Solutions Manual

to accompany

Introduction to Nuclear Engineering 3/e

By

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These solutions are the product of many people including the late John Lamarsh and his students as well as the students at Penn State who used this text. I wish to thank all of them including the graduate assistants who worked with me to develop the course on which this 3rd edition is based.

Tony

Chapter 2

1. How many neutrons and protons are there in the nuclei of the following atoms:
   (a) ^{7}Li,
   (b) ^{24}Mg,
   (c) ^{135}Xe,
   (d) ^{209}Bi,
   (e) ^{222}Rn?

(a) $Z = 3$, $N = 4$; (b) $Z = 12$, $N = 12$; (c) $Z = 54$, $N = 81$; (d) $Z = 83$, $N = 126$; (e) $Z = 86$, $N = 136$.

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3. How many atoms are there in 10 g of ^{12}C?

10 g = 10/12 mole and contains $(10/12) \times 0.6022 \times 10^{24} = 0.502 \times 10^{24}$ atoms.

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5. When H_2 gas is formed from naturally occurring hydrogen, what percentages of the molecules have molecular weights of approximately 2, 3, and 4?

The molecules in question are $^1H^1H$, $^1H^2H$ or $^2H^1H$, and $^2H^2H$. Let $\gamma(1)$ and $\gamma(2)$ be the fractions of naturally-occurring hydrogen that are ^1H and ^2H. Combining atoms pulled successively from a large volume of hydrogen, the fraction that has weight 2 is $\gamma(1) \times \gamma(1) = (0.99985)^2 = 0.9997 = 99.97\%$. The fraction with weight 4 is $\gamma(2) \times \gamma(2) = 2.25 \times 10^{-8} = 2.25 \times 10^{-6}\%$. Weight 3 can be formed in two ways. The fraction of 3 is $\gamma(1)\gamma(2) + \gamma(2)\gamma(1) = 2\gamma(1)\gamma(2) = 2 \times 0.99985 \times 0.00015 = 3.00 \times 10^{-4} = 0.03\%$. Note that the fraction that have molecular weights of either 2, 3, or 4 is $\gamma^2(1) + 2\gamma(1)\gamma(2) + \gamma^2(2) = (\gamma(1) + \gamma(2))^2 = 1$.

7. A beaker contains 50 g of ordinary (i.e., naturally occurring) water.
   (a) How many moles of water are present?
   (b) How many hydrogen atoms?
   (c) How many deuterium atoms?

(a) Number of moles = 50/18.015 = 2.775.
(b) No. of molecules = $2.775 N_A = 1.671 \times 10^{24}$.
There are 2 atoms of H per molecule, so no. of atoms of H = $2 \times 1.671 \times 10^{24} = 3.342 \times 10^{24}$.
(c) No. of ^2H atoms = $1.5 \times 10^{-4} \times 3.342 \times 10^{24} = 5.013 \times 10^{20}$.

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9. Compute the mass of a proton in amu.

Mass of proton = 1.67265×10^{-27} kg
1 amu = 1.66057×10^{-27} Kg. Mass of proton in amu is

$$m_p(\text{amu}) = \frac{1.67265 \times 10^{-27}\,\text{Kg}}{1.66057 \times 10^{-27}\,\text{Kg/amu}}$$

$$= 1.0073 \text{ amu}$$

11. Show that 1 amu is numerically equal to the reciprocal of Avogadro's number.

Avogadro's number is $.6022 \times 10^{24}$

$$\frac{1}{.6022 \times 10^{24}\ \text{atoms/gm mole}}$$

$$= 1.66057 \times 10^{-27}\,\text{Kg}$$

13. Using Eq. (2.3), estimate the density of nuclear matter in g/cm^3; in Kg/m^3. Take the mass of each nucleon to be approximately 1.5×10^{-24} g.

The volume of the nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (1.25 \times 10^{-13})^3 A.$$

The nucleon density is then

$$\frac{A}{V} = \frac{3 \times 10^{39}}{4\pi (1.25)^3} \text{ nucleons/cm}^3;$$

and the mass density is

$$\rho = \frac{3 \times 10^{39}}{4\pi (1.25)^3} \times 1.5 \times 10^{-24} = 1.96 \times 10^{14} \text{ g/cm}^3.$$

$$= 1.96 \times 10^{11} \text{ kg/m}^3$$

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15. The complete combustion of 1 kg of bituminous coal releases about 3×10^7 J in heat energy. The conversion of 1 g of mass into energy is equivalent to the burning of how much coal?

1 gram is equivalent to $1 \times (3 \times 10^{10})^2 = 9 \times 10^{20}$ ergs $= 9 \times 10^{13}$ joules. From Table I.8, $1 \text{ J} = 9.478 \times 10^{-4}$ Btu. Thus $1\text{g} \sim 9 \times 10^{13} \times 9.478 \times 10^{-4} = 8.530 \times 10^{10}$ Btu. This is the heat released from $8.530 \times 10^{10} / 13000 \times 2000 = 3280$ tons of coal.

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17. Compute the neutron–proton mass difference in MeV.

Using values from Table II.1, $\Delta M = M_n - M_p = 1.008665 - 1.007277 = 0.001388$ amu. Since 1 amu = 931.481 MeV, $\Delta M = 0.001388 \times 931.481 = 1.293$ MeV.

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19. Derive Eq. (2.18). [*Hint:* Square both sides of Eq. (2.5) and solve for mv.]

From Eq. (2.5),

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$m^2 = \frac{m_0^2}{1 - v^2/c^2}$$

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4.$$

But $mc^2 = E_{tot}$, $m_0c^2 = E_{rest}$, and $mv = p$.

Thus,

$$p = \frac{1}{c}\sqrt{E_{tot}^2 - E_{rest}^2}.$$

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21. Using the result derived in Problem 2.20, calculate the speed of a 1-MeV electron, one with a kinetic energy of 1 MeV.

For an electron, $E_{rest} = 0.511$ MeV, and if its kinetic energy is 1 MeV, then $E_{tot} = 1.511$ MeV.

From prob. 2.16,

$$v = c\sqrt{1 - \left(\frac{0.511}{1.511}\right)^2} = 0.941c = 2.82 \times 10^{10} \text{ cm/sec.}$$

$$= 2.82 \times 10^8 \text{ m/sec}$$

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23. Show that the wavelength of a relativistic particle is given by

$$\lambda = \lambda_C \frac{m_e c^2}{\sqrt{E_{\text{total}}^2 - E_{\text{rest}}^2}},$$

where $\lambda_C = h/m_e c = 2.426 \times 10^{-10}$ cm is called the *Compton wavelength.*

From Eq. (2.19),

$$\lambda = \frac{hc}{\sqrt{E_{tot}^2 - E_{rest}^2}} = \frac{h}{m_e c}\frac{m_e c^2}{\sqrt{E_{tot}^2 - E_{rest}^2}}$$

$$= \lambda_C \frac{m_e c^2}{\sqrt{E_{tot}^2 - E_{rest}^2}}$$

25. An electron moves with a kinetic energy equal to its rest-mass energy. Calculate the electron's

(a) total energy in units of $m_e c^2$;
(b) mass in units of m_e;
(c) speed in units of c;
(d) wavelength in units of the Compton wavelength.

(a) $E_{tot} = 2m_e c^2$. (b) $mc^2 = E_{tot} = 2m_e c^2$, and

$m = 2m_e$. (c) From prob. 2.16,

$$v = c\sqrt{1-\left(\frac{m_e c^2}{2m_e c^2}\right)^2} = 0.866c.$$

(d) From prob. 2.19,

$$\lambda = \lambda_c \frac{m_e c^2}{\sqrt{(2m_e c^2)^2-(m_e c^2)^2}} = 0.577\lambda_c$$

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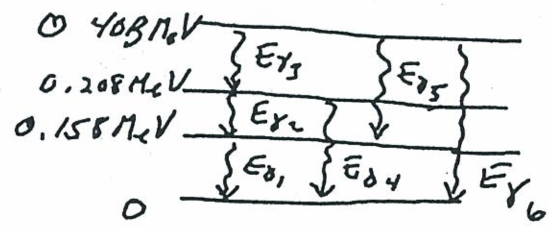
27. The first three excited states of the nucleus of ^{199}Hg are at 0.158 MeV, 0.208 MeV, and 0.403 MeV above the ground state. If all transitions between these states and ground occurred, what energy γ-rays would be observed?

^{199}Hg



$\gamma_1 = 0.158$ MeV
$\gamma_2 = 0.05$ MeV
$\gamma_3 = 0.195$ MeV
$\gamma_4 = 0.208$ MeV
$\gamma_5 = 0.245$ MeV
$\gamma_6 = 0.403$ MeV

29. Tritium (^3H) decays by negative beta decay with a half-life of 12.26 years. The atomic weight of ^3H is 3.016.

(a) To what nucleus does ^3H decay?

(b) What is the mass in grams of 1 mCi of tritium?

(a) Tritium ^3H consists of a proton and two neutrons. Beta decay results in transformation of a n into a proton plus an electron. The product has 2 protons and 1 neutron and is thus an isotope of helium

$$^3H \rightarrow {}^3He + \beta^-$$

(b) Activity, α, is λn. For tritium, the decay constant λ is

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

$$\lambda = \frac{\ln 2}{12.26\,\text{yr} \times \frac{365\,\text{days}}{\text{yr}} \times \frac{24\,\text{hr}}{\text{day}} \times \frac{3600\,\text{sec}}{\text{hr}}}$$

$$= 1.793 \times 10^{-9}\,\text{sec}^{-1}$$

$$\alpha = 10^{-3}\,\text{Ci} \times 3.7 \times 10^{10}\,\frac{\text{dis/sec}}{\text{Ci}} = 3.7 \times 10^{7}\,\text{dis/sec}$$

$$n_{^3H} = \frac{\alpha}{\lambda}$$

$$= \frac{3.7 \times 10^{7}\,\text{dis/sec}}{1.793 \times 10^{-9}\,\text{sec}^{-1}}$$

$$= 2.064 \times 10^{16}\,\text{atoms}$$

$$\text{Mass} = n_{^3H} \times \text{mass Tritium atom}$$

$= 2.064 \times 10^{14}$ atoms $\times \frac{3 g/g\text{atom}}{.6022 \times 10^{24} \text{atoms}/g\text{-atom}}$

$= 1.03 \times 10^{-7} g$

31. Carbon tetrachloride labeled with ^{14}C is sold commercially with an activity of 10 millicuries per millimole (10 mCi/mM). What fraction of the carbon atoms is ^{14}C?

Carbon tetrachloride is CCl_4 There is one atom of carbon per molecule. In 1mM there are 1mM of carbon atoms or $.6022 \times 10^{24}$ atoms/mole $\times 10^{-3}$ mole/mM = $.6022 \times 10^{21}$ atoms C/mM. If the sample contains 10 mCi then there are $3.7 \times 10^{10} \frac{\text{dis/sec}}{\text{Ci}} \times 10^{-3}$ Ci/mCi or 3.7×10^{7} dis/sec due to ^{14}C. The activity $\alpha = \lambda N$.

$\lambda = 0.693/T_{1/2}$. $T_{1/2} = 5736 y \times \frac{365 d}{y} \times \frac{24 hrs}{d} \times \frac{3600 sec}{hr} = 1.809 \times 10^{11} s$

$\lambda = 3.83 \times 10^{-12} sec^{-1}$, $N = \frac{\alpha}{\lambda}$ $\frac{3.7 \times 10^{7}}{3.83 \times 10^{-12}} = 9.66 \times 10^{18}$ atoms

The fraction of carbon that is carbon 14 in 1mM is then $9.66 \times 10^{18} / .6022 \times 10^{21} = 1.6 \times 10^{-2}$ or 1.6%.

33. After the initial cleanup effort at Three Mile Island, approximately 400,000 gallons of radioactive water remained in the basement of the containment building of the Three Mile Island Unit 2 nuclear plant. The principal sources of this radioactivity were ^{137}Cs at 156 $\mu Ci/cm^3$ and ^{134}Cs at 26 $\mu Ci/cm^3$. How many atoms per cm^3 of these radionuclides were in the water at that time?

$\alpha(^{137}Cs) = 156 \mu Ci/cm^3$ and $t_{1/2}^{137} =$ 30.2 yr

$$\lambda_{137} = \frac{\ln 2}{t_{1/2}^{137}} = \frac{\ln 2}{30.2 yr} = 0.0230 yr^{-1}$$

$\alpha(^{134}Cs) = 26 \mu Ci/cm^3$ and $t_{1/2}^{134} =$ 2.06 yr.

$$\lambda_{134} = \frac{\ln 2}{t_{1/2}^{134}} = \frac{\ln 2}{2.06 yr} = 0.3365 yr^{-1}$$

$$\alpha_{137} = \lambda_{137} n_{137}$$

$$n_{137} = \frac{\alpha_{137}}{\lambda_{137}} = \frac{(156 \times 10^{-6} Ci/cm^3)(3.7 \times 10^{10} dis/sec\text{-}Ci)}{(0.0230 yr^{-1})} \times \left(\frac{3.15 \times 10^7 sec}{yr}\right)$$

$$= 7.91 \times 10^{15} atoms/cm^3$$

$$n_{134} = \frac{\alpha_{134}}{\lambda_{134}} = \frac{(26 \times 10^{-6} Ci/cm^3)(3.7 \times 10^{10} \frac{d}{sec\text{-}Ci})}{0.3365 yr^{-1}} \times \left(\frac{3.15 \times 10^7 s}{yr}\right)$$

$$= 9.01 \times 10^{13} atoms/cm^3$$

35. Polonium-210 decays to the ground state of ^{206}Pb by the emission of a 5.305-MeV α-particle with a half-life of 138 days. What mass of ^{210}Po is required to produce 1 MW of thermal energy from its radioactive decay?

$$^{210}Po \longrightarrow {}^{206}Pb + \alpha$$

$\alpha(^{210}Po)$ = # of disint/sec

$\mathcal{E}$ = energy released/disint = 5.305 MeV

$$\text{Power} = \alpha \mathcal{E} = P$$

$$\alpha = \lambda_{210} n_{210}$$

$$n_{210} = \frac{P}{\lambda_{210} \mathcal{E}}$$

$$\lambda_{210} = \frac{\ln 2}{t_{1/2}^{210}} = \frac{\ln 2}{138\,\text{da.}} \left(\frac{1\,\text{da.}}{24\,\text{hrs}}\right)\left(\frac{\text{hr}}{3600\,\text{sec}}\right)$$

$$= 5.81 \times 10^{-8}\,\text{sec}^{-1}$$

$$n^{210} = \frac{1\,\text{MW}}{(5.81 \times 10^{-8}\,\text{sec}^{-1})(5.305\,\text{MeV})} \times \left(\frac{10^6\,\text{W}}{1\,\text{MW}}\right)\left(\frac{1\,\text{J/s}}{1\,\text{W}}\right)\left(\frac{\text{MeV}}{10^6\,\text{eV}}\right)\frac{\text{eV}}{1.6 \times 10^{-19}\,\text{J}}$$

$$\text{mass}\ ^{210}P = \frac{n^{210} M_{210}}{N_A}$$

$$= \frac{(2.03 \times 10^{25}\,\text{atoms})(210\,\text{g/mole})}{(6.02 \times 10^{23}\,\text{atoms/mole})}$$

$$= 7.08 \times 10^3\,\text{g} = 7.08\,\text{Kg.}$$

37. Since the half-life of ^{235}U (7.13×10^8 years) is less than that of ^{238}U (4.51×10^9 years), the isotopic abundance of ^{235}U has been steadily decreasing since the earth was formed about 4.5 billion years ago. How long ago was the isotopic abundance of ^{235}U equal to 3.0 *a/o*, the enrichment of the uranium used in many nuclear power plants?

$$t_{1/2}^{235} = 7.13 \times 10^9 \text{ yr}$$

$$\lambda^{235} = \ln 2 / t_{1/2}^{235} = 9.72 \times 10^{-10} \text{ yr}^{-1}$$

$$t_{1/2}^{238} = 4.51 \times 10^9 \text{ yr}$$

$$\lambda^{238} = \ln 2 / t_{1/2}^{238} = 1.54 \times 10^{-10} \text{ yr}^{-1}$$

$$n^{235}(t) = n_0^{235} e^{-\lambda^{235} t}$$

$$n^{238}(t) = n_0^{238} e^{-\lambda^{238} t}$$

At $t = 0$ isotopic abundance $^{235}U = 0.03 = \gamma_{235}$

Since n^{235} and $n^{238} >> n^{234}$

$$\gamma_{235} = \frac{n_0^{235}}{n_0^{234} + n_0^{235} + n_0^{238}} = \frac{n_0^{235}}{n_0^{235} + n_0^{238}} = 0.03$$

$$n_0^{235} = .03\, n_0^{235} + .03\, n_0^{238}$$

or $0.97\, n_0^{235} = .03\, n_0^{238}$

$$\Rightarrow \frac{n_0^{235}}{n_0^{238}} = \frac{.03}{0.97} = 0.03093 \text{ at } t = 0$$

Today $\gamma_{235} = .0072 = \dfrac{n^{235}}{n^{235} + n^{238}}$

$$\Rightarrow 0.9928\, n^{235} = 0.0072\, n^{238}$$

$$\frac{n^{238}}{n^{235}} = \frac{0.9928}{0.0072} = \frac{1}{0.00725}$$

Dividing the 2 decay equations yields

$$\frac{n^{235}(t)}{n^{238}(t)} = \frac{n_0^{235}}{n_0^{238}} e^{(\lambda^{238}-\lambda^{235})t}$$

Solving for t

$$t = \frac{1}{\lambda^{238}-\lambda^{235}} \ln\left(\frac{n^{235}(t)/n^{238}(t)}{n_0^{235}/n_0^{238}}\right)$$

$$t = \frac{1}{1.54\times10^{-10}\,yr^{-1} - 9.72\times10^{-10}\,yr^{-1}} \ln\left(\frac{0.00725}{0.03093}\right)$$

$$t = 1.77\times10^{9}\,yr$$

39. Consider the chain decay

$$A \rightarrow B \rightarrow C \rightarrow,$$

with no atoms of B present at $t = 0$.

(a) Show that the activity of B rises to a maximum value at the time t_m given by

$$t_m = \frac{1}{\lambda_B - \lambda_A} \ln\left(\frac{\lambda_B}{\lambda_A}\right),$$

at which time the activities of A and B are equal.

(b) Show that, for $t < t_m$, the activity of B is less than that of A, whereas the reverse is the case for $t > t_m$.

a. From Eq. 2.33, the activity of B is given by:

$$\alpha_B = \alpha_{B0} e^{-\lambda_B t} + \frac{\alpha_{A0}\lambda_B}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

Assuming no initial atoms of B drops the first term since $\alpha_{B0} = 0$ then. The activity of B is a maximum (or maximum) when $\frac{d\alpha_B}{dt} = 0$. Taking the derivative w.r.t. time of α_B gives:

$$\frac{d\alpha_B}{dt} = \frac{\alpha_{A0}\lambda_B}{\lambda_B - \lambda_A}\left(-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}\right) = 0$$

rearranging

$$e^{-(\lambda_A - \lambda_B)t} = \frac{\lambda_B}{\lambda_A}$$

taking ln of both sides and solve for t_m gives

$$t_m = \frac{1}{\lambda_B - \lambda_A}\left(\ln\frac{\lambda_B}{\lambda_A}\right)$$

b.) For $t < t_m$, $\frac{d\alpha_B}{dt}$ has to be > 0 since at $t = t_m$ $\frac{d\alpha_B}{dt} = 0$ and α_B is a maximum. For

$\frac{d\alpha_B}{dt} > 0$ then for $\frac{d\alpha_B}{dt} > 0$ need $\frac{dn_B}{dt} > 0$

since $\alpha_B = \lambda_B n_B$ But $\frac{dn_B}{dt} = -\lambda_B n_B + \lambda_A n_A$

$= -\alpha_B + \alpha_B$

Then $\alpha_B > \alpha_B$. For $\frac{d\alpha_B}{dt} < 0$ reverse is true

41.. Show that the abundance of ^{234}U can be explained by assuming that this isotope originates solely from the decay of ^{238}U.

^{238}U and ^{234}U are nuclides in the Uranium Series. The isotopes in the first portion of the series are ^{238}U, ^{234}Th, ^{234m}Pa, ^{234}Pa, and ^{234}U. The half-life of ^{238}U is 4.51×10^9 yrs The half-lives of the other isotopes are much shorter than ^{238}U Thus, the decay constant of ^{238}U is much, much less than the decay constants of the other members of the series. The daughter products of ^{238}U are in secular equilibrium and all decay at the rate set by ^{238}U. One can show that

$$\lambda_{238} N_{238} = \lambda_{234} N_{234}$$

or

$$\frac{\lambda_{238}}{\lambda_{234}} = \frac{N_{234}}{N_{238}}.$$

and

$$\frac{T_{1/2\,234}}{T_{1/2\,238}} = \frac{N_{234}}{N_{238}}$$

Since $T_{1/2\,234} = 2.48 \times 10^5$ yrs

$$\frac{2.48 \times 10^5 \text{ yrs}}{4.51 \times 10^9 \text{ yrs}} = 5.5 \times 10^{-5}$$

From Appendix II table II.2 the abundance of ^{234}U is 0.0055 a/o and of ^{238}U 99.27 a/o The ratio is then 5.5×10^{-5}. Thus ^{234}U can be assumed to originate from the decay of ^{238}U.

43. According to U.S. Nuclear Regulatory Commission regulations, the maximum permissible concentration of radon-222 in air in equilibrium with its short-lived daughters is 3 pCi/liter for nonoccupational exposure. This corresponds to how many atoms of radon-222 per cm^3?

An activity of 3 pCi/liter = 3×10^{-12} Ci/liter. From the definition of Ci = 3.7×10^{10} dis/sec then the concentration of 3 pCi/liter yields 11.1×10^{-1} dis/sec or 1.11 dis/sec-liter. This is the ^{222}Rn activity in a liter. But the activity is $\lambda_{Rn} N_{Rn}$ then the number of atoms in a liter that yield 1.11 dis/sec is

$$N_{Rn} = \frac{1.11 \text{ dis/sec}}{\lambda_{Rn}}$$

From the chart of the nuclides, $T_{1/2} = 3.8$ dy and thus $\lambda = 2.11\times10^{-6}\, sec^{-1}$. The number of atoms is $\underline{5.26\times10^5}$ /liter

45. Complete the following reactions and calculate their Q values. [*Note:* The atomic weight of ^{14}C is 14.003242.]

(a) $^4He(p, d)$
(b) $^9Be(\alpha, n)$
(c) $^{14}N(n, p)$
(d) $^{115}In(d, p)$
(e) $^{207}Pb(\gamma, n)$

(a) $^4He(p,d)$

$$^4_2He + ^1_1p \rightarrow ^3_2He + ^2_1d$$

$$Q = [(M_{He} + m_p) - (M_{^3He} + m_d)]\, 931\, MeV/amu$$

Use mass hydrogen for m_p to account for e^- on deuterium

$$= [(4.002604 + 1.007825) - (3.01603 + 2.014102)] \times 931$$

$$= -18.3\, MeV$$

(b) $^{9}Be(\alpha, n)$

$$^{9}_{4}Be + ^{4}_{2}\alpha \rightarrow ^{12}_{6}C + ^{1}_{0}n$$

$$Q = [(M_{Be} + m_{\alpha}) - (M_{c} + m_{n})] 931 \text{ MeV}$$

Use mass of helium for α particle

$$= [(9.01219 + 4.002603) - (12.000 + 1.008665)] 931$$

$$= 5.705 \text{ MeV}$$

(c) $^{14}N(n, p)$

$$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}p$$

$$Q = [(M_{N} + m_{n}) - (M_{C} + m_{p})] 931 \text{ MeV/amu}$$

Use mass of hydrogen for proton

$$= [(14.003074 + 1.008665) - (14.003242 + 1.007825)] \times 931$$

$$= 6.256 \text{ MeV}$$

(d) $^{115}In(d, p)$

$$^{115}_{49}In + ^{2}_{1}D \longrightarrow ^{116}_{49}In + ^{1}_{1}p$$

Use mass of hydrogen for proton

$$Q = [(M_{In} + m_{D}) - (M_{In} + m_{H})] 931 \text{ MeV/amu}$$

$$= [(114.90387 + 2.014102) - (115.905 + 1.007825)] \times 931$$

$$= 4.547 \text{ MeV}$$

(e) $^{207}Pb(\gamma, n)$

$$^{207}_{82}Pb + ^{0}_{0}\gamma \longrightarrow ^{206}_{82}Pb + ^{1}_{0}n$$

$$Q = [(M_{Pb} + 0) - (M_{Pb} + m_{n})] 931 \text{ MeV/amu}$$

$$= (206.981558 + 0) - (205.974468 + 1.008665)] 931$$

49. The atomic weight of ^{206}Pb is 205.9745. Using the data in Problem 2.35, calculate the atomic weight of ^{210}Po. [*Caution:* See Problem 2.46]

$$Q = [M_{Po} - (M_{Pb} + m_\alpha)] \times 931 \text{ MeV/amu}$$

The energy is divided between the α particle which is 5.305 MeV and the recoil energy of the Pb atom. To determine the weight of Po need total Q value. Can show the KE_{Pb} is given by $\frac{m_\alpha}{M_{Pb}} KE_\alpha$

$$Q = KE_\alpha \left(1 + \frac{m_\alpha}{M_{Pb}}\right) \simeq 5.305\left(1 + \frac{4}{206}\right) = 5.408 \text{ MeV}$$

$$M_{Po} = M_{Pb} + m_\alpha + \frac{Q}{931 \text{ MeV}} = 205.9745 + 4.002604 + \frac{5.40}{931}$$

$$= 209.983 \text{ amu}$$

51. Consider the reaction

$$^6Li(\alpha, \text{p})^9\text{Be}.$$

Using atomic mass data, compute:
(a) the total binding energy of ^{6}Li, ^{9}Be, and ^{4}He;
(b) the Q value of the reaction using the results of part (a).

$$BE = [Z\, M(^1H) + N M_n - M(\text{atom})]\, 931 \text{ MeV/amu}$$

$M(^1H) = 1.007825$ $\quad M_{Li} = 6.015125$ $\quad M_{He} = 4.002604$

$M_n = 1.008665$ $\quad M_{Be} = 9.012186$

Evaluating for each

$BE_{Li} = 31.993$ $\quad BE_{Be} = 58.163$ $\quad BE_{He} = 28.296$

$$Q = BE_{Be} - (BE_{Li} + BE_\alpha)$$

$$= -2.126 \text{ MeV}$$

53. Using the mass formula, compute the binding energy per nucleon for the nuclei in Problem 2.52. Compare the results with those obtained in that problem.

Example.

^{12}C

$$M = NM_n + ZM_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \zeta(A-2Z)/A + \delta$$

for ^{12}C $\quad Z = 6$ and $N = 6$

$$= 6 \times 939.573 + 6 \times 938.280 - 15.56 \times 12 + 17.23(12)^{2/3}$$

$$+ 0.697(6)^2/(12)^{1/3} + 23.285(12-12)/12$$

$$- 12$$

The last term is negative since both Z and N are even

$$= 1.1594 \times 10^4$$

Dividing by 931 MeV/amu

$= 11.986$ which compares to 12.000

57. Calculate the atom density of graphite having density of 1.60 g/cm³.

$$N = \frac{\rho N_A}{M} \quad \text{For graphite, } M = 12.01115$$

$$N = \frac{1.60\ g/cm^3}{12.01115\ grams/mole} \times .6022 \times 10^{24}\ atoms/mole = 8.0219 \times 10^{22}\ \frac{atoms}{cm^3}$$

59. What is the atom density of ^{235}U in uranium enriched to 2.5 *a/o* in this isotope if the physical density of the uranium is 19.0 g/cm³?

$$N_i = \frac{\gamma_i \rho N_A}{100 M}$$

Use for M, the mass of uranium from Table 11.3

$$= \frac{(2.5)(19.0\ g/cm^3)(.6022 \times 10^{24}\ atoms/mole)}{100 \times 238.03\ g/mole}$$

$$= 1.201 \times 10^{21}\ atoms/cm^3$$

61. It has been proposed to use uranium carbide (UC) for the initial fuel in certain types of breeder reactors, with the uranium enriched to 25 w/o. The density of UC is 13.6 g/cm^3.
(a) What is the atomic weight of the uranium?
(b) What is the atom density of the ^{235}U?

(a) From Eq. 2.65

$$\frac{1}{M} = \frac{1}{100}\left(\frac{25}{235.0439} + \frac{75}{238.0508}\right)$$

$$M = 237.292$$

(b) The # of UC atoms is given by

$$w\frac{\rho N_A}{M(UC)}$$

where w is w/o of uranium in the uranium carbide, the w/o uranium is

$$w/o = \frac{M_u \times 100}{M_u + M_c} = \frac{237.292}{237.292 + 12.000} = 95.19\%$$

Weight of uranium in each cc is then

$$\rho_u = w/o \text{ uranium} \times \rho_{uc}$$

$$= 12.95 \text{ g/cc}$$

$$\# \text{ of } {}^{235}\text{U atoms/cc} = \frac{w/o_u \, \rho_u \, N_A}{100 \, M_{u235}} = \frac{25 \times 12.95 \times .6022 \times 10^{24}}{100 \times 235.0439}$$

$$= 1.89 \times 10^{23} \text{ atoms/cc}$$

63. The fuel for a certain breeder reactor consists of pellets composed of mixed oxides, UO_2 and PuO_2, with the PuO_2 comprising approximately 30 w/o of the mixture. The uranium is essentially all ^{238}U, whereas the plutonium contains the following isotopes: ^{239}Pu (70.5 w/o), ^{240}Pu (21.3 w/o), ^{241}Pu (5.5 w/o), and ^{242}Pu (2.7 w/o). Calculate the number of atoms of each isotope per gram of the fuel.

PuO_2 comprises 30 w/o of the fuel or each gram of fuel contains .3 gm of PuO_2. The w/o of the isotopes are ^{239}Pu = 70.5 w/o, ^{240}Pu = 21.3 w/o, ^{241}Pu = 5.5 w/o and ^{242}Pu = 2.7 w/o. The number density per gram of fuel is then the respective w/o times the w/o that is Pu times .3

w/o of PuO_2 that is Pu = $\frac{M_{Pu}}{M_{Pu} + 2M_O}$

$$\frac{1}{M_{Pu}} = \frac{1}{100}\left[\frac{70.5}{239.052175} + \frac{21.3}{240.053836} + \frac{5.5}{241.056873} + \frac{2.7}{242.058765}\right]$$

$M_{Pu} = 239.455$

w/o Pu is $100 \times \frac{239.455}{31.9984 + 239.455} = .882 \times 100$

the atoms/gm is then

$$\frac{w/o \text{ ith isotope}}{100} \times \frac{(.3) \times (.882) \times .6022 \times 10^{24}}{M_i}$$

where M_i = mass of ith Pu isotope and w_i is the w/o of the ith isotope