

# **An Introduction to Stochastic Modeling**

## **Fourth Edition**

*Instructor Solutions Manual*

**Mark A. Pinsky**

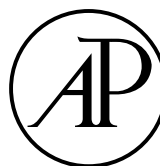
*Department of Mathematics  
Northwestern University  
Evanston, Illinois*

**Samuel Karlin**

*Department of Mathematics  
Stanford University  
Stanford, California*



AMSTERDAM • BOSTON • HEIDELBERG • LONDON  
NEW YORK • OXFORD • PARIS • SAN DIEGO  
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO  
Academic Press is an imprint of Elsevier



Academic Press is an imprint of Elsevier  
225 Wyman Street, Waltham, MA 02451, USA  
The Boulevard, Langford Lane, Kidlington, Oxford, OX5 1GB, UK

© 2011 Elsevier Inc. All rights reserved.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or any information storage and retrieval system, without permission in writing from the publisher. Details on how to seek permission, further information about the Publisher's permissions policies and our arrangements with organizations such as the Copyright Clearance Center and the Copyright Licensing Agency, can be found at our website: [www.elsevier.com/permissions](http://www.elsevier.com/permissions)

This book and the individual contributions contained in it are protected under copyright by the Publisher (other than as may be noted herein).

#### Notices

Knowledge and best practice in this field are constantly changing. As new research and experience broaden our understanding, changes in research methods, professional practices, or medical treatment may become necessary.

Practitioners and researchers must always rely on their own experience and knowledge in evaluating and using any information, methods, compounds, or experiments described herein. In using such information or methods they should be mindful of their own safety and the safety of others, including parties for whom they have a professional responsibility.

To the fullest extent of the law, neither the Publisher nor the authors, contributors, or editors, assume any liability for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions, or ideas contained in the material herein.

ISBN: 978-0-12-385232-8

For information on all Academic Press publications,  
visit our website at [www.elsevierdirect.com](http://www.elsevierdirect.com)

Typeset by: diacriTech, India

Working together to grow  
libraries in developing countries

[www.elsevier.com](http://www.elsevier.com) | [www.bookaid.org](http://www.bookaid.org) | [www.sabre.org](http://www.sabre.org)

ELSEVIER

BOOK AID  
International

Sabre Foundation

# Contents

<b>Chapter 1</b> .....	<b>1</b>
<b>Chapter 2</b> .....	<b>7</b>
<b>Chapter 3</b> .....	<b>12</b>
<b>Chapter 4</b> .....	<b>24</b>
<b>Chapter 5</b> .....	<b>31</b>
<b>Chapter 6</b> .....	<b>41</b>
<b>Chapter 7</b> .....	<b>52</b>
<b>Chapter 8</b> .....	<b>57</b>
<b>Chapter 9</b> .....	<b>61</b>
<b>Chapter 10</b> .....	<b>65</b>
<b>Chapter 11</b> .....	<b>68</b>



# Chapter 1

**2.1**  $E[\mathbf{1}\{A_1\}] = Pr\{A_1\} = \frac{1}{13}$ . Similarly,  $E[\mathbf{1}\{A_k\}] = Pr\{A_k\} = \frac{1}{13}$  for  $k = 1, \dots, 13$ . Then, because the expected value of a sum is always the sum of the expected values,  $E[N] = E[\mathbf{1}\{A_1\}] + \dots + E[\mathbf{1}\{A_{13}\}] = \frac{1}{13} + \dots + \frac{1}{13} = 1$ .

**2.2** Let  $X$  be the first number observed and let  $Y$  be the second. We use the identity  $(\sum x_i)^2 = \sum x_i^2 + \sum_{i \neq j} x_i x_j$  several times.

$$E[X] = E[Y] = \frac{1}{N} \sum x_i;$$

$$Var[X] = Var[Y] = \frac{1}{N} \sum x_i^2 - \left(\frac{1}{N} \sum x_i\right)^2 = \frac{(N-1) \sum x_i^2 - \sum_{i \neq j} x_i x_j}{N^2};$$

$$E[XY] = \frac{\sum_{i \neq j} x_i x_j}{N(N-1)};$$

$$Cov[X, Y] = E[XY] - E[X]E[Y] = \frac{\sum_{i \neq j} x_i x_j - (N-1) \sum x_i^2}{N^2(N-1)}$$

$$\tilde{n}_{X,Y} = \frac{Cov[X, Y]}{\sigma_X \sigma_Y} = -\frac{1}{N-1}.$$

**2.3** Write  $S_r = \xi_1 + \dots + \xi_r$  where  $\xi_k$  is the number of additional samples needed to observe  $k$  distinct elements, assuming that  $k-1$  distinct elements have already been observed. Then, defining  $p_k = Pr\{\xi_k = 1\} = 1 - \frac{k-1}{N}$  we have  $Pr\{\xi_k = n\} = p_k(1-p_k)^{n-1}$  for  $n = 1, 2, \dots$  and  $E[\xi_k] = \frac{1}{p_k}$ . Finally,  $E[S_r] = E[\xi_1] + \dots + E[\xi_r] = \frac{1}{p_1} + \dots + \frac{1}{p_r}$  will verify the given formula.

**2.4** Using an obvious notation, the event  $\{N = n\}$  is equivalent to either  $\overbrace{HTH \dots HTH}^{n-1}$  or  $\overbrace{THT \dots THH}^{n-1}$  so  $Pr\{N = n\} = 2 \times \left(\frac{1}{2}\right)^{n-1} \times \frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}$  for  $n = 2, 3, \dots$ ;  $Pr\{N \text{ is even}\} = \sum_{n=2,4,\dots} \left(\frac{1}{2}\right)^{n-1} = \frac{2}{3}$  and  $Pr\{N \leq 6\} = \sum_{n=2}^6 \left(\frac{1}{2}\right)^{n-1} = \frac{31}{32}$ .  
 $Pr\{N \text{ is even and } N \leq 6\} = 5 \sum_{m=2,4,6} \left(\frac{1}{2}\right)^{m-1} = \frac{21}{32}$ .

**2.5** Using an obvious notation, the probability that A wins on the  $2n+1$  trial is  $Pr\left\{\overbrace{A^c B^c \dots A^c B^c A}^{n \text{ losses}}\right\} = [(1-p)(1-q)]^n p$ ,  
 $n = 0, 1, \dots$ .  $Pr\{A \text{ wins}\} = \sum_{n=0}^{\infty} [(1-p)(1-q)]^n p = \frac{p}{1-(1-p)(1-q)}$ .  $Pr\{A \text{ wins on } 2n+1 \text{ play} | A \text{ wins}\} = (1-\pi)\pi^n$  where  $\pi = (1-p)(1-q)$ .  $E[\#\text{trials} | A \text{ wins}] = \sum_{n=0}^{\infty} (2n+1)(1-\pi)\pi^n = 1 + \frac{2\pi}{1-\pi} = \frac{1+(1-p)(1-q)}{1-(1-p)(1-q)} = \frac{2}{1-(1-p)(1-q)} - 1$ .

**2.6** Let  $N$  be the number of losses and let  $S$  be the sum. Then  $Pr\{N = n, S = k\} = \left(\frac{1}{6}\right)^{n-1} \left(\frac{5}{6} p_k\right)$  where  $p_3 = p_{11} = p_4 = p_{10} = \frac{1}{15}$ ;  $p_5 = p_9 = p_6 = p_8 = \frac{2}{15}$  and  $p_7 = \frac{3}{15}$ . Finally  $Pr\{S = k\} = \sum_{n=1}^{\infty} Pr\{N = n, S = k\} = p_k$ . (It is *not* a correct argument to simply say  $Pr\{S = k\} = Pr\{\text{Sum of 2 dice} = k | \text{Dice differ}\}$ . Compare with Exercise II, 2.1.)

**2.7** We are given that (\*)  $Pr\{U > u, W > w\} = [1 - F_U(u)][1 - F_W(w)]$  for all  $u, w$ . According to the definition for independence we wish to show that  $Pr\{U \leq u, W \leq w\} = F_U(u)F_W(w)$  for all  $u, w$ . Taking complements and using the addition law

$$Pr\{U \leq u, W \leq w\} = 1 - Pr\{U > u \text{ or } W > w\}$$

$$= 1 - [Pr\{U > u\} + Pr\{W > w\} - Pr\{U > u, W > w\}]$$

$$= 1 - [(1 - F_U(u)) + (1 - F_W(w)) - (1 - F_U(u))(1 - F_W(w))]$$

$$= F_U(u)F_W(w) \text{ after simplification.}$$

**2.8 (a)**  $E[Y] = E[a + bX] = \int (a + bx)dF_X(x) = a \int dF_X(x) + b \int x dF_X(x) = a + bE[X] = a + b\mu$ . In words, (a) implies that the expected value of a constant times a random variable is the constant times the expected value of the random variable. So  $E[b^2(X - \mu)^2] = b^2E[(X - \mu)^2]$ .

$$(b) \text{Var}[Y] = E[(Y - E\{Y\})^2] = E[(a + bX - a - b\mu)^2] = E[b^2(X - \mu)^2] = b^2E[(X - \mu)^2] = b^2\sigma^2$$

**2.9** Use the usual sums of numbers formula (See I, 6 if necessary) to establish

$$\sum_{k=1}^n k(n-k) = \frac{1}{6}n(n+1)(n-1); \text{ and}$$

$$\sum_{k=1}^n k^2(n-k) = n \sum k^2 - \sum k^3 = \frac{1}{12}n^2(n+1)(n-1), \text{ so}$$

$$E[X] = \frac{2}{n(n-1)} \sum k(n-k) = \frac{1}{3}(n+1)$$

$$E[X^2] = \frac{3}{n(n-1)} \sum k^2(n-k) = \frac{1}{6}n(n+1), \text{ and}$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = \frac{1}{18}(n+1)(n-2).$$

**2.10** Observe, for example,  $Pr\{Z = 4\} = Pr\{X = 3, Y = 1\} = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$ , using independence. Continuing in this manner,

$z$	1	2	3	4	5	6
$Pr\{Z = z\}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$

**2.11** Observe, for example,  $Pr\{W = z\} = Pr\{U = 0, V = 2\} + Pr\{U = 1, V = 1\} = \frac{1}{6} + \frac{1}{6} + \frac{1}{3}$ . Continuing in this manner, arrive at

$w$	1	2	3	4
$Pr\{W = w\}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

**2.12** Changing any of the random variables by adding or subtracting a constant will not affect the covariance. Therefore, by replacing  $U$  with  $U - E[U]$ , if necessary, etc, we may assume, without loss of generality that all of the means are zero. Because the means are zero,

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y] = E[XY] = E[UV - UW + VW - W^2] = -E[W^2] = -\sigma^2. (E[UV] = E[U]E[V] = 0, \text{ etc.})$$

**2.13**  $Pr\{v < V, U \leq u\} = Pr\{v < X \leq u, v < Y \leq u\}$   
 $= Pr\{v < X \leq u\} Pr\{v < Y \leq u\}$  (by independence)

$$= (u - v)^2$$

$$= \iint_{(u', v') : v < v' \leq u' \leq u} f_{u,v}(u', v') du' dv'$$

$$= \int_v^u \left\{ \int_{v'}^u f_{u,v}(u', v') du' \right\} dv'.$$

The integrals are removed from the last expression by successive differentiation, first w.r.t.  $v$  (changing sign because  $v$  is a lower limit) than w.r.t.  $u$ . This tells us

$$f_{u,v}(u, v) = -\frac{d}{du} \frac{d}{dv} (u - v)^2 = 2 \text{ for } 0 < v \leq u \leq 1.$$

**3.1**  $Z$  has a discrete uniform distribution on  $0, 1, \dots, 9$ .

**3.2** In maximizing a continuous function, we often set the derivative equal to zero. In maximizing a function of a discrete variable, we equate the ratio of successive terms to one. More precisely,  $k^*$  is the smallest  $k$  for which  $\frac{p^{(k+1)}}{p^{(k)}} < 1$ , or, the smallest  $k$  for which  $\frac{n-k}{k+1} \left( \frac{p}{1-p} \right) < 1$ . Equivalently, (b)  $k^* = \lceil (n+1)p \rceil$  where  $\lceil x \rceil =$  greatest integer  $\leq x$ . for (a) let  $n \rightarrow \infty, p \rightarrow 0, \lambda = np$ . Then  $k^* = \lceil \lambda \rceil$ .

**3.3** Recall that  $e^\lambda = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots$  and  $e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \dots$  so that  $\sinh \lambda \equiv \frac{1}{2}(e^\lambda - e^{-\lambda}) = \lambda + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \dots$ . Then  $\Pr\{X \text{ is odd}\} = \sum_{k=1,3,5,\dots} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda} \sinh(\lambda) = \frac{1}{2}(1 - e^{-2\lambda})$ .

$$\begin{aligned} \mathbf{3.4} \quad E[V] &= \sum_{k=0}^{\infty} \frac{1}{k+1} \frac{\lambda^k e^{-\lambda}}{k!} = \frac{e^{-\lambda}}{\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{(k+1)!} \\ &= \frac{1}{\lambda} e^{-\lambda} (e^\lambda - 1) = \frac{1}{\lambda} (1 - e^{-\lambda}). \end{aligned}$$

$$\begin{aligned} \mathbf{3.5} \quad E[XY] &= E[X(N-X)] = NE[X] - E[X^2] \\ &= N^2 p - [Np(1-p) + N^2 p^2] = N^2 p(1-p) - Np(1-p) \end{aligned}$$

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y] = -Np(1-p).$$

**3.6** Your intuition should suggest the correct answers: (a)  $X_1$  is binomially distributed with parameters  $M$  and  $\pi_1$ ; (b)  $N$  is binomial with parameters  $M$  and  $\pi_1 + \pi_2$ ; and (c)  $X_1$ , given  $N = n$ , is conditionally binomial with parameters  $n$  and  $p = \pi_1 / (\pi_1 + \pi_2)$ . To derive these correct answers formally, begin with

$$\Pr\{X_1 = i, X_2 = j, X_3 = k\} = \frac{M!}{i!j!k!} \pi_1^i \pi_2^j \pi_3^k; i+j+k = M.$$

Since  $k = M - (i+j)$

$$\Pr\{X_1 = i, X_2 = j\} = \frac{M!}{i!j!(M-i-j)!} \pi_1^i \pi_2^j \pi_3^{M-i-j}; 0 \leq i+j \leq M.$$

$$\begin{aligned} \mathbf{(a)} \quad \Pr\{X_1 = i\} &= \sum_j \Pr\{X_1 = i, X_2 = j\} \\ &= \frac{M!}{i!(M-i)!} \pi_1^i \sum_{j=0}^{M-i} \frac{(M-i)!}{j!(M-i-j)!} \pi_2^j \pi_3^{M-i-j} \\ &= \binom{M}{i} \pi_1^i (\pi_2 + \pi_3)^{M-i}, i = 0, 1, \dots, M. \end{aligned}$$

**(b)** Observe that  $N = n$  if and only if  $X_3 = M - n$ . Apply the results of (a) to  $X_3$ :

$$\Pr\{N = n\} = \Pr\{X_3 = M - n\} = \frac{M!}{n!(M-n)!} (\pi_1 + \pi_2)^n \pi_3^{M-n}$$

$$\begin{aligned} \mathbf{(c)} \quad \Pr\{X_1 = k | N = n\} &= \frac{\Pr\{X_1 = k, X_2 = n - k\}}{\Pr\{N = n\}} \\ &= \frac{\frac{M!}{k!(M-n)!(n-k)!} \pi_1^k \pi_2^{n-k} \pi_3^{M-n}}{\frac{M!}{n!(M-n)!} (\pi_1 + \pi_2)^n \pi_3^{M-n}} \\ &= \frac{n!}{k!(n-k)!} \left( \frac{\pi_1}{\pi_1 + \pi_2} \right)^k \left( \frac{\pi_2}{\pi_1 + \pi_2} \right)^{n-k}, k = 0, 1, \dots, n. \end{aligned}$$

$$\begin{aligned}
 3.7 \Pr\{Z = n\} &= \sum_{k=0}^n \Pr\{X = k\}\Pr\{Y = n - k\} \\
 &= \sum_{k=0}^n \frac{\mu^k e^{-\mu} \nu^{(n-k)} e^{-\nu}}{k!(n-k)!} = e^{-(\mu+\nu)} \frac{1}{n!} \sum_{k=0}^n \frac{n!}{k!(n-k)!} \mu^k \nu^{n-k} \\
 &= \frac{e^{-(\mu+\nu)} (\mu + \nu)^n}{n!} \quad (\text{Using binomial formula.})
 \end{aligned}$$

$Z$  is Poisson distributed, parameter  $\mu + \nu$ .

**3.8 (a)**  $X$  is the sum of  $N$  independent Bernoulli random variables, each with parameter  $p$ , and  $Y$  is the sum of  $M$  independent Bernoulli random variables each with the same parameter  $p$ .  $Z$  is the sum of  $M + N$  independent Bernoulli random variables, each with parameter  $p$ .

**(b)** By considering the ways in which a committee of  $n$  people may be formed from a group comprised of  $M$  men and  $N$  women, establish the identity  $\binom{M+N}{n} = \sum_{k=0}^n \binom{N}{k} \binom{M}{n-k}$ .

Then

$$\begin{aligned}
 \Pr\{Z = n\} &= \sum_{k=0}^n \Pr\{X = k\}\Pr\{Y = n - K\} \\
 &= \sum_{k=0}^n \binom{N}{k} p^k (1-p)^{N-k} \binom{M}{n-k} p^{n-k} (1-p)^{M-n+k} \\
 &= \binom{M+N}{n} p^n (1-p)^{M+N-n} \text{ for } n = 0, 1, \dots, M+N.
 \end{aligned}$$

Note:

$$\binom{N}{k} = 0 \text{ for } k > N.$$

$$\begin{aligned}
 3.9 \Pr\{X + Y = n\} &= \sum_{k=0}^n \Pr\{X = k, Y = n - k\} = \sum_{k=0}^n (1-\pi)\pi^k (1-\pi)\pi^{n-k} \\
 &= (1-\pi)^2 \pi^n \sum_{k=0}^n 1 = (n+1)(1-\pi)^2 \pi^n \text{ for } n \geq 0.
 \end{aligned}$$

**3.10**

$k$	Binomial $n = 10 \ p = .1$	Binomial $n = 100 \ p = .01$	Poisson $\lambda = 1$
0	.349	.366	.368
1	.387	.370	.368
2	.194	.185	.184

$$3.11 \Pr\{U = u, W = 0\} = \Pr\{X = u, Y = u\} = (1-\pi)^2 \pi^{2u}, u \geq 0.$$

$$\Pr\{U = u, W = w > 0\} = \Pr\{X = u, Y = u + w\} + \Pr\{Y = u, X = u + w\} = 2(1-\pi)^2 \pi^{2u+w}$$

$$\Pr\{U = u\} = \sum_{w=0}^{\infty} \Pr\{U = u, W = w\} = \pi^{2u} (1-\pi^2).$$

$$\Pr\{W = 0\} = \sum_{w=0}^{\infty} \Pr\{U = u, W = 0\} = (1-\pi)^2 / (1-\pi^2).$$

$$\Pr\{W = w > 0\} = 2 \left[ (1-\pi)^2 / (1-\pi)^2 (1-\pi^2) \right] \pi^w, \text{ and}$$

$$\Pr\{U = u, W = w\} = \Pr\{U = u\}\Pr\{W = w\} \text{ for all } u, w.$$



**3.12** Let  $X$  = number of calls to switch board in a minute.  $Pr\{X \geq 7\} = 1 - \sum_{k=0}^6 \frac{4^k e^{-4}}{k!} = .111$ .

**3.13** Assume that inspected items are independently defective or good. Let  $X$  = # of defects in sample.

$$Pr\{X = 0\} = (.95)^{10} = .599$$

$$Pr\{X = 1\} = 10(.95)^9(.05) = .315$$

$$Pr\{X \geq 2\} = 1 - (.599 + .315) = .086.$$

**3.14 (a)**  $E[Z] = \frac{1-p}{p} = 9$ ,  $Var[Z] = \frac{1-p}{p^2} = 90$

**(b)**  $Pr\{Z > 10\} = (.9)^{10} = .349$ .

**3.15**  $Pr\{X \leq 2\} = \left(1 + 2 + \frac{2^2}{2}\right)e^{-2} = 5e^{-2} = .677$ .

**3.16 (a)**  $p_0 = 1 - b \sum_{k=1}^{\infty} (1-p)^k = 1 - b\left(\frac{1-p}{p}\right)$ .

**(b)** When  $b = p$ , then  $p_k$  is given by (3.4).

When  $b = \frac{p}{1-p}$ , then  $p_k$  is given by (3.5).

**(c)**  $Pr\{N = n > 0\} = Pr\{X = 0, Z = n\} + Pr\{X = 1, Z = n - 1\}$   
 $= (1 - \alpha)p(1 - p)^n + \alpha p(1 - p)^{n-1}$   
 $= [(1 - \alpha)p + \alpha p / (1 - p)](1 - p)^n$

So  $b = (1 - \alpha)p + \alpha p / (1 - p)$ .

$$4.1 \ E[e^{\lambda Z}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\frac{1}{2}z^2 + \lambda z} dz = e^{\frac{1}{2}\lambda^2} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\frac{1}{2}(z-\lambda)^2} dz \right\} = e^{\frac{1}{2}\lambda^2}.$$

**4.2 (a)**  $PrW > \frac{1}{\theta} = e^{-\theta/\theta} = e^{-1} = .368 \dots$

**(b)** Mode = 0.

**4.3**  $X - \theta$  and  $Y - \theta$  are both uniform over  $\left[-\frac{1}{2}, \frac{1}{2}\right]$ , independent of  $\theta$ , and  $W = X - Y = (X - \theta) - (Y - \theta)$ . Therefore the distribution of  $W$  is independent of  $\theta$  and we may determine it assuming  $\theta = 0$ . Also, the density of  $W$  is symmetric since that of both  $X$  and  $Y$  are.

$$Pr\{W > w\} = Pr\{X > Y + w\} = \frac{1}{2}(1 - w)^2, \quad w > 0$$

So  $f_w(w) = 1 - w$  for  $0 \leq w \leq 1$  and  $f_w(w) = 1 - |w|$  for  $-1 \leq w \leq +1$

**4.4**  $\mu_c = .010$ ;  $\sigma_c^2 = (.005)^2$ ,  $Pr\{C < 0\} = Pr\left\{\frac{C - .010}{.005} < \frac{-.010}{.005}\right\} = Pr\{Z < -2\} = .0228$ .

**4.5**  $Pr\{Z < Y\} = \int_0^{\infty} \left\{ \int_x^{\infty} 3e^{-3y} dy \right\} 2e^{-2x} dx = \frac{2}{5}$ .

**5.1**  $Pr\{N > k\} = Pr\{X_1 \leq \xi, \dots, X_k \leq \xi\} = [F(\xi)]^k, k = 0, 1, \dots$

$Pr\{N = k\} = Pr\{N > k - 1\} - Pr\{N > k\} = [1 - F(\xi)]F(\xi)^{k-1}, k = 1, 2, \dots$

**5.2**  $Pr\{Z > z\} = Pr\{X_1 > z, \dots, X_n > z\} = Pr\{X_1 > z\} \cdots Pr\{X_n > z\}$   
 $= e^{-\lambda z} \cdots e^{-\lambda z} = e^{-n\lambda z}, z > 0$ .

$Z$  is exponentially distributed, parameter  $n\lambda$ .

**5.3**  $Pr\{X > k\} = \sum_{l=k+1}^{\infty} p(1-p)^l = p(1-p)^{k+1}, k = 0, 1, \dots$

$$E[X] = \sum_{k=0}^{\infty} Pr\{X > k\} = \frac{1-p}{p}.$$

**5.4** Write  $V = V^+ - V^-$  when  $V^+ = \max\{V, 0\}$  and  $V^- = \max\{-V, 0\}$ . Then  $Pr\{V^+ > v\} = 1 - F_v(v)$  and  $Pr\{V^- > v\} = F_v(-v)$  for  $v > 0$ . Use (5.3) on  $V^+$  and  $V^-$  together with  $E[V] = E[V^+] - E[V^-]$ . Mean does not exist if  $E[V^+] = E[V^-] = \infty$ .

**5.5**  $E[W^2] = \int_0^\infty P\{W^2 > t\} dt = \int_0^\infty [1 - F_w(\sqrt{t})] dt = \int_0^\infty 2y[1 - F_w(y)] dy$  by letting  $y = \sqrt{t}$ .

**5.6**  $Pr\{V > t\} = \int_t^\infty \lambda e^{-\lambda v} dv = e^{-\lambda t}$ ;  $E[V] = \int_0^\infty Pr\{V > t\} dt = \frac{1}{\lambda} \int_0^\infty \lambda e^{-\lambda t} dt = \frac{1}{\lambda}$ .

**5.7**  $Pr\{V > v\} = Pr\{X_1 > v, \dots, X_n > v\} = Pr\{X_1 > v\} \cdots Pr\{X_n > v\}$   
 $= e^{-\lambda_1 v} \cdots e^{-\lambda_n v} = e^{-(\lambda_1 + \cdots + \lambda_n)v}$ ,  $v > 0$ .

$V$  is exponentially distributed with parameter  $\sum_i \lambda_i$ .

**5.8**

<b>Spares</b>	3	2	1	0
<b>A</b>				
<b>B</b>				
<b>Mean</b>	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$

Expected flash light operating duration =  $\frac{1}{2\lambda} + \frac{1}{2\lambda} + \frac{1}{2\lambda} + \frac{1}{2\lambda} = \frac{2}{\lambda} = 2$  Expected battery operating durations!