radius})^2 = \pi(70)^2$. Area of square end is x^2.
Equating areas, we have $x^2 = \pi(70)^2$.
Thus $x = \pm\sqrt{\pi(70)^2} = \pm 70\sqrt{\pi}$. Since x must be positive, $x = 70\sqrt{\pi} \approx 124$ mm.

7. Let w = "width" (in meters) of the pavement. Then $5 - w$ is the radius of the circular flower bed.



Thus

$$\pi r^2 = A$$
$$\pi(5-w)^2 = 60$$
$$w^2 - 10w + 25 = \frac{60}{\pi}$$
$$w^2 - 10w + \left(25 - \frac{60}{\pi}\right) = 0$$

$a = 1$, $b = -10$, $c = 25 - \frac{60}{\pi}$

$$w = \frac{-b \pm \sqrt{100 - 4(1)\left(25 - \frac{60}{\pi}\right)}}{2} \approx 9.37,\ 0.63$$

Since $0 < w < 5$, $w \approx 0.63$ m.

8. Let q = required number of units.
Profit = Total Revenue – Total Cost
$150,000 = 50q - (25q + 500,000)$
$150,000 = 25q - 500,000$
$650,000 = 25q$, from which
$q = 26,000$

9. Let q = number of tons for \$560,000 profit.
Profit = Total Revenue – Total Cost
$560,000 = 134q - (82q + 120,000)$
$560,000 = 52q - 120,000$
$680,000 = 52q$
$\frac{680,000}{52} = q$
$q \approx 13,076.9 \approx 13,077$ tons.

10. Let x = amount at 4% and
$120,000 - x$ = amount at 5%.

$$0.04x + 0.05(120,000 - x) = 0.045(120,000)$$
$$-0.01x + 6000 = 5400$$
$$-0.01x = -600$$
$$x = 60,000$$

The investment consisted of \$60,000 at 5% and \$60,000 at 4%.

11. Let x = amount at 6% and
$20,000 - x$ = amount at $7\frac{1}{2}$%.
$x(0.06) + (20,000 - x)(0.075) = 1440$
$-0.015x + 1500 = 1440$
$-0.015x = -60$
$x = 4000$, so $20,000 - x = 16,000$. Thus the investment should be \$4000 at 6% and \$16,000 at $7\frac{1}{2}$%.

12. Following the procedure in Example 34 we obtain the total value at the end of the second year to be $1,000,000(1+r)^2$.
So at the end of the third year, the accumulated amount will be $1,000,000(1+r)^2$ plus the interest on this, which is $1,000,000(1+r)^2 r$.
Thus the total value at the end of the third year will be $1,000,000(1+r)^2 + 1,000,000(1+r)^2 r$
$= 1,000,000(1+r)^3$.
This must equal \$1,125,800.

$$1,000,000(1+r)^3 = 1,125,800$$
$$(1+r)^3 = \frac{1,125,800}{1,000,000} = 1.1258$$
$$1 + r \approx 1.04029$$
$$r \approx 0.04029$$

Thus $r \approx 0.04029 \approx 4\%$.

13. Let p = selling price. Then profit = $0.2p$.
selling price = cost + profit
$p = 3.40 + 0.2p$
$0.8p = 3.40$
$p = \frac{3.40}{0.8} = \$4.25$

14. Let n = number of bookings.
$0.90n = 81$
$n = 90$ seats booked

15. Following the procedure in Example 34 we obtain

$$3,000,000(1+r)^2 = 3,245,000$$
$$(1+r)^2 = \frac{649}{600}$$
$$1 + r = \pm\sqrt{\frac{649}{600}}$$
$$r = -1 \pm \sqrt{\frac{649}{600}}$$
$$r \approx -2.04 \text{ or } 0.04$$

We choose $r \approx 0.04 = 4\%$.

16. Let n = number of people polled.
$0.20p = 700$
$p = \frac{700}{0.20} = 3500$

17. Let x be Aramex's profit in the first half of 2009. The profit realized in the same period of 2010 is equal to the profit in 2009 plus an extra 10%, so we have

$x + 0.1x = 102$

$1.1x = 102$

$x = \frac{102}{1.1} \approx 92.73$

Hence, the profit realized by Aramex in the first six months of 2009 was approximately 92.73 million dirhams.

18. Yearly salary before strike = (7.50)(8)(260) = \$15,600
Lost wages = (7.50)(8)(46) = \$2760
Let P be the required percentage increase (as a decimal).
$P(15,600) = 2760$

$P = \frac{2760}{15,600} \approx 0.177 = 17.7\%$

19. Let q = number of cartridges sold to break even.
total revenue = total cost

$21.95q = 14.92q + 8500$

$7.03q = 8500$

$q \approx 1209.10$

1209 cartridges must be sold to approximately break even.

20. Let v = total annual vision-care expenses (in dollars) covered by program. Then
$35 + 0.80(v - 35) = 100$
$0.80v + 7 = 100$
$0.80v = 93$
$v = \$116.25$

21. a. $0.031c$

b. $c - 0.031c = 600,000,000$
$0.969c = 600,000,000$
$c \approx 619,195,046$
Approximately 619,195,046 bars will have to be made.

22. Revenue = (number of units sold)(price per unit)
Thus

$400 = q\left[\frac{80-q}{4}\right]$

$1600 = 80q - q^2$

$q^2 - 80q + 1600 = 0$

$(q - 40)^2 = 0$

$q = 40$ units

23. If I = interest, P = principal, r = rate, and t = time, then $I = Prt$. To triple an investment of P at the end of t years, the interest earned during that time must equal $2P$. Thus
$2P = P(0.045)t$
$2 = 0.045t$

$t = \frac{2}{0.045} \approx 44.4$ years

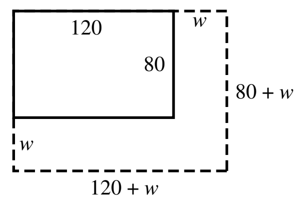
24. Let q = required number of units. Equate the incomes of each proposal.

$5000 + 0.50q = 50,000$

$0.50q = 45,000$

$q = 90,000$ units

25. Let w = width of strip. The original area is 80(120) and the new area is $(120 + w)(80 + w)$.



Thus
$(120 + w)(80 + w) = 2(80)(120)$

$9600 + 200w + w^2 = 19,200$

$w^2 + 200w - 9600 = 0$

$(w + 240)(w - 40) = 0$
$w = -240$ or $w = 40$
We choose $w = 40$ m.

26. Let x = original value of the blue-chip investment, then $3,100,000 - x$ is the original value of the glamour stocks. Then the current value of the blue-chip stock is $x + \frac{1}{10}x$, or $\frac{11}{10}x$.
For the glamour stocks the current value is

$(3,100,000 - x) - \frac{1}{10}(3,100,000 - x)$, which

simplifies to $\frac{9}{10}(3,100,000 - x)$.

Thus for the current value of the portfolio,

$\frac{11}{10}x + \frac{9}{10}(3,100,000 - x) = 3,240,000$

$11x + 27,900,000 - 9x = 32,400,000$
$2x = 4,500,000$
$x = 2,250,000$
Thus the current value of the blue chip investment is $\frac{11}{10}(2,250,000)$ or \$2,475,000.

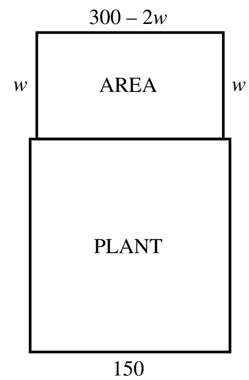
27. Let n = number of \$20 increases. Then at the rental charge of $400 + 20n$ dollars per unit, the number of units that can be rented is $50 - 2n$. The total of all monthly rents is $(400 + 20n)(50 - 2n)$, which must equal 20,240.
$20,240 = (400 + 20n)(50 - 2n)$
$20,240 = 20,000 + 200n - 40n^2$
$40n^2 - 200n + 240 = 0$
$n^2 - 5n + 6 = 0$
$(n - 2)(n - 3) = 0$
$n = 2, 3$
Thus the rent should be either
\$400 + 2(\$20) = \$440 or \$400 + 3(\$20) = \$460.

28. $10,000 = 800p - 7p^2$
$7p^2 - 800p + 10,000 = 0$
$$p = \frac{800 \pm \sqrt{640,000 - 280,000}}{14}$$
$$= \frac{800 \pm \sqrt{360,000}}{14} = \frac{800 \pm 600}{14}$$
For $p > 50$ we choose $p = \frac{800 + 600}{14} = \$100.$

29. Let p be the percentage change in market value.
$$(1 + 0.15)\left(\frac{P}{E}\right) = \frac{(1 + p)P}{(1 - 0.10)E}$$
$$1.15 = \frac{1 + p}{0.90}$$
$1.035 = 1 + p$
$p = 0.035 = 3.5\%$
The market value increased by 3.5%.

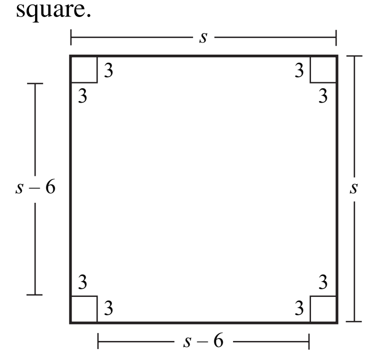
30. $2p^2 - 3p = 20 - p^2$
$3p^2 - 3p - 20 = 0$
$a = 3, b = -3, c = -20$
$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$
$$= \frac{3 \pm \sqrt{249}}{6}$$
$p \approx 3.130$ or $p \approx -2.130$
The equilibrium price is $p \approx 3.13$.

31. Let w = width (in meters) of enclosed area. Then length of enclosed area is
$300 - w - w = 300 - 2w$.



Thus
$w(300 - 2w) = 11,200$
$2w(150 - w) = 11,200$
$w(150 - w) = 5600$
$0 = w^2 - 150w + 5600$
$0 = (w - 80)(w - 70)$
Hence $w = 80, 70$. If $w = 70$, then length is $300 - 2w = 300 - 2(70) = 160$. Since the building has length of only 150 m, we reject $w = 70$. If $w = 80$, then length is $300 - 2w = 300 - 2(80) = 140$. Thus the dimensions are 80 m by 140 m.

32. Let s = length in centimeters of side of original square.



Considering the volume of the box, we have
(length)(width)(height) = volume
$(s - 4)(s - 4)(2) = 50$
$(s - 4)^2 = 25$
$s - 4 = \pm\sqrt{25} = \pm 5$
$s = 4 \pm 5$
Hence $s = -1, 9$. We reject $s = -1$ and choose $s = 9$. The dimensions are 9 cm. by 9 cm.

33. Original volume = $(10)(5)(2) = 100 \text{ cm}^3$
Volume increase = $0.50(100) = 50 \text{ cm}^3$
Volume of new sweet = $100 + 50 = 150 \text{ cm}^3$
Let x = number of centimeters that the length and width are each increased. Then

$2(x+10)(x+5)=150$
$x^2+15x+50=75$
$x^2+15x-25=0$
$a = 1, b = 15, c = -25$
$$x=\frac{-15\pm\sqrt{15^2-4(1)(-25)}}{2}\approx 1.51, -16.51$$
We reject −16.51 as impossible. The new length is approximately 11.51 cm, and the new width is approximately 6.51 cm.

34. Volume of old style candy
$=\pi(7.1)^2(2.1)-\pi(2)^2(2.1)$
$=97.461\pi \text{ mm}^3$
Let r = inner radius (in millimeters) of new style candy. Considering the volume of the new style candy, we have
$\pi(7.1)^2(2.1)-\pi r^2(2.1)=0.78(97.461\pi)$
$29.84142\pi=2.1\pi r^2$
$14.2102=r^2$
$r\approx\pm 3.7696$
Since r is a radius, we choose r = 3.77 mm.

35. Let x = amount of loan. Then the amount actually received is $x - 0.16x$. Hence,
$x-0.16x=195,000$
$0.84x=195,000$
$x\approx 232,142.86$
To the nearest thousand, the loan amount is \$232,000. In the general case, the amount received from a loan of L with a compensating balance of p% is $L-\frac{p}{100}L$.
$$L-\frac{p}{100}L=E$$
$$\frac{100-p}{100}L=E$$
$$L=\frac{100E}{100-p}$$

36. Let n = number of machines sold over 600. Then the commission on each of 600 + n machines is 40 + 0.04n. Equating total commissions to 30,800 we obtain
$(600 + n)(40 + 0.04n) = 30,800$
$24,000+24n+40n+0.04n^2=30,800$
$0.02n^2+32n-3400=0$
$$n=\frac{-32\pm\sqrt{1024+272}}{0.04}=\frac{-32\pm 36}{0.04}$$
We choose $n=\frac{-32+36}{0.04}=100$. Thus the number of machines that must be sold is 600 + 100 = 700.

37. Let q = number of units of product sold last year and q + 2000 = the number sold this year. Then the revenue last year was 3q and this year it is 3.5(q + 2000). By the definition of margin of profit, it follows that
$$\frac{7140}{3.5(q+2000)}=\frac{4500}{3q}+0.02$$
$$\frac{2040}{q+2000}=\frac{1500}{q}+0.02$$
$2040q = 1500(q + 2000) + 0.02q(q + 2000)$
$2040q=1500q+3,000,000+0.02q^2+40q$
$0=0.02q^2-500q+3,000,000$
$$q=\frac{500\pm\sqrt{250,000-240,000}}{0.04}$$
$$=\frac{500\pm\sqrt{10,000}}{0.04}$$
$$=\frac{500\pm 100}{0.04}$$
= 10,000 or 15,000
So that the margin of profit this year is not greater than 0.15, we choose q = 15,000. Thus 15,000 units were sold last year and 17,000 this year.

38. Let q = number of units of B and q + 25 = number of units of A produced.
Each unit of B costs $\frac{1000}{q}$, and each unit of A costs $\frac{1500}{q+25}$. Therefore,
$$\frac{1500}{q+25}=\frac{1000}{q}+2$$
$1500q = 1000(q + 25) + 2(q)(q + 25)$
$0=2q^2-450q+25,000$
$0=q^2-225q+12,500$
$0 = (q - 100)(q - 125)$
q = 100 or q = 125
If q = 100, then q + 25 = 125; if q = 125, q + 25 = 150. Thus the company produces either 125 units of A and 100 units of B, or 150 units of A and 125 units of B.

39. To find the year in which Bahrain's birth rate will be 10, we replace y by 10 in the equation $y = -0.268x + 565.179$ and solve for x, as follows:

$$10 = -0.268x + 565.179$$
$$0.268x = 555.179$$
$$x = \frac{555.179}{0.268} = 2071.56$$

The birth rate of Bahrain (per 1000) will be 10 by the middle of the year 2071.

Apply It 1.4

4. Because 549,130 is 549.13 thousand, we solve the inequality $131.18 + 32.15t \geq 549.13$ for t:

$$131.18 + 32.15t \geq 549.13$$
$$32.15t \geq 549.13 - 131.18 = 417.95$$
$$t \geq \frac{417.95}{32.15} = 13$$

Hence the number of internet users in Libya will be at least 549,130 from the year 2015 onward.

5. Since $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, and $x_4 \geq 0$, we have the inequalities

$$150 - x_4 \geq 0$$
$$3x_4 - 210 \geq 0$$
$$x_4 + 60 \geq 0$$
$$x_4 \geq 0$$

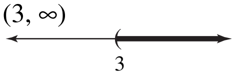
Problems 1.4

1. $5x > 15$

$x > \frac{15}{5}$

$x > 3$

$(3, \infty)$

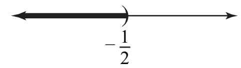


2. $4x < -2$

$x < \frac{-2}{4}$

$x < -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right)$

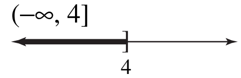


3. $5x - 11 \leq 9$

$5x \leq 20$

$x \leq 4$

$(-\infty, 4]$

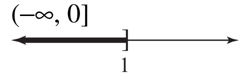


4. $5x \leq 0$

$x \leq \frac{0}{5}$

$x \leq 0$

$(-\infty, 0]$

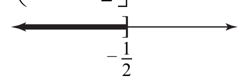


5. $-4x \geq 2$

$x \leq \frac{2}{-4}$

$x \leq -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right]$

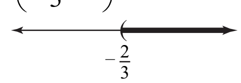


6. $3z + 2 > 0$

$3z > -2$

$z > -\frac{2}{3}$

$\left(-\frac{2}{3}, \infty\right)$

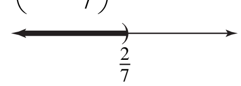


7. $5 - 7s > 3$

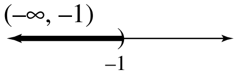
$-7s > -2$

$s < \frac{2}{7}$

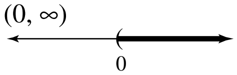
$\left(-\infty, \frac{2}{7}\right)$



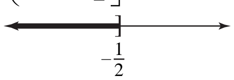
8. $4s - 1 < -5$
$4s < -4$
$s < -1$
$(-\infty, -1)$



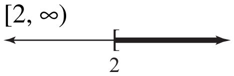
9. $3 < 2y + 3$
$0 < 2y$
$0 < y$
$y > 0$
$(0, \infty)$



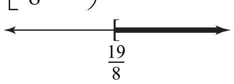
10. $4 \le 3 - 2y$
$1 \le -2y$
$-\frac{1}{2} \ge y$
$y \le -\frac{1}{2}$
$\left(-\infty, -\frac{1}{2}\right]$



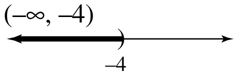
11. $t + 6 \le 2 + 3t$
$4 \le 2t$
$2 \le t$
$t \ge 2$
$[2, \infty)$



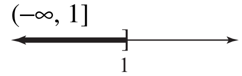
12. $-3 \ge 8(2 - x)$
$-3 \ge 16 - 8x$
$8x \ge 19$
$x \ge \frac{19}{8}$
$\left[\frac{19}{8}, \infty\right)$



13. $8(x + 1) + 1 < 3(2x) + 1$
$8x + 9 < 6x + 1$
$2x < -8$
$x < -4$
$(-\infty, -4)$

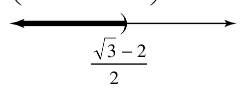


14. $5 - (x + 2) \le 2(2 - x)$
$5 - x - 2 \le 4 - 2x$
$x \le 1$
$(-\infty, 1]$

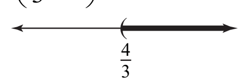


15. $2(4x - 2) > 4(2x + 1)$
$8x - 4 > 8x + 4$
$-4 > 4$, which is false for all x.
Thus the solution set is $\varnothing$.

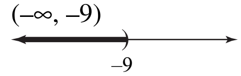
16. $x + 2 < \sqrt{3} - x$
$2x < \sqrt{3} - 2$
$x < \frac{\sqrt{3} - 2}{2}$
$\left(-\infty, \frac{\sqrt{3} - 2}{2}\right)$



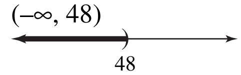
17. $\sqrt{2}(x + 2) > \sqrt{8}(3 - x)$
$\sqrt{2}(x + 2) > 2\sqrt{2}(3 - x)$
$x + 2 > 2(3 - x)$
$x + 2 > 6 - 2x$
$3x > 4$
$x > \frac{4}{3}$
$\left(\frac{4}{3}, \infty\right)$



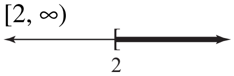
18. $-\frac{2}{3}x > 6$
$-x > 9$
$x < -9$
$(-\infty, -9)$



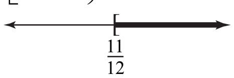
19. $\frac{5}{6}x < 40$
$5x < 240$
$x < 48$
$(-\infty, 48)$



20. $\frac{5y+2}{4} \le 2y-1$
$5y+2 \le 8y-4$
$-3y \le -6$
$y \ge 2$
$[2, \infty)$



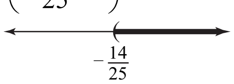
21. $\frac{3y-2}{3} \ge \frac{1}{4}$
$12y-8 \ge 3$
$12y \ge 11$
$y \ge \frac{11}{12}$
$\left[\frac{11}{12}, \infty\right)$



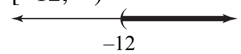
22. $-3x+1 \le -3(x-2)+1$
$-3x+1 \le -3x+7$
$1 \le 7$, which is true for all x. The solution is $-\infty < x < \infty$.
$(-\infty, \infty)$

23. $0x \le 0$
$0 \le 0$, which is true for all x. The solution is $-\infty < x < \infty$.
$(-\infty, \infty)$

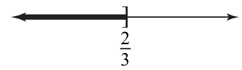
24. $\frac{5(3t+1)}{3} > \frac{2t-4}{6} + \frac{t}{2}$
$10(3t+1) > 2t-4+3t$
$30t+10 > 5t-4$
$25t > -14$
$t > -\frac{14}{25}$
$\left(-\frac{14}{25}, \infty\right)$



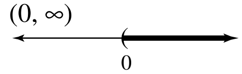
25. $2x+13 \ge \frac{1}{3}x-7$
$6x+39 \ge x-21$
$5x \ge -60$
$x \ge -12$
$[-12, \infty)$



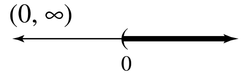
26. $3x - \frac{1}{3} \le \frac{5}{2}x$
$18x-2 \le 15x$
$3x \le 2$
$x \le \frac{2}{3}$
$\left(-\infty, \frac{2}{3}\right]$



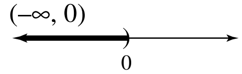
27. $\frac{2}{3}r < \frac{5}{6}r$
$4r < 5r$
$0 < r$
$r > 0$
$(0, \infty)$



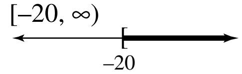
28. $\frac{7}{4}t > -\frac{8}{3}t$
$21t > -32t$
$53t > 0$
$t > 0$
$(0, \infty)$



29. $y + \frac{y}{2} < \frac{y}{3} + \frac{y}{5}$
$30y+15y < 10y+6y$
$29y < 0$
$y < 0$
$(-\infty, 0)$



30. $9-0.1x \le \frac{2-0.01x}{0.2}$
$1.8-0.02x \le 2-0.01x$
$-0.01x \le 0.2$
$x \ge -20$
$[-20, \infty)$



31. $\frac{3y-1}{-3} < \frac{5(y+1)}{-3}$

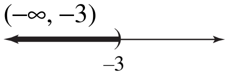
$3y-1 > 5y+5$

$-6 > 2y$

$-3 > y$

$y < -3$

$(-\infty, -3)$



32. $12(50) < S < 12(150)$

$600 < S < 1800$

33. $t \geq 3$ and $t \leq 5$ or $3 \leq t \leq 5$.

34. The measures of the acute angles of a right triangle sum to 90°.

If x is the measure of one acute angle, the other angle has measure $90 - x$.

$x < 3(90 - x) + 10$

$x < 270 - 3x + 10$

$4x < 280$

$x < 70$

The measure of the angle is less than 70°.

35. Let d be the number of disks. The stereo plus d disks will cost $219 + 18.95d$.

219 + 18.95d • 360

18.95*d* • 141

$d \leq \frac{141}{18.95} \approx 7.44$

The student can buy at most 7 disks.

Problems 1.5

1. Let q = number of units sold.

Profit > 0

Total revenue – Total cost > 0

$20q - (15q + 600{,}000) > 0$

$5q - 600{,}000 > 0$

$5q > 600{,}000$

$q > 120{,}000$

Thus at least 120,001 units must be sold.

2. Let q = number of units sold.

Total revenue – Total cost = Profit

We want Profit > 0.

$7.40q - [(2.50 + 4)q + 5000] > 0$

$0.9q - 5000 > 0$

$0.9q > 5000$

$q > \frac{5000}{0.9} = 5555\frac{5}{9}$

Thus at least 5556 units must be sold.

3. Let x = number of miles driven per year.

If the auto is leased, the annual cost is $12(420) + 0.06x$.

If the auto is purchased, the annual cost is

$4700 + 0.08x$. We want Rental cost ≤ Purchase cost.
$12(420) + 0.06x \le 4700 + 0.08x$
$5040 + 0.06x \le 4700 + 0.08x$
$340 \le 0.02x$
$17{,}000 \le x$
The number of miles driven per year must be at least 17,000.

4. Let q = number of clocks produced during regular work week, so $11{,}000 - q$ = number produced in overtime. Then
$2q + 3(11{,}000 - q) \le 25{,}000$
$-q + 33{,}000 \le 25{,}000$
$8000 \le q$
At least 8000 clocks must be produced during the regular workweek.

5. Let q = number of magazines printed. Then the cost of publication is $1.30q$. The number of magazines sold is $0.80q$. The revenue from dealers is $(1.50)(0.80q)$. If fewer than 100,000 magazines are sold, the only revenue is from the sales to dealers, while if more than 100,000 are sold, there are advertising revenues of $0.20(1.50)(0.80q - 100{,}000)$. Thus,

$$\text{Revenue} = \begin{cases} 1.5(0.8)q & \text{if} \quad 0.8q \le 100{,}000 \\ 1.5(0.8)q + 0.2(1.5)(0.8q - 100{,}000) & \text{if} \quad 0.8q > 100{,}000 \end{cases}$$

$$= \begin{cases} 1.2q & q \le 125{,}000 \\ 1.44q - 30{,}000 & q > 125{,}000 \end{cases}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$= \begin{cases} 1.2q - 1.3q & q \le 125{,}000 \\ 1.44q - 30{,}000 - 1.3q & q > 125{,}000 \end{cases}$$

$$= \begin{cases} -0.1q & q \le 125{,}000 \\ 0.14q - 30{,}000 & q > 125{,}000 \end{cases}$$

Clearly, the profit is negative if fewer than 125,001 magazines are printed.

$$0.14q - 30{,}000 \ge 0$$
$$0.14q \ge 30{,}000$$
$$q \ge 214{,}286$$

Thus, at least 214,286 magazines must be printed in order to avoid a loss.

6. Let L be current liabilities. Then

$$\text{Current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$3.8 = \frac{570{,}000}{L}$$

$3.8L = 570{,}000$
$L = \$150{,}000$
Let x = amount of money they can borrow, where $x \ge 0$.

$$\frac{570{,}000 + x}{150{,}000 + x} \ge 2.6$$

$570{,}000 + x \ge 390{,}000 + 2.6x$
$180{,}000 \ge 1.6x$
$112{,}500 \ge x$
Thus current liabilities are \$150,000 and the maximum amount they can borrow is \$112,500.

7. Let q be the number of units sold this month at \$4.00 each. Then $2500 - q$ will be sold at \$4.50 each. Then
Total revenue $\geq 10{,}750$
$4q + 4.5(2500 - q) \geq 10{,}750$
$-0.5q + 11{,}250 \geq 10{,}750$
$500 \geq 0.5q$
$1000 \geq q$
The maximum number of units that can be sold this month is 1000.

8. Revenue = (no. of units)(price per unit)
$$q\left(\frac{200}{q}+3\right) > 9000$$
$$200+3q > 9000$$
$$3q > 8800$$
$$q > 2933.\overline{3}$$
At least 2934 units must be sold.

9. For $t < 40$, we want
income on hourly basis
> income on per-job basis
$$9t > 320+3(40-t)$$
$$9t > 440-3t$$
$$12t > 440$$
$$t > 36.7 \text{ hr}$$

10. Let s = yearly sales. With the first method, the salary is $35{,}000 + 0.03s$, and with the second method it is $0.05s$.
$$35{,}000+0.03s > 0.05s$$
$$35{,}000 > 0.02s$$
$$1{,}750{,}000 > s$$
The first method is better for yearly sales less than \$1,750,000.

11. Let x = accounts receivable. Then
$$\text{Acid test ratio} = \frac{450{,}000+x}{398{,}000}$$
$$1.3 \leq \frac{450{,}000+x}{398{,}000}$$
$517{,}400 \leq 450{,}000 + x$
$x \geq 67{,}400$
The company must have at least \$67,400 in accounts receivable.

Apply It 1.6

6. $|w-500| \leq 13$

Problems 1.6

1. $|-13| = 13$

2. $\left|2^{-1}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$

3. $|8-2| = |6| = 6$

4. $\left|\frac{-3-5}{2}\right| = \left|\frac{-8}{2}\right| = |-4| = 4$

5. $\left|2\left(-\frac{7}{2}\right)\right| = |-7| = 7$

6. $|3 - 5| - |5 - 3| = |-2| - |2| = 2 - 2 = 0$

7. $|x| < 4$, $-4 < x < 4$

8. $|x| < 10$, $-10 < x < 10$

9. Because $3-\sqrt{10} < 0$,
$\left|3-\sqrt{10}\right| = -\left(3-\sqrt{10}\right) = \sqrt{10}-3$.

10. Because $\sqrt{5}-2 > 0$, $\left|\sqrt{5}-2\right| = \sqrt{5}-2$.

11. **a.** $|x-7| < 3$

b. $|x-2| < 3$

c. $|x-7| \leq 5$

d. $|x-7| = 4$

e. $|x+4| < 2$

f. $|x| < 3$

g. $|x| > 6$

h. $|x-105| < 3$

i. $|x-850| < 100$

12. $|f(x) - L| < \varepsilon$

13. $|p_1 - p_2| \le 9$

14. $|x - \mu| < 3\sigma$
$-3\sigma < x - \mu < 3\sigma$
$\mu - 3\sigma < x < \mu + 3\sigma$

15. $|x| = 7$
$x = \pm 7$

16. $|-x| = 2$
$-x = 2$ or -2
$x = \pm 2$

17. $\left|\frac{x}{5}\right| = 7$
$\frac{x}{5} = \pm 7$
$x = \pm 35$

18. $\left|\frac{5}{x}\right| = 12$
$\frac{5}{x} = \pm 12$
$x = \pm\frac{5}{12}$

19. $|x - 5| = 9$
$x - 5 = \pm 9$
$x = 5 \pm 9$
$x = 14$ or $x = -4$

20. $|4 + 3x| = 6$
$4 + 3x = \pm 6$
$3x = -4 \pm 6$
$3x = -10$ or 2
$x = -\frac{10}{3}$ or $x = \frac{2}{3}$

21. $|5x - 2| = 0$
$5x - 2 = 0$
$x = \frac{2}{5}$

22. $|7 - 4x| = 5$
$7 - 4x = \pm 5$
$-4x = -7 \pm 5$
$-4x = -2$ or -12
$x = \frac{1}{2}$ or $x = 3$

23. $|5 - 3x| = 7$
$5 - 3x = \pm 7$
$-3x = -5 \pm 7$
$-3x = 2$ or -12
$x = -\frac{2}{3}$ or $x = 4$

24. $|x| < M$
$-M < x < M$
$(-M, M)$
Note that $M > 0$ is required.

25. $|-x| < 3$
$|x| < 3$
$-3 < x < 3$
$(-3, 3)$

26. $\left|\frac{x}{4}\right| > 2$
$\frac{x}{4} < -2$ or $\frac{x}{4} > 2$
$x < -8$ or $x > 8$, so the solution is $(-\infty, -8) \cup (8, \infty)$.

27. $\left|\frac{x}{3}\right| > \frac{1}{2}$
$\frac{x}{3} < -\frac{1}{2}$ or $\frac{x}{3} > \frac{1}{2}$
$x < -\frac{3}{2}$ or $x > \frac{3}{2}$, so the solution is $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

28. $|x + 7| < 3$
$-3 < x + 7 < 3$
$-10 < x < -4$
$(-10, -4)$

29. $|2x - 17| < -4$
Because $-4 < 0$, the solution set is $\varnothing$.

30. $\left|x-\frac{1}{2}\right|>\frac{1}{2}$

$x-\frac{1}{2}<-\frac{1}{2}$ or $x-\frac{1}{2}>\frac{1}{2}$

$x<0$ or $x>1$

$(-\infty, 0) \cup (1, \infty)$

31. $|1-3x|>2$

$1-3x>2$ or $1-3x<-2$

$-3x>1$ or $-3x<-3$

$x<-\frac{1}{3}$ or $x>1$

The solution is $\left(-\infty, -\frac{1}{3}\right)\cup(1, \infty)$.

32. $|5-8x|\le 1$

$-1 \le 5-8x \le 1$

$-6 \le -8x \le -4$

$\frac{3}{4} \ge x \ge \frac{1}{2}$, which may be rewritten as

$\frac{1}{2} \le x \le \frac{3}{4}$.

The solution is $\left[\frac{1}{2}, \frac{3}{4}\right]$.

33. $|3x-2|\ge 0$ is true for all x because $|a|\ge 0$ for all a. Thus $-\infty < x < \infty$, or $(-\infty,\infty)$.

34. $\left|\frac{3x-8}{2}\right|\ge 4$

$\frac{3x-8}{2}\le -4$ or $\frac{3x-8}{2}\ge 4$

$3x-8\le -8$ or $3x-8\ge 8$

$3x\le 0$ or $3x\ge 16$

$x\le 0$ or $x\ge \frac{16}{3}$

The solution is $(-\infty, 0]\cup\left[\frac{16}{3}, \infty\right)$.

35. $\left|\frac{x-7}{3}\right|\le 5$

$-5\le \frac{x-7}{3}\le 5$

$-15\le x-7\le 15$

$-8\le x\le 22$

$[-8, 22]$

36. $|d-35.2\text{ m}|\le 20\text{ cm}$ or $|d-35.2|\le 0.20$

37. Let T_1 and T_2 be the temperatures of the two chemicals.

$5\le |T_1-T_2|\le 10$

38. $|x-\mu|>h\sigma$

Either $x-\mu<-h\sigma$, or $x-\mu>h\sigma$. Thus either $x<\mu-h\sigma$ or $x>\mu+h\sigma$, so the solution is $(-\infty, \mu-h\sigma)\cup(\mu+h\sigma, \infty)$.

39. $|x-0.01|\le 0.005$

Chapter 1 Review Problems

1. $3x-1\ge 2(x-3)$

$3x-1\ge 2x-6$

$x\ge -5$

$[-5, \infty)$

2. $2x-(7+x)\le x$

$2x-7-x\le x$

$-7\le 0$, which is true for all x, so $-\infty<x<\infty$, or $(-\infty, \infty)$.

3. $-(5x+2)<-(2x+4)$

$-5x-2<-2x-4$

$-3x<-2$

$x>\frac{2}{3}$

$\left(\frac{2}{3}, \infty\right)$

4. $-2(x+6)>x+4$

$-2x-12>x+4$

$-3x>16$

$x<-\frac{16}{3}$

$\left(-\infty, -\frac{16}{3}\right)$

5. $3p(1-p)>3(2+p)-3p^2$

$3p-3p^2>6+3p-3p^2$

$0>6$, which is false for all x. The solution set is $\varnothing$.

6. $2\left(6-\frac{5}{2}p\right)<7$

$12-5p<7$

$-5p<-5$

$p>1$

$(1, \infty)$

7. $\frac{x+5}{3}-\frac{1}{2}\le 2$

$2(x+5)-3(1)\le 6(2)$

$2x+10-3\le 12$

$2x\le 5$

$x\le\frac{5}{2}$

$\left(-\infty, \frac{5}{2}\right]$

8. $\frac{x}{3}-\frac{x}{4}>\frac{x}{5}$

$20x-15x>12x$

$5x>12x$

$0>7x$

$0>x$

$(-\infty, 0)$

9. $\frac{1}{4}s-3\le\frac{1}{8}(3+2s)$

$2s-24\le 3+2s$

$0\le 27$, which is true for all s. Thus $-\infty<s<\infty$, or $(-\infty,\infty)$.

10. $\frac{1}{3}(t+2)\ge\frac{1}{4}$

$4(t+2)\ge 3t+48$

$4t+8\ge 3t+48$

$t\ge 40$

$[40, \infty)$

11. $|4-3x|=9$

$4-3x=\pm 9$

$-3x=-4\pm 9$

$-3x=5$ or $-3x=-13$

$x=-\frac{5}{3}$ or $x=\frac{13}{3}$

12. $\left|\frac{5x-6}{13}\right|=0$

$\frac{5x-6}{13}=0$

$5x-6=0$

$x=\frac{6}{5}$

13. $|2z-3|<5$

$-5<2z-3<5$

$-2<2z<8$

$-1<z<4$

$(-1, 4)$

14. $4<\left|\frac{2}{3}x+5\right|$

$\frac{2}{3}x+5<-4$ or $\frac{2}{3}x+5>4$

$\frac{2}{3}x<-9$ or $\frac{2}{3}x>-1$

$x<-\frac{27}{2}$ or $x>-\frac{3}{2}$

The solution is $\left(-\infty, -\frac{27}{2}\right)\cup\left(-\frac{3}{2}, \infty\right)$.

15. $|3-2x|\ge 4$

$3-2x\ge 4$ or $3-2x\le -4$

$-2x\ge 1$ or $-2x\le -7$

$x\le -\frac{1}{2}$ or $x\ge\frac{7}{2}$

The solution is $\left(-\infty, -\frac{1}{2}\right]\cup\left[\frac{7}{2}, \infty\right)$.

16. Let p = selling price, c = cost. Then

$p-0.40p=c$

$0.6p=c$

$p=\frac{c}{0.6}=\frac{5c}{3}=c+\left(\frac{2}{3}\right)c$

Thus the profit is $\frac{2}{3}$, or $66\frac{2}{3}\%$, of the cost.

17. Let x be the number of issues with a decline, and $x+48$ be the number of issues with an increase. Then

$x+(x+48)=1132$

$2x=1084$

$x=542$

18. Let x = purchase amount excluding tax.

$x+0.065x=3039.29$

$1.065x=3039.29$

$x=2853.79$

Thus tax is $3039.29-2853.79=\$185.50$.

19. Let q units be produced at A and $10,000 - q$ at B.
Cost at A + Cost at B • 117,000
$[5q + 30,000] + [5.50(10,000 - q) + 35,000]$
• 117,000
$-0.5q + 120,000$ • 117,000
$-0.5q$ • -3000
q • 6000
Thus at least 6000 units must be produced at plant A.

20. Total volume of old tanks

$$= \pi(10)^2(25) + \pi(20)^2(25)$$
$$= 2500\pi + 10,000\pi$$
$$= 12,500\pi \text{ ft}^3$$

Let r be the radius (in feet) of the new tank. Then

$$\frac{4}{3}\pi r^3 = 12,500\pi$$
$$r^3 = 9375$$
$$r = \sqrt[3]{9375} \approx 21.0858$$

The radius is approximately 21.0858 feet.

21. Let c = operating costs

$$\frac{c}{236,460} < 0.90$$
$$c < \$212,814$$

Chapter Test

1. $z + 3(z - 4) = 5;\ \frac{17}{4},\ 4$

Set $z = \frac{17}{4}$:

$$\frac{17}{4} + 3\left(\frac{17}{4} - 4\right) \stackrel{?}{=} 5$$
$$\frac{17}{4} + \frac{51}{4} - 12 \stackrel{?}{=} 5$$
$$5 = 5$$

Set $z = 4$:

$$4 + 3(4 - 4) \stackrel{?}{=} 5$$
$$4 + 0 \stackrel{?}{=} 5$$
$$4 \neq 5$$

Thus, $\frac{17}{4}$ satisfies the equation, but 4 does not.

2. Multiplying both sides by $x - 2$; equivalence not guaranteed

3. $3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x$

Multiplying both sides by 5 gives

$$15x + x - 25 = 1 + 25x$$
$$16x - 25 = 1 + 25x$$
$$-9x = 26$$
$$x = -\frac{26}{9}$$

4. $\frac{1}{x+1} + \frac{2}{x-3} = \frac{-6}{3-2x}$

$$(x-3)(2x-3) + 2(x+1)(2x-3) = 6(x+1)(x-3)$$
$$2x^2 - 9x + 9 + 4x^2 - 2x - 6 = 6x^2 - 12x - 18$$
$$6x^2 - 11x + 3 = 6x^2 - 12x - 18$$
$$x = -21$$

5. $S = P(1 + rt)$

$$S = P + Prt$$
$$S - P = r(Pt)$$
$$r = \frac{S - P}{Pt}$$

6.

$$F = \frac{vf}{334.8}$$
$$495 = \frac{v(2500)}{334.8}$$
$$165,726 = 2500v$$
$$v = \frac{165,726}{2500} = 66.2904$$

Since the car is traveling at 66.2904 km/h on a 65 km/h highway, the officer can claim that you were speeding.

7. Let P be the amount in the account one year ago. Then the interest earned is $0.073P$ and $P + 0.073P = 1257$.

$$1.073P = 1257$$
$$P = \frac{1257}{1.073} \approx 1171.48$$

The amount in the account one year ago was \$1171.48, and the interest earned is \$1171.48(0.073) = \$85.52.

8. $-x^2 + 3x + 10 = 0$

$$x^2 - 3x - 10 = 0$$
$$(x - 5)(x + 2) = 0$$

$x - 5 = 0$ or $x + 2 = 0$
$x = 5$ or $x = -2$

9. $u^2 - u = 1$

$u^2 - u - 1 = 0$

$a = 1, b = -1, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$$

$x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

10. $2\left(\frac{1}{x+4}\right)^2 + 7\left(\frac{1}{x+4}\right) + 3 = 0$

Let $w = \frac{1}{x+4}$. Then

$2w^2 + 7w + 3 = 0$

$(2w + 1)(w + 3) = 0$

$w = -\frac{1}{2}, -3$

Thus, $\frac{1}{x+4} = -\frac{1}{2}$ or $\frac{1}{x+4} = -3$.

$x = -6, -\frac{13}{3}$

11. $\frac{A}{A+12} d = \frac{A+1}{24} d.$

Dividing both sides by d and then multiplying both sides by $24(A + 12)$ gives

$24A = (A + 12)(A + 1)$

$24A = A^2 + 13A + 12$

$0 = A^2 - 11A + 12$

From the quadratic formula,

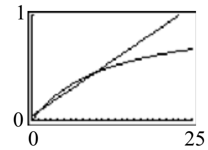
$A = \frac{11 \pm \sqrt{121-48}}{2} = \frac{11 \pm \sqrt{73}}{2}.$

$A = \frac{11+\sqrt{73}}{2} \approx 10$ or $A = \frac{11-\sqrt{73}}{2} \approx 1.$

The doses are the same at 1 year and 10 years.

$c = d$ in Cowling's rule when $\frac{A+1}{24} = 1,$ which occurs when $A = 23$. Thus, adulthood is achieved at age 23 according to Cowling's rule.

$c = d$ in Young's rule when $\frac{A}{A+12} = 1,$ which is never true. Thus, adulthood is never reached according to Young's rule.



Young's rule prescribes less than Cowling's for ages less than one year and greater than 10 years. Cowling's rule prescribes less for ages between 1 and 10.

12. Let n = number of grams in each part. Then we have

$4n + 5n = 145$

$9n = 145$

$n = 16\frac{1}{9}$

Thus there should be $4\left(16\frac{1}{9}\right) = 64\frac{4}{9}$ grams of A and $5\left(16\frac{1}{9}\right) = 80\frac{5}{9}$ grams of B.

13. Total revenue = variable cost + fixed cost

$100\sqrt{q} = 2q + 1200$

$50\sqrt{q} = q + 600$

$2500q = q^2 + 1200q + 360{,}000$

$0 = q^2 - 1300q + 360{,}000$

$0 = (q - 400)(q - 900)$

$q = 400$ or $q = 900$

14. Let n = number of shares.

total investment = $5000 + 20n$

$0.04(5000) + 0.50n = 0.03(5000 + 20n)$

$200 + 0.50n = 150 + 0.60n$

$-0.10n = -50$

$n = 500$

500 shares should be bought.

15. To have supply = demand,

$2p - 10 = 200 - 3p$

$5p = 210$

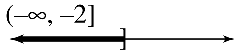
$p = 42$

16. Let n = number of acres sold. Then $n + 20$ acres were originally purchased at a cost of $\frac{7200}{n+20}$ each. The price of each acre sold was $30 + \left[\frac{7200}{n+20}\right]$. Since the revenue from selling n acres is \$7200 (the original cost of the parcel), we have

$$n\left[30+\frac{7200}{n+20}\right]=7200$$
$$n\left[\frac{30n+600+7200}{n+20}\right]=7200$$
$n(30n + 600 + 7200) = 7200(n + 20)$
$$30n^2+7800n=7200n+144{,}000$$
$$30n^2+600n-144{,}000=0$$
$$n^2+20n-4800=0$$
$(n + 80)(n - 60) = 0$
$n = 60$ acres (since $n > 0$), so 60 acres were sold.

17. $3(2 - 3x) > 4(1 - 4x)$
$6 - 9x > 4 - 16x$
$7x > -2$
$$x>-\frac{2}{7}$$
$$\left(-\frac{2}{7},\infty\right)$$


18. $$\frac{1-t}{2}<\frac{3t-7}{3}$$
$3(1-t)<2(3t-7)$
$3-3t<6t-14$
$-9t<-17$
$$t>\frac{17}{9}$$
$$\left(\frac{17}{9},\infty\right)$$


19. $0.1(0.03x + 4) \geq 0.02x + 0.434$
$0.003x + 0.4 \geq 0.02x + 0.434$
$-0.017x \geq 0.034$
$x \leq -2$
$(-\infty, -2]$


20. Let N = required number of shirts. Then
Total revenue = $3.5N$ and
Total cost = $1.3N + 0.4N + 6500$.
$$\text{Profit} > 0$$
$$3.5N-(1.3N+0.4N+6500)>0$$
$$1.8N-6500>0$$
$$1.8N>6500$$
$$N>3611.1$$
At least 3612 shirts must be sold.

21. Let x = amount at $6\frac{3}{4}\%$ and $30{,}000 - x$ = amount at 5%. Then interest at $6\frac{3}{4}\%$ + interest at 5% $\geq$ interest at $6\frac{1}{2}\%$
$x(0.0675) + (30{,}000 - x)(0.05) \geq (0.065)(30{,}000)$
$0.0175x + 1500 \geq 1950$
$0.0175x \geq 450$
$x \geq 25{,}714.29$
Thus at least \$25,714.29 must be invested at $6\frac{3}{4}\%$.

22. $|7x+3|=x$
Here we must have $x \geq 0$.
$7x + 3 = x$ or $-(7x + 3) = x$
$6x = -3$ $\quad$ $-7x - 3 = x$
$x=-\frac{1}{2}<0$ $\quad$ $x=-\frac{3}{8}<0$
There is no solution.

23. $$\left|\frac{3x-8}{2}\right|\geq 4$$
$\frac{3x-8}{2}\leq -4$ or $\frac{3x-8}{2}\geq 4$
$3x - 8 \leq -8$ or $3x - 8 \geq 8$
$3x \leq 0$ or $3x \geq 16$
$x \leq 0$ or $x\geq\frac{16}{3}$
The solution is $(-\infty, 0]\cup\left[\frac{16}{3},\infty\right)$.

24. Let x be the first even integer. Then the second and third consecutive even integers are $x + 2$ and $x + 4$. We want these three numbers to satisfy the equation

$$x+2(x+2)=4(x+4)$$

Expanding and solving for x, we get

$$x+2x+4=4x+16$$
$$4-16=4x-2x-x$$
$$-12=x$$

Hence, the integers are -12, -10, and -8.

25. To find the year in which electricity production will be 25.5 billion kWh, we substitute 25.5 for E in the equation $E = 1.0366x - 2064.185$ and solve for x:

$$25.5 = 1.0366x - 2064.185$$
$$2089.685 = 1.0366x$$
$$x = \frac{2089.685}{1.0366} \approx 2015.9$$

Therefore the electricity production in Qatar will be 25.5 billion kWh around the year 2016.

26. Revenue = (number of units sold) × (price per unit). So

$$400 = q\left(\frac{80-q}{4}\right)$$
$$1600 = 80q - q^2$$
$$q^2 - 80q + 1600 = 0$$
$$(q-40)^2 = 0$$
$$q = 40$$

For the revenue to be \$400, 40 units need to be sold.

27. Given $\overline{M} = \dfrac{Q(Q+10)}{44}$, we have

$$44\overline{M} = Q^2 + 10Q$$
$$Q^2 + 10Q - 44\overline{M} = 0$$

Using the quadratic formula with $a = 1$, $b = 10$, and $c = -44\overline{M}$, we get

$$Q = \frac{-10 \pm \sqrt{100 - 4(1)(-44\overline{M})}}{2}$$
$$= \frac{-10 \pm 2\sqrt{25 + 44\overline{M}}}{2}$$
$$= -5 \pm \sqrt{25 + 44\overline{M}}$$

Thus, $-5 + \sqrt{25 + 44\overline{M}}$ is a root.

Explore & Extend—Chapter 1

1. Here $m = 120$ and $M = 2\frac{1}{2}(60) = 150$. For LP, $r = 2$, so the first t minutes take up $\frac{t}{2}$ of the 120 available minutes. For SP, $r = 1$, so the remaining $150 - t$ minutes take up $\frac{150-t}{1}$ of the 120 available.

$$\frac{t}{2} + \frac{150-t}{1} = 120$$
$$t + 300 - 2t = 240$$
$$-t = -60$$
$$t = 60$$

Switch after 1 hour.

2. Here $m = 120$ and $M = 2\frac{1}{2}(60) = 150$. For EP, $r = 3$, so the first t minutes will take up $\frac{t}{3}$ of the 120 available minutes. For SP, $r = 1$, so the remaining $150 - t$ minutes take up $\frac{150-t}{1}$ of the 120 available.

$$\frac{t}{3} + \frac{150-t}{1} = 120$$
$$t + 450 - 3t = 360$$
$$-2t = -90$$
$$t = 45$$

Switch after 45 minutes.

3. Use the reasoning in Exercise 1, with M unknown and $m = 120$.

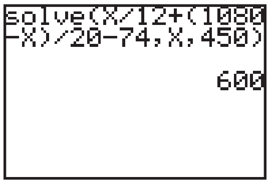
$$\frac{t}{2} + \frac{M-t}{1} = 120$$
$$t + 2M - 2t = 240$$
$$-t = 240 - 2M$$
$$t = 2M - 240$$

The switch should be made after $2M - 240$ minutes.

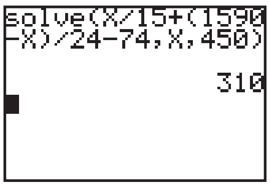
4. Use the reasoning in Exercise 2, with M unknown and $m = 120$.

$$\frac{t}{3} + \frac{M-t}{1} = 120$$
$$t + 3M - 3t = 360$$
$$-2t = 360 - 3M$$
$$t = \frac{1}{2}(3M - 360)$$

The switch should be made after
$\frac{1}{2}(3M - 360)$ minutes.

5. 

$x = 600$



$x = 310$

6. Both equations represent audio being written onto 74-minute CDs. In the first equation, 18 hours (1080 minutes) are being written to a CD using a combination of 12-to-1 and 20-to-1 compression ratios. Here, *x* gives the maximum amount of audio (600 minutes or 10 hours) that can be written using the 12-to-1 compression ratio. In the second equation, 26.5 hours (1590 minutes) is being written using 15-to-1 and 24-to-1 compression ratios. A maximum of 310 minutes can be written at 15-to-1.

7. The first *t* minutes use $\frac{t}{R}$ of the *m* available minutes, the remaining $M - t$ minutes use $\frac{M-t}{r}$ of the *m* available.

$$\frac{t}{R} + \frac{M-t}{r} = m$$
$$\frac{t}{R} + \frac{M}{r} - \frac{t}{r} = m$$
$$t\left(\frac{1}{R} - \frac{1}{r}\right) = m - \frac{M}{r}$$
$$t\left(\frac{r-R}{rR}\right) = \frac{mr-M}{r}$$
$$t = \frac{R(mr-M)}{r-R}$$

Chapter 2

Apply It 2.1

1. **a.** The formula for the area of a circle is πr^2, where r is the radius.
$a(r) = \pi r^2$

   b. The domain of $a(r)$ is all real numbers.

   c. Since a radius cannot be negative or zero, the domain for the function, in context, is $r > 0$.

2. **a.** The formula relating distance, time, and speed is $d = rt$ where d is the distance, r is the speed, and t is the time. This can also be written as $t = \frac{d}{r}$. When $d = 300$, we have $t(r) = \frac{300}{r}$.

   b. The domain of $t(r)$ is all real numbers except 0.

   c. Since speed is not negative, the domain for the function, in context, is $r > 0$.

   d. Replacing r by x: $t(x) = \frac{300}{x}$.
Replacing r by $\frac{x}{2}$: $t\left(\frac{x}{2}\right) = \frac{300}{\frac{x}{2}} = \frac{600}{x}$.
Replacing r by $\frac{x}{4}$: $t\left(\frac{x}{4}\right) = \frac{300}{\frac{x}{4}} = \frac{1200}{x}$.

   e. When the speed is reduced (divided) by a constant, the time is scaled (multiplied) by the same constant; $t\left(\frac{r}{c}\right) = \frac{300c}{r}$.

3. **a.** If the price is \$18.50 per large pizza, $p = 18.5$.
$18.5 = 26 - \frac{q}{40}$
$-7.5 = -\frac{q}{40}$
$300 = q$
At a price of \$18.50 per large pizza, 300 pizzas are sold each week.

   b. If 200 large pizzas are being sold each week, $q = 200$.
$p = 26 - \frac{200}{40}$
$p = 26 - 5$
$p = 21$
The price is \$21 per pizza if 200 large pizzas are being sold each week.

   c. To double the number of large pizzas sold, use $q = 400$.
$p = 26 - \frac{400}{40}$
$p = 26 - 10$
$p = 16$
To sell 400 large pizzas each week, the price should be \$16 per pizza.

4. Revenue = price · quantity = pq
From the table, the weekly revenue is:
$pq = 500 \cdot 11 = 5500$
$pq = 600 \cdot 14 = 8400$
$pq = 700 \cdot 17 = 11{,}900$
$pq = 800 \cdot 20 = 16{,}000$

Problems 2.1

1. The functions are not equal because $f(x) \geq 0$ for all values of x, while $g(x)$ can be less than 0. For example, $f(-2) = \sqrt{(-2)^2} = \sqrt{4} = 2$ and $g(-2) = -2$, thus $f(-2) \neq g(-2)$.

2. The functions are different because they have different domains. The domain of $G(x)$ is $[-1, \infty)$ (all real numbers ≥ -1) because you can only take the square root of a non-negative number, while the domain of $H(x)$ is all real numbers.

3. The functions are not equal because they have different domains. $h(x)$ is defined for all non-zero real numbers, while $k(x)$ is defined for all real numbers.

4. The functions are equal. For $x = 3$ we have $f(3) = 2$ and $g(3) = 3 - 1 = 2$, hence $f(3) = g(3)$. For $x \neq 3$, we have
$f(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3} = x - 1.$
Note that we can cancel the $x - 3$ because we are assuming $x \neq 3$ and so $x - 3 \neq 0$. Thus for

$x \neq 3$, $f(x) = x - 1 = g(x)$.
$f(x) = g(x)$ for all real numbers and they have the same domains, thus the functions are equal.

5. The denominator is zero when $x = 1$. Any other real number can be used for x.
Answer: all real numbers except 1

6. Any real number can be used for x.
Answer: all real numbers

7. For $\sqrt{x-3}$ to be real, $x - 3 \geq 0$, so $x \geq 3$.
Answer: all real numbers ≥ 3

8. For $\sqrt{z-1}$ to be real, $z - 1 \geq 0$, so $z \geq 1$. We exclude values of z for which $\sqrt{z-1} = 0$, so $z - 1 = 0$, thus $z = 1$.
Answer: all real numbers > 1

9. Any real number can be used for z.
Answer: all real numbers

10. We exclude values of x for which
$x + 3 = 0$
$x = -3$
Answer: all real numbers except –3

11. We exclude values of x where
$2x + 7 = 0$
$2x = -7$
$x = -\frac{7}{2}$
Answer: all real numbers except $-\frac{7}{2}$

12. We exclude values of y for which $y^2 - 4y + 4 = 0$. $y^2 - 4y + 4 = (y-2)^2$, so we exclude values of y for which $y - 2 = 0$, thus $y = 2$.
Answer: all real numbers except 2.

13. We exclude values of x for which
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3, 2$$
Answer: all real numbers except –3 and 2

14. $r^2 + 1$ is never 0.
Answer: all real numbers

15. $f(x) = 2x + 1$
$f(0) = 2(0) + 1 = 1$
$f(3) = 2(3) + 1 = 7$
$f(-4) = 2(-4) + 1 = -7$

16. $H(s) = 5s^2 - 3$
$H(4) = 5(4)^2 - 3 = 80 - 3 = 77$
$H\left(\sqrt{2}\right) = 5\left(\sqrt{2}\right)^2 - 3 = 10 - 3 = 7$
$H\left(\frac{2}{3}\right) = 5\left(\frac{2}{3}\right)^2 - 3 = \frac{20}{9} - 3 = -\frac{7}{9}$

17. $F(x) = -7x + 1$
$F(s) = -7s + 1$
$F(t + 1) = -7(t + 1) + 1 = -7t - 6$
$F(x + 3) = -7(x + 3) + 1 = -7x - 20$

18. $\gamma(u) = 2u^2 - u$
$\gamma(-2) = 2(-2)^2 - (-2) = 8 + 2 = 10$
$\gamma(2v) = 2(2v)^2 - (2v) = 8v^2 - 2v$
$\gamma(x+a) = 2(x+a)^2 - (x+a)$
$= 2x^2 + 4ax + 2a^2 - x - a$

19. $h(v) = \frac{1}{\sqrt{v}}$
$h(16) = \frac{1}{\sqrt{16}} = \frac{1}{4}$
$h\left(\frac{1}{4}\right) = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$
$h(1-x) = \frac{1}{\sqrt{1-x}}$

20. $f(x) = x^2 + 2x + 1$
$f(1) = 1^2 + 2(1) + 1 = 1 + 2 + 1 = 4$
$f(-1) = (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$
$f(x+h) = (x+h)^2 + 2(x+h) + 1$
$= x^2 + 2xh + h^2 + 2x + 2h + 1$

21. $H(x) = (x+4)^2$
$H(0) = (0+4)^2 = 16$
$H(2) = (2+4)^2 = 6^2 = 36$
$H(t-4) = [(t-4)+4]^2 = t^2$

22. $k(x)=\dfrac{x-5}{x^2+1}$

$k(5)=\dfrac{5-5}{5^2+1}=0$

$k(2x)=\dfrac{2x-5}{(2x)^2+1}=\dfrac{2x-5}{4x^2+1}$

$k(x+h)=\dfrac{(x+h)-5}{(x+h)^2+1}=\dfrac{x+h-5}{x^2+2xh+h^2+1}$

23. $k(x)=\sqrt{x-3}$

$k(4)=\sqrt{4-3}=\sqrt{1}=1$

$k(3)=\sqrt{3-3}=\sqrt{0}=0$

$k(x+1)-k(x)=\sqrt{(x+1)-3}-\sqrt{x-3}$
$=\sqrt{x-2}-\sqrt{x-3}$

24. $f(x)=x^{4/3}$

$f(0)=0^{4/3}=0$

$f(64)=64^{4/3}=\left(\sqrt[3]{64}\right)^4=(4)^4=256$

$f\left(\dfrac{1}{8}\right)=\left(\dfrac{1}{8}\right)^{4/3}=\left(\sqrt[3]{\dfrac{1}{8}}\right)^4=\left(\dfrac{1}{2}\right)^4=\dfrac{1}{16}$

25. $g(x)=x^{2/5}$

$g(32)=32^{2/5}=\left(\sqrt[5]{32}\right)^2=(2)^2=4$

$g(-64)=(-64)^{2/5}=\left(\sqrt[5]{-64}\right)^2$

$=\left(\sqrt[5]{-32}\sqrt[5]{2}\right)^2=\left(-2\sqrt[5]{2}\right)^2=4\sqrt[5]{4}$

$g(t^{10})=(t^{10})^{2/5}=t^4$

26. $f(x)=4x-5$

a. $f(x+h)=4(x+h)-5=4x+4h-5$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{(4x+4h-5)-(4x-5)}{h}=\dfrac{4h}{h}=4$

27. $f(x)=\dfrac{x}{3}$

a. $f(x+h)=\dfrac{x+h}{3}$

b. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{x+h}{3}-\frac{x}{3}}{h}=\dfrac{\frac{h}{3}}{h}=\dfrac{1}{3}$

28. $f(x)=3x^2-2x-1$

a. $f(x+h)=3(x+h)^2-2(x+h)-1$
$=3(x^2+2xh+h^2)-2x-2h-1$
$=3x^2+6xh+3h^2-2x-2h-1$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{(3x^2+6xh+3h^2-2x-2h-1)-(3x^2-2x-1)}{h}$

$=\dfrac{6xh+3h^2-2h}{h}$

$=6x+3h-2$

29. $f(x)=x^2+2x$

a. $f(x+h)=(x+h)^2+2(x+h)$
$=x^2+2xh+h^2+2x+2h$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{(x^2+2xh+h^2+2x+2h)-(x^2+2x)}{h}$

$=\dfrac{2xh+h^2+2h}{h}=2x+h+2$

30. $f(x)=3-2x+4x^2$

a. $f(x+h)=3-2(x+h)+4(x+h)^2$
$=3-2x-2h+4(x^2+2xh+h^2)$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{3-2x-2h+4x^2+8xh+4h^2-(3-2x+4x^2)}{h}$

$=\dfrac{-2h+8xh+4h^2}{h}$

$=-2+8x+4h$

31. $f(x)=x^3$

a. $f(x+h)=(x+h)^3=x^3+3x^2h+3xh^2+h^3$

b. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}=\dfrac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$

32. $f(x)=\dfrac{x+8}{x}$

a. $f(x+h)=\dfrac{(x+h)+8}{x+h}=\dfrac{x+h+8}{x+h}$

b. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{x+h+8}{x+h}-\frac{x+8}{x}}{h}=\dfrac{x(x+h)\left(\frac{x+h+8}{x+h}-\frac{x+8}{x}\right)}{x(x+h)h}=\dfrac{x(x+h+8)-(x+h)(x+8)}{x(x+h)h}$

$=\dfrac{x^2+xh+8x-x^2-hx-8x-8h}{x(x+h)h}=\dfrac{-8h}{x(x+h)h}=-\dfrac{8}{x(x+h)}$

33. $f(x)=\dfrac{1}{x-1}$

a. $f(x+h)=\dfrac{1}{x+h-1}$

b. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{1}{x+h-1}-\frac{1}{x-1}}{h}=\dfrac{\frac{x-1-(x+h-1)}{(x-1)(x+h-1)}}{h}=\dfrac{-1}{(x-1)(x+h-1)}$

34. $\dfrac{f(3+h)-f(3)}{h}=\dfrac{[5(3+h)+3]-[5(3)+3]}{h}$

$=\dfrac{[15+5h+3]-[15+3]}{h}$

$=\dfrac{18+5h-18}{h}$

$=\dfrac{5h}{h}$

$=5$

35. $\dfrac{f(x)-f(2)}{x-2}=\dfrac{2x^2-x+1-(8-2+1)}{x-2}$

$=\dfrac{2x^2-x+1-7}{x-2}$

$=\dfrac{2x^2-x-6}{x-2}$

$=2x+3$

36. $9y-3x-4=0$

The equivalent form $y=\dfrac{3x+4}{9}$ shows that for each input x there is exactly one output, $\dfrac{3x+4}{9}$. Thus y is a function of x. Solving for x gives $x=\dfrac{9y-4}{3}$. This shows that for each input y there is exactly one output, $\dfrac{9y-4}{3}$. Thus x is a function of y.

37. $x^4-1+y=0$

The equivalent form $y=-x^4+1$ shows that for each input x there is exactly one output, $-x^4+1$. Thus y is a function of x. Solving for x gives

$x = \pm\sqrt[4]{1-y}$. If, for example, $y = -15$, then $x = \pm 2$, so x is not a function of y.

38. $y = 7x^2$

For each input x, there is exactly one output $7x^2$. Thus y is a function of x. Solving for x gives $x = \pm\sqrt{\frac{y}{7}}$. If, for example, $y = 7$, then $x = \pm 1$, so x is not a function of y.

39. $x^2 + y^2 = 1$

Solving for y we have $y = \pm\sqrt{1-x^2}$. If $x = 0$, then $y = \pm 1$, so y is not a function of x. Solving for x gives $x = \pm\sqrt{1-y^2}$. If $y = 0$, then $x = \pm 1$, so x is not a function of y.

40. a. $f(a) = a^2a^3 + a^3a^2 = a^5 + a^5 = 2a^5$

b.
$$\begin{aligned} f(ab) &= a^2(ab)^3 + a^3(ab)^2 \\ &= a^2a^3b^3 + a^3a^2b^2 \\ &= a^5b^3 + a^5b^2 \\ &= a^5b^2(b+1) \end{aligned}$$

41. Yes, because corresponding to each input r there is exactly one output, πr^2.

42. Depreciation at the end of t years is $0.02t(30{,}000)$, so value V of machine is $V = f(t) = 30{,}000 - 0.02t(30{,}000)$, or $V = f(t) = 30{,}000(1 - 0.02t)$.

43. Yes; for each input q there corresponds exactly one output, $1.25q$, so P is a function of q. The dependent variable is P and the independent variable is q.

44. Charging \$600,000 per film corresponds to $p = 600{,}000$.

$$600{,}000 = \frac{1{,}200{,}000}{q}$$

$$q = 2$$

The actor will star in 2 films per year. To star in 4 films per year the actor should charge $p = \frac{1{,}200{,}000}{4} = \$300{,}000$ per film.

45. The function can be written as $q = 48p$. At \$8.39 per kg, the supply is $q = 48(8.39) = 402.72$ kg per week. At \$19.49 per kg, the supply is $q = 48(19.49) = 935.52$ kg per week. The amount supplied increases as the price increases.

46. a. $f(0) = 1 - 1 = 0$

b.
$$\begin{aligned} f(100) &= 1 - \left(\frac{200}{300}\right)^3 \\ &= 1 - \left(\frac{2}{3}\right)^3 \\ &= 1 - \frac{8}{27} \\ &= \frac{19}{27} \end{aligned}$$

c.
$$\begin{aligned} f(800) &= 1 - \left(\frac{200}{1000}\right)^3 \\ &= 1 - \left(\frac{1}{5}\right)^3 \\ &= 1 - \frac{1}{125} \\ &= \frac{124}{125} \end{aligned}$$

d. Solve

$$0.5 = 1 - \left(\frac{200}{200+t}\right)^3$$

$$\left(\frac{200}{200+t}\right)^3 = 0.5$$

$$\frac{200}{200+t} = \sqrt[3]{0.5}$$

$$200 = 200\sqrt[3]{0.5} + t\sqrt[3]{0.5}$$

$$t = \frac{200 - 200\sqrt[3]{0.5}}{\sqrt[3]{0.5}} \approx 51.98$$

Half the group was discharged after 52 days.

47. a. $f(1000) = \frac{\left(\sqrt[3]{1000}\right)^4}{2500} = \frac{10^4}{2500} = \frac{10{,}000}{2500} = 4$

b. $f(2000) = \frac{\left[\sqrt[3]{1000(2)}\right]^4}{2500} = \frac{\left(10\sqrt[3]{2}\right)^4}{2500}$

$$= \frac{10{,}000\sqrt[3]{2^4}}{2500} = 4\sqrt[3]{2^3 \cdot 2} = 8\sqrt[3]{2}$$

c. $f(2I_0) = \dfrac{(2I_0)^{4/3}}{2500} = \dfrac{2^{4/3} I_0^{4/3}}{2500}$

$= 2\sqrt[3]{2}\left[\dfrac{I_0^{4/3}}{2500}\right] = 2\sqrt[3]{2}\, f(I_0)$

Thus $f(2I_0) = 2\sqrt[3]{2}\, f(I_0)$, which means that doubling the intensity increases the response by a factor of $2\sqrt[3]{2}$.

48. $P(1) = 1 - \frac{1}{2}(1-0.344)^0 = 1 - \frac{1}{2}(1) = \frac{1}{2}$

$P(2) = 1 - \frac{1}{2}(1-0.344)^1 = 1 - \frac{1}{2}(0.656) = 0.672$

49. a. Domain: 3000, 2900, 2300, 2000
$f(2900) = 12$, $f(3000) = 10$

b. Domain: 10, 12, 17, 20
$g(10) = 3000$, $g(17) = 2300$

50. a. –18.97

b. –581.77

c. –18.51

51. a. –5.13

b. 2.64

c. –17.43

52. a. 1,997,723.57

b. 1,287,532.35

c. 2,964,247.40

53. a. 7.89

b. 63.85

c. 1.21

Apply It 2.2

5. a. Let n = the number of visits and $p(n)$ be the premium amount.
$p(n) = 125$

b. The premiums do not change regardless of the number of doctor visits.

c. This is a constant function.

6. a. The degree of each term of $d(t)$ is a nonnegative integer, so $d(t)$ is a polynomial.

b. The leading term is $0.0022t^2$, which has power 2, so the degree of $d(t)$ is 2.

c. The leading coefficient is 0.0022.

7. The cost C of using n units of electricity is

$$C(n) = \begin{cases} 0.03n & \text{if } n \le 3000 \\ 0.03\times 3000 + 0.09(n-3000) & \text{if } 3000 < n \le 5000 \\ 0.03\times 3000 + 0.09\times 2000 + 0.16(n-5000) & \text{if } n > 5000 \end{cases}$$

$$= \begin{cases} 0.03n & \text{if } n \le 3000 \\ 0.09n - 180 & \text{if } 3000 < n \le 5000 \\ 0.16n - 530 & \text{if } n > 5000 \end{cases}$$

8. Think of the bookshelf having 7 slots, from left to right. You have a choice of 7 books for the first slot. Once a book has been put in the first slot, you have 6 choices for which book to put in the second slot, etc. The number of arrangements is $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040$.

Problems 2.2

1. yes

2. $f(x) = \frac{x^3 + 7x - 3}{3} = \frac{1}{3}x^3 + \frac{7}{3}x - 1,$ which is a polynomial function.

3. no

4. yes

5. yes

6. yes

7. no

8. $g(x) = 4x^{-4} = \frac{4}{x^4}$, which is a rational function.

9. all real numbers

10. all real numbers

11. all real numbers

12. all x such that $1 \le x \le 3$

13. **a.** 2

    b. 9

14. **a.** 7

    b. 1

15. **a.** 0

    b. 9

16. $f(x) = 8$
    $f(2) = 8$
    $f(t + 8) = 8$
    $f\left(-\sqrt{17}\right) = 8$

17. $F(12) = 2$
    $F\left(-\sqrt{3}\right) = -1$
    $F(1) = 0$
    $F\left(\frac{18}{5}\right) = 2$

18. $g(x) = |x - 3|$
    $g(10) = |10 - 3| = |7| = 7$
    $g(3) = |3 - 3| = |0| = 0$
    $g(-3) = |-3 - 3| = |-6| = 6$

19. $G(8) = 8 - 1 = 7$
    $G(3) = 3 - 1 = 2$
    $G(-1) = 3 - (-1)^2 = 2$
    $G(1) = 3 - (1)^2 = 2$

20. $f(3) = 4$
    $f(-4) = 3$
    $f(0) = 4$

21. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

22. $(3 - 3)! = 0! = 1$

23. $(4 - 2)! = 2! = 2 \cdot 1 = 2$

24. $$\begin{aligned} 6! \cdot 2! &= (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1) \\ &= (720)(2) \\ &= 1440 \end{aligned}$$

25. $\frac{n!}{(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = n$

26. $$\begin{aligned} \frac{8!}{5!(8-5)!} &= \frac{8!}{5! \cdot 3!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} \\ &= \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} \\ &= 8 \cdot 7 \\ &= 56 \end{aligned}$$

27. Let i = the passenger's income and
    $c(i)$ = the cost for the ticket.
    $c(i) = 6.24$
    This is a constant function.

28. Let w = the width of the prism, then
    $w + 3$ = the length of the prism, and
    $2w - 1$ = the height of the prism. The formula for the volume of a rectangular prism is
    V = length · width · height.
    $V(w) = (w + 3)(w)(2w - 1) = 2w^3 + 5w^2 - 3w$
    This is a cubic function.

29. **a.** $C = 850 + 3q$

    b. $$\begin{aligned} 1600 &= 850 + 3q \\ 750 &= 3q \\ q &= 250 \end{aligned}$$

30. The interest is Prt, so principal and interest amount to $f(t) = P + Prt$, or $f(t) = P(1 + rt)$. Since $f(t) = at + b$ where a (= Pr) and b (= P) are constants, f is a linear function of t.

31. For a committee of five, there are 5 choices for who will be member A. For each choice of member A, there are 4 choices for member G. Once members A and G have been chosen, there are 3 choices for member M, two choices for member N, then one choice for member S once members A, G, M, and N have been chosen. Thus, there are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ ways to label the members.

32. $P(5) = \dfrac{5!\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0}{5!(0!)} = \dfrac{5!\left(\frac{1}{1024}\right)(1)}{5!(1)} = \dfrac{1}{1024}$

33. $P(2) = \dfrac{3!\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^1}{2!(1!)} = \dfrac{6\left(\frac{1}{16}\right)\left(\frac{3}{4}\right)}{2(1)} = \dfrac{9}{64}$

34. a. all T such that $30 \le T \le 39$

b. $f(30) = \dfrac{1}{24}(30) + \dfrac{11}{4} = \dfrac{5}{4} + \dfrac{11}{4} = \dfrac{16}{4} = 4$

$f(36) = \dfrac{1}{24}(36) + \dfrac{11}{4} = \dfrac{6}{4} + \dfrac{11}{4} = \dfrac{17}{4}$

$f(39) = \dfrac{4}{3}(39) - \dfrac{175}{4} = 52 - \dfrac{175}{4} = \dfrac{33}{4}$

35. a. 742.50

b. –20.28

c. 1218.60

36. a. 1182.74

b. 4985.27

c. 252.15

37. a. 19.12

b. –62.94

c. 57.69

38. a. 2.21

b. 9.98

c. –14.52

Apply It 2.3

9. The customer's price is
$(c \circ s)(x) = c(s(x)) = c(x+3) = 2(x+3)$
$= 2x + 6$

10. $g(x) = (x+3)^2$ can be written as
$g(x) = a(l(x)) = (a \circ l)(x)$ where $a(x) = x^2$ and $l(x) = x + 3$. Then $l(x)$ represents the length of the sides of the square, while $a(x)$ is the area of a square with side of length x.

Problems 2.3

1. $f(x) = x + 3$, $g(x) = x + 5$

a. $(f+g)(x) = f(x) + g(x)$
$= (x+3) + (x+5)$
$= 2x + 8$

b. $(f + g)(0) = 2(0) + 8 = 8$

c. $(f-g)(x) = f(x) - g(x)$
$= (x+3) - (x+5)$
$= -2$

d. $(fg)(x) = f(x)g(x)$
$= (x+3)(x+5)$
$= x^2 + 8x + 15$

e. $(fg)(-2) = (-2)^2 + 8(-2) + 15 = 3$

f. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+3}{x+5}$

g. $(f \circ g)(x) = f(g(x))$
$= f(x+5)$
$= (x+5) + 3$
$= x + 8$

h. $(f \circ g)(3) = 3 + 8 = 11$

i. $(g \circ f)(x) = g(f(x))$
$= g(x+3)$
$= (x+3) + 5$
$= x + 8$

j. $(g \circ f)(3) = 3 + 8 = 11$

2. $f(x) = 2x,\ g(x) = 6 + x$

a. $(f+g)(x) = f(x)+g(x) = (2x)+(6+x) = 3x+6$

b. $(f-g)(x) = f(x)-g(x) = (2x)-(6+x) = x-6$

c. $(f-g)(4) = (4) - 6 = -2$

d. $(fg)(x) = f(x)g(x) = 2x(6+x) = 12x+2x^2$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x}{6+x}$

f. $\frac{f}{g}(2) = \frac{2(2)}{6+2} = \frac{4}{8} = \frac{1}{2}$

g. $(f\circ g)(x) = f(g(x)) = f(6+x) = 2(6+x) = 12+2x$

h. $(g\circ f)(x) = g(f(x)) = g(2x) = 6+2x$

i. $(g\circ f)(2) = 6+2(2) = 6+4 = 10$

3. $f(x) = x^2-1,\ \ g(x) = x^2+x$

a. $(f+g)(x) = f(x)+g(x) = (x^2-1)+(x^2+x) = 2x^2+x-1$

b. $(f-g)(x) = f(x)-g(x) = (x^2-1)-(x^2+x) = -x-1$

c. $(f-g)\left(-\frac{1}{2}\right) = \frac{1}{2}-1 = -\frac{1}{2}$

d. $(fg)(x) = f(x)g(x) = (x^2-1)(x^2+x) = x^4+x^3-x^2-x$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2-1}{x^2+x} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{x-1}{x},\ x\neq -1$

f. $\frac{f}{g}\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}-1}{-\frac{1}{2}} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

g. $(f\circ g)(x) = f(g(x)) = f(x^2+x) = (x^2+x)^2-1 = x^4+2x^3+x^2-1$

h. $(g\circ f)(x) = g(f(x)) = g(x^2-1) = (x^2-1)^2+(x^2-1) = x^4-2x^2+1+x^2-1 = x^4-x^2$

i. $(g\circ f)(-3) = (-3)^4-(-3)^2 = 72$

4. $f(x) = x^2+1,\ g(x) = 5$

a. $(f+g)(x) = f(x)+g(x) = (x^2+1)+5 = x^2+6$

b. $(f+g)\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2+6 = \frac{58}{9}$

c. $(f-g)(x) = f(x)-g(x) = (x^2+1)-5 = x^2-4$

d. $(fg)(x) = f(x)g(x) = (x^2+1)(5) = 5x^2+5$

e. $(fg)(7) = 5(7^2) + 5 = 245 + 5 = 250$

f. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2+1}{5}$

g. $(f \circ g)(x) = f(g(x)) = f(5) = 5^2 + 1 = 26$

h. $(f \circ g)(12{,}003) = 26$

i. $(g \circ f)(x) = g(f(x)) = g(x^2+1) = 5$

5. $f(g(2)) = f(4 - 4) = f(0) = 0 + 6 = 6$
$g(f(2)) = g(12 + 6) = g(18) = 4 - 36 = -32$

6.
$$\begin{aligned}(f \circ g)(p) &= f(g(p))\\ &= f\left(\frac{p-2}{3}\right)\\ &= \frac{4}{\frac{p-2}{3}}\\ &= \frac{12}{p-2}\end{aligned}$$

$$(g \circ f)(p) = g(f(p)) = g\left(\frac{4}{p}\right) = \frac{\frac{4}{p}-2}{3} = \frac{4-2p}{3p}$$

7.
$$\begin{aligned}(F \circ G)(t) &= F(G(t))\\ &= F\left(\frac{2}{t-1}\right)\\ &= \left(\frac{2}{t-1}\right)^2 + 7\left(\frac{2}{t-1}\right) + 1\\ &= \frac{4}{(t-1)^2} + \frac{14}{t-1} + 1\end{aligned}$$

$$\begin{aligned}(G \circ F)(t) &= G(F(t))\\ &= G(t^2+7t+1)\\ &= \frac{2}{(t^2+7t+1)-1}\\ &= \frac{2}{t^2+7t}\end{aligned}$$

8.
$$\begin{aligned}(F \circ G)(t) &= F(G(t))\\ &= F(2t^2-2t+1)\\ &= \sqrt{2t^2-2t+1}\end{aligned}$$

$$\begin{aligned}(G \circ F)(t) &= G(F(t))\\ &= G\left(\sqrt{t}\right)\\ &= 2\left(\sqrt{t}\right)^2 - 2\left(\sqrt{t}\right) + 1\\ &= 2t - 2\sqrt{t} + 1\end{aligned}$$

9.
$$\begin{aligned}(f \circ g)(v) &= f(g(v))\\ &= f\left(\sqrt{v+2}\right)\\ &= \frac{1}{\left(\sqrt{v+2}\right)^2+1}\\ &= \frac{1}{v+2+1}\\ &= \frac{1}{v+3}\end{aligned}$$

$$\begin{aligned}(g \circ f)(v) &= g(f(v))\\ &= g\left(\frac{1}{v^2+1}\right)\\ &= \sqrt{\frac{1}{v^2+1}+2}\\ &= \sqrt{\frac{1+2(v^2+1)}{v^2+1}}\\ &= \sqrt{\frac{2v^2+3}{v^2+1}}\end{aligned}$$

10.
$$\begin{aligned}(f \circ f)(x) &= f(f(x))\\ &= f(x^2+2x-1)\\ &= (x^2+2x-1)^2 + 2(x^2+2x-1) - 1\\ &= x^4 + 4x^3 + 4x^2 - 2\end{aligned}$$

11. Let $g(x) = 11x$ and $f(x) = x - 7$. Then
$h(x) = g(x) - 7 = f(g(x))$

12. Let $g(x) = x^2 - 2$ and $f(x) = \sqrt{x}$. Then
$h(x) = \sqrt{x^2-2} = \sqrt{g(x)} = f(g(x))$

13. Let $g(x) = x^2 + x + 1$ and $f(x) = \frac{3}{x}$. Then
$$h(x) = \frac{3}{x^2 - x + 1} = \frac{3}{g(x)} = f(g(x))$$

14. Let $g(x) = 9x^3 - 5x$ and $f(x) = x^3 - x^2 + 11$.
Then $h(x) = [g(x)]^3 - [g(x)]^2 + 11 = f(g(x))$

15. Let $g(x) = \frac{x^2 - 1}{x + 3}$ and $f(x) = \sqrt[4]{x}$.
Then $h(x) = \sqrt[4]{g(x)} = f(g(x))$.

16. Let $g(x) = 3x - 5$ and $f(x) = \frac{2 - x}{x^2 + 2}$. Then
$h(x) = \frac{2 - (3x - 5)}{(3x - 5)^2 + 2} = f(g(x))$.

17. a. The revenue is \$9.75 per pound of coffee sold, so $r(x) = 9.75x$.

b. The expenses are $e(x) = 4500 + 4.25x$.

c. Profit = revenue – expenses.
$(r - e)(x) = 9.75x - (4500 + 4.25x)$
$= 5.5x - 4500$.

18. $v(x) = \frac{4}{3}\pi(3x - 1)^3$ can be written as

$v(x) = f(l(x)) = (f \circ l)(x)$ where $f(x) = \frac{4}{3}\pi x^3$ and $l(x) = 3x - 1$. Then $l(x)$ represents the radius of the sphere, while $f(x)$ is the volume of a sphere with radius x.

19.
$$\begin{aligned}(g \circ f)(m) &= g(f(m)) \\ &= g\left(\frac{40m - m^2}{4}\right) \\ &= 40\left(\frac{40m - m^2}{4}\right) \\ &= 10(40m - m^2) \\ &= 400m - 10m^2\end{aligned}$$
This represents the total revenue received when the total output of m employees is sold.

20.
$$\begin{aligned}(f \circ g)(E) &= f(g(E)) \\ &= f(7202 + 0.29E^{3.68}) \\ &= 0.45(7202 + 0.29E^{3.68} - 1000)^{0.53} \\ &= 0.45(6202 + 0.29E^{3.68})^{0.53}\end{aligned}$$
This represents status based on years of education.

21. a. 14.05

b. 1169.64

22.a. –0.13

b. 18.85

23. a. 194.47

b. 0.29

24. a. 0.45

b. 1.61

Problems 2.4

1. $f^{-1}(x) = \frac{x}{3} - \frac{7}{3}$

2. $g^{-1}(x) = \frac{x}{5} + \frac{3}{5}$

3. $F^{-1}(x) = 2x + 14$

4. $f^{-1}(x) = \frac{\sqrt{x}}{4} + \frac{5}{4}$

5. $r(A) = \sqrt{\frac{A}{\pi}}$

6. $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$

7. $f(x) = 5x + 12$ is one-to-one, for if $f(x_1) = f(x_2)$ then $5x_1 + 12 = 5x_2 + 12$, so $5x_1 = 5x_2$ and thus $x_1 = x_2$.

8. $g(x) = (3x + 4)^2$ is not one-to-one, because $g(x_1) = g(x_2)$ does not imply $x_1 = x_2$. For example, $g\left(-\frac{1}{3}\right) = g\left(-\frac{7}{3}\right) = 9$.

9. $h(x) = (5x + 12)^2$, for $x \ge -\frac{5}{12}$, is one-to-one.
If $h(x_1) = h(x_2)$ then $(5x_1 + 12)^2 = (5x_2 + 12)^2$.
Since $x \ge -\frac{5}{12}$ we have $5x + 12 \ge 0$, and thus $(5x_1 + 12)^2 = (5x_2 + 12)^2$ only if $5x_1 + 12 = 5x_2 + 12$, and hence $x_1 = x_2$.

10. $F(x) = |x-9|$ is not one-to-one, because $F(x_1) = F(x_2)$ does not imply $x_1 = x_2$. For example, $F(8) = F(10) = 1$.

11. The inverse of $f(x) = (4x-5)^2$ for $x \geq \frac{5}{4}$ is $f^{-1}(x) = \frac{\sqrt{x}}{4} + \frac{5}{4}$, so to find the solution, we find $f^{-1}(23)$.

$f^{-1}(23) = \frac{\sqrt{23}}{4} + \frac{5}{4}$

The solution is $x = \frac{\sqrt{23}}{4} + \frac{5}{4}$.

12. The inverse of $f(x) = 2x^3 + 1$ is $f^{-1} = \sqrt[3]{\frac{x-1}{2}}$, so the solution is $f^{-1}(129) = 4$.

13. From $p = \frac{1,200,000}{q}$, we get $q = \frac{1,200,000}{p}$. Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = \frac{1,200,000}{p}$, $p > 0$.

$$\begin{aligned} p(q(p)) &= p\left(\frac{1,200,000}{p}\right) \\ &= \frac{1,200,000}{\frac{1,200,000}{p}} \\ &= 1,200,000 \cdot \frac{p}{1,200,000} \\ &= p \end{aligned}$$

Similarly, $q(p(q)) = q$, so the functions are inverses.

14. From $p = \frac{q}{48}$, we get $q = 48p$. Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = 48p$, $p > 0$.

$q(p(q)) = q\left(\frac{q}{48}\right) = 48 \cdot \frac{q}{48} = q$

$p(q(p)) = p(48p) = \frac{48p}{48} = p$

Thus, $p(q)$ and $q(p)$ are inverses.

15. yes, it is one-to-one.

Apply It 2.5

11. Let y = the amount of money in the account. Then, after one month, $y = 7250 - (1 \cdot 600) = \$6650$, and after two months $y = 7250 - (2 \cdot 600) = \$6050$. Thus, in general, if we let x = the number of months during which Amal spends from this account, $y = 7250 - 600x$. To identify the x-intercept, we set $y = 0$ and solve for x.

$y = 7250 - 600x$

$0 = 7250 - 600x$

$600x = 7250$

$x = 12\frac{1}{12}$

The x-intercept is $\left(12\frac{1}{12}, 0\right)$.

Therefore, after 12 months and approximately 2.5 days Amal will deplete her savings. To identify the y-intercept, we set $x = 0$ and solve for y.

$y = 7250 - 600x$

$y = 7250 - 600(0)$

$y = 7250$

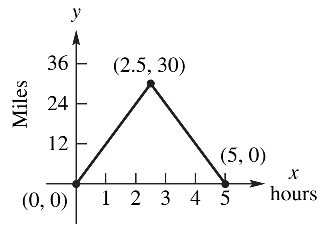
The y-intercept is (0, 7250).

Therefore, before any months have gone by, Amal has \$7250 in her account.

12. Let y = the cost to the customer and let x = the number of rides he or she takes. Since the cost does not change, regardless of the number of rides taken, the equation $y = 12.6$ represents this situation. The graph of $y = 12.6$ is a horizontal line whose y-intercept is (0, 12.6). Since the line is parallel to the x-axis, there is no x-intercept.

13. The formula relating distance, time, and speed is $d = rt$, where d is the distance, r is the speed, and t is the time. Let x = the time spent biking (in hours). Then, $12x$ = the distance traveled. Yassine bikes $12 \cdot 2.5 = 30$ km and then turns around and bikes the same distance back to the rental shop. Therefore, we can represent the distance from the turn-around point at any time x as $|30 - 12x|$. Similarly, the distance from the rental shop at any time x can be represented by the function $y = 30 - |30 - 12x|$.

x	0	1	2	2.5	3	4	5
y	0	12	24	30	24	12	0

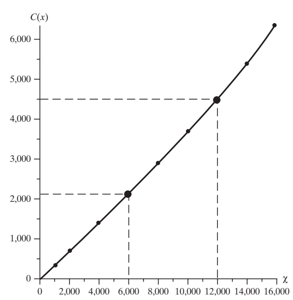


14. The monthly cost C of using x IG of water is

$$C(x)=\begin{cases}0.35x & \text{if } x\le 6000\\ 0.35\times 6000+0.4(x-6000) & \text{if } 6000<x\le 12000\\ 0.35\times 6000+0.4\times 6000+0.46(x-12000) & \text{if } x>12000\end{cases}$$

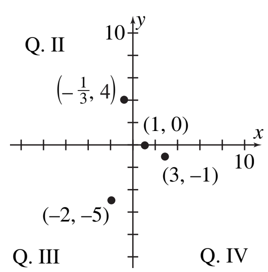
$$=\begin{cases}0.35x & \text{if } x\le 6000\\ 0.4x-300 & \text{if } 6000<x\le 12000\\ 0.46x-1020 & \text{if } x>12000\end{cases}$$

The graph of $C(x)$ is as follows:

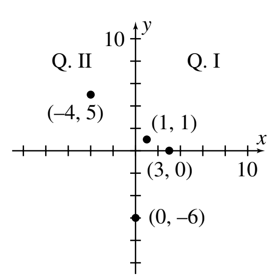


Problems 2.5

1.



2.



3. a. $f(0)=1, f(2)=2, f(4)=3, f(-2)=0$

b. Domain: all real numbers

c. Range: all real numbers

d. $f(x)=0$ for $x=-2$. So a real zero is -2.

4. a. $f(0)=2, f(2)=0$

b. Domain: all $x\ge 0$

c. Range: all $y\ge 2$

d. $f(x)=0$ for $x=2$. So a real zero is 2.

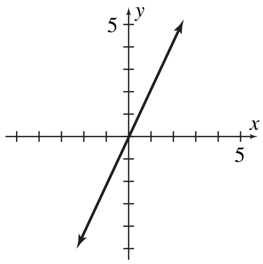
5. a. $f(0)=0, f(1)=1, f(-1)=1$

b. Domain: all real numbers

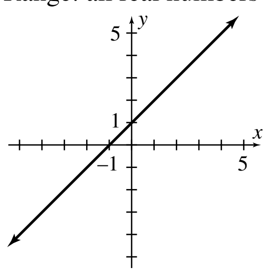
c. Range: all nonnegative real numbers

d. $f(x)=0$ for $x=0$. So a real zero is 0.

6. $y = 2x$
If $y = 0$, then $x = 0$. If $x = 0$, then $y = 0$.
Intercept: (0, 0)
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



7. $y = x + 1$
If $y = 0$, then $x = -1$.
If $x = 0$, then $y = 1$.
Intercepts: (–1, 0), (0, 1)
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

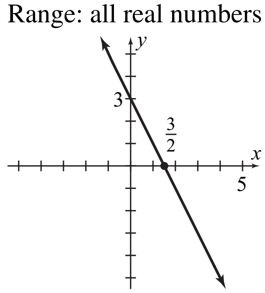


8. $y = 3 - 2x$

If $y = 0$, then $0 = 3 - 2x$, $x = \frac{3}{2}$.

If $x = 0$, then, $y = 3$. Intercepts: $\left(\frac{3}{2}, 0\right)$, (0, 3)

y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

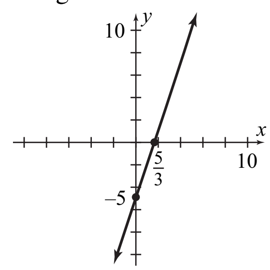


9. $y = 3x - 5$

If $y = 0$, then $0 = 3x - 5$, $x = \frac{5}{3}$.

If $x = 0$, then $y = -5$. Intercepts: $\left(\frac{5}{3}, 0\right)$, (0, –5)

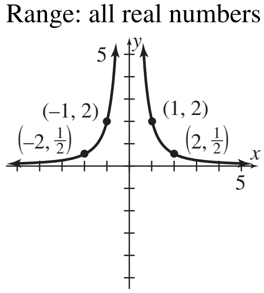
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



10. $y = \frac{2}{x^2}$

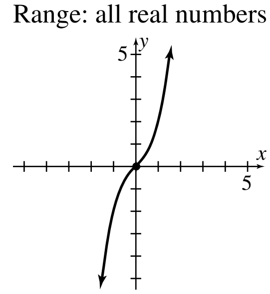
If $y = 0$, then $0 = \frac{2}{x^2}$, which has no solution.

Thus there is no x-intercept. Because $x \neq 0$,
Not one-to-one.
Domain: all real numbers except 0
Range: all real numbers > 0



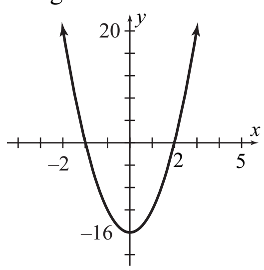
11. $y = x^3 + x$

If $y = 0$, then $0 = x^3 + x = x(x^2 + 1)$, $x = 0$. If
$x = 0$, then $y = 0$.
Intercept: (0, 0)
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

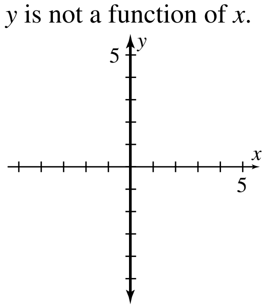


12. $y = 4x^2 - 16$

If $y = 0$, then $0 = 4x^2 - 16 = 4(x^2 - 4)$,
$0 = 4(x+2)(x-2)$, $x = \pm 2$.
If $x = 0$, then $y = -16$.
Intercepts: $(\pm 2, 0)$, $(0, -16)$
y is a function of x. Not one-to-one.
Domain: all real numbers
Range: all real numbers ≥ -16

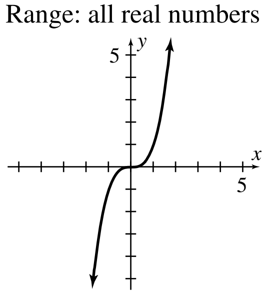


13. $x = 0$
If $y = 0$, then $x = 0$. If $x = 0$, then y can be any real number. Intercepts: every point on y-axis
y is not a function of x.

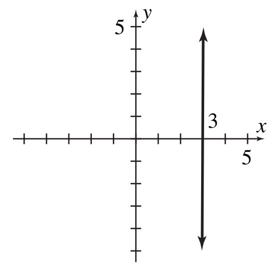


14. $y = x^3$

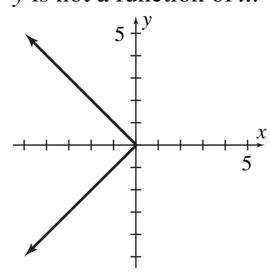
If $y = 0$, then $0 = x^3$, $x = 0$. If $x = 0$, then $y = 0$.
Intercept: (0, 0). y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



15. $x = 3$ is a vertical line.
Intercept: (3, 0)
y is not a function of x.

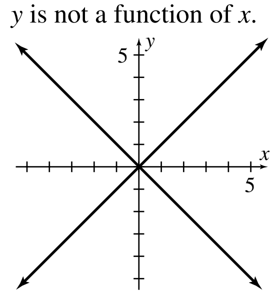


16. $x = -|y|$
If $y = 0$, then $x = 0$. If $x = 0$, then $0 = -|y|$, $y = 0$.
Intercept: (0, 0)
y is not a function of x.

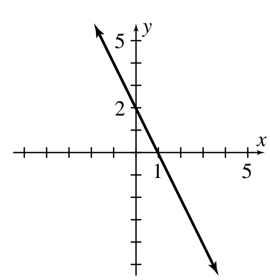


17. $x^2 = y^2$

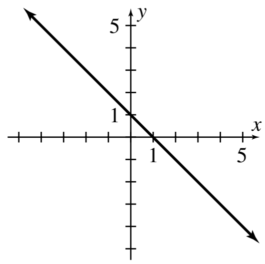
If $y = 0$, then $x^2 = 0, x = 0$. If $x = 0$, then
$0 = y^2$, $y = 0$. Intercept: (0, 0)
y is not a function of x.



18. $2x + y - 2 = 0$
If $y = 0$, then $2x - 2 = 0$, $x = 1$. If $x = 0$, then $y - 2 = 0$, $y = 2$. Intercepts: (1, 0), (0, 2)
Note that $y = 2 - 2x$. y is a function of x.
One-to-one.
Domain: all real numbers
Range: all real numbers

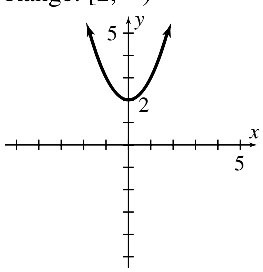


19. $x + y = 1$
If $y = 0$, then $x = 1$. If $x = 0$, then $y = 1$.
Intercepts: (1, 0), (0, 1)
Note that $y = 1 - x$.
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



20. $u = f(v) = 2 + v^2$

If $u = 0$, then $0 = 2 + v^2$. No intercept.
If $v = 0$, then $u = 2$.
Intercept: (0, 2)
Domain: all real numbers
Range: $[2, \infty)$



21. $f(x) = 5 - 2x^2$. If $f(x) = 0$, then $0 = 5 - 2x^2$

$$2x^2 = 5$$

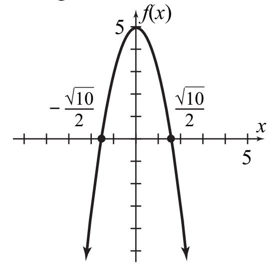
$$x^2 = \frac{5}{2}$$

$$x = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{10}}{2}.$$

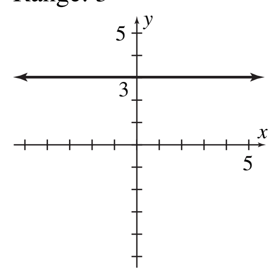
If $x = 0$, then $f(x) = 5$.

Intercepts: $\left(\pm\frac{\sqrt{10}}{2}, 0\right), (0, 5)$

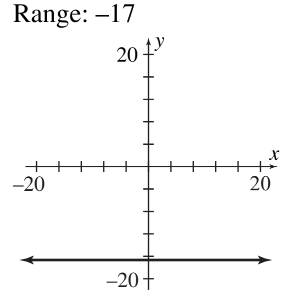
Domain: all real numbers
Range: all real numbers ≤ 5



22. $y = h(x) = 3$
Because y cannot be 0, there is no x-intercept. If $x = 0$, then $y = 3$. Intercept: (0, 3)
Domain: all real numbers
Range: 3

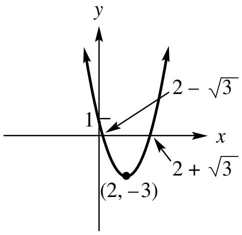


23. $g(s) = -17$
Because $g(s)$ cannot be 0, there is no s-intercept.
If $s = 0$, then $g(s) = -17$.
Intercept: (0, –17)
Domain: all real numbers
Range: –17



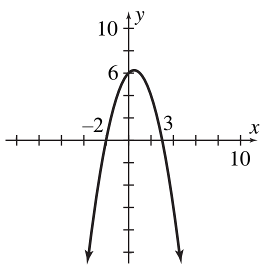
24. $y = h(x) = x^2 - 4x + 1$

If $y = 0$, then $0 = x^2 - 4x + 1$, and by the quadratic formula, $x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$. If $x = 0$, then $y = 1$. Intercepts: $(2 \pm \sqrt{3}, 0), (0, 1)$
Domain: all real numbers
Range: all real numbers ≥ -3



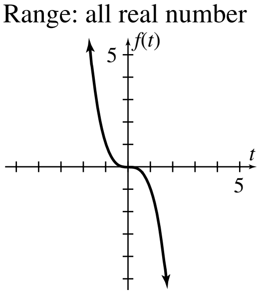
25. $y = f(x) = -x^2 + x + 6$

If $y = 0$, then $0 = -x^2 + x + 6 = (x-3)(x+2)$, so $x = 3, -2$. If $x = 0$, $y = 6$.
Intercepts: (3, 0), (–2, 0), (0, 6)
Domain: all real numbers
Range: $\left(-\infty, \frac{25}{4}\right)$



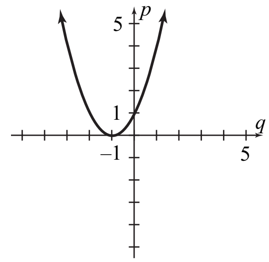
26. $f(t) = -t^3$

If $f(t) = 0$, then $0 = -t^3$, $t = 0$.
If $t = 0$, then $f(t) = 0$. Intercept: (0, 0)
Domain: all real numbers
Range: all real number



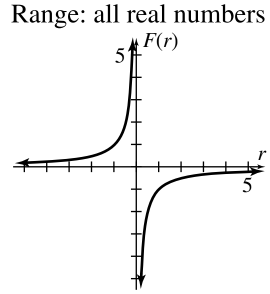
27. $p = h(q) = 1 + 2q + q^2$

If $p = 0$, then $1 + 2q + q^2 = 0$, $(1+q)^2 = 0$, so $q = -1$. If $q = 0$ then $p = 1$.
Intercepts: (–1, 0), (0, 1)
Domain: all real numbers
Range: all real numbers ≥ 0



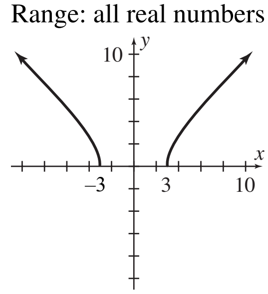
28. $F(r) = -\frac{1}{r}$

If $F(r) = 0$, then $0 = -\frac{1}{r}$, which has no solution.
Because $r \neq 0$, there is no vertical-axis intercept.
Intercept: none.
Domain: all real numbers ≠ 0
Range: all real numbers ≠ 0



29. $s = f(t) = \sqrt{t^2 - 9}$

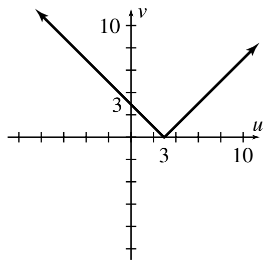
Note that for $\sqrt{t^2-9}$ to be a real number, $t^2 - 9 \geq 0$, so $t^2 \geq 9$, and $|t| \geq 3$. If $s = 0$, then $0 = \sqrt{t^2-9}$, $0 = t^2 - 9$, or $t = \pm 3$. Because $|t| \geq 3$, we know $t \neq 0$, so no s-intercept exists.
Intercepts: (–3, 0), (3, 0)
Domain: all real numbers $t \leq -3$ and ≥ 3
Range: all real numbers ≥ 0



30. $v = H(u) = |u - 3|$

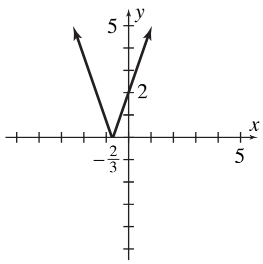
If $v = 0$, then $0 = |u-3|$, $u - 3 = 0$, so $u = 3$.

If $u = 0$, then $v = |-3| = 3$.
Intercepts: (3, 0), (0, 3).
Domain: all real numbers
Range: all real numbers ≥ 0



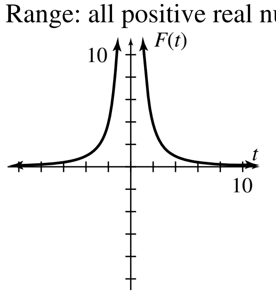
31. $f(x) = |3x + 2|$

If $f(x) = 0$, then $0 = |3x + 2|$ and $x = -\frac{2}{3}$.
If $x = 0$, $f(x) = |2| = 2$.
Intercepts: $\left(-\frac{2}{3}, 0\right)$, (0, 2)
Domain: all real numbers
Range: all real numbers ≥ 0



32. $F(t) = \frac{16}{t^2}$

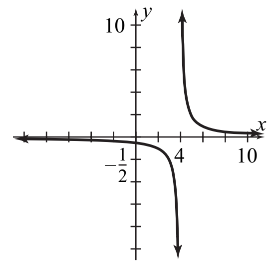
If $F(t) = 0$, then $0 = \frac{16}{t^2}$, which has no solution.
Because $t \neq 0$, there is no vertical-axis intercept.
No intercepts
Domain: all nonzero real numbers
Range: all positive real numbers



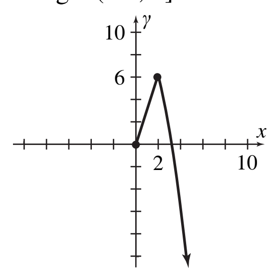
33. $y = f(x) = \frac{2}{x-4}$
Note that the denominator is 0 when $x = 4$. Thus $x \neq 4$. If $y = 0$, then $0 = \frac{2}{x-4}$, which has no solution. If $x = 0$, then $y = -\frac{1}{2}$.

Intercept: $\left(0, -\frac{1}{2}\right)$

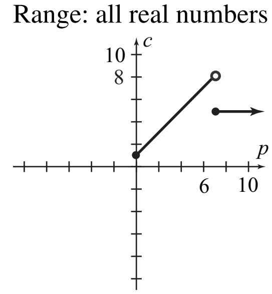
Domain: all real numbers except 4
Range: all real numbers except 0



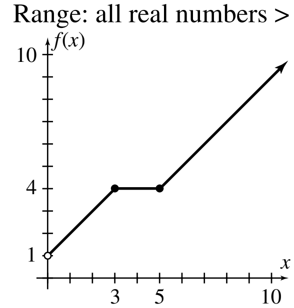
34. Domain: $[0, \infty)$
Range: $(-\infty, 6]$



35. Domain: all real numbers ≥ 0
Range: all real numbers $1 \leq c < 8$



36. Domain: all positive real numbers
Range: all real numbers > 1



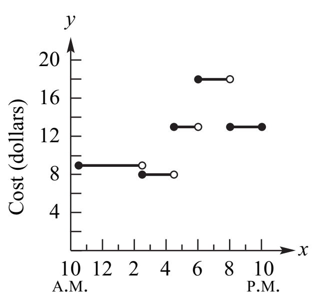
37. From the vertical-line test, the graphs that represent functions of x are (a), (b), and (d).

38. From the horizontal line test, the graphs which represent one-to-one functions of x are (c) and (d).

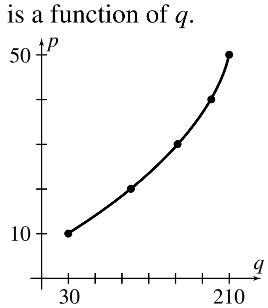
39. Let y = credit card balance.
Then $y = 9200 - 325x$.
If $y = 0$, then $x = \frac{9200}{325} \approx 28.31$—the time to pay off the debt (29 months).
If $x = 0$, then $y = 9200$—the amount originally owed.

40. The cost of an item as a function of the time of day, x is

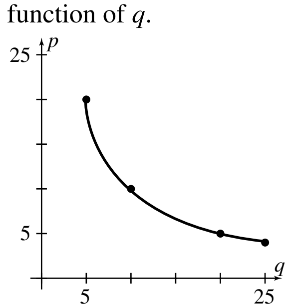
$$y = \begin{cases} 9, \text{ if } 10{:}30 \text{ A.M.} \le x < 2{:}30 \text{ P.M.} \\ 8, \text{ if } 2{:}30 \text{ P.M.} \le x < 4{:}30 \text{ P.M.} \\ 13, \text{ if } 4{:}30 \text{ P.M.} \le x < 6{:}00 \text{ P.M.} \\ 18, \text{ if } 6{:}00 \text{ P.M.} \le x < 8{:}00 \text{ P.M.} \\ 13, \text{ if } 8{:}00 \text{ P.M.} \le x \le 10{:}00 \text{ P.M.} \end{cases}$$



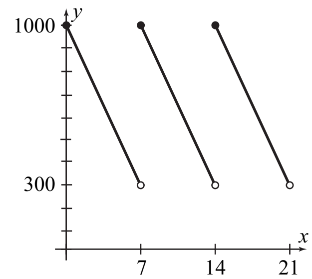
41. As price increases, quantity supplied increases; p is a function of q.



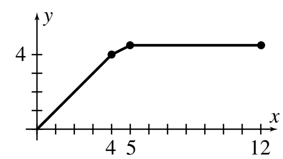
42. As price decreases, quantity increases; p is a function of q.



43.



44.



45. 0.39

46. –0.50, 0.57

47. –0.61, –0.04

48. 0.62, 1.73, 4.65

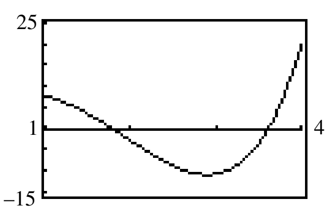
49. –1.12

50. No real zeros

51. –1.70, 0

52. –0.49, 0.52, 1.25

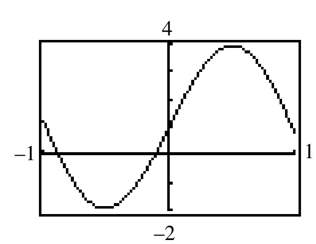
53.



a. maximum value of $f(x)$: 19.60

b. minimum value of $f(x)$: –10.86

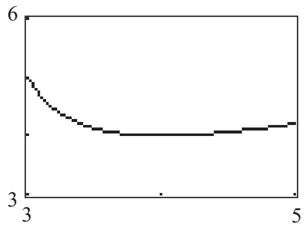
54.



a. maximum value of $f(x)$: 3.94

b. minimum value of $f(x)$: –1.94

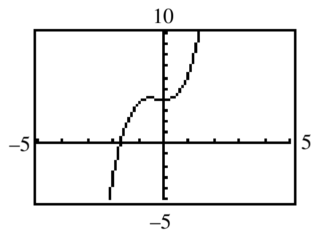
55.



a. maximum value of $f(x)$: 5

b. minimum value of $f(x)$: 4

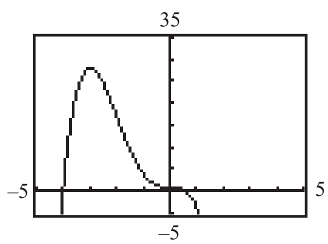
56.



a. range: $(-\infty, \infty)$

b. intercepts: (–1.73, 0), (0, 4)

57.

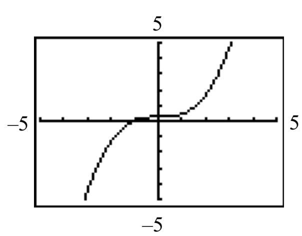


a. maximum value of $f(x)$: 28

b. range: $(-\infty, 28]$

c. real zeros: –4.02, 0.60

58.

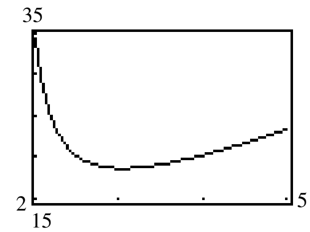


a. range: $(-\infty, \infty)$

b. intercepts: (0, 0.29), (–1.03, 0)

c. real zero: −1.03

59.



a. maximum value of $f(x)$: 34.21

b. minimum value of $f(x)$: 18.68

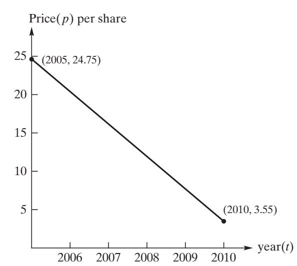
c. range: [18.68, 34.21]

d. no intercept

60. The points of intersection are where the two graphs cross each other. From the diagram, the coordinates of these two points are estimated to be (1989.5, 10.1) and (1992.9, 10.6). This means that around the middle of year 1989, imports to and exports from Kuwait were the same amount, approximately \$10.1 billion. Similarly, near the end of 1992, imports and exports were both about \$10.6 billion.

Apply It 2.6

15. The line describing the relationship between the price per share and the year passes through the points (2005, 24.75) and (2010, 3.55).



The slope of the line is

$$m = \frac{3.55 - 24.75}{2010 - 2005} = -4.24$$

This is a negative number, which means that the share price was decreasing at a rate of \$4.24 per year.

16. If enrollment is growing by 14 students per year, the number S of students in the program can be described by the equation of a line whose slope is 14. Since the program had 50 students when $t = 3$ (in the third year), the line passes through the point (3, 50). Using the point-slope form of the equation of a line with $m = 14$ and $(t_1, S_1) = (3, 50)$, we get

$$S - S_1 = m(t - t_1)$$
$$S - 50 = 14(t - 3)$$
$$S - 50 = 14t - 42$$
$$S = 14t + 8$$

17. Let F stand for the temperature in degrees Fahrenheit and C for the temperature in degrees Celsius. The line passing through the points (25, 77) and (17, 62.6) has slope

$$m = \frac{62.6 - 77}{17 - 25} = 1.8$$

Using the point-slope form of the equation of a line with $m = 1.8$ and $(C_1, F_1) = (25, 77)$, we get

$$F - F_1 = m(C - C_1)$$
$$F - 77 = 1.8(C - 25)$$
$$F - 77 = 1.8C - 45$$
$$F = 1.8C + 32$$

18. To find the slope and y-intercept, let $a = 1000$, then write the equation in slope-intercept form.

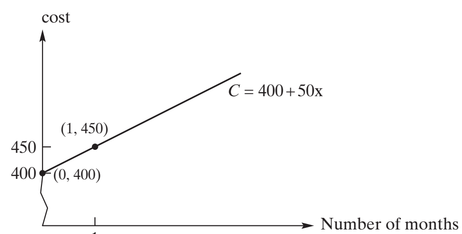
$$y = \frac{1}{24}(t + 1)a$$
$$y = \frac{1}{24}(t + 1)1000$$
$$y = \frac{1000}{24}t + \frac{1000}{24}$$
$$y = \frac{125}{3}t + \frac{125}{3}$$

Thus the slope, m, is $\frac{125}{3}$ and the y-intercept, b, is $\frac{125}{3}$.

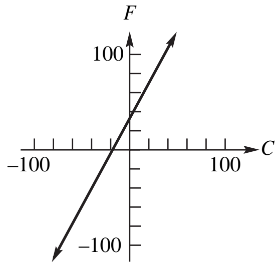
19. The total cost of club membership is given by the fixed one-time fee of \$400 plus the monthly fees of \$50. For x months, the total comes to

$$C = 400 + 50x$$

The graph is a line that passes through (0, 400) and (1, 450).



20.



To convert Celsius to Fahrenheit, locate the Celsius temperature on the horizontal axis, move vertically to the line, then move horizontally to read the Fahrenheit temperature of the vertical axis.

21. Right angles are formed by perpendicular lines. The slopes of the sides of the triangle are:

$\overline{AB}\left\{ m = \frac{0-0}{6-0} = \frac{0}{6} = 0 \right.$

$\overline{BC}\left\{ m = \frac{7-0}{7-6} = \frac{7}{1} = 7 \right.$

$\overline{AC}\left\{ m = \frac{7-0}{7-0} = \frac{7}{7} = 1 \right.$

Since none of the slopes are negative reciprocals of each other, there are no perpendicular lines. Therefore, the points do not define a right triangle.

Problems 2.6

1. $m = \frac{10-2}{7-3} = \frac{8}{4} = 2$

2. $m = \frac{10-3}{-2-5} = \frac{7}{-7} = -1$

3. $m = \frac{-3-(-2)}{8-6} = \frac{-1}{2} = -\frac{1}{2}$

4. $m = \frac{-4-(-4)}{3-2} = \frac{0}{1} = 0$

5. The difference in the *x*-coordinates is 5 – 5 = 0, so the slope is undefined.

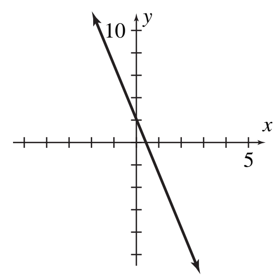
6. $m = \frac{6-(-4)}{3-0} = \frac{10}{3}$

7. $m = \frac{-2-(-2)}{4-5} = \frac{0}{-1} = 0$

8. $m = \frac{0-(-7)}{9-1} = \frac{7}{8}$

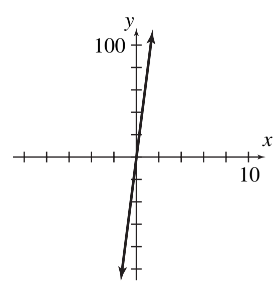
9.

$$\begin{aligned} y-7 &= -5[x-(-1)] \\ y-7 &= -5(x+1) \\ y-7 &= -5x-5 \\ 5x+y-2 &= 0 \end{aligned}$$



10.

$$\begin{aligned} y-0 &= 75(x-0) \\ y &= 75x \\ 75x-y &= 0 \end{aligned}$$

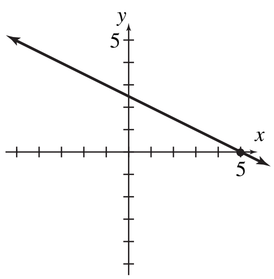


11. $y-5=-\frac{1}{2}[x-(-5)]$

$2(y-5)=-(x+5)$

$2y-10=-x-5$

$x+2y-5=0$

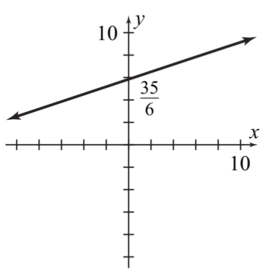


12. $y-5=\frac{1}{3}\left[x-\left(-\frac{5}{2}\right)\right]$

$6(y-5)=2\left[x+\frac{5}{2}\right]$

$6y-30=2x+5$

$2x-6y+35=0$



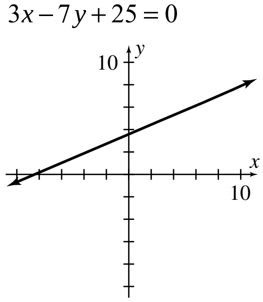
13. $m=\frac{4-1}{1-(-6)}=\frac{3}{7}$

$y-4=\frac{3}{7}(x-1)$

$7(y-4)=3(x-1)$

$7y-28=3x-3$

$3x-7y+25=0$

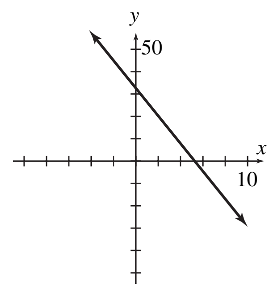


14. $m=\frac{2-(-4)}{5-6}=\frac{6}{-1}=-6$

$y-2=-6(x-5)$

$y-2=-6x+30$

$6x+y-32=0$

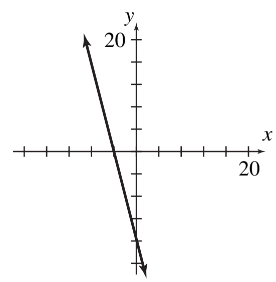


15. $m=\frac{-8-(-4)}{-2-(-3)}=\frac{-4}{1}=-4$

$y-(-4)=-4[x-(-3)]$

$y+4=-4x-12$

$4x+y+16=0$



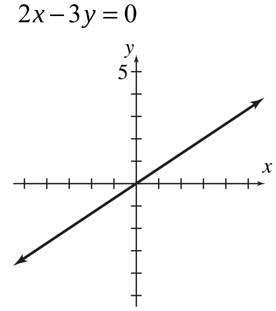
16. $m=\frac{-2-0}{-3-0}=\frac{2}{3}$

$y-0=\frac{2}{3}(x-0)$

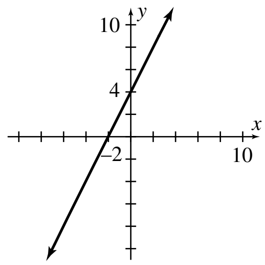
$y=\frac{2}{3}x$

$3y=2x$

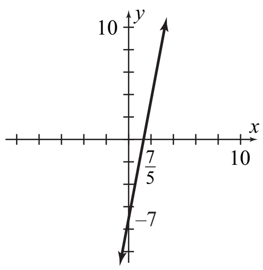
$2x-3y=0$



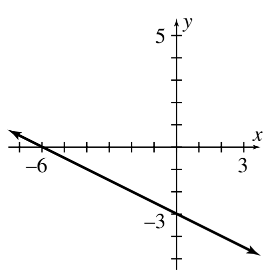
17. $y = 2x + 4$
$2x - y + 4 = 0$



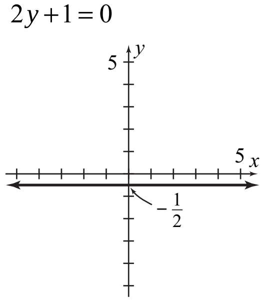
18. $y = 5x - 7$
$5x - y - 7 = 0$



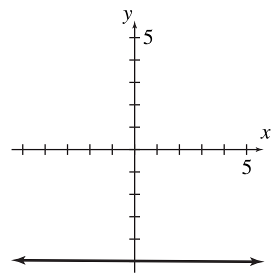
19. $y = -\frac{1}{2}x - 3$
$2y = 2\left(-\frac{1}{2}x - 3\right)$
$2y = -x - 6$
$x + 2y + 6 = 0$



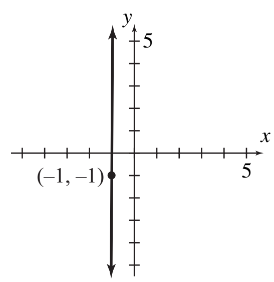
20. $y = 0x - \frac{1}{2}$
$y = -\frac{1}{2}$
$2y = -1$
$2y + 1 = 0$



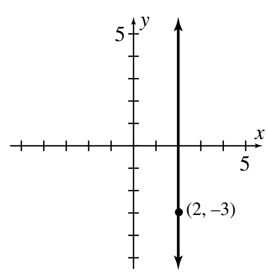
21. A horizontal line has the form $y = b$. Thus $y = -5$, or $y + 5 = 0$.



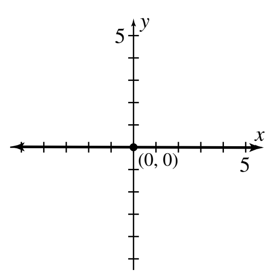
22. A vertical line has the form $x = a$. Thus $x = -1$, or $x + 1 = 0$.



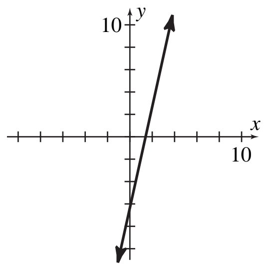
23. A vertical line has the form $x = a$. Thus $x = 2$, or $x - 2 = 0$.



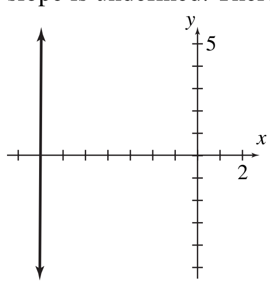
24. A horizontal line has the form $y = b$. Thus $y = 0$.



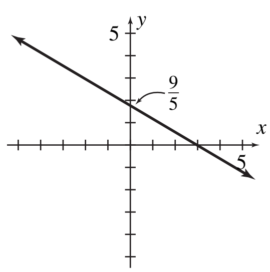
25. $y = 4x - 6$ has the form $y = mx + b$, where $m = 4$ and $b = -6$.



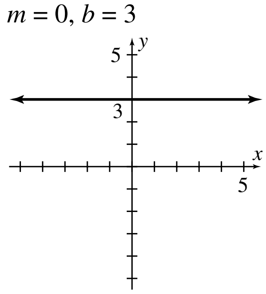
26. $x + 9 = 2$ or $x = -7$, is a vertical line. Thus the slope is undefined. There is no y-intercept.



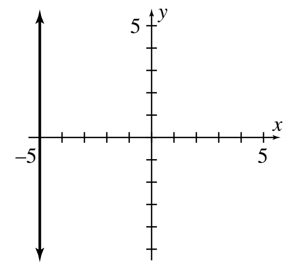
27.
$$3x + 5y - 9 = 0$$
$$5y = -3x + 9$$
$$y = -\frac{3}{5}x + \frac{9}{5}$$
$$m = -\frac{3}{5},\ b = \frac{9}{5}$$



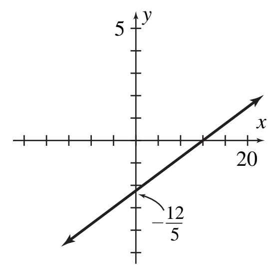
28.
$$y + 4 = 7$$
$$y = 3$$
$$y = 0x + 3$$
$$m = 0,\ b = 3$$



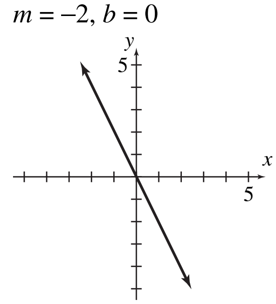
29. $x = -5$ is a vertical line. Thus the slope is undefined. There is no y-intercept.



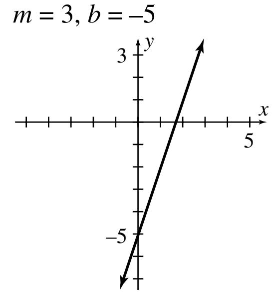
30.
$$x - 9 = 5y + 3$$
$$5y = x - 12$$
$$y = \frac{1}{5}x - \frac{12}{5}$$
$$m = \frac{1}{5},\ b = -\frac{12}{5}$$



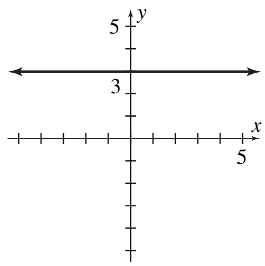
31.
$$y = -2x$$
$$y = -2x + 0$$
$$m = -2,\ b = 0$$



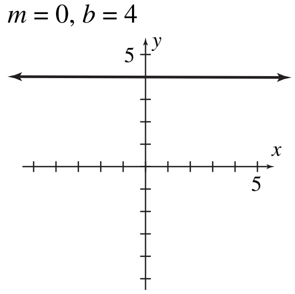
32.
$$y - 7 = 3(x - 4)$$
$$y - 7 = 3x - 12$$
$$y = 3x - 5$$
$$m = 3,\ b = -5$$



33. $y = 3$
$y = 0x + 3$
$m = 0,\ b = 3$



34. $6y - 24 = 0$
$y = 4$
$y = 0x + 4$
$m = 0,\ b = 4$



35. $2x = 5 - 3y$, or $2x + 3y - 5 = 0$ (general form)
$3y = -2x + 5$, or $y = -\frac{2}{3}x + \frac{5}{3}$ (slope-intercept form)

36. $5x - 2y = 10$, or $5x - 2y - 10 = 0$ (general form)
$-2y = -5x + 10$, or $y = \frac{5}{2}x - 5$ (slope-intercept form)

37. $4x + 9y - 5 = 0$ is a general form.
$9y = -4x + 5$, or $y = -\frac{4}{9}x + \frac{5}{9}$ (slope-intercept form)

38. $3(x-4) - 7(y+1) = 2$
$3x - 12 - 7y - 7 = 2$
$3x - 7y - 21 = 0$ (general form)
$-7y = -3x + 21$, or $y = \frac{3}{7}x - 3$ (slope-intercept form)

39.
$$-\frac{x}{2} + \frac{2y}{3} = -4\frac{3}{4}$$
$$12\left(-\frac{x}{2} + \frac{2y}{3}\right) = 12\left(-\frac{19}{4}\right)$$
$$-6x + 8y = -57$$
$$6x - 8y - 57 = 0 \text{ (general form)}$$
$$-8y = -6x + 57$$
$$y = \frac{3}{4}x - \frac{57}{8} \text{ (slope-intercept form)}$$

40. $y = \frac{1}{300}x + 8$ is in slope-intercept form.
$$300y = 300\left(\frac{1}{300}x + 8\right)$$
$$300y = x + 2400$$
$$x - 300y + 2400 = 0 \text{ (general form)}$$

41. The lines $y = -5x + 7$ and $y = -5x - 3$ have the same slope, –5. Thus they are parallel.

42. The lines $y = 4x + 3$ and $y = 5 + 4x$ (or $y = 4x + 5$) have the same slope, 4. Thus they are parallel.

43. The lines $y = 5x + 2$ and $-5x + y - 3 = 0$ (or $y = 5x + 3$) have the same slope, 5. Thus they are parallel.

44. The line $y = x$ has slope $m_1 = 1$ and the line $y = -x$ has slope $m_2 = -1$. $m_1 = -\frac{1}{m_2}$ so the lines are perpendicular.

45. The line $x + 3y + 5 = 0$ $\left(\text{or } y = -\frac{1}{3}x - \frac{5}{3}\right)$ has slope $m_1 = -\frac{1}{3}$ and the line $y = -3x$ has slope $m_2 = -3$. Since $m_1 \neq m_2$ and $m_1 \neq -\frac{1}{m_2}$, the lines are neither parallel nor perpendicular.

46. The line $x + 2y = 0$ $\left(\text{or } y = -\frac{1}{2}x\right)$ has slope $m_1 = -\frac{1}{2}$ and the line $x + 4y - 4 = 0$ (or $\left.y = -\frac{1}{4}x + 1\right)$ has slope $m_2 = -\frac{1}{4}$. Since

$m_1 \neq m_2$ and $m_1 \neq -\frac{1}{m_2}$, the lines are neither parallel nor perpendicular.

47. The line $y = 3$ is horizontal and the line $x = -\frac{1}{3}$ is vertical, so the lines are perpendicular.

48. Both lines are vertical and thus parallel.

49. The line $3x + y = 4$ (or $y = -3x + 4$) has slope $m_1 = -3$, and the line $x - 3y + 1 = 0$ $\left(\text{or } y = \frac{1}{3}x + \frac{1}{3}\right)$ has slope $m_2 = \frac{1}{3}$. Since $m_2 = -\frac{1}{m_1}$, the lines are perpendicular.

50. The line $x - 2 = 3$ (or $x = 5$) is vertical and the line $y = 2$ is horizontal, so the lines are perpendicular.

51. The slope of $y = 4x + 3$ is 4, so the slope of a line parallel to it must also be 4. An equation of the desired line is $y - 3 = 4(x - 2)$ or $y = 4x - 5$.

52. $x = -4$ is a vertical line. A line parallel to $x = -4$ has the form $x = a$. Since the line must pass through $(2, -8)$, its equation is $x = 2$.

53. $y = 2$ is a horizontal line. A line parallel to it has the form $y = b$. Since the line must pass through (2, 1) its equation is $y = 1$.

54. The slope of $y = 3 + 2x$ is 2, so the slope of a line parallel to it must also be 2. An equation of the desired line is $y - (-4) = 2(x - 3)$, or $y = 2x - 10$.

55. The slope of $y = 3x - 5$ is 3, so the slope of a line perpendicular to it must have slope $-\frac{1}{3}$. An equation of the desired line is $y - 4 = -\frac{1}{3}(x - 3)$, or $y = -\frac{1}{3}x + 5$.

56. The line $3x + 2y - 4 = 0$, or $y = -\frac{3}{2}x + 2$, has slope $-\frac{3}{2}$. A line perpendicular to it must have slope $\frac{2}{3}$. An equation of the desired line is $y - 1 = \frac{2}{3}(x - 3)$, or $y = \frac{2}{3}x - 1$.

57. $y = -3$ is a horizontal line, so the perpendicular line must be vertical with equation of the form $x = a$. Since that line passes through (5, 2), its equation is $x = 5$.

58. The line $3y = -\frac{2x}{5} + 3$ $\left(\text{or } y = -\frac{2x}{15} + 1\right)$ has slope $-\frac{2}{15}$, so the slope of a line perpendicular to it must have slope $\frac{15}{2}$. An equation of the desired line is $y - (-5) = \frac{15}{2}(x - 4)$ or $y = \frac{15}{2}x - 35$.

59. The line $2x + 3y + 6 = 0$ has slope $-\frac{2}{3}$, so the slope of a line parallel to it must also be $-\frac{2}{3}$. An equation of the desired line is $y - (-5) = -\frac{2}{3}[x - (-7)]$, or $y = -\frac{2}{3}x - \frac{29}{3}$.

60. The y-axis is vertical. A parallel line is also vertical and has an equation of the form $x = a$. Since it passes through $(-4, 10)$, its equation is $x = -4$.

61. $(-1, -2)$, $(4, 1)$

$$m = \frac{1 - (-2)}{4 - (-1)} = \frac{3}{5}$$

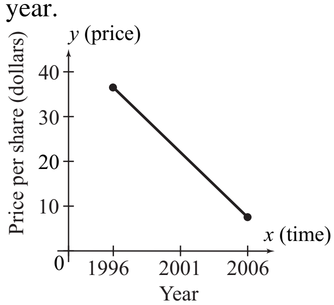
Point-slope form: $y - 1 = \frac{3}{5}(x - 4)$. When the x-coordinate is 3,

$$y - 1 = \frac{3}{5}(3 - 4)$$
$$y - 1 = \frac{3}{5}(-1)$$
$$y - 1 = -\frac{3}{5}$$
$$y = \frac{2}{5}$$

Thus the point is $\left(3, \frac{2}{5}\right)$.

62. $m = 3$, $b = 1$
Slope-intercept form: $y = 3x + 1$. The point $(-1, -2)$ lies on the line if its coordinates satisfy the equation. If $x = -1$ and $y = -2$, then $-2 = 3(-1) + 1$ or $-2 = -2$, which is true. Thus $(-1, -2)$ lies on the line.

63. Let x = the time (in years) and y = the price per share. Then,
In 1996: $x_1 = 1996$ and $y_1 = 37$
In 2006: $x_2 = 2006$ and $y_2 = 8$
The slope is
$$m = \frac{8-37}{2006-1996} = \frac{-29}{10} = -2.9$$
The stock price dropped an average of \$2.90 per year.



64. The number of goals scored increased as a function of time (in months). The given points are $(x_1, y_1) = (3, 14)$ and $(x_2, y_2) = (5, 20)$.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20-14}{5-3} = \frac{6}{2} = 3$$
Using the point-slope form with $m = 3$ and $(x_1, y_1) = (3, 14)$ gives
$$y - y_1 = m(x - x_1)$$
$$y - 14 = 3(x - 3)$$
$$y - 14 = 3x - 9$$
$$y = 3x + 5$$

65. The owner's profits increased as a function of time. Let x = the time (in years) and let y = the profit (in dollars). The given points are $(x_1, y_1) = (0, -100,000)$ and $(x_2, y_2) = (5, 40,000)$.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40,000-(-100,000)}{5-0} = \frac{140,000}{5}$$
$= 28,000$
Using the point-slope form with $m = 28,000$ and $(x_1, y_1) = (0, -100,000)$ gives
$$y - y_1 = m(x - x_1)$$
$$y - (-100,000) = 28,000(x - 0)$$
$$y + 100,000 = 28,000x$$
$$y = 28,000x - 100,000$$

66. Solve the equation for t.
$L = 1.53t - 6.7$
$L + 6.7 = 1.53t$
$$\frac{(L+6.7)}{1.53} = t$$
$0.65L + 4.38 = t$
The slope is approximately 0.65 and the y-intercept is approximately 4.38.

67. A general linear form of $d = 184 + t$ is $-t + d - 184 = 0$.

68. **a.** Using the points $(3.5, -1.5)$ and $(0.5, 0.5)$ gives a slope of $m = \frac{-1.5-0.5}{3.5-0.5} = -\frac{2}{3}$.
An equation is $y - 0.5 = -\frac{2}{3}(x - 0.5)$ or
$y = -\frac{2}{3}x + \frac{5}{6}$.

b. Using the points $(0.5, 0.5)$ and $(-1, -2.5)$ gives a slope of $m = \frac{-2.5-0.5}{-1-0.5} = \frac{-3}{-1.5} = 2$.
An equation is $y - 0.5 = 2(x - 0.5)$ or
$y = 2x - \frac{1}{2}$.

These two paths are not perpendicular to each other because the slopes are not negative reciprocals of each other.

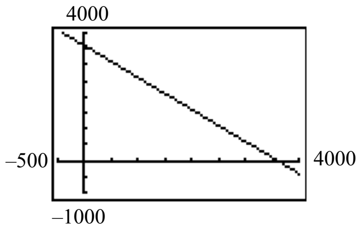
69. The slopes of the sides of the figure are:
$\overline{AB}\left\{ m = \frac{4-0}{0-0} = \frac{4}{0} = \text{undefined (vertical)} \right.$
$\overline{CD}\left\{ m = \frac{7-3}{2-2} = \frac{4}{0} = \text{undefined (vertical)} \right.$
$\overline{AC}\left\{ m = \frac{3-0}{2-0} = \frac{3}{2} \right.$
$\overline{BD}\left\{ m = \frac{7-4}{2-0} = \frac{3}{2} \right.$
Since $\overline{AB}$ is parallel to $\overline{CD}$ and $\overline{AC}$ is parallel to $\overline{BC}$, $ABCD$ is a parallelogram.

70. Let x = the distance traveled and let y = the altitude. The path of descent is a straight line with a slope of –1 and y-intercept of 3600. Therefore, using the slope-intercept form with $m = -1$ and $b = 3600$ gives

$y = mx + b$

$y = (-1)x + 3600$

$y = -x + 3600$

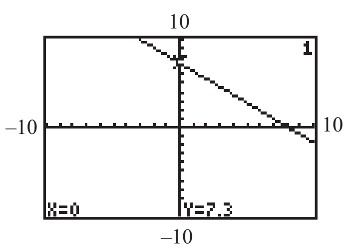


If the airport is located 3800 meters from where the plane begins its landing approach, the plane will crash 200 meters short of the airport.

71. The line has slope 59.82 and passes through (6, 1128.50). Thus $C - 1128.50 = 59.82(T - 6)$ or
$C = 59.82T + 769.58$.

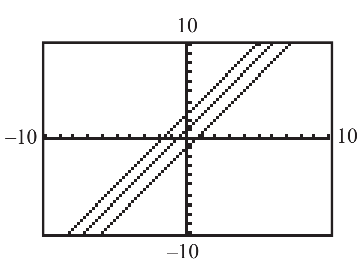
72. The line has slope 50,000 and passes through (5, 330,000). Thus $R - 330{,}000 = 50{,}000(T - 5)$ or $R = 50{,}000T + 80{,}000$.

73.



The graph of the equation $y = -0.9x - 7.3$ shows that when $x = 0$, $y = 7.3$. Thus, the y-intercept is 7.3.

74.



The lines are parallel, which is expected because they have the same slope, 1.5.

75. The slope is 7.1.

76. The line passes through $(a, 0)$ and $(0, b)$, so

$m = \frac{b-0}{0-a} = -\frac{b}{a}$. Thus

$$y - b = -\frac{b}{a}(x - 0)$$

$$y - b = -\frac{b}{a}x$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Apply It 2.7

22. Let x = the number of bottles of Strawberry Milk that are produced and let y = the number of pots of Fruit Yogurt that are produced. Then, the equation $8x + 14y = 1000$ describes all possible production levels of the two products.

23. If the price p and quantity q are linearly related, then the points (2, 23) and (1.5, 18.75) must lie on the line. The slope of the line is

$$m = \frac{18.75 - 23}{1.5 - 2} = 8.5$$

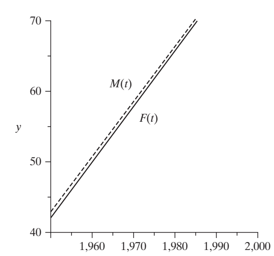
Hence the equation of the line (using the point-slope form) is

$$p - 23 = 8.5(q - 2)$$

$$p = 8.5q + 6$$

This is the supply equation.

24.



The life expectancy of a man born in 1966 is

$$M(1966) = 0.779 \times .1966 - 1473.9272 = 55.42$$

Solving $F(t) = 0.793t - 1505.1991 = 55.42$ gives

$$0.7934t = 55.42 + 1505.1991$$
$$t = \frac{1560.6191}{0.7934} \approx 1967$$

So 1967 is the birth year of a woman who would be expected to die at the same age as a man born in 1966.

25. If $t = 0$ corresponds to 2005, then 2007 is $t = 2$.

The line describing the amount of crude oil production passes through the points (0, 83.1) and (2, 86.21). Its slope is $m = \frac{86.21-83.1}{2-0} = 1.555$. Using the point-slope form, we get the equation

$$y - 83.1 = 1.555(x - 0)$$
$$y = 1.555x + 83.1$$

Similarly, the line describing the amount of imported crude oil passes through (0, 89.13) and (2, 87.3). Its slope is $m = \frac{87.3-89.13}{2-0} = -0.915$. Therefore we have the equation

$$y - 89.13 = -0.915(x - 0)$$
$$y = -0.915x + 89.13$$

Thus, the amounts of oil produced and imported are given, respectively, by

$$y = 1.555x + 83.1 \quad \text{and} \quad y = -0.915x + 89.13$$

26. a. Let x be the number of years after 1995. Then the line describing the number of shops in terms of x passes through the points (0, 1500) and (9, 5000). Its slope is $m = \frac{5000-1500}{9} \approx 389$. Using the point-slope form, we get the equation

$$y - 1500 = 389(x - 0)$$
$$y = 389x + 1500$$

b. The year 2018 corresponds to $x = 23$, so the number of shops in this year is predicted to be

$$y = 389 \times 23 + 1500 = 10,447$$

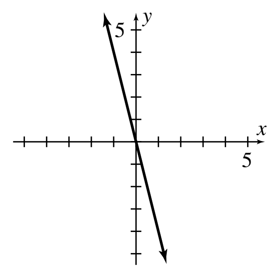
c. To find the year in which the number of shops in Dubai will be 20,000, replace y by 20000 in the equation $y = 389x + 1500$ and solve for x:

$$20000 = 389x + 1500$$
$$389x = 18500$$
$$x = 47.55$$

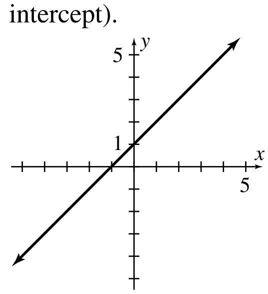
Hence, there are expected to be 20,000 shops in Dubai around the middle of the year 2042.

Problems 2.7

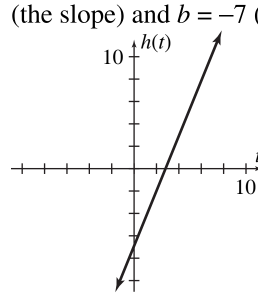
1. $y = f(x) = -4x = -4x + 0$ has the form $f(x) = ax + b$ where $a = -4$ (the slope) and $b = 0$ (the vertical-axis intercept).



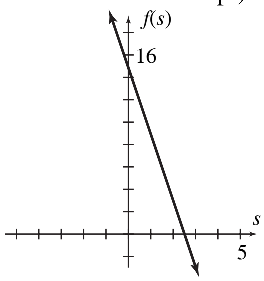
2. $y = f(x) = x + 1$ has the form $f(x) = ax + b$ where $a = 1$ (the slope) and $b = 1$ (the vertical-axis intercept).



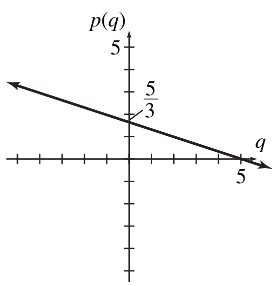
3. $h(t) = 5t - 7$ has the form $h(t) = at + b$ with $a = 5$ (the slope) and $b = -7$ (the vertical-axis intercept).



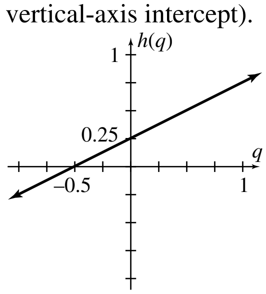
4. $f(s) = 3(5 - 2s) = 15 - 6s$ has the form $f(s) = as + b$ where $a = -6$ (slope) and $b = 15$ (the vertical-axis intercept).



5. $p(q) = \frac{5-q}{3} = \frac{5}{3} - \frac{1}{3}q$ has the form $p(q) = aq + b$ where $a = -\frac{1}{3}$ (the slope) and $b = \frac{5}{3}$ (the vertical-axis intercept).



6. $h(q) = 0.5q + 0.25$ has the form $h(q) = aq + b$ with $a = 0.5$ (the slope) and $b = 0.25$ (the vertical-axis intercept).



7. $f(x) = ax + b = 4x + b$. Since $f(2) = 8$, $8 = 4(2) + b$, $8 = 8 + b$, $b = 0 \Rightarrow f(x) = 4x$.

8. Let $y = f(x)$. The points (0, 3) and (4, –5) lie on the graph of f. $m = \frac{-5-3}{4-0} = -2$. Thus $y - 3 = -2(x - 0)$, so $y = -2x + 3 \Rightarrow f(x) = -2x + 3$.

9. Let $y = f(x)$. The points (1, 2) and (–2, 8) lie on the graph of f. $m = \frac{8-2}{-2-1} = -2$. Thus $y - 2 = -2(x - 1)$, so $y = -2x + 4 \Rightarrow f(x) = -2x + 4$.

10. $f(x) = ax + b = -5x + b$.
Since $f\left(\frac{1}{4}\right) = 9$, we have

$$9 = -5\left(\frac{1}{4}\right) + b$$

$$b = 9 + \frac{5}{4} = \frac{41}{4}$$

so $f(x) = -5x + \frac{41}{4}$.

11. $f(x) = ax + b = -\frac{2}{3}x + b$. Since $f\left(-\frac{2}{3}\right) = -\frac{2}{3}$, we have

$$-\frac{2}{3} = -\frac{2}{3}\left(-\frac{2}{3}\right) + b$$

$$b = -\frac{2}{3} - \frac{4}{9} = -\frac{10}{9},$$

so $f(x) = -\frac{2}{3}x - \frac{10}{9}$.

12. Let $y = f(x)$. The points (1, 1) and (2, 2) lie on the graph of f. $m = \frac{2-1}{2-1} = 1$.
Thus $y - 1 = 1(x - 1) \Rightarrow y = x$, so $f(x) = x$.

13. Let $y = f(x)$. The points (–2, –1) and (–4, –3) lie on the graph of f. $m = \frac{-3+1}{-4+2} = 1$. Thus $y + 1 = 1(x + 2)$, so $y = x + 1 \Rightarrow f(x) = x + 1$.

14. $f(x) = ax + b = 0.01x + b$. Since $f(0.1) = 0.01$, we have $0.01 = (0.01)(0.1) + b \Rightarrow b = 0.009$ $\Rightarrow f(x) = 0.01x + 0.009$.

15. The points (60, 15.30) and (35, 19.30) lie on the graph of the equation, which is a line.
$m = \frac{19.30 - 15.30}{35 - 60} = -\frac{4}{25}$. Hence an equation of the line is $p - 15.30 = -\frac{4}{25}(q - 60)$, which can

be written $p=-\frac{4}{25}q+24.9$. When $q = 40$, then

$p=-\frac{4}{25}(40)+24.9=\$18.50$.

16. The line passes through (26,000, 12) and (10,000, 18), so
$m=\frac{18-12}{10,000-26,000}=-0.000375$. Then
$p - 18 = -0.000375(q - 10,000)$ or
$p = -0.000375q + 21.75$.

17. The line passes through (3000, 940) and (2200, 740), so $m=\frac{740-940}{2200-3000}=0.25$. Then
$p - 740 = 0.25(q - 2200)$ or $p = 0.25q + 190$.

18. The points (50, 35) and (35, 30) lie on the graph of the equation, which is a line.
$m=\frac{30-35}{35-50}=\frac{-5}{-15}=\frac{1}{3}$. Hence an equation of the line is

$p-35=\frac{1}{3}(q-50)$

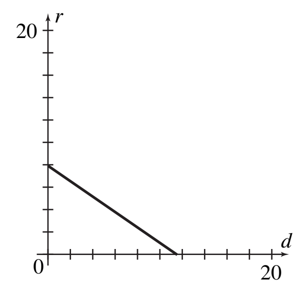
$p=\frac{1}{3}q+\frac{55}{3}$

19. The line passing through (10, 40) and (20, 70) has slope $\frac{70-40}{20-10}=3$, so an equation for the line is
$c - 40 = 3(q - 10)$
$c = 3q + 10$
If $q = 35$, then $c = 3(35) + 10 = 105 + 10 = \$115$.

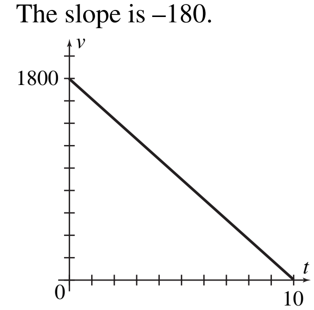
20. The line passing through (100, 89) and (200, 93) has slope $\frac{93-89}{200-100}=0.04$ so an equation for the line is
$c-89=0.04(x-100)$
$c=0.04x+85$

21. If x = the number of kilowatt hours used in a month, then $f(x)$ = the total monthly charges for x kilowatt hours of electricity. If $f(x)$ is a linear function it has the form $f(x) = ax + b$. The problem states that $f(380) = 51.65$. Since 12.5 cents are charged per kilowatt hour used, $a = 0.125$.
$f(x) = ax + b$
$51.65 = 0.125(380) + b$
$51.65 = 47.5 + b$
$4.15 = b$
Hence, $f(x) = 0.125x + 4.15$ is a linear function that describes the total monthly charges for any number of kilowatt hours x.

22. The number of curative units from d cubic centimeters of the drug is $210d$, and the number of curative units from r minutes of radiation is $305r$. Thus
$210d + 305r = 2410$
$42d + 61r = 482$



23. Each year the value decreases by 0.10(1800). After t years the total decrease is $0.10(1800)t$. Thus
$v = 1800 - 0.10(1800)t$
$v = -180t + 1800$
The slope is –180.



24. The line has slope –120 and passes through (4, 340). Thus $y - 340 = -120(x - 4)$ or
$y = f(x) = -120x + 820$.

25. The line has slope 53,000 and passes through (6, 1,183,000). Thus
$y - 1,183,000 = 53,000(x - 6)$ or
$y = f(x) = 53,000x + 865,000$.
In thousands of dollars, we have
$y = f(x) = 53x + 865$.

26. The line has slope $\frac{245,000}{15} = \frac{49,000}{3}$ and y-intercept 245,000. So
$y = f(x) = \frac{49,000}{3}x + 245,000.$

27. If x = the number of hours of service, then $f(x)$ = the price of x hours of service. Let $y = f(x)$. $f(1) = 159$ and $f(3) = 287$, so (1, 159) and (3, 287) lie on the graph of f which has slope $a = \frac{287-159}{3-1} = 64.$ Using (1, 159), we get $y - 159 = 64(x - 1)$ or $y = 64x + 95$, so $f(x) = 64x + 95$.

28. **a.** Suppose r = respiratory rate, l = wool length, and (l, r) lies on the graph, which is a line. The points (2, 160) and (4, 125) are on the line, so its slope is $\frac{125-160}{4-2} = -\frac{35}{2}$. Thus

$$r - 160 = -\frac{35}{2}(l - 2)$$

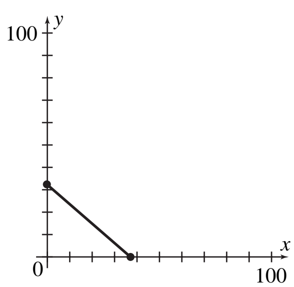
$$r = -\frac{35}{2}l + 195$$

b. If $l = 1$, then $r = -\frac{35}{2}(1) + 195 = 177.5$

29. At \$200/ton, x tons cost $200x$, and at \$2000/acre, y acres cost $2000y$. Hence the required equation is $200x + 2000y = 20,000$, which can be written as $x + 10y = 100$.

30. $P = 7x + 8y$ where $x, y \geq 0$.

a. $260 = 7x + 8y$



b. Since the equation can be written $y = -\frac{7}{8}x + 32.5,$ slope $= -\frac{7}{8}.$

c. $860 = 7x + 8y$. Since the equation can be written $y = -\frac{7}{8}x + 107.5,$ slope $= -\frac{7}{8}.$

d. Solving $P = 7x + 8y$ for y gives $y = -\frac{7}{8}x + \frac{P}{8}.$ Thus any isoprofit line has slope $-\frac{7}{8}$, and lines with the same slope are parallel. Hence isoprofit lines are always parallel.

31. **a.**

$$m = \frac{100-65}{100-56} = \frac{35}{44}$$

$$y - 100 = \frac{35}{44}(x - 100)$$

$$y = \frac{35}{44}x - \frac{3500}{44} + 100$$

$$y = \frac{35}{44}x + \frac{225}{11}$$

b.

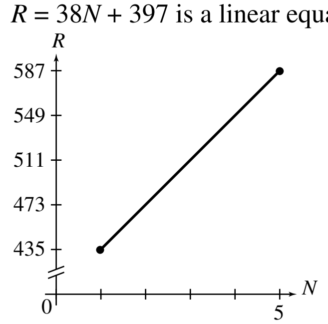
$$62 = \frac{35}{44}x + \frac{225}{11}$$

$$\frac{35}{44}x = 62 - \frac{225}{11}$$

$$x = \frac{1828}{35} \approx 52.2$$

52.2 is the lowest passing score on original scale.

32. $R = 38N + 397$ is a linear equation. Slope = 38.



33. $p = f(t) = at + b$, $f(5) = 0.32$, a = slope = 0.059.

a. $p = f(t) = 0.059t + b$. Since $f(5) = 0.32$, $0.32 = 0.059(5) + b$, $0.32 = 0.295 + b$, so $b = 0.025$. Thus $p = 0.059t + 0.025$.

b. When $t = 9$, then $p = 0.059(9) + 0.025 = 0.556$.

34. $w = f(d) = ad + b, f(0) = 21,$

$a = \text{slope } = \frac{6.3}{10} = 0.63.$ Thus

$w = f(d) = 0.63d + b$. Since $f(0) = 21$, we have $20 = 0.63(0) + b$, so $b = 21$. Hence $w = 0.63d + 21$.

When $d = 55$, then

$w = 0.63(55) + 21 = 34.65 + 21 = 55.65$ kg.

35. a. $m = \frac{t_2 - t_1}{c_2 - c_1} = \frac{80-68}{172-124} = \frac{12}{48} = \frac{1}{4}.$

$t - 68 = \frac{1}{4}(c - 124)$, $t - 68 = \frac{1}{4}c - 31$, or

$t = \frac{1}{4}c + 37.$

b. Since c is the number of chirps per minute, then $\frac{1}{4}c$ is the number of chirps in $\frac{1}{4}$ minute or 15 seconds. Thus from part (a), to estimate temperature add 37 to the number of chirps in 15 seconds.

Apply It 2.8

27. $P(x)$ is a quadratic function with $a = -0.083$, $b = 0.39$ and $c = 0.09$. Since $a < 0$, the graph is a parabola that opens downward.

The vertex is at $x = -\frac{b}{2a} = -\frac{0.39}{2(-0.083)} = 2.35,$

and the y-value is

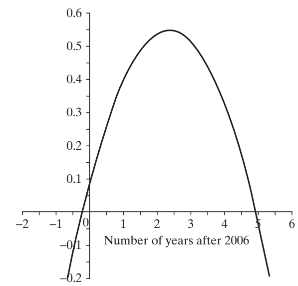
$$P(2.35) = -0.083(2.35)^2 + 0.39(2.35) + 0.09 \approx 0.55.$$

The y-intercept is (0, 0.09).

The x-intercepts are where $P(x) = 0$. Using the quadratic formula, we find

$$x = \frac{-0.39 \pm \sqrt{(0.39)^2 - 4(-0.083)(0.09)}}{2(-0.083)} \approx -0.22, 4.92$$

So the x-intercepts are (–0.22, 0) and (4.92, 0).



28. In the quadratic function $h(t) = -16t^2 + 32t + 28$, $a = -16$, $b = 32$, and $c = 8$. Since $a < 0$, the parabola opens downward. The x-coordinate of the vertex is

$-\frac{b}{2a} = -\frac{32}{2(-16)} = 1$. The y-coordinate of the vertex is

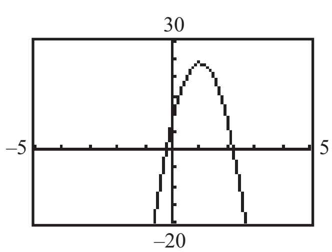
$h(1) = -16\left(1^2\right) + 32(1) + 8 = 24$. Thus, the vertex is (1, 24). Since $c = 8$, the y-intercept is (0, 8). To find the x-intercepts we set $y = h(t) = 0$.

$$0 = -16t^2 + 32t + 8$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-32 \pm \sqrt{32^2 - 4(-16)(8)}}{2(-16)}$$

$$= \frac{-32 \pm \sqrt{1536}}{-32} = \frac{-32 \pm 16\sqrt{6}}{-32} = 1 \pm \frac{\sqrt{6}}{2}$$

Thus, the x-intercepts are $\left(1 + \frac{\sqrt{6}}{2}, 0\right)$ and $\left(1 - \frac{\sqrt{6}}{2}, 0\right)$.



Problems 2.8

1. $f(x) = 5x^2$ has the form $f(x) = ax^2 + bx + c$ where $a = 5$, $b = 0$, and $c = 0 \Rightarrow$ quadratic.

2. $g(x) = \frac{1}{2x^2 - 4}$ cannot be put in the form $g(x) = ax^2 + bx + c$ where $a \neq 0 \Rightarrow$ not quadratic.

3. $g(x) = 7 - 6x$ cannot be put in the form $g(x) = ax^2 + bx + c$ where $a \neq 0 \Rightarrow$ not quadratic.

4. $k(v) = 2v^2(v^2 + 2) = 2v^4 + 4v^2$ cannot be put in the form $k(v) = av^2 + bv + c$ where $a \neq 0 \Rightarrow$ not quadratic.

5. $h(q) = (3 - q)^2 = 9 - 6q + q^2$ has form $h(q) = aq^2 + bq + c$ where $a = 1$, $b = -6$, and $c = 9 \Rightarrow$ quadratic.

6. $f(t) = 2t(3 - t) + 4t = -2t^2 + 10t$ has the form $f(t) = at^2 + bt + c$ where $a = -2$, $b = 10$, and $c = 0 \Rightarrow$ quadratic.

7. $f(s) = \frac{s^2 - 9}{2} = \frac{1}{2}s^2 - \frac{9}{2}$ has the form $f(s) = as^2 + bs + c$ where $a = \frac{1}{2}$, $b = 0$, and $c = -\frac{9}{2} \Rightarrow$ quadratic.

8. $g(t) = \left(t^2 - 1\right)^2 = t^4 - 2t^2 + 1$ cannot be put in the form $g(t) = at^2 + bt + c$ where $a \neq 0 \Rightarrow$ not quadratic.

9. $y = f(x) = 3x^2 + 5x + 1$
   $a = 3$, $b = 5$, $c = 1$

   a. $-\frac{b}{2a} = -\frac{5}{2(3)} = -\frac{5}{6}$

   $f\left(-\frac{5}{6}\right) = 3\left(-\frac{5}{6}\right)^2 + 5\left(-\frac{5}{6}\right) + 1 = -\frac{13}{12}$

   Vertex: $\left(-\frac{5}{6}, -\frac{13}{12}\right)$

   b. $a = 3 > 0$, so the vertex corresponds to the lowest point.

10. $y = f(x) = 8x^2 + 4x - 1$
    $a = 8$, $b = 4$, $c = -1$

    a. $-\frac{b}{2a} = -\frac{4}{2 \cdot 8} = -\frac{1}{4}$

    $f\left(-\frac{1}{4}\right) = 8\left(-\frac{1}{4}\right)^2 + 4\left(-\frac{1}{4}\right) - 1 = -\frac{3}{2}$

    Vertex: $\left(-\frac{1}{4}, -\frac{3}{2}\right)$

    b. $a = 8 > 0$, so the vertex corresponds to the lowest point.

11. $y = x^2 + x - 6$
    $a = 1$, $b = 1$, $c = -6$

    a. $c = -6$. Thus the y-intercept is -6.

    b. $x^2 + x - 6 = (x - 2)(x + 3) = 0$, so $x = 2, -3$.
    x-intercepts: 2, -3

    c. $-\frac{b}{2a} = -\frac{1}{2}$

    $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 6 = -\frac{25}{4}$

    Vertex: $\left(-\frac{1}{2}, -\frac{25}{4}\right)$

12. $y = f(x) = 5 - x - 3x^2$
    $a = -3$, $b = -1$, $c = 5$

    a. $c = 5$. Thus the y-intercept is 5.

b. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(-3)(5)}}{2(-3)}$

$= \dfrac{1 \pm \sqrt{61}}{-6}$

$= \dfrac{-1 \pm \sqrt{61}}{6}$

x-intercepts: $\dfrac{-1+\sqrt{61}}{6}, \dfrac{-1-\sqrt{61}}{6}$

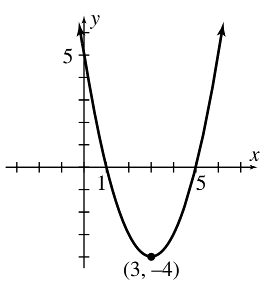
c. $-\dfrac{b}{2a} = -\dfrac{-1}{2(-3)} = -\dfrac{1}{6}$

$f\left(-\dfrac{1}{6}\right) = 5 - \left(-\dfrac{1}{6}\right) - 3\left(-\dfrac{1}{6}\right)^2 = \dfrac{61}{12}$

Vertex: $\left(-\dfrac{1}{6}, \dfrac{61}{12}\right)$

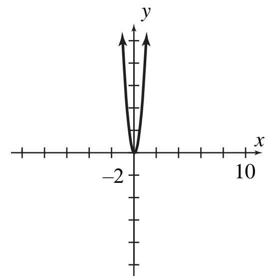
13. $y = f(x) = x^2 - 6x + 5$
$a = 1$, $b = -6$, $c = 5$
Vertex: $-\dfrac{b}{2a} = -\dfrac{-6}{2 \cdot 1} = 3$

$f(3) = 3^2 - 6(3) + 5 = -4$
Vertex = (3, –4)
y-intercept: $c = 5$
x-intercepts: $x^2 - 6x + 5 = (x-1)(x-5) = 0$, so
$x = 1, 5$.
Range: all $y \ge -4$



14. $y = f(x) = 9x^2$
$a = 9$, $b = 0$, $c = 0$
Vertex: $-\dfrac{b}{2a} = -\dfrac{0}{2(9)} = 0$

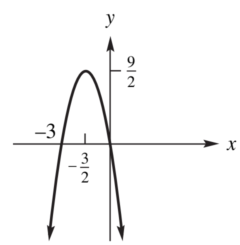
$f(0) = 9(0)^2 = 0$
Vertex = (0, 0)
y-intercept: $c = 0$
x-intercepts: $9x^2 = 0$, so $x = 0$.
Range: all $y \ge 0$



15. $y = g(x) = -2x^2 - 6x$
$a = -2$, $b = -6$, $c = 0$
Vertex: $-\dfrac{b}{2a} = -\dfrac{-6}{2(-2)} = -\dfrac{6}{4} = -\dfrac{3}{2}$

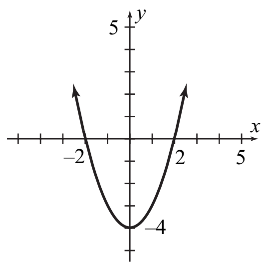
$f\left(\dfrac{-3}{2}\right) = -2\left(\dfrac{-3}{2}\right)^2 - 6\left(\dfrac{-3}{2}\right) = \dfrac{-9}{2} + 9 = \dfrac{9}{2}$

Vertex: $\left(-\dfrac{3}{2}, \dfrac{9}{2}\right)$
y-intercept: $c = 0$
x-intercepts: $-2x^2 - 6x = -2x(x+3) = 0$, so
$x = 0, -3$.
Range: all $y \le \dfrac{9}{2}$



16. $y = f(x) = x^2 - 4$
$a = 1$, $b = 0$, $c = -4$
Vertex: $-\dfrac{b}{2a} = -\dfrac{0}{2 \cdot 1} = 0$

$f(0) = 0^2 - 4 = -4$
Vertex = (0, –4)
y-intercept: $c = -4$
x-intercepts: $x^2 - 4 = (x+2)(x-2) = 0$, so
$x = -2, 2$.
Range: all $y \ge 4$



17. $s = h(t) = t^2 + 6t + 9$

$a = 1$, $b = 6$, $c = 9$

Vertex: $-\dfrac{b}{2a} = -\dfrac{6}{2\cdot 1} = -3$

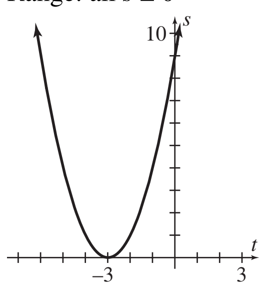
$h(-3) = (-3)^2 + 6(-3) + 9 = 0$

Vertex = (–3, 0)

s-intercept: $c = 9$

t-intercepts: $t^2 + 6t + 9 = (t+3)^2 = 0$, so $t = -3$.

Range: all $s \geq 0$



18. $s = h(t) = 2t^2 + 3t - 2$

$a = 2$, $b = 3$, $c = -2$

Vertex: $-\dfrac{b}{2a} = -\dfrac{3}{2\cdot 2} = -\dfrac{3}{4}$

$h\left(-\dfrac{3}{4}\right) = 2\left(-\dfrac{3}{4}\right)^2 + 3\left(-\dfrac{3}{4}\right) - 2$

$= \dfrac{9}{8} - \dfrac{9}{4} - 2 = -\dfrac{25}{8}$

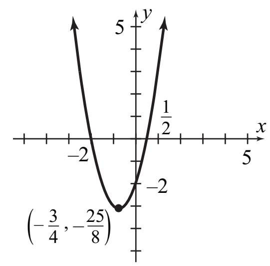
Vertex $= \left(-\dfrac{3}{4}, -\dfrac{25}{8}\right)$

s-intercept: $c = -2$

t-intercepts: $2t^2 + 3t - 2 = (2t-1)(t+2) = 0$, so

$t = \dfrac{1}{2}$, –2.

Range: all $s \geq -\dfrac{25}{8}$



19. $y = f(x) = -5 + 3x - 3x^2$

$a = -3$, $b = 3$, $c = -5$

Vertex: $-\dfrac{b}{2a} = -\dfrac{3}{2(-3)} = \dfrac{1}{2}$

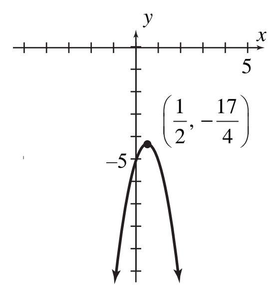
$f\left(\dfrac{1}{2}\right) = -5 + 3\left(\dfrac{1}{2}\right) - 3\left(\dfrac{1}{2}\right)^2 = -\dfrac{17}{4}$

Vertex $= \left(\dfrac{1}{2}, -\dfrac{17}{4}\right)$

y-intercept: $c = -5$

x-intercepts: Because the parabola opens downward ($a < 0$) and the vertex is below the x-axis, there is no x-intercept.

Range: $y \leq -\dfrac{17}{4}$



20. $y = H(x) = 1 - x - x^2$

$a = -1$, $b = -1$, $c = 1$

Vertex: $-\dfrac{b}{2a} = -\dfrac{-1}{2(-1)} = -\dfrac{1}{2}$

$f\left(-\dfrac{1}{2}\right) = 1 - \left(-\dfrac{1}{2}\right) - \left(-\dfrac{1}{2}\right)^2 = \dfrac{5}{4}$

Vertex $= \left(-\dfrac{1}{2}, \dfrac{5}{4}\right)$

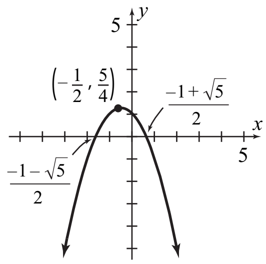
y-intercept: $c = 1$

x-intercepts: Solving $1 - x - x^2 = 0$ by the quadratic formula gives

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)(1)}}{2(-1)} = \frac{1 \pm \sqrt{5}}{-2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Range: all $y \le \frac{5}{4}$



21. $t = f(s) = s^2 - 8s + 14$

$a = 1$, $b = -8$, $c = 14$

Vertex: $-\frac{b}{2a} = -\frac{-8}{2 \cdot 1} = 4$

$f(4) = 4^2 - 8(4) + 14 = -2$

Vertex = (4, –2)

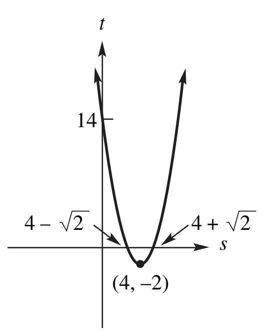
t-intercept: $c = 14$

s-intercepts: Solving $s^2 - 8s + 14 = 0$ by the quadratic formula:

$$s = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{8}}{2} = \frac{8 \pm 2\sqrt{2}}{2} = 4 \pm \sqrt{2}$$

Range: all $t \ge -2$



22. $t = f(s) = s^2 + 6s + 11$

$a = 1$, $b = 6$, $c = 11$

Vertex: $-\frac{b}{2a} = -\frac{6}{2 \cdot 1} = -3$

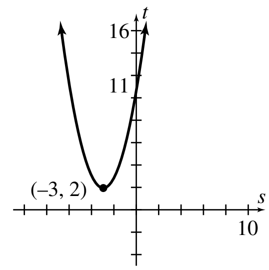
$f(-3) = (-3)^2 + 6(-3) + 11 = 2$

Vertex: (–3, 2)

t-intercept: $c = 11$

s-intercepts: Because the parabola opens upward ($a > 0$) and the vertex is above the s-axis, there is no s-intercept.

Range: all $t \ge 2$



23. $f(x) = 49x^2 - 10x + 17$

Since $a = 49 > 0$, the parabola opens upward and $f(x)$ has a minimum value that occurs when $x = -\frac{b}{2a} = -\frac{-10}{2 \cdot 49} = \frac{5}{49}$. The minimum value is

$$f\left(\frac{5}{49}\right) = 49\left(\frac{5}{49}\right)^2 - 10\left(\frac{5}{49}\right) + 17 = \frac{808}{49}.$$

24. $f(x) = -7x^2 - 2x + 6$

Since $a = -7 < 0$, the parabola opens downward and $f(x)$ has a maximum value that occurs when

$$x = -\frac{b}{2a} = -\frac{-2}{2(-7)} = -\frac{1}{7}$$

The maximum value is

$$f\left(-\frac{1}{7}\right) = -7\left(-\frac{1}{7}\right)^2 - 2\left(-\frac{1}{7}\right) + 6 = \frac{43}{7}.$$

25. $f(x) = 4x - 50 - 0.1x^2$

Since $a = -0.1 < 0$, the parabola opens downward and $f(x)$ has a maximum value that occurs when $x = -\frac{b}{2a} = -\frac{4}{2(-0.1)} = 20$. The maximum value is

$f(20) = 4(20) - 50 - 0.1(20)^2 = -10$.

26. $f(x) = x(x+3) - 12 = x^2 + 3x - 12$

Because $a = 1 > 0$, the parabola opens upward and $f(x)$ has a minimum value that occurs when $x = -\frac{b}{2a} = -\frac{3}{2 \cdot 1} = -\frac{3}{2}$. The minimum value is

$$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 12 = -\frac{57}{4}$$

27. $f(x) = x^2 - 2x + 4$

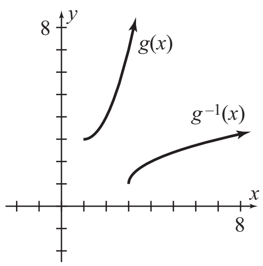
$a = 1$, $b = -2$, $c = 4$

$$v = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$$

The restricted function is $g(x) = x^2 - 2x + 4$, $x \geq 1$. From the quadratic formula applied to $x^2 - 2x + 4 - y = 0$, we get

$$x = \frac{2 \pm \sqrt{4 - 4(1)(4 - y)}}{2(1)} = 1 \pm \sqrt{1 - (4 - y)}$$

So the inverse of $g(x)$ is $g^{-1}(x) = 1 + \sqrt{x - 3}$, $x \geq 3$.



28. $f(x) = -x^2 + 4x - 3$

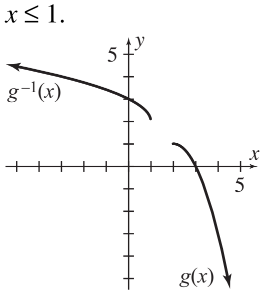
$a = -1$, $b = 4$, $c = -3$

$$v = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

The restricted function is $g(x) = -x^2 + 4x - 3$, $x \geq 2$. From the quadratic formula applied to $-x^2 + 4x - 3 - y = 0$, we get

$$x = \frac{-4 \pm \sqrt{16 - 4(-1)(-3 - y)}}{2(-1)}$$
$$= 2 \pm (-1)\sqrt{4 + (-3 - y)}$$

So the inverse of $g(x)$ is $g^{-1}(x) = 2 + \sqrt{1 - x}$, $x \leq 1$.



29. If we express the revenue r as a function of the quantity produced q, we obtain

$r = pq$

$r = (100 - 10q)q$

$r = 100q - 10q^2$

This is a quadratic function with $a = -10$, $b = 100$, and $c = 0$. Since $a < 0$, the graph of the function is a parabola that opens downward, and r is maximum at the vertex (q, r).

$$q = -\frac{b}{2a} = -\frac{100}{2(-10)} = 5$$

$r = 100(5) - 10(5)^2 = 250$

Thus, the maximum revenue that the manufacturer can receive is \$250, which occurs at a production level of 5 units.

30. If we express the revenue r as a function of the quantity produced q, we obtain

$r = pq$

$r = (0.85 - 0.00045q)q$

$r = 0.85q - 0.00045q^2$

This is a quadratic function with $a = -0.00045$, $b = 0.85$, and $c = 0$. Since $a < 0$, the graph of the function is a parabola that opens downward, and r is a maximum at the vertex (q, r).

$$q = -\frac{b}{2a} = -\frac{0.85}{2(-0.00045)} = \frac{8500}{9} \approx 944$$

$r = 0.85(944) - 0.00045(944)^2 = 401.39$

Thus, the maximum revenue that the manufacturer can receive is \$401.39, which occurs at a production level of 944 units.

31. If we express the revenue r as a function of the quantity produced q, we obtain

$r = pq$

$r = (2400 - 6q)q$

$r = 2400q - 6q^2$

This is a quadratic function with $a = -6$, $b = 2400$, and $c = 0$. Since $a < 0$, the graph of the function is a parabola that opens downward, and r is maximum at the vertex (q, r).

$$q = -\frac{b}{2a} = -\frac{2400}{2(-6)} = 200$$

$r = 2400(200) - 6(200)^2 = 240{,}000$

Thus, the maximum revenue that the manufacturer can receive is \$240,000, which occurs at a production level of 200 units.

32. $f(n) = \frac{10}{9}n(12 - n) = \frac{40}{3}n - \frac{10}{9}n^2$, where $0 \leq n \leq 12$. Since $a = -\frac{10}{9} < 0$, $f(n)$ has a maximum value that occurs at the vertex.

$$-\frac{b}{2a} = -\frac{\frac{40}{3}}{2\left(-\frac{10}{9}\right)} = 6$$

The maximum value of $f(n)$ is

$f(6) = \frac{40}{3}(6) - \frac{10}{9}(6)^2 = 80 - 40 = 40$, which corresponds to 40,000 households.

33. In the quadratic function

$P(x) = -x^2 + 18x + 144,$

$a = -1$, $b = 18$, and $c = 144$. Since $a < 0$, the graph of the function is a parabola that opens downward. The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{18}{2(-1)} = 9$. The y-coordinate of the vertex is $P(9) = -\left(9^2\right) + 18(9) + 144 = 225$.

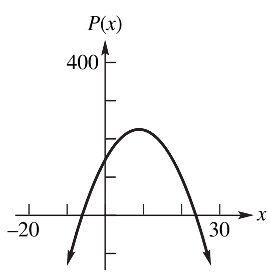
Thus, the vertex is (9, 225). Since $c = 144$, the y-intercept is (0, 144). To find the x-intercepts, let $y = P(x) = 0$.

$0 = -x^2 + 18x + 144$

$0 = -\left(x^2 - 18x - 144\right)$

$0 = -(x - 24)(x + 6)$

Thus, the x-intercepts are (24, 0) and (–6, 0).



34. If $k = 3$, then

$y = kx^2$

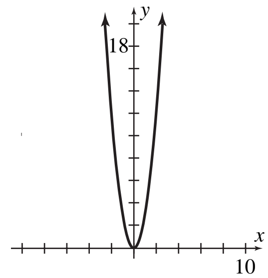
$y = 3x^2$

This is a quadratic equation with $a = 3$, $b = 0$ and $c = 0$. Since $a > 0$, the graph of the function is a parabola that opens upward. The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{0}{2(3)} = 0$.

The y-coordinate is

$y = 3(0)^2 = 0$

Thus, the vertex is (0, 0).



35. $f(P) = -\frac{1}{50}P^2 + 2P + 20$, where $0 \le P \le 100$.

Because $a = -\frac{1}{50} < 0$, $f(P)$ has a maximum value that occurs at the vertex.

$-\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{50}\right)} = 50$. The maximum value of $f(P)$ is

$f(50) = \frac{-1}{50}(50)^2 + 2(50) + 20 = 70$ grams.

36. $s = -4.9t^2 + 62.3t + 1.8$

Since $a = -4.9 < 0$, s has a maximum value that occurs at the vertex where

$t = -\frac{b}{2a} = -\frac{62.3}{2(-4.9)} = \frac{62.3}{9.8} = \frac{89}{14} \approx 6.36$ sec.

When $t = \frac{89}{14}$, then

$s = -4.9\left(\frac{89}{14}\right)^2 + 62.3\left(\frac{89}{14}\right) + 1.8$

$= 199.825$ meters.

37. $h(t) = -5 \cdot t^2 + 25 \cdot t + 9$

Since $a = -5 < 0$, $h(t)$ has a maximum that occurs at the vertex where $t = \frac{-b}{2a}$,

$t = -\frac{25}{2(-5)} = 2.5$ and $h(2.5) = 40.25$ meters.

38. $h(t) = -5\,t^2 + 5\,t + 1.75$

Since $a = -5 < 0$, $h(t)$ has a maximum that occurs at the vertex where $t = \frac{-b}{2a}$,

$t = -\frac{5}{2(-5)} = 0.5$ and $h(0.5) = 3$ meters.

39. $h(t) = -5t^2 + 27t + 6$

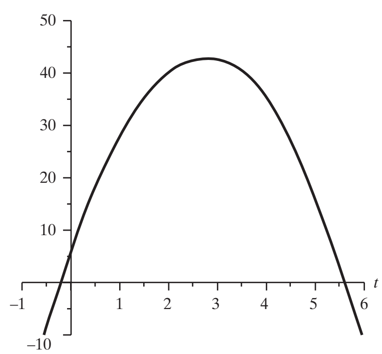
Since $a = -5 < 0$, $h(t)$ has a maximum that occurs at the vertex where $t = \frac{-b}{2a}$,

$t = -\frac{27}{2(-5)} = 3$ and $h(3) = 42$ meters.

The vertex is (3, 42), y-intercept is (0, 6) and t-intercepts are given by $-5t^2 + 27t + 6 = 0$.

So t-intercepts are

$\left(\frac{27+\sqrt{849}}{10}, 0\right)$ and $\left(\frac{27-\sqrt{849}}{10}, 0\right)$.



40. $A = x(11 - x) = 11x - x^2$, so A is a quadratic function of x where $a = -1 < 0$. A has maximum value at the vertex where

$x = -\frac{b}{2a} = -\frac{11}{2(-1)} = \frac{11}{2}$.

41. The area is given by

$A = x(150 - 2x) = 150x - 2x^2$

Since $a = -2 < 0$, A has a maximum when

$x = -\frac{b}{2a} = -\frac{150}{2(-2)} = 37.5$, and the side opposite to the highway $150 - 2(37.5) = 75$. Thus the dimensions are 37.5 meters and 75 meters.

42. Let x, y be two numbers whose sum is 78. Thus $x + y = 78$ and $y = 78 - x$. Their product is then $p(x) = x(78 - x) = 78x - x^2$. Since $a = -1 < 0$, $p(x)$ has a maximum value that occurs at the vertex where $x = -\frac{b}{2a} = -\frac{78}{2(-1)} = 39$ and $y = 78 - x = 78 - 39 = 39$. Thus, two numbers whose sum is 78 and whose product is a maximum are 39 and 39.

Chapter 2 Review Problems

1. Denominator is 0 when

$x^2 - 6x + 5 = 0$
$(x-1)(x-5) = 0$
$x = 1, 5$

Domain: all real numbers except 1 and 5.

2. all real numbers

3. all real numbers

4. all real numbers

5. $s - 5 \geq 0$
$s \geq 5$

Domain: all real numbers s such that $s \geq 5$.

6. $f(x) = 2x^2 - 3x + 5$

$f(0) = 2(0)^2 - 3(0) + 5 = 5$

$f(-2) = 2(-2)^2 - 3(-2) + 5 = 8 + 6 + 5 = 19$

$f(5) = 2(5)^2 - 3(5) + 5 = 50 - 15 + 5 = 40$

$f(\pi) = 2\pi^2 - 3\pi + 5$

7. $h(x) = 7$; all function values are 7.
Answer: 7, 7, 7, 7

8. $G(x) = \sqrt[4]{x-3}$

$G(3) = \sqrt[4]{3-3} = \sqrt[4]{0} = 0$

$G(19) = \sqrt[4]{19-3} = \sqrt[4]{16} = 2$

$G(t+1) = \sqrt[4]{(t+1)-3} = \sqrt[4]{t-2}$

$G(x^3) = \sqrt[4]{x^3 - 3}$

9. $F(x) = \frac{x-3}{x+4}$

$F(-1) = \frac{-1-3}{-1+4} = -\frac{4}{3}$

$F(0) = \frac{0-3}{0+4} = -\frac{3}{4}$

$F(5) = \frac{5-3}{5+4} = \frac{2}{9}$

$F(x+3) = \frac{(x+3)-3}{(x+3)+4} = \frac{x}{x+7}$

10. $H(t)=\frac{(t-2)^3}{5}$

$H(-1)=\frac{(-1-2)^3}{5}=-\frac{27}{5}$

$H(0)=\frac{(0-2)^3}{5}=-\frac{8}{5}$

$H\left(\frac{1}{3}\right)=\frac{\left(\frac{1}{3}-2\right)^3}{5}=\frac{\left(-\frac{5}{3}\right)^3}{5}=\left(-\frac{125}{27}\right)\left(\frac{1}{5}\right)=-\frac{25}{27}$

$H(x^2)=\frac{(x^2-2)^3}{5}$

11. $f\left(-\frac{1}{2}\right)=-\left(-\frac{1}{2}\right)+1=\frac{1}{2}+1=\frac{3}{2}$

$f(0)=0^2+1=1$

$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2+1=\frac{1}{4}+1=\frac{5}{4}$

$f(5)=5^3-99=125-99=26$

$f(6)=6^3-99=216-99=117$

12. **a.** $f(x+h)=3-7(x+h)=3-7x-7h$

b. $\frac{f(x+h)-f(x)}{h}=\frac{(3-7x-7h)-(3-7x)}{h}$

$=\frac{-7h}{h}=-7$

13. **a.** $f(x+h)=11(x+h)^2+4$

$=11x^2+22xh+11h^2+4$

b. $\frac{f(x+h)-f(x)}{h}$

$=\frac{(11x^2+22xh+11h^2+4)-(11x^2+4)}{h}$

$=\frac{22xh+11h^2}{h}=22x+11h$

14. **a.** $f(x+h)=\frac{7}{(x+h)+1}=\frac{7}{x+h+1}$

b. $\frac{f(x+h)-f(x)}{h}=\frac{\frac{7}{x+h+1}-\frac{7}{x+1}}{h}$

$=\frac{\frac{7(x+1)-7(x+h+1)}{(x+h+1)(x+1)}}{h}$

$=\frac{-7h}{(x+h+1)(x+1)h}$

$=\frac{-7}{(x+h+1)(x+1)}$

15. $f(x)=3x-1$, $g(x)=2x+3$

a. $(f+g)(x)=f(x)+g(x)=(3x-1)+(2x+3)$
$=5x+2$

b. $(f+g)(4)=5(4)+2=22$

c. $(f-g)(x)=f(x)-g(x)=(3x-1)-(2x+3)$
$=x-4$

d. $(fg)(x)=f(x)g(x)=(3x-1)(2x+3)$
$=6x^2+7x-3$

e. $(fg)(1)=6(1)^2+7(1)-3=10$

f. $\frac{f}{g}(x)=\frac{f(x)}{g(x)}=\frac{3x-1}{2x+3}$

g. $(f\circ g)(x)=f(g(x))=f(2x+3)$
$=3(2x+3)-1=6x+8$

h. $(f\circ g)(5)=6(5)+8=38$

i. $(g\circ f)(x)=g(f(x))=g(3x-1)$
$=2(3x-1)+3=6x+1$

16. $f(x)=\frac{1}{x^2}$, $g(x)=x+1$

$(f\circ g)(x)=f(g(x))=f(x+1)=\frac{1}{(x+1)^2}$

$(g\circ f)(x)=g(f(x))=g\left(\frac{1}{x^2}\right)=\frac{1}{x^2}+1=\frac{1+x^2}{x^2}$

17. $f(x)=\sqrt{x+2},\ \ g(x)=x^3$

$(f\circ g)(x)=f(g(x))=f(x^3)=\sqrt{x^3+2}$

$(g\circ f)(x)=g(f(x))=g\left(\sqrt{x+2}\right)=\left(\sqrt{x+2}\right)^3$

$=(x+2)^{3/2}$

18. $f(x)=2,\ g(x)=3$

$(f\circ g)(x)=f(g(x))=f(3)=2$

$(g\circ f)(x)=g(f(x))=g(2)=3$

19. $y=4+x^2$

Intercepts: If $y = 0$, then $0=4+x^2$, which is never true.
If $x = 0$, then $y = 4$.
Testing for symmetry gives:

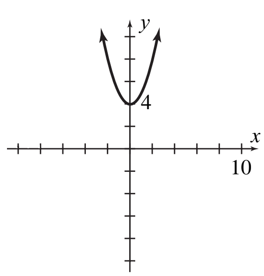
x-axis: $-y=4+x^2$
$y=-4-x^2$, which is not the original equation.

y-axis: $y=4+(-x)^2$
$y=4+x^2$, which is the original equation.

origin: $-y=4+(-x)^2$
$y=-4-x^2$, which is not the original equation.

line $y = x$: (a, b) on graph, then $b=4+a^2$ and
$a=\pm\sqrt{b-4}\neq 4+b^2$ for all b, so
(b, a) is not on the graph.

Answer: (0, 4); symmetry about y-axis.



20. $y = 3x - 7$

Intercepts: If $y = 0$, then $0 = 3x - 7$, or $x=\dfrac{7}{3}$.

If $x = 0$, then $y = -7$.
Testing for symmetry gives:

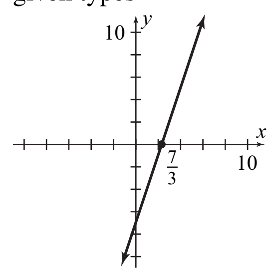
x-axis: $-y = 3x - 7$
$y = -3x + 7$, which is not the original equation.

y-axis: $y = 3(-x) - 7$
$y = -3x - 7$, which is not the original equation.

origin: $-y = 3(-x) - 7$
$y = 3x + 7$, which is not the original equation.

line $y = x$: (a, b) on graph, then $b = 3a - 7$ and
$a=\frac{1}{3}(b+7)\neq 3b-7$ for all b, so
(b, a) is not on the graph.

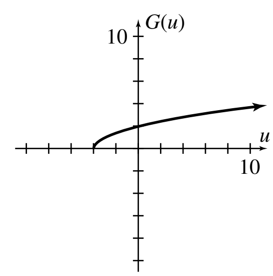
Answer: $(0, -7)$, $\left(\dfrac{7}{3}, 0\right)$; no symmetry of the given types



21. $G(u)=\sqrt{u+4}$

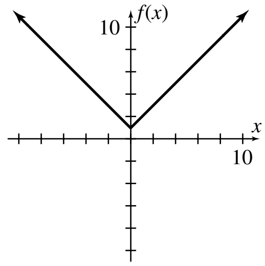
If $G(u) = 0$, then $0=\sqrt{u+4}$.
$0 = u + 4$,
$u = -4$

If $u = 0$, then $G(u)=\sqrt{4}=2$.
Intercepts: (0, 2), (–4, 0)
Domain: all real numbers u such that $u \geq -4$
Range: all real numbers ≥ 0



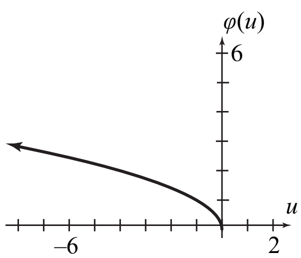
22. $f(x)=|x|+1$

If $f(x) = 0$, then $0=|x|+1$.
$|x|=-1$, which has no solution.
If $x = 0$, then $f(x) = 1$.
Intercept: (0, 1)
Domain: all real numbers
Range: all real numbers ≥ 1

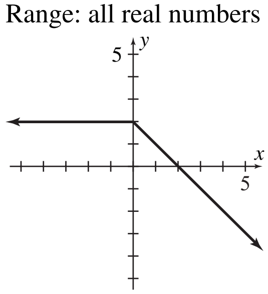


23. $v = \phi(u) = \sqrt{-u}$

If $\phi(u) = 0$, then $0 = \sqrt{-u}$,
$u = 0$.
If $u = 0$, $\phi(u) = 0$.
Intercept: (0, 0)
Domain: all reals ≤ 0
Range: all reals ≥ 0



24. Domain: all real numbers.
Range: all real numbers ≤ 2



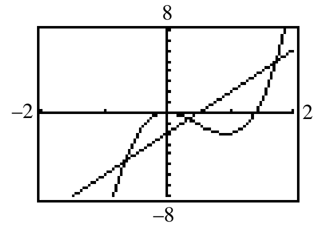
25. For 2006, $t = 5$. Hence
$S = 150{,}000 + 3000(5) = \$165{,}000$.
S is a function of t.

26. From the vertical-line test, the graphs that represent functions of x are (a) and (c).

27. **a.** 729

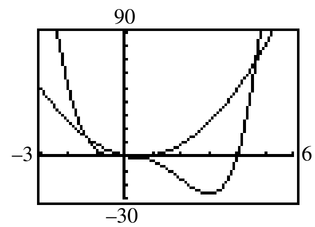
b. 359.43

28.



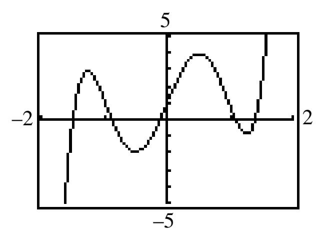
–0.67; 0.34, 1.73

29.



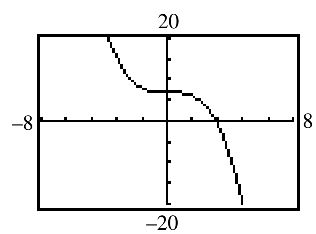
–1.38, 4.68

30.



–1.50, –0.88, –0.11, 1.09, 1.40

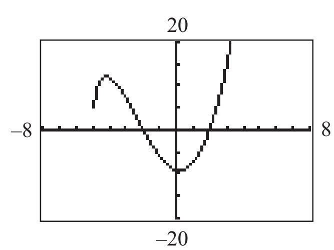
31.



a. $(-\infty, \infty)$

b. (1.92, 0), (0, 7)

32.



a. −9.03

b. all real numbers ≥ −9.03

c. −5, ±2.

33. Solving $\frac{k-5}{3-2}=4$ gives $k - 5 = 4$, $k = 9$.

34. The equation $\frac{4-4}{5-k}=0$ is true for any real number $k \neq 5$.

35. (−2, 3) and (0, −1) lie on the line, so $m=\frac{-1-3}{0-(-2)}=-2.$ Slope-intercept form: $y = mx + b \Rightarrow y = -2x - 1$. A general form: $2x + y + 1 = 0$.

36.
$$y-3=3(x-8)$$
$$y-3=3x-24$$
$$y=3x-21,$$
which is slope-intercept form.
$$y=3x-21$$
$$-y=-3x+21$$
$$3x-y-21=0,$$
which is a general form.

37. Slope of a vertical line is undefined, so slope-intercept form does not exist. An equation of the vertical line is $x = 3$. General form: $x - 3 = 0$.

38. Slope of a horizontal line is 0. Thus
$y - 4 = 0[x - (-2)]$
$y - 4 = 0$,
so slope-intercept form is $y = 4$. A general form is $y - 4 = 0$.

39. The line $2y + 5x = 2$ $\left(\text{or } y=-\frac{5}{2}x+1\right)$ has slope $-\frac{5}{2},$ so the line perpendicular to it has slope $\frac{2}{5}$. Since the y-intercept is −3, the equation is $y=\frac{2}{5}x-3.$ A general form is $2x - 5y - 15 = 0$.

40. The line has slope $\frac{13-7}{4-2}=\frac{6}{2}=3,$ so an equation of the line is $y - 7 = 3(x - 2)$. If $x = 3$, then
$$y-7=3(3-2)$$
$$y-7=3$$
$$y=10$$
Thus (3, 11) does not lie on the line.

In Problems 41–45, $m_1 =$ slope of first line, and $m_2 =$ slope of second line.

41. $x + 4y + 2 = 0$ $\left(\text{or } y=-\frac{1}{4}x-\frac{1}{2}\right)$ has slope $m_1=-\frac{1}{4}$ and $8x - 2y - 2 = 0$ (or $y = 4x - 1$) has slope $m_2 = 4$. Since $m_1=-\frac{1}{m_2}$, the lines are perpendicular to each other.

42. $y - 2 = 2(x - 1)$ (or $y = 2x$) has slope $m_1 = 2$, and $2x + 4y - 3 = 0$ $\left(\text{or } y=-\frac{1}{2}x+\frac{3}{4}\right)$ has slope $m_2=-\frac{1}{2}$. Since $m_1=-\frac{1}{m_2}$, the lines are perpendicular.

43. $x - 3 = 2(y + 4)$ $\left(\text{or } y=\frac{1}{2}x-\frac{11}{2}\right)$ has slope $m_1=\frac{1}{2}$, and $y = 4x + 2$ has slope $m_2 = 4$. Since $m_1 \neq m_2$ and $m_1 \neq -\frac{1}{m_2}$, the lines are neither parallel nor perpendicular to each other.

44. $2x + 7y - 4 = 0$ $\left(\text{or } y=-\frac{2}{7}x+\frac{4}{7}\right)$ has slope $m_1=-\frac{2}{7},$ and $6x + 21y = 90$ $\left(\text{or } y=-\frac{2}{7}x+\frac{30}{7}\right)$ has slope $m_2=-\frac{2}{7}.$ Since $m_1 = m_2$, the lines are parallel.

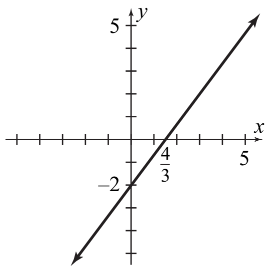
45. $y = 7x$ has slope $m_1 = 7,$ and $y = 7$ has slope $m_2 = 0$. Since $m_1 \neq m_2$ and $m_1 \neq -\frac{1}{m_2}$, the lines are neither parallel nor perpendicular.

46. $3x - 2y = 4$

$-2y = -3x + 4$

$y = \frac{3}{2}x - 2$

$m = \frac{3}{2}$

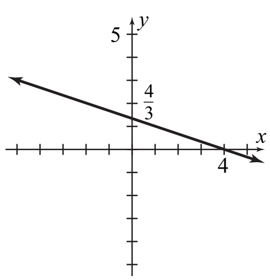


47. $x = -3y + 4$

$3y = -x + 4$

$y = -\frac{1}{3}x + \frac{4}{3}$

$m = -\frac{1}{3}$

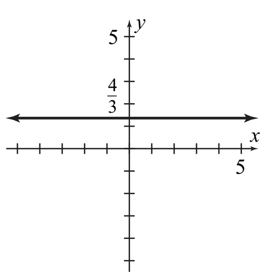


48. $4 - 3y = 0$

$-3y = -4$

$y = \frac{4}{3}$

$m = 0$

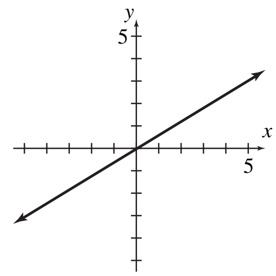


49. $3x - 5y = 0$

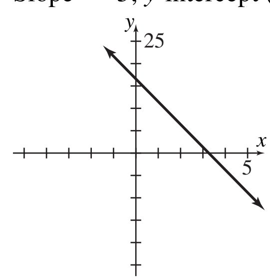
$-5y = -3x$

$y = \frac{3}{5}x$

$m = \frac{3}{5}$



50. $y = f(x) = 17 - 5x$ has the linear form $f(x) = ax + b$, where $a = -5$ and $b = 17$. Slope $= -5$; y-intercept $(0, 17)$.



51. $s = g(t) = 5 - 3t + t^2$ has the quadratic form

$g(t) = at^2 + bt + c$, where $a = 1$, $b = -3$, $c = 5$.

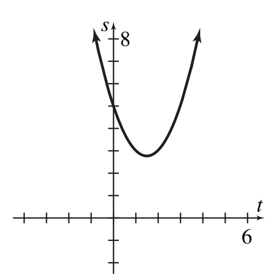
Vertex: $-\frac{b}{2a} = -\frac{-3}{2(1)} = \frac{3}{2}$

$g\left(\frac{3}{2}\right) = 5 - 3\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = \frac{11}{4}$

$\Rightarrow$ Vertex $= \left(\frac{3}{2}, \frac{11}{4}\right)$

s-intercept: $c = 5$

t-intercepts: Because the parabola opens upward ($a > 0$) and the vertex is above the t-axis, there is no t-intercept.

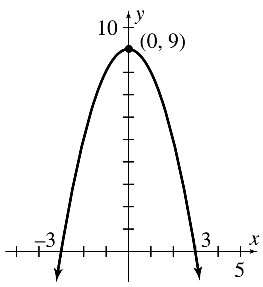


52. $y = f(x) = 9 - x^2$ has the quadratic form $f(x) = ax^2 + bx + c$, where $a = -1$, $b = 0$ and $c = 9$.

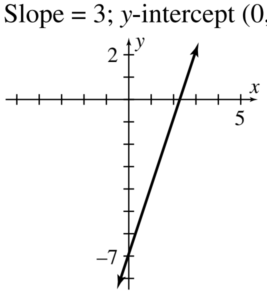
Vertex: $-\frac{b}{2a} = -\frac{0}{2(-1)} = 0$

$f(0) = 9 - 0^2 = 9$

$\Rightarrow$ Vertex = (0, 9)
y-intercept: $c = 9$
x-intercepts: $9 - x^2 = (3 - x)(3 + x) = 0$, so $x = 3, -3$.



53. $y = f(x) = 3x - 7$ has the linear form $f(x) = ax + b$, where $a = 3$, $b = -7$.
Slope = 3; y-intercept (0, –7)

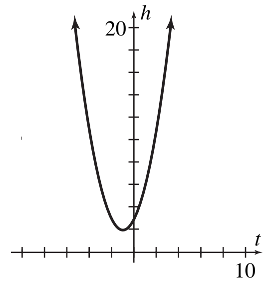


54. $y = h(t) = 3 + 2t + t^2$ has the quadratic form $h(t) = at^2 + bt + c$, where $a = 1$, $b = 2$, and $c = 3$.

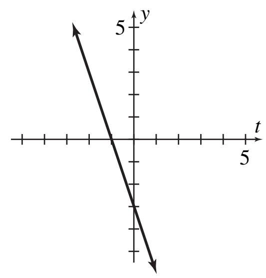
Vertex: $-\frac{b}{2a} = -\frac{2}{2(1)} = -1$

$h(-1) = 3 + 2(-1) + (-1)^2 = 2$

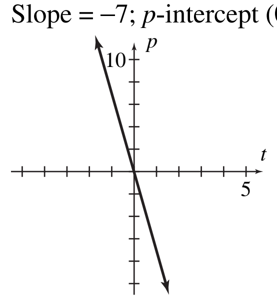
$\Rightarrow$ Vertex = (–1, 2)
y-intercept: $c = 3$
Since $3 + 2t + t^2 > 0$ for all t, there are no t-intercepts.



55. $y = k(t) = -3 - 3t$ has the linear form $k(t) = at + b$, where $a = -3$, $b = -3$.
Slope = –3, y-intercept (0, –3)



56. $p = g(t) = -7t$ has the linear form $g(t) = at + b$, where $a = -7$ and $b = 0$.
Slope = –7; p-intercept (0, 0)

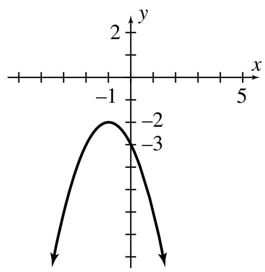


57. $y = F(x) = -\left(x^2 + 2x + 3\right) = -x^2 - 2x - 3$ has the quadratic form $F(x) = ax^2 + bx + c$, where $a = -1$, $b = -2$, and $c = -3$

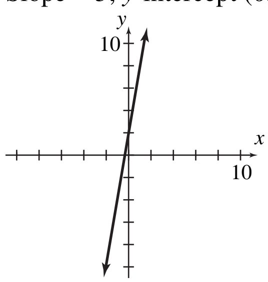
Vertex: $-\frac{b}{2a} = -\frac{-2}{2(-1)} = -1$

$F(-1) = -\left[(-1)^2 + 2(-1) + 3\right] = -2$

$\Rightarrow$ Vertex = (–1, –2)
y-intercept: $c = -3$
x-intercepts: Because the parabola opens downward ($a < 0$) and the vertex is below the x-axis, there is no x-intercept.



58. $y = f(x) = 5x + 2$ has the linear form $f(x) = ax + b$, where $a = 5,\ b = 2$.
Slope = 5; y-intercept (0, 2)



59. Slope is $\frac{-4}{3} \Rightarrow f(x) = ax + b = -\frac{4}{3}x + b$. Since $f(1) = 5$,

$5 = -\frac{4}{3}(1) + b$

$b = \frac{19}{3}$

Thus $f(x) = -\frac{4}{3}x + \frac{19}{3}$.

60. The slope of f is $\frac{5-8}{2-(-1)} = \frac{-3}{3} = -1$. Thus
$f(x) = ax + b = -x + b$. Since $f(2) = 5$,
$5 = -2 + b$
$b = 7$
Thus $f(x) = -x + 7$.

61. Let p_1 and p_2 be the prices (in dollars) of the two items before the discount. The sum of the prices is $p_1 + p_2$, which is equal to 7. After the 7% discount, the prices become $0.93p_1$ and $0.93p_2$, so their sum is $0.93p_1 + 0.93p_2$, which is supposed to equal 5.25.

Therefore, we have the system

$$\begin{cases} p_1 + p_2 = 7 & (1) \\ 0.93p_1 + 0.93p_2 = 5.25 & (2) \end{cases}$$

Multiplying equation (1) by 0.93 gives

$$\begin{cases} 0.93p_1 + 0.93p_2 = 6.51 \\ 0.93p_1 + 0.93p_2 = 5.25 \end{cases}$$

and subtracting the equations results in 0 = 1.26, which is clearly false. This indicates that the system does not have a solution, i.e. the scenario is not possible.

62. **a.** $R = aL + b$. If $L = 0$, then $R = 1310$. Thus we have $1310 = 0 \cdot L + b$, or $b = 1310$. So $R = aL + 1310$. Since $R = 1460$ when $L = 2$,
$1460 = a(2) + 1310$
$150 = 2a$
$a = 75$
Thus $R = 75L + 1310$.

b. If $L = 1$, then
$R = 75(1) + 1310 = 1385$ milliseconds.

c. Since $R = 75L + 1310$, the slope is 75. The slope gives the change in R for each 1-unit increase in L. Thus the time necessary to travel from one level to the next level is 75 milliseconds.

63. $y_{TR} = 16q$; $y_{TC} = 8q + 10{,}000$. Letting $y_{TR} = y_{TC}$ gives
$16q = 8q + 10{,}000$
$8q = 10{,}000$
$q = 1250$
If $q = 1250$, then $y_{TR} = 16(1250) = 20{,}000$.
Thus the break-even point is (1250, 20,000) or 1250 units, $20,000.

64. $C = aF + b$. The points (32, 0) and (212, 100) lie on the graph of the function. Thus its slope is $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$, so $C = \frac{5}{9}F + b$. Since

$C = 0$ when $F = 32$, $0 = \frac{5}{9}(32) + b$, so

$b = -\frac{160}{9}$. Thus $C = \frac{5}{9}F - \frac{160}{9}$ or

$C = \frac{5}{9}(F - 32)$. When

$F = 50$, then $C = \frac{5}{9}(50 - 32) = \frac{5}{9}(18) = 10$.

65. Equating L-values gives

$$0.0183-\frac{0.0042}{p}=0.0005+\frac{0.0378}{p}$$
$$0.0178=\frac{0.042}{p}$$
$$0.0178p=0.042$$
$$p\approx 2.36$$

The equilibrium pollution level is about 2.36 tons per square kilometer.

Chapter Test

1. For $\sqrt{4x+3}$ to be real,
$4x+3\ge 0$
$4x\ge -3$
$x\ge -\frac{3}{4}$

Answer: all real numbers $\ge -\frac{3}{4}$

2. We exclude all values of x for which
$$3x^2-5x-2=0$$
$$(3x+1)(x-2)=0$$
$$x=-\frac{1}{3},\ 2$$

Answer: all real numbers except 2 and $-\frac{1}{3}$

3. $G(x)=2-x^2$

$G(-8)=2-(-8)^2=2-64=-62$

$G(u)=2-u^2$

$G(u^2)=2-(u^2)^2=2-u^4$

4. Weekly excess of income over expenses is $7200-4900=2300$.
After t weeks the excess accumulates to $2300t$.
Thus the value of V of the business at the end of t weeks is given by $V=f(t)=50{,}000+2300t$.

5. a. 3

b. 7

6. $F(3)=3^2-3(3)+1=1$
$F(-3)=2(-3)-5=-11$
$F(2)$ is not defined.

7. The cost for buying n tickets is
$$c(n)=\begin{cases}9.5n & 0\le n<12\\ 8.75n & 12\le n\end{cases}$$

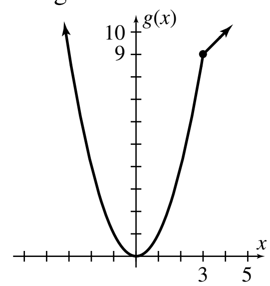
8. a. $f(0)=0, f(2)=1, f(3)=3, f(4)=2$

b. Domain: all x such that $0\le x\le 4$

c. Range: all y such that $0\le y\le 3$

d. $f(x)=0$ for $x=0$. So a real zero is 0.

9. Domain: all real numbers
Range: all real numbers ≥ 0



10. For $\sqrt{x}$ to be real, x must be nonnegative. For the denominator $x-1$ to be different from 0, x cannot be 1. Both conditions are satisfied by all nonnegative numbers except 1.
Domain: all nonnegative real numbers except 1.

11. $h(u)=\frac{\sqrt{u+4}}{u}$

$h(5)=\frac{\sqrt{5+4}}{5}=\frac{\sqrt{9}}{5}=\frac{3}{5}$

$h(-4)=\frac{\sqrt{-4+4}}{-4}=\frac{0}{-4}=0$

$h(x)=\frac{\sqrt{x+4}}{x}$

$h(u-4)=\frac{\sqrt{(u-4)+4}}{u-4}=\frac{\sqrt{u}}{u-4}$

12. $f(4)=4+16=20$
$f(-2)=-3$
$f(0)=-3$
$f(1)$ is not defined.

13. a. $f(x+h)=3(x+h)^2+(x+h)-2$

b. $\frac{f(x+h)-f(x)}{h}$
$= \frac{3(x+h)^2+(x+h)-2-(3x^2+x-2)}{h}$
$= \frac{3x^2+6xh+3h^2+x+h-2-3x^2-x+2}{h}$
$= \frac{6xh+3h^2+h}{h}$
$= 6x+3h+1$

14. $f(x) = -x^2,\ g(x) = 3x-2$

a. $(f+g)(x) = f(x)+g(x) = -x^2+3x-2$

b. $(f-g)(x) = f(x)-g(x)$
$= -x^2-(3x-2)$
$= -x^2-3x+2$

c. $(f-g)(-3) = -(-3)^2-3(-3)+2 = 2$

d. $(fg)(x) = f(x)g(x)$
$= (-x^2)(3x-2)$
$= -3x^3+2x^2$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{-x^2}{3x-2}$

f. $\frac{f}{g}(2) = \frac{-(2)^2}{3(2)-2} = -1$

g. $(f \circ g)(x) = f(g(x))$
$= f(3x-2)$
$= -(3x-2)^2$
$= -9x^2+12x-4$

h. $(g \circ f)(x) = g(f(x))$
$= g(-x^2)$
$= 3(-x^2)-2$
$= -3x^2-2$

i. $(g \circ f)(-4) = -3(-4)^2-2 = -48-2 = -50$

15. $f(x) = \frac{x-2}{3},\ g(x) = \frac{1}{\sqrt{x}}$

$(f \circ g)(x) = f(g(x)) = \frac{\frac{1}{\sqrt{x}}-2}{3} = \frac{1-2\sqrt{x}}{3\sqrt{x}}$

$(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{\frac{x-2}{3}}} = \sqrt{\frac{3}{x-2}}$

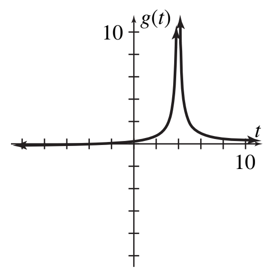
16. $y = g(t) = \frac{2}{|t-4|}$

If $y = 0$, then $0 = \frac{2}{|t-4|}$, which has no solution.

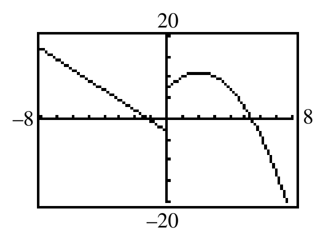
If $t = 0$, then $y = \frac{2}{4} = \frac{1}{2}$.

Intercept: $\left(0, \frac{1}{2}\right)$

Domain: all real numbers t such that $t \neq 4$
Range: all real numbers > 0



17.



$(-\infty, \infty)$

18. Slope of $y = 3x - 4$ is $m = 3$, so slope of parallel line is also $m = 3$. Thus
$y - (-1) = 3[x - (-1)]$
$y + 1 = 3x + 3$,
Slope-intercept form: $y = 3x + 2$. General form: $3x - y + 2 = 0$.

19. $-3y + 5x = 7$ $\left(\text{or } y = \frac{5}{3}x - \frac{7}{3}\right)$ has slope $\frac{5}{3}$.

Thus the line perpendicular to it has slope $-\frac{3}{5}$ and its equation is $y - 2 = -\frac{3}{5}(x-1)$, or $y = -\frac{3}{5}x + \frac{13}{5}$. A general form is $3x + 5y - 13 = 0$.

20. $y = 5x + 2$ has slope 5, and $10x - 2y = 3$ $\left(\text{or } y = 5x - \frac{3}{2}\right)$ has slope 5. Since the two slopes are equal the lines are parallel.

21. $y = F(x) = (2x-1)^2 = 4x^2 - 4x + 1$ has the quadratic form $F(x) = ax^2 + bx + c$, where $a = 4$, $b = -4$, $c = 1$.

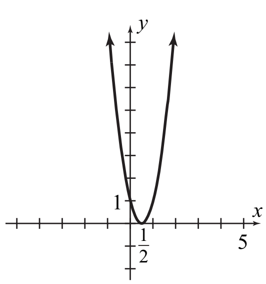
Vertex: $-\frac{b}{2a} = -\frac{-4}{2 \cdot 4} = \frac{1}{2}$

$$F\left(\frac{1}{2}\right) = \left[2\left(\frac{1}{2}\right) - 1\right]^2 = 0$$

$\Rightarrow$ Vertex $= \left(\frac{1}{2}, 0\right)$

y-intercept: $c = 1$

x-intercepts: $(2x-1)^2 = 0$, so $x = \frac{1}{2}$



22. $r = pq = (200 - 2q)q = 200q - 2q^2$, which is a quadratic function with $a = -2$, $b = 200$, $c = 0$. Since $a < 0$, r has a maximum value when $q = -\frac{b}{2a} = -\frac{200}{-4} = 50$ units. If $q = 50$, then $r = [200 - 2(50)](50) = \$5000$.

23. Let t be the number of years since Mehdi bought his car. Assuming linear depreciation, the slope is $\frac{32000 - 53000}{3} = -7000$. We also know that at $t = 0$, the function value is 53000, which is the y-intercept of the line. So a linear function for the value of the car is

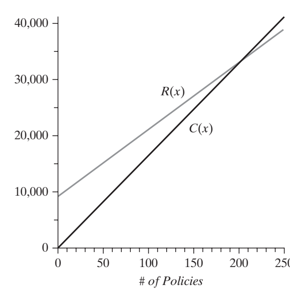
$$f(t) = -7000t + 53000$$

24. **a.** At the break-even point, Total Revenue = Total Cost:

$$R(x) = C(x)$$
$$700x = 250x + 2000$$
$$450x = 2000$$
$$x = \frac{40}{9} \approx 4.444$$

So the break-even point occurs at around 4444 policies.

b.



c. $R(6) = 4200$ and $C(6) = 3500$.

25. **a.** Let x be the dollar amount of sales made each month.

Total salary = base salary + commission.

For Option 1, 5% commission is earned only on sales over \$3000, and there will be no commission if the amount of sales made in a month is \$3000 or less. The total amount of commission earned is therefore given by the piecewise-defined function

$$\begin{cases} 0 & \text{if } x \le 3000 \\ 0.05(x - 3000) & \text{if } x > 3000 \end{cases}$$

So the salary function is

$$S_1(x) = \begin{cases} 500 & \text{if } x \le 3000 \\ 500 + 0.05(x-3000) & \text{if } x > 3000 \end{cases}$$
$$= \begin{cases} 500 & \text{if } x \le 3000 \\ 350 + 0.05x & \text{if } x > 3000 \end{cases}$$

For Option 2, 3% commission is earned on all sales, so the total amount of commission earned is $0.03x$, and we have the salary function

$$S_2(x) = 400 + 0.03x$$

b. Option 1: To make \$4000,

$$350 + 0.05x = 4000$$
$$0.05x = 3650$$
$$x = 73000$$

So Ahmad must make \$73,000 worth of sales.

Option 2: To make \$4000,

$$400 + 0.03x = 4000$$
$$0.03x = 3600$$
$$x = 120000$$

So Ahmad must make \$120,000 worth of sales.

c. For $x \le 3000$, the value of $S_2(x)$ is at most $300 + 0.03 \times 3000 = 490$, which is less than $S_1(x) = 500$. For $x > 3000$, the graph of $S_1(x)$ rises faster (i.e. has a greater slope) than the graph of $S_2(x)$. Hence the graph of $S_1(x)$ always lies above the graph of $S_2(x)$, so Ahmad will be better off with Option 1.

26. Rewrite $\begin{cases} p = 65q & \text{(supply)} \\ p = -q^2 + 2850 & \text{(demand)} \end{cases}$

as $\begin{cases} p - 65q = 0 & (1) \\ p + q^2 = 2850 & (2) \end{cases}$

Subtract (1) from (2) to get

$$q^2 + 65q = 2850$$
$$q^2 + 65q - 2850 = 0$$
$$(q-30)(q+95) = 0$$
$$q = 30 \text{ or } -95$$

Since q is a quantity of goods, it must be nonnegative. So $q = 30$ is the equilibrium quantity, and the corresponding value of p is $p = 65 \times 30 = 1950$, which means that the equilibrium price is \$1950.

27. If x is the number of unsold seats, then there are $100 - x$ passengers booked on the flight. Each passenger paid a fare of \$$(1200 + 100x)$, so the total revenue is

$$R(x) = (100 - x)(1200 + 100x)$$
$$= -100x^2 + 8800x + 120000$$

28. Total Profit = Total Revenue − Total Cost

$$P(x) = R(x) - C(x)$$
$$= (110x - x^2) - (300 + 6x)$$
$$= -x^2 + 104x - 300$$

This is a quadratic function whose graph is a parabola that opens downward. The maximum occurs at its vertex, for which

$$x = -\frac{b}{2a} = -\frac{104}{-2} = 52$$

The maximum value is $P(52) = 2404$.

Explore & Extend—Chapter 2

1. $P_1(1351) = 40 + 0.43 \times (1351 - 25) = 610.18$

$P_*(1351) = 250$

so the person would lose $610.18 - 250 = 360.18$ riyals by using plan P_1.

2. From the graph, P_2 and P_3 intersect at the point where $P_2 = 65 + 0.41(t - 100)$ and $P_3 = 110$. This occurs when

$65 + 0.41(t - 100) = 110$

$0.41t = 110 + 41 - 65 = 86$

$t = \frac{86}{0.41} \approx 209.76$

Thus, plan P_2 is better for t between 83.14 and 209.76 minutes.

3. P_3 and P_4 intersect at the point where $P_3 = 110 + 0.37(t - 250)$ and $P_4 = 190$. This occurs when

$110 + 0.37(t - 250) = 190$

$0.37t = 190 + 92.5 - 110 = 172.5$

$t = \frac{172.5}{0.37} \approx 466.22$

Thus, plan P_3 is better for t between 209.76 and 466.22 minutes.

Chapter 3

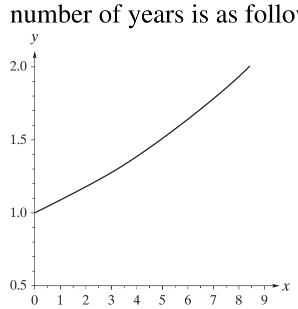
Apply It 3.1

1. The shapes of the graphs are the same. The value of A scales the value of any point by A.

2. Let P be the amount of money invested and r the rate at which the investment grows. Then after one year, the investment has increased by Pr and the total amount of money invested becomes $P + Pr = P(1 + r)$. Here $r = 8.6\% = 0.086$, so the factor by which P increases in the first year is $1 + r = 0.086$. During the second year, the investment increases from $P(1 + r)$ to $P(1 + r) + rP(1 + r) = P(1 + r)(1 + r) = P(1 + r)^2$, and the multiplicative factor is $(1 + r)^2 = (1 + 0.086)^2 = (1.086)^2 = 1.1794$. Continuing in this way, we find that the factor of increase in the third year is $(1.086)^3 = 1.2808$, and the factor of increase in the fourth year is $(1.086)^4 = 1.3910$. This pattern is shown in the table below:

Year	Multiplicative Increase	Expression as a Power
0	1	1.086^0
1	1.086	1.086^1
2	1.1794	1.086^2
3	1.2808	1.086^3
4	1.3910	1.086^4

The growth of the investment is exponential with base $1 + r = 1 + 0.086 = 1.086$. The graph of the multiplicative increase as a function of the number of years is as follows:

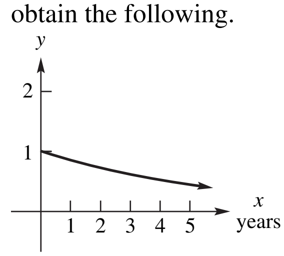


From the graph, the investment will double (i.e. the multiplicative increase reaches 2) in approximately 8.5 years.

3. If V = the value of the car and r = the annual rate at which V depreciates, then after 1 year the value of the car is $V - rV = V(1 - r)$. Since $r = 0.15$, the factor by which V decreases for the first year is $1 - r = 1 - 0.15 = 0.85$. Similarly, after the second year the value of the car is $V(1-r) - r[V(1-r)] = V(1-r)^2$. Again, since $r = 0.15$, the multiplicative decrease for the second year is $(1-r)^2 = (1-0.15)^2 = 0.72$. This pattern will continue as shown in the table.

Year	Multiplicative Decrease	Expression
0	1	0.85^0
1	0.85	0.85^1
2	0.72	0.85^2
3	0.61	0.85^3

Thus, the depreciation is exponential with a base of $1 - r = 1 - 0.15 = 0.85$. If we graph the multiplicative decrease as a function of years, we obtain the following.



4. Let t = the time for which Jihan's sister has been saving; then since Jihan is 3 years behind, $t - 3$ = the time for which Jihan has been saving. Therefore, if $y = 1.08^t$ represents the multiplicative increase in Jihan's sister's account $y = 1.08^{t-3}$ represents the multiplicative increase in Jihan's account. A graph showing the projected increase in Jihan's money will have the same shape as the graph of the projected increase in his sister's account, but will be shifted 3 units to the right.

5. $S = P(1+r)^n$

   $S = 2000(1+0.13)^5 = 2000(1.13)^5 \approx 3684.87$

   The value of the investment after 5 years will be \$3684.87. The interest earned over the first 5 years is $3684.87 - 2000 = \$1684.87$.