CHAPTER 2

INDEX AND CLASSIFICATION PROPERTIES OF SOILS

2-1. *From memory, draw a phase diagram (like Fig. 2.2, but don't look first!). The "phases" have a Volume side and Mass side. Label all the parts.*

SOLUTION: Refer to Figure 2.2.

2-2. *From memory, write out the definitions for water content, void ratio, dry density, wet or moist density, and saturated density.*

SOLUTION: Refer to Section 2.2.

2-3. *Assuming a value of ρ_s = 2.7 Mg/m^3, take the range of saturated density in Table 2.1 for the six soil types and calculate/estimate the range in void ratios that one might expect for these soils.*

SOLUTION: Create a spreadsheet using input values from Table 2.1 and Eq. 2.18.

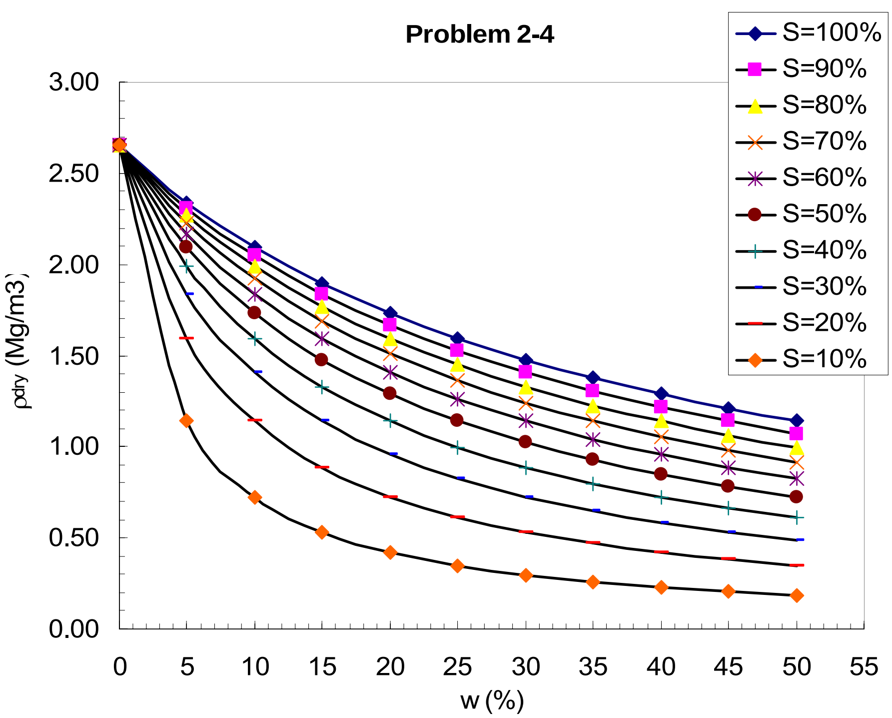
(Given)		(see Eq. 2.18)	
ρ' - min (Mg/m^3)	ρ' - max (Mg/m^3)	e_{max}	e_{min}
0.9	1.4	0.89	0.21
0.4	1.1	3.25	0.55
1.1	1.4	0.55	0.21
0.9	1.2	0.89	0.42
0.0	0.1	∞	16.00
0.3	0.8	4.67	1.13

2-4. *Prepare a spreadsheet plot of dry density in Mg/m³ as the ordinate versus water content in percent as the abscissa. Assume ρ_s = 2.65 Mg/m³ and vary the degree of saturation, S, from 100% to 40% in 10% increments. A maximum of 50% water content should be adequate.*

SOLUTION: Solve Eq. 2.12 and Eq. 2.15 for $\rho_d = f(\rho_s, w, S, G_s)$, or use Eq. 5.1.

$$\rho_{dry} = \frac{\rho_w S}{w + \frac{\rho_w}{\rho_s} S}$$

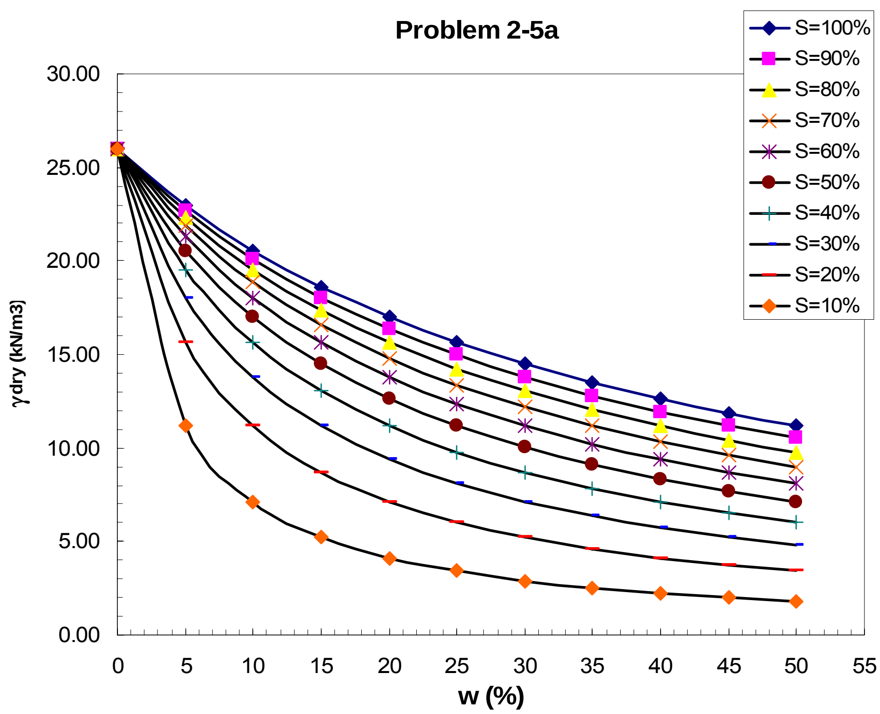
S =	100	90	80	70	60	50	40	30	20	10
w (%)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)	ρ_{dry} (Mg/m³)
0.0	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65
5.0	2.34	2.31	2.27	2.23	2.17	2.09	1.99	1.84	1.59	1.14
10.0	2.09	2.05	1.99	1.92	1.84	1.73	1.59	1.41	1.14	0.73
15.0	1.90	1.84	1.77	1.69	1.59	1.48	1.33	1.14	0.89	0.53
20.0	1.73	1.67	1.59	1.51	1.41	1.29	1.14	0.96	0.73	0.42
25.0	1.59	1.53	1.45	1.36	1.26	1.14	1.00	0.83	0.61	0.35
30.0	1.48	1.41	1.33	1.24	1.14	1.02	0.89	0.73	0.53	0.30
35.0	1.37	1.31	1.23	1.14	1.04	0.93	0.80	0.65	0.47	0.26
40.0	1.29	1.22	1.14	1.05	0.96	0.85	0.73	0.58	0.42	0.23
45.0	1.21	1.14	1.06	0.98	0.89	0.78	0.67	0.53	0.38	0.21
50.0	1.14	1.07	1.00	0.92	0.83	0.73	0.61	0.49	0.35	0.19



2-5a *Prepare a graph like that in Problem 2.4, only use dry density units of kN/m³ and pounds per cubic feet.*

SOLUTION: From Eq. 2.12 and Eq. 2.15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$

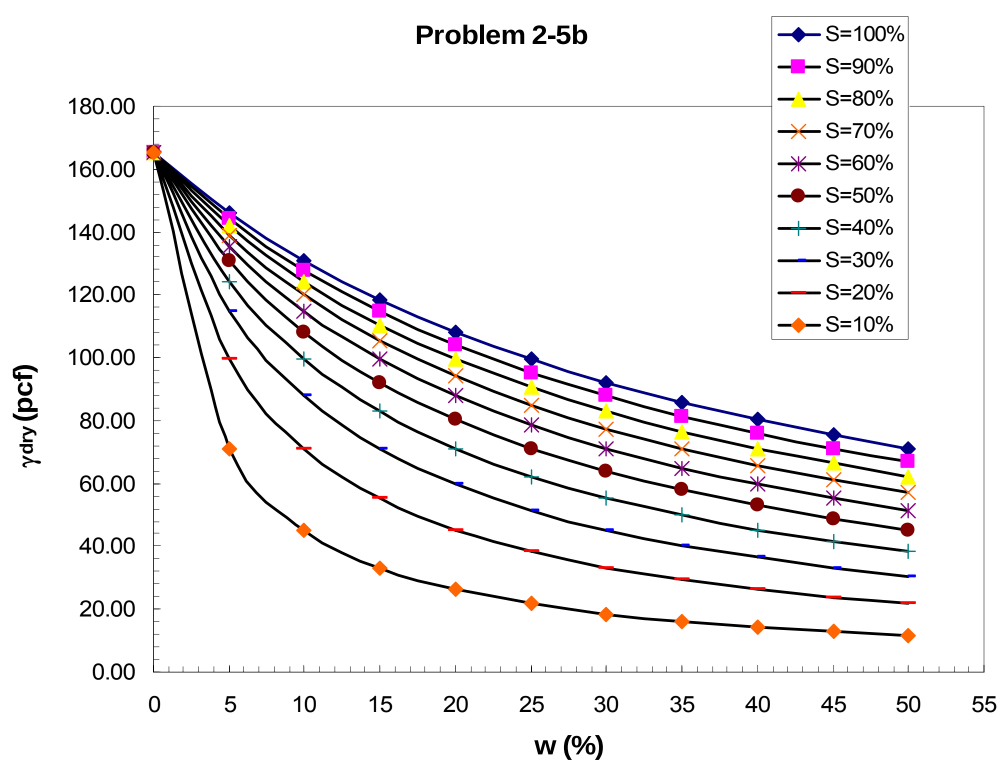
S =	100	90	80	70	60	50	40	30	20	10
w (%)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)	γ_{dry} (kN/m³)
0.0	26.00	26.00	26.00	26.00	26.00	26.00	26.00	26.00	26.00	26.00
5.0	22.95	22.66	22.30	21.86	21.29	20.55	19.53	18.03	15.64	11.18
10.0	20.55	20.08	19.53	18.86	18.03	16.99	15.64	13.80	11.18	7.12
15.0	18.60	18.03	17.37	16.58	15.64	14.48	13.04	11.18	8.70	5.23
20.0	16.99	16.36	15.64	14.79	13.80	12.62	11.18	9.40	7.12	4.13
25.0	15.64	14.97	14.22	13.36	12.35	11.18	9.79	8.10	6.03	3.41
30.0	14.48	13.80	13.04	12.17	11.18	10.04	8.70	7.12	5.23	2.90
35.0	13.49	12.80	12.04	11.18	10.21	9.11	7.83	6.35	4.61	2.53
40.0	12.62	11.94	11.18	10.34	9.40	8.33	7.12	5.73	4.13	2.24
45.0	11.86	11.18	10.44	9.62	8.70	7.68	6.53	5.23	3.73	2.01
50.0	11.18	10.52	9.79	8.99	8.10	7.12	6.03	4.80	3.41	1.82



2-5b *Prepare a graph like that in Problem 2.4, only use dry density units of pounds per cubic feet.*

SOLUTION: $\gamma_{dry} = \dfrac{G_s \gamma_w S}{S + G_s w}$

S =	100	90	80	70	60	50	40	30	20	10
w (%)	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**	**γ_{dry} (pcf)**
0.0	165.36	165.36	165.36	165.36	165.36	165.36	165.36	165.36	165.36	165.36
5.0	146.01	144.14	141.86	139.04	135.45	130.72	124.21	114.70	99.46	71.12
10.0	130.72	127.75	124.21	119.95	114.70	108.08	99.46	87.80	71.12	45.30
15.0	118.33	114.70	110.47	105.47	99.46	92.12	82.94	71.12	55.35	33.24
20.0	108.08	104.07	99.46	94.11	87.80	80.27	71.12	59.77	45.30	26.25
25.0	99.46	95.25	90.45	84.96	78.59	71.12	62.25	51.54	38.34	21.69
30.0	92.12	87.80	82.94	77.43	71.12	63.85	55.35	45.30	33.24	18.48
35.0	85.79	81.44	76.58	71.12	64.95	57.92	49.83	40.41	29.33	16.09
40.0	80.27	75.93	71.12	65.77	59.77	53.00	45.30	36.48	26.25	14.26
45.0	75.42	71.12	66.39	61.16	55.35	48.85	41.53	33.24	23.75	12.79
50.0	71.12	66.89	62.25	57.16	51.54	45.30	38.34	30.53	21.69	11.60

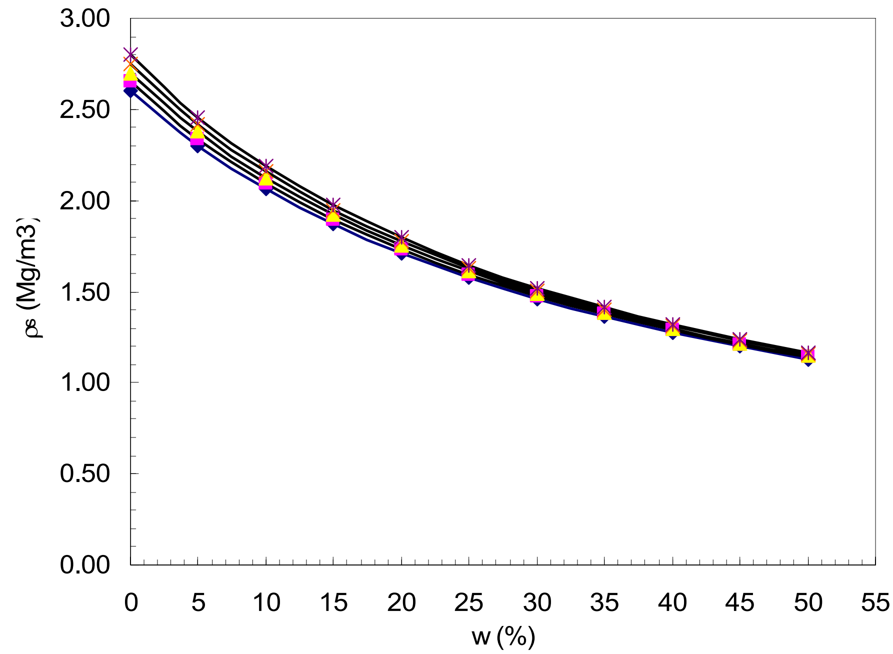


2.6. *Prepare a graph like that in Problem 2.4, only for S = 100% and vary the density of solids from 2.60 to 2.80 Mg/m^3^. You decide the size of the increments you need to "satisfactorily" evaluate the relationship as ρ_s varies. Prepare a concluding statement of your observations.*

SOLUTION: From Eq. 2.12 and Eq. 2.15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$

Note: The relationship between ρ_{dry} and w is not overly sensitive to ρ_s.

ρ_s =	2.6	2.65	2.7	2.75	2.8
w (%)	**ρ_{dry} (Mg/m^3)**	**ρ_{dry} (Mg/m^3)**	**ρ_{dry} (Mg/m^3)**	**ρ_{dry} (Mg/m^3)**	**ρ_{dry} (Mg/m^3)**
0.0	2.60	2.65	2.70	2.75	2.80
5.0	2.30	2.34	2.38	2.42	2.46
10.0	2.06	2.09	2.13	2.16	2.19
15.0	1.87	1.90	1.92	1.95	1.97
20.0	1.71	1.73	1.75	1.77	1.79
25.0	1.58	1.59	1.61	1.63	1.65
30.0	1.46	1.48	1.49	1.51	1.52
35.0	1.36	1.37	1.39	1.40	1.41
40.0	1.27	1.29	1.30	1.31	1.32
45.0	1.20	1.21	1.22	1.23	1.24
50.0	1.13	1.14	1.15	1.16	1.17



2.7. *The dry density of a compacted sand is 1.87 Mg/m^3 and the density of the solids is 2.67 Mg/m^3. What is the water content of the material when saturated?*

SOLUTION:

From Eq. 2-12 and Eq. 2-15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$; Note: S = 100%

Solving for w: $w = \rho_w S\left(\dfrac{1}{\rho_{dry}} - \dfrac{1}{\rho_s}\right) = (1\,Mg/m^3)(100\%)\left(\dfrac{1}{1.87\,Mg/m^3} - \dfrac{1}{2.67 Mg/m^3}\right) = \underline{\underline{16.0\%}}$

2.8. *A soil that is completely saturated has a total density of 2045 kg/m^3 and a water content of 24%. What is the density of the solids? What is the dry density of the soil?*

SOLUTION:

a) Solve using equations or phase diagrams:

$$\rho_{dry} = \frac{\rho_t}{(1+w)} = \frac{2045}{(1+0.24)} = \underline{\underline{1649.2\,kg/m^3}}$$

$$\rho_{sat} = \left(1 - \frac{\rho_w}{\rho_s}\right)\rho_{dry} + \rho_w$$

$$\rho_s = \frac{\rho_w \rho_{dry}}{\rho_{dry} + \rho_w - \rho_{sat}} = \frac{(1000)(1649.2)}{(1649.2 + 1000 - 2045)} = \underline{\underline{2729.6\,kg/m^3}}$$

b) Solve using phase diagram relationships:

$$\text{assume } V_t = 1.0$$

$$M_t = 2045\,kg$$

$$\frac{M_w}{M_s} = 0.24 \rightarrow M_w = 0.24 M_s$$

$$M_t = M_w + M_s \rightarrow 2045 = 0.24 M_s + M_s \rightarrow \underline{M_s = 1649.19\,kg}$$

$$\rho_w = 1000 = \frac{M_w}{V_w} \rightarrow V_w = \frac{0.24 M_s}{1000} = \underline{0.3958\,m^3}$$

$$\rho_s = \frac{M_s}{V_s} = \frac{1649.19}{1 - 0.3958} = \underline{\underline{2729.6\,kg/m^3}}$$

$$\rho_{dry} = \frac{M_s}{V_t} = \frac{1649.19}{1} = \underline{\underline{1649.2\,kg/m^3}}$$

2.9 *What is the water content of a fully saturated soil with a dry density of 1.72 Mg/m³? Assume* ρ_s *= 2.72 Mg/m³.*

SOLUTION:

From Eq. 2-12 and Eq. 2-15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$; Note: S = 100%

Solving for w: $w = \rho_w S \left(\dfrac{1}{\rho_{dry}} - \dfrac{1}{\rho_s} \right) = (1\,Mg/m^3)(100\%)\left(\dfrac{1}{1.72\,Mg/m^3} - \dfrac{1}{2.72\,Mg/m^3} \right) = \underline{\underline{21.4\%}}$

2.10. *A dry quartz sand has a density of 1.68 Mg/m³. Determine its density when the degree of saturation is 75%. The density of solids for quartz is 2.65 Mg/m³.*

SOLUTION:

Recognize that ρ_{dry} (initial) = ρ_{dry} (final); $S_{(final)}$ = 75%

From Eq. 2-12 and Eq. 2-15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$

Solving for w: $w = \rho_w S \left(\dfrac{1}{\rho_{dry}} - \dfrac{1}{\rho_s} \right) = (1\,Mg/m^3)(75\%)\left(\dfrac{1}{1.68\,Mg/m^3} - \dfrac{1}{2.65\,Mg/m^3} \right) = \underline{16.34\%}$

final $\rho_t = \rho_{dry}(1 + w) = 1.68(1 + 0.1634) = \underline{\underline{1.95\,Mg/m^3}}$

2.11. *The dry density of a soil is 1.60 Mg/m³ and the solids have a density of 2.65 Mg/m³. Find the (a) water content, (b) void ratio, and (c) total density when the soil is saturated.*

SOLUTION:

Given: S = 100%

From Eq. 2-12 and Eq. 2-15: $\rho_{dry} = \dfrac{\rho_w S}{w + \dfrac{\rho_w}{\rho_s} S}$

(a) Solving for w: $w = \rho_w S \left(\dfrac{1}{\rho_{dry}} - \dfrac{1}{\rho_s} \right) = (1\,Mg/m^3)(100\%)\left(\dfrac{1}{1.60\,Mg/m^3} - \dfrac{1}{2.65\,Mg/m^3} \right) = \underline{\underline{24.76\%}}$

(b) From Eq. 2.15: $e = \dfrac{w\rho_s}{S\rho_w} = \dfrac{(24.76)(2.65)}{(100)(1.0)} = \underline{\underline{0.656}}$

(c) $\rho_t = \rho_{dry}(1 + w) = 1.60(1 + 0.2476) = \underline{\underline{1.996 = 2.00\,Mg/m^3}}$

2.12. *A natural deposit of soil is found to have a water content of 20% and to be 90% saturated. What is the void ratio of this soil?*

SOLUTION:

w = 20% and S = 90%; assume $G_s = 2.70$

From Eq. 2.15: $e = \dfrac{w\rho_s}{S\rho_w} = \dfrac{(20.0)(2.70)}{(90)(1.0)} = \underline{\underline{0.60}}$

2.13. *A chunk of soil has a wet weight of 62 lb and a volume of 0.56 ft^3. When dried in an oven, the soil weighs 50 lb. If the specific gravity of solids G_s = 2.64, determine the water content, wet unit weight, dry unit weight, and void ratio of the soil.*

SOLUTION: Solve using phase diagram relationships.

(a) $W_w = W_t - W_s = 62 - 50 = 12 \text{ lb}$

$w = \dfrac{W_w}{W_s} \times 100\% = \dfrac{12(100)}{50} = \underline{\underline{24.0\%}}$

(b) $\gamma_t = \dfrac{W_t}{V_t} = \dfrac{62}{0.56} = \underline{\underline{110.7 \text{ pcf}}}$

(c) $\gamma_{dry} = \dfrac{W_s}{V_t} = \dfrac{50}{0.56} = \underline{\underline{89.29 \text{ pcf}}}$

(d) $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{12}{62.4} = 0.1923 \text{ ft}^3$

$V_s = \dfrac{W_s}{G_s\gamma_w} = \dfrac{50}{(2.64)(62.4)} = 0.3035 \text{ ft}^3$

$V_v = V_t - V_v = 0.56 - 0.3035 = 0.2565$

$e = \dfrac{V_v}{V_s} = \dfrac{0.2565}{0.3035} = 0.8451 = \underline{\underline{0.84}}$

2.14. *In the lab, a container of saturated soil had a mass of 113.27 g before it was placed in the oven and 100.06 g after the soil had dried. The container alone had a mass of 49.31 g. The specific gravity of solids is 2.80. Determine the void ratio and water content of the original soil sample.*

SOLUTION: Solve using phase diagram relationships.

$M_s = 100.06 - 49.31 = 50.75 \text{ g}$

$M_w = 113.27 - 100.06 = 13.21 \text{ g}$

(a) $w = \dfrac{M_w}{M_s} \times 100\% = \dfrac{13.21(100)}{50.75} = 26.03 = \underline{\underline{26.0\%}}$

(b) From Eq. 2.15: $e = \dfrac{\rho_s w}{\rho_w S} = \dfrac{2.80(26.03)}{(1)(100)} = 0.7288 = \underline{\underline{0.73}}$

2.15. *The natural water content of a sample taken from a soil deposit was found to be 12.0%. It has been calculated that the maximum density for the soil will be obtained when the water content reaches 22.0%. Compute how many grams of water must be added to each 1000 g of soil (in its natural state) in order to increase the water content to 22.0%.*

SOLUTION:

Natural state

$$w = 0.12 = \frac{M_w}{M_s} \quad \rightarrow M_w = (0.12)M_s$$

$M_t = M_s + M_w = M_s + 0.12M_s = 1.12M_s = 1000 \text{ g}$

$M_s = 892.857 \text{ g}$

$M_w = (0.12)(892.857) = 107.143 \text{ g}$

Target state

(Note: M_s does not change between natural state and target state)

$M_w = w \times M_s = (0.22)(892.857) = 196.429 \text{ g}$

additional water necessary = $196.429 - 107.143 = 89.286 = \underline{\underline{89.29 \text{ g}}}$

2.16. *A cubic meter of dry quartz sand (G_s = 2.65) with a porosity of 60% is immersed in an oil bath having a density of 0.92 g/cm^3. If the sand contains 0.27 m^3 of entrapped air, how much force is required to prevent it from sinking? Assume that a weightless membrane surrounds the specimen. (Prof. C. C. Ladd.)*

SOLUTION:

$V_t = 1 \text{ m}^3 = 1{,}000{,}000 \text{ cm}^3$

$V_v = n \times V_t = (0.6)(1.0) = 0.60 \text{ m}^3$

$V_s = V_t - V_v = 1.0 - 0.60 = 0.40 \text{ m}^3$

$M_s = G_s \times \rho_s \times V_s = (2.65)(1000 \text{ kg/m}^3)(0.40 \text{ m}^3) = 1060 \text{ kg} = M_t$

$$\rho_t = \frac{M_t}{V_t} = \frac{1060}{1.0} = 1060 \text{ kg/m}^3$$

$\rho_{buoy} = \rho_t - \rho_{oil} = 1060 - 920 = 140 \text{ kg/m}^3$

$\gamma_{buoy} = \rho_{buoy} \times g = (140)(9.81) = 1373.4 \text{ N/m}^3$

Force = $\gamma_{buoy} \times (\text{entrapped air}) = 1373.4 \text{ N/m}^3 \times 0.27 \text{ m}^3 = \underline{\underline{370.8 \text{ N}}}$

2.17. A soil sample taken from a borrow pit has a natural void ratio of 1.15. The soil will be used for a highway project where a total of 100,000 m^3 of soil is needed in its compacted state; its compacted void ratio is 0.73. How much volume has to be excavated from the borrow pit to meet the job requirements?

SOLUTION:

$$V_s = \frac{V_v}{e} = \frac{V_t - V_s}{e} \rightarrow V_s = \frac{V_t}{e+1}$$

Embankment

$$V_t = 100{,}000 \text{ m}^3$$

$$V_{s(emb)} = \frac{100{,}000}{0.73+1} = 57{,}803.47 \text{ m}^3$$

Borrow Pit

$$V_{s(borr)} = V_{s(emb)} = \frac{V_t}{e+1}$$

$$V_{t(borr)} = V_s \times (e_{borr} + 1) = (57{,}803.47)(1.15+1) = \underline{\underline{124{,}277 \text{ m}^3}}$$

2.18. *A sample of moist soil was found to have the following characteristics:*

Total volume: 0.01456 m^3

Total mass: 25.74 kg

Mass after oven drying: 22.10 kg

Specific gravity of solids: 2.69

Find the density, unit weight, void ratio, porosity, and degree of saturation for the moist soil.

SOLUTION:

(a) $\rho_t = \dfrac{25.74}{0.01456} = 1767.857 = \underline{\underline{1768 \text{ kg/m}^3}}$

(b) $\gamma_t = \rho_t \times g = (1767.857)(9.81) = 17{,}342.68 \text{ N/m}^3 = \underline{\underline{17.34 \text{ kN/m}^3}}$

(c) $V_s = \dfrac{M_s}{G_s \rho_w} = \dfrac{22.10}{(2.69)(1000)} = 0.00822 \text{ m}^3$

$$V_v = 0.01456 - 0.00822 = 0.006344 \text{ m}^3$$

$$e = \frac{0.006344}{0.00822} = 0.7718 = \underline{\underline{0.77}}$$

(d) $n = \dfrac{0.7718}{1+0.7718} \times 100 = 43.56 = \underline{\underline{43.6\%}}$

(e) $M_w = 25.74 - 22.10 = 3.64$

$$V_w = \frac{3.64}{1000} = 0.00364$$

$$S = \frac{0.00364}{0.006344} \times 100 = 57.377 = \underline{\underline{57.4\%}}$$

2.19. *A gray silty clay (CL) is sampled from a depth of 12.5 feet. The "moist" soil was extruded from a 6-inch-high brass liner with an inside diameter of 2.83 inches and weighed 777 grams. (a) Calculate the wet density in pounds per cubic feet. (b) A small chunk of the original sample had a wet weight of 140.9 grams and weighed 85.2 grams after drying. Compute the water content, using the correct number of significant figures. (c) Compute the dry density in Mg/m³ and the dry unit weight in kN/m³.*

SOLUTION:

$$\text{(a)}\quad V_t = \frac{\pi(2.83)^2}{4}\times 6 = 37.741\,\text{in}^3,\quad \gamma_t = \frac{M_t}{V_t} = \frac{(777\text{ g})\left(1\text{ lb}/453.6\text{ g}\right)}{\left(37.741\,\text{in}^3\right)\left(1\text{ ft}/12\text{ in}\right)^3} = 78.429 = \underline{\underline{78.4\text{ pcf}}}$$

$$\text{(b)}\quad M_w = M_t - M_s = 140.9 - 85.2 = 55.7\text{ g},\quad w = \frac{M_w}{M_s}\times 100\% = \frac{55.7}{85.2} = 65.38 = \underline{\underline{65.4\%}}$$

$$\text{(c)}\quad \gamma_{dry} = \frac{\gamma_t}{(1+w)} = \frac{78.4}{(1+0.654)} = 47.40\text{ pcf}$$

$$\rho_{dry} = \left(47.4\,\text{lb}/\text{ft}^3\right)\left(1\text{ ft}/0.3048\text{ m}\right)^3\left(0.4536\text{ kg}/1\text{ lb}\right) = 759.288\,\text{kg}/\text{m}^3 = \underline{\underline{0.759\,\text{Mg}/\text{m}^3}}$$

$$\gamma_{dry} = (759.288)(9.81) = 7448.6\,\text{N}/\text{m}^3 = \underline{\underline{7.45\,\text{kN}/\text{m}^3}}$$

2.20. *A cylindrical soil specimen is tested in the laboratory. The following properties were obtained:*

Sample diameter	*3 inches*
Sample length	*6 inches*
Wt. before drying in oven	*2.95 lb*
Wt. after drying in oven	*2.54 lb*
Oven temperature	*110°C*
Drying time	*24 hours*
Specific gravity of solids	*2.65*

What is the degree of saturation of this specimen?

SOLUTION:

$$V_t = \frac{\pi(3)^2}{4}\times 6 = 42.4115\text{ in}^3 = 0.02454\text{ ft}^3$$

$$V_s = \frac{W_s}{G_s\gamma_w} = \frac{2.54}{(2.65)(62.4)} = 0.01536\text{ ft}^3 = 26.542\text{ in}^3$$

$$V_v = V_t - V_s = 42.4115 - 26.542 = 15.869\text{ in}^3 = 0.009184\text{ ft}^3$$

$$W_w = W_t - W_s = 2.95 - 2.54 = 0.41\,\text{lb}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.41}{62.4} = 0.00657\text{ ft}^3 = 11.354\text{ in}^3$$

$$S = \frac{V_w}{V_v}\times 100\% = \frac{11.354}{15.869}\times 100 = \underline{\underline{71.5\%}}$$

2.21 A sample of saturated silt is 10 cm in diameter and 2.5 cm thick. Its void ratio in this state is 1.35, and the specific gravity of solids is 2.70. The sample is compressed to a 2-cm thickness without a change in diameter.

(a) Find the density of the silt sample, in prior to being compressed.
(b) Find the void ratio after compression and the change in water content that occurred from initial to final state.

SOLUTION:

(a) $V_t = \dfrac{\pi(10)^2}{4} \times 2.5 = 196.350 \text{ cm}^3$

$S = 1 = \dfrac{V_w}{V_v} \rightarrow V_w = V_v$

$V_v = e \times V_s = 1.35V_s$

$V_t = V_v + V_s = 1.35V_s + V_s = 2.35V_s = 196.350 \rightarrow V_s = 83.553 \text{ cm}^3$

$V_v = (1.35)(83.553) = 112.797 \text{ cm}^3 = V_w$

$M_w = \rho_w \times V_w = (1 \text{ g/cm}^3)(112.797) = 112.797 \text{ g}$

$M_s = G_s \times V_s \times \rho_w = (2.70)(83.553)(1 \text{ g/cm}^3) = 225.594 \text{ g}$

$M_t = 112.797 + 225.594 = 338.391 \text{ g}$

$\rho_t = \dfrac{M_t}{V_t} = \dfrac{338.391}{196.35} = 1.723 \text{ g/cm}^3 = \underline{\underline{1723 \text{ kg/m}^3}}$

(b) $V_{t-2} = \dfrac{\pi(10)^2}{4} \times 2.0 = 157.08 \text{ cm}^3$

$V_s = 83.553 \text{ cm}^3$ (no change)

$V_v = V_t - V_s = 157.08 - 83.553 = 73.527 \text{ cm}^3$

$e_{final} = \dfrac{73.527}{83.553} = \underline{\underline{0.88}}$

(c) $w_{initial} = \dfrac{112.797}{225.594} \times 100\% = \underline{50.0\%}$

<u>final conditions</u>

$V_w = V_v = 73.527 \text{ cm}^3$; $M_w = 73.527 \text{ g}$; $M_s = 225.594 \text{ g}$ (no change)

$w_{final} = \dfrac{73.527}{225.594} \times 100\% = \underline{32.6\%}$

$\Delta w = 50.0 - 32.6 = \underline{\underline{17.4\%}}$

2.22. *A sample of sand has the following properties: total mass M_t = 160 g; total volume V_t =80 cm^3; water content w = 20%; specific gravity of solids G_s =2.70. How much would the sample volume have to change to get 100% saturation, assuming the sample mass M_t stayed the same?*

SOLUTION:

$M_w = w \times M_s = (0.20)M_s$

$M_t = M_s + 0.20M_s = 160 \text{ g}$

$M_s = 133.33 \text{ g}$

$M_w = (0.20)(133.33) = 26.667 \text{ g}; \quad V_w = (1 \, {}^{g}\!/\!_{cm^3})M_w = 26.667 \text{ cm}^3$

$$V_s = \frac{M_s}{G_s \rho_w} = \frac{133.33}{2.70} = 49.383 \text{ cm}^3; \quad V_v = 80 - 49.383 = 30.617 \text{ cm}^3$$

$$S_i = \frac{26.667}{30.617} \times 100\% = \underline{87.10\%}$$

$\underline{\text{Desired condition: } S = 100\%}$

V_t changes, but V_s and M_s remain the same

$M_w = 26.667 \text{ g}; \quad V_w = 26.667 \text{ cm}^3$

$V_v = V_w \quad (\text{when } S = 100\%)$

$V_t = V_s + V_v = 49.383 + 26.667 = \underline{76.053 \text{ cm}^3}$

$\Delta V = 80 - 76.053 = \underline{\underline{3.95 \text{ cm}^3}}$

2.23. Draw a phase diagram and begin to fill in the blanks: A soil specimen has total volume of 80,000 mm^3 and weighs 145 g. The dry weight of the specimen is 128 g, and the density of the soil solids is 2.68 Mg/m^3. Find the: (a) water content, (b) void ratio, (c) porosity, (d) degree of saturation, (e) wet density, and (f) dry density. Give the answers to parts (e) and (f) in both SI and British engineering units.

SOLUTION:

(a) $M_w \, 145 - 128 = 17 \text{ g}$

$$w = \frac{M_w}{M_s} \times 100\% = \frac{17}{128} \times 100 = 13.281 = \underline{\underline{13.3\%}}$$

(b) $V_s = \dfrac{M_s}{G_s \rho_w} = \dfrac{128}{(2.68)(1)} = 47.7612 \text{ cm}^3 = 47{,}761.2 \text{ mm}^3$

$$V_v = 80{,}000 - 47{,}761.2 = 32{,}238.8 \text{ mm}^3$$

$$e = \frac{32{,}238.8}{47{,}761.2} = \underline{\underline{0.67}}$$

(c) $n = \dfrac{32{,}238.8}{80{,}000} \times 100\% = \underline{\underline{40.3\%}}$

(d) $V_w = M_w \times \rho_w = (17)(1) = 17 \text{ cm}^3 = 17{,}000 \text{ mm}^3$

$$S = \frac{17{,}000}{32{,}238.8} \times 100\% = \underline{\underline{52.7\%}}$$

(e) $\rho_t = \dfrac{145}{80} = 1.8125 \text{ g/cm}^3 = \underline{\underline{1812.5 \text{ kg/m}^3}}$

$$\rho_t = 1812.5 \text{ kg/m}^3 \times \left(\frac{1}{16.018}\right) = \underline{\underline{113.2 \text{ lbm/ft}^3}} \quad \text{(see Appendix A)}$$

(f) $\rho_{dry} = \dfrac{1812.5}{(1 + 0.13281)} = \underline{\underline{1600.0 \text{ kg/m}^3}}$

$$\rho_{dry} = 1600.0 \text{ kg/m}^3 \times \left(\frac{1}{16.018}\right) = \underline{\underline{99.9 \text{ lbm/ft}^3}} \quad \text{(see Appendix A)}$$

2.24. *A sample of soil plus container weighs 397.6 g when the initial water content is 6.3%. The container weighs 258.7 g. How much water needs to be added to the original specimen if the water content is to be increased by 3.4%? After U.S. Dept. of Interior (1990).*

SOLUTION:

$$M_t = 397.6 - 258.7 = 138.9 \text{ g}$$
$$M_w = 0.063M_s$$
$$M_t = 138.9 = M_w + M_s = 0.063M_s + M_s = 1.063M_s$$
$$M_s = 130.668 \text{ g}$$
$$\Delta w = \frac{\Delta M_w}{M_s} = 0.034$$
$$\Delta M_w = \Delta w \times M_s = (0.034)(130.668) = \underline{\underline{4.44 \text{ g}}}$$

2.25. *A water-content test was made on a sample of clayey silt. The weight of the wet soil plus container was 18.46 g, and the weight of the dry soil plus container was 15.03 g. Weight of the empty container was 7.63 g. Calculate the water content of the sample.*

SOLUTION:

$$M_s = 15.03 - 7.63 = 7.40 \text{ g}$$
$$M_w = 18.46 - 15.03 = 3.43 \text{ g}$$
$$\text{(a)}\quad w = \frac{M_w}{M_s} \times 100\% = \frac{3.43(100)}{7.40} = 46.351 = \underline{\underline{46.3\%}}$$

2.26. *A soil sample is dried in a microwave oven to determine its water content. From the data below, evaluate the water content and draw conclusions. The oven-dried water content is 23.7%. The mass of the dish is 146.30 grams. After U.S. Dept. of Interior (1990).*

SOLUTION:

$M_t = 231.62 - 146.3 = 85.32$ g

$M_w = (0.237)M_s$

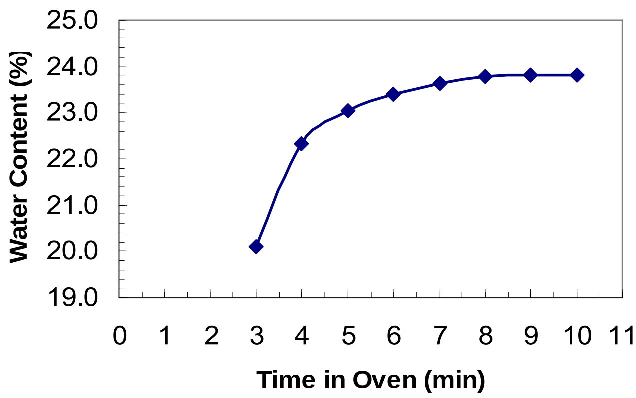
$85.32 = 0.237M_s + M_s$

$M_s = 68.9733$ g (this value is constant throughout drying period)

$$\text{column 5} = \frac{231.62 - \text{column 3}}{68.9733} \times 100\%$$

CONCLUSION: The loss of additional water in the soil sample becomes negligible after 8 to 10 minutes in the microwave oven used in the experiment.

GIVEN			CALCULATED	
Time in Oven (min)	**Total Oven Time (min)**	**Mass of Soil + Dish (g)**	**Mass of Water (g)**	**Water Content (%)**
0	0	231.62	--	--
3	3	217.75	13.87	20.11
1	4	216.22	15.40	22.33
1	5	215.72	15.90	23.05
1	6	215.48	16.14	23.40
1	7	215.32	16.30	23.63
1	8	215.22	16.40	23.78
1	9	215.19	16.43	23.82
1	10	215.19	16.43	23.82



2.27. *The mass of a sample of silty clay soil plus container is 18.43 g and the weight of the dry soil plus container is 13.67 g. The container weighs 8.84 g. Compute the water content of the sample.*

SOLUTION:

$M_s = 13.67 - 8.84 = 4.83 \text{ g}$

$M_w = 18.43 - 13.67 = 4.76 \text{ g}$

(a) $w = \frac{M_w}{M_s} \times 100\% = \frac{4.76(100)}{4.83} = \underline{\underline{98.5\%}}$

2.28. *A specimen of fully saturated clay soil that weighs 1389 g in its natural state weighs 982 g after drying. What is the natural water content of the soil?*

SOLUTION:

$M_w = 1389 - 982 = 407 \text{ g}$

(a) $w = \frac{M_w}{M_s} \times 100\% = \frac{407(100)}{982} = \underline{\underline{41.4\%}}$

2.29. *The volume of water in a sample of moist soil is 0.24 m^3. The volume of solids V_s is 0.25 m^3. Given that the density of soil solids ρ_s is 2600 kg/m^3, find the water content.*

SOLUTION:

$M_s = \rho_s \times V_s = (2600)(0.25) = 650 \text{ kg}$

$M_w = \rho_w \times V_w = (1000)(0.24) = 240 \text{ kg}$

$w = \frac{240}{650} \times 100\% = \underline{\underline{36.9\%}}$

2.30. *For the soil sample of Problem 2.29, compute (a) the void ratio and (b) the porosity.*

SOLUTION: Assume S = 100%

(a) $e = \frac{G_s w}{S} = \frac{(2.6)(0.3692)}{(1)} = \underline{\underline{0.96}}$

(b) $n = \frac{0.96}{1 + 0.96} \times 100 = \underline{\underline{50.0\%}}$

2.31. *For the soil sample of Problem 2.29, compute (a) the total or wet density and (b) the dry density. Give your answers in Mg/m^3, kg/m^3, and lbf/ft^3.*

SOLUTION: Assume S = 100%

(a) $V_t = 0.25 + 0.24 = 0.49 \text{ m}^3$

$$\rho_t = \frac{710}{0.49} = \underline{\underline{1448.98\ ^{kg}/_{m^3}}} = \underline{\underline{1.45\ ^{Mg}/_{m^3}}} = \underline{\underline{90.4\ ^{lbm}/_{ft^3}}}$$

(b) $$\rho_{dry} = \frac{650}{0.49} = \underline{\underline{1326.53\ ^{kg}/_{m^3}}} = \underline{\underline{1.33\ ^{Mg}/_{m^3}}} = \underline{\underline{82.8\ ^{lbm}/_{ft^3}}}$$

2.32. *A 592-cm^3 volume of moist sand weighs 1090 g. Its dry weight is 920 g and the density of solids is 2680 kg/m^3. Compute the void ratio, porosity, water content, degree of saturation, and total density in kg/m^3.*

SOLUTION:

$$\rho_s = 2680\ ^{kg}/_{m^3} = 2.68\ ^{g}/_{cm^3}$$

$$V_s = \frac{920}{2.68} = 343.284 \text{ cm}^3$$

$$V_v = 592 - 343.284 = 248.716 \text{ cm}^3$$

(a) $$e = \frac{V_v}{V_s} = \frac{248.716}{343.284} = 0.7245 = \underline{\underline{0.72}}$$

(b) $$n = \frac{0.7245}{1 + 0.7245} \times 100\% = \underline{\underline{42.0\%}}$$

(c) $M_w = 1090 - 920 = 170 \text{ g}$

$$V_w = \frac{170}{\rho_w} = 170 \text{ cm}^3$$

$$S = \frac{V_w}{V_v} \times 100\% = \frac{170}{248.716} \times 100 = \underline{\underline{68.4\%}}$$

(d) $$\rho_t = \frac{1090}{592} = 1.841\ ^{g}/_{cm^3} = \underline{\underline{1841\ ^{kg}/_{m^3}}}$$

2.33. *The saturated density γ_{sat} of a soil is 137 lbf/ft^3. Find the buoyant density of this soil in both lbf/ft^3 and kg/m^3.*

SOLUTION:

$$\gamma' = 137 - 62.4 = \underline{\underline{74.6\ ^{lbf}/_{ft^3}}}$$

$$\rho' = \left(74.6\ ^{lbf}/_{ft^3}\right)\left(\frac{16.018\ ^{kg}/_{m^3}}{1\ ^{lbm}/_{ft^3}}\right) = \underline{\underline{1195\ ^{kg}/_{m^3}}}$$

2.34. *A sand is composed of solid constituents having a density of 2.68 Mg/m3. The void ratio is 0.58. Compute the density of the sand when dry and when saturated and compare it with the density when submerged.*

SOLUTION:

Assume $V_s = 1\,m^3$

$V_v = 0.58, \quad V_t = 1 + 0.58 = 1.58\ m^3$

$$\rho_{dry} = \frac{2.68}{1.58} = 1.6962 = \underline{\underline{1.70\ ^{Mg}/_{m^3}}}$$

For S = 100%; $V_w = V_v = 0.58\ m^3$

$M_w = (1)(0.58) = 0.58\ Mg$

$M_t = 2.68 + 0.58 = 3.26\ Mg$

$$\rho_{sat} = \frac{3.26}{1.58} = \underline{\underline{2.06\ ^{Mg}/_{m^3}}}$$

$$\rho' = 2.06 - 1.0 = \underline{\underline{1.06\ ^{Mg}/_{m^3}}}$$

2.35. *A sample of natural glacial till was taken from below the groundwater table. The water content was found to be 52%. Estimate the wet density, dry density, buoyant density, porosity, and void ratio. Clearly state any necessary assumptions.*

SOLUTION:

Assume $V_s = 1\,m^3$, Assume $G_s = 2.7$

$M_s = G_s \times V_s \times \rho_s = (2.7)(1)(1) = 2.7\text{ Mg}, \quad M_w = w \times M_s = (0.52)(2.7) = 1.404\text{ Mg}$

$M_t = 2.7 + 1.404 = 4.104\text{ Mg}$

$S = \dfrac{V_w}{V_v} = 1 \rightarrow V_w = V_v, \quad V_w = \dfrac{M_w}{\rho_w} = 1.404\text{ m}^3 = V_v$

$V_t = 1.404 + 1.0 = 2.404\text{ m}^3$

(a) $\rho_t = \dfrac{4.104}{2.404} = \underline{\underline{1.71\ \text{Mg/m}^3}}$

(b) $\rho_{dry} = \dfrac{2.7}{2.404} = \underline{\underline{1.12\ \text{Mg/m}^3}}$

(c) $\rho' = 1.71 - 1.0 = \underline{\underline{0.71\ \text{Mg/m}^3}}$

(d) $n = \dfrac{1.404}{2.404} \times 100\% = \underline{\underline{58.4\%}}$

(e) $e = \dfrac{1.404}{1.0} = \underline{\underline{1.4}}$

2.36. *A 1-m^3 sample of moist soil weighs 2000 kg. The water content is 10%. Assume ρ_s = 2.70 Mg/m^3. With this information, fill in all blanks in the phase diagram of Fig. P2.36.*

SOLUTION:

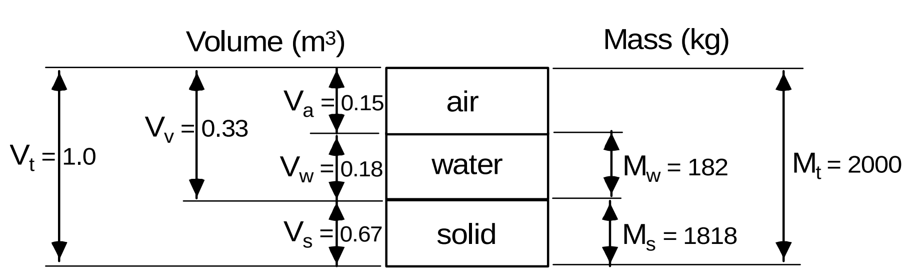
$M_w = M_s \times w = 0.10M_s$

$M_t = 2000 = M_s + 0.10M_s = 1.10M_s \;\rightarrow\; M_s = \underline{1818.18\text{ kg}}$

$M_w = (0.10)(1818.18) = \underline{181.82\text{ kg}}$

$V_s = \dfrac{M_s}{\rho_s} = \dfrac{1818.18}{2700} = \underline{0.673\text{ m}^3}, \quad V_w = \dfrac{181.82}{1000} = \underline{0.181\,\text{m}^3}$

$V_v = 1 - 0.673 = \underline{0.327\text{ m}^3}, \quad V_a = 0.327 - 0.181 = \underline{0.146\text{ m}^3}$



2.37. *For the information given in Problem 2.36, calculate (a) the void ratio, (b) the porosity, and (c) the dry density.*

SOLUTION:

(a) $e = \dfrac{0.327}{0.673} = 0.4859 = \underline{\underline{0.49}}$

(b) $n = \dfrac{0.327}{1.0} \times 100 = \underline{\underline{32.7\%}}$

(c) $\rho_{dry} = \dfrac{1818.18}{1.0} = \underline{\underline{1818\ ^{kg}/_{m^3}}}$

2.38. *The void ratio of clay soil is 0.6 and the degree of saturation is 75%. Assuming the density of the solids is 2710 kg/m*^{3}*, compute (a) the water content and (b) dry and wet densities in both SI and British engineering units.*

SOLUTION:

Assume $V_s = 1.0\ m^3$

$V_v = e \times V_s = (0.6)(1.0) = 0.6\ m^3, \quad V_w = S \times V_v = (0.75)(0.6) = 0.45\ m^3$

$V_t = 0.6 + 1.0 = 1.6\ m^3$

$M_w = \rho_w \times V_w = (1000)(0.45) = 450\ kg, \quad M_s = \rho_s \times V_s = (2710)(1.0) = 2710\ kg$

$M_t = 450 + 2710 = 3160\ kg$

(a) $w = \dfrac{M_w}{M_s} \times 100 = \dfrac{450}{2710} \times 100 = \underline{\underline{16.6\ \%}}$

(b) $\rho_{dry} = \dfrac{M_s}{V_t} = \dfrac{2710}{1.6} = 1693.75 = \underline{\underline{1694\ ^{kg}/_{m^3}}}$

$\rho_{dry} = \left(1694\ ^{kg}/_{m^3}\right)\left(\dfrac{1\ ^{lbm}/_{ft^3}}{16.018\ ^{kg}/_{m^3}}\right) = \underline{\underline{105.8\ ^{lbm}/_{ft^3}}}$

$\rho_t = \dfrac{M_t}{V_t} = \dfrac{3160}{1.6} = \underline{\underline{1975\ ^{kg}/_{m^3}}}$

$\rho_t = (1975)\left(1/16.018\right) = \underline{\underline{123.3\ ^{lbm}/_{ft^3}}}$

2.39. *A specimen of saturated glacial clay has a water content of 38%. On the assumption that ρ_s = 2.70 Mg/m³, compute the void ratio, porosity, and saturated density.*

SOLUTION:

$w = 38\%,\ S = 100\%,\ \rho_s = 2.70\ \mathrm{Mg/m^3}$

(a) From Eq. 2.15: $e = \dfrac{w\rho_s}{S\rho_w} = \dfrac{(38.0)(2.70)}{(100)(1.0)} = 1.026 = \underline{\underline{1.03}}$

(b) $n = \dfrac{e}{1+e} \times 100 = \dfrac{1.026}{1+1.026} \times 100 = \underline{\underline{50.6\ \%}}$

(c) From Eq. 2.17: $\rho_{sat} = \dfrac{\rho_s + \rho_w e}{1+e} = \dfrac{(2.70)+(1.0)(1.026)}{1+1.026} = 1.8391 = \underline{\underline{1.84\ \mathrm{Mg/m^3}}}$

2.40. *The values of minimum e and maximum e for a pure silica sand were found to be 0.50 and 0.70, respectively. What is the corresponding range in the saturated density in kg/m³?*

SOLUTION:

Eq. 2.17: $\rho_{sat} = \dfrac{\rho_s + \rho_w e}{1+e}$

maximum: $\rho_{sat} = \dfrac{(2.70)+(1.0)(0.50)}{1+0.50} = \underline{\underline{2.13\ \mathrm{Mg/m^3}}}$

minimum: $\rho_{sat} = \dfrac{(2.70)+(1.0)(0.70)}{1+0.70} = \underline{\underline{2.00\ \mathrm{Mg/m^3}}}$

2.41. *Calculate the maximum possible porosity and void ratio for a collection of (a) tennis balls (assume they are 64.14 mm in diameter) and (b) tiny ball bearings 0.3 mm in diameter.*

SOLUTION:

Three-dimensional particle arrangement of equal spheres has been studied in depth by mathematicians, statisticians, and materials scientists since the 1600s. A quick internet search on packing of equal spheres will reveal numerous mathematical theories and approaches for estimating the densest and loosest possible packing. In general, the loosest arrangement of equal spheres yields a void fraction of about 0.48, regardless of sphere size. As an aside, the densest possible packing of equal-size spheres yields a solids volume of about:

$V_s = \pi / \sqrt{18} = 0.7405$. (These values are approximate – there is not a unified consensus in the literature.)

Loosest packing

For $V_t = 1.0, \quad V_v = 0.48, \quad$ and $\quad V_s = 1 - 0.48 = 0.52$

thus;

$$n_{max} = \frac{V_v}{V_t} \times 100 = \frac{0.48}{1.0} \times 100 = \underline{\underline{48\ \%}}$$

$$e_{max} = \frac{V_v}{V_s} = \frac{0.48}{0.52} = \underline{\underline{0.92}}$$

Densest packing (not required in problem statement)

$V_s = 0.7405, \quad V_v = 1.0 - 0.7405 = 0.2595$

thus;

$$n_{min} = \frac{0.2595}{1.0} \times 100 = \underline{\underline{26\%}}$$

$$e_{min} = \frac{0.2595}{0.7405} = \underline{\underline{0.35}}$$

2.42. *A plastic-limit test has the following results:*

Wet weight + container = 23.12 g

Dry weight + container = 20.84 g

Container weight = 1.46 g

Compute the PL of the soil. Can the plastic limit be evaluated by a one-point method?

SOLUTION:

$W_w = 23.12 - 20.84 = 2.28\ \text{g}, \quad W_s = 20.84 - 1.46 = 19.38\ \text{g}$

$$PL = \frac{W_w}{W_s} \times 100 = \frac{2.28}{19.38} \times 100 = \underline{\underline{11.8}}$$

The plastic limit cannot be evaluated using a one-point method.

2.43. *During a plastic-limit test, the following data was obtained for one of the samples:*

Wet weight + container = 23.13 g

Dry weight + container = 19.12 g
Container weight = 1.50 g

What is the PL of the soil?

SOLUTION:

$$W_w = 23.13 - 19.12 = 4.01\,\text{g}, \quad W_s = 19.12 - 1.50 = 17.62\text{ g}$$

$$PL = \frac{W_w}{W_s} \times 100 = \frac{4.01}{17.62} \times 100 = \underline{\underline{22.8}}$$

2.44. *The degree of saturation of a cohesive soil is 100%. The clay when wet weighs 1489 g and after drying weighs only 876 g. Find the water content of the soil. Draw a phase diagram and properly label it.*

SOLUTION:

$$S = 100\%, \quad M_t = 1489\text{ g}, \quad M_s = 876\text{ g}$$

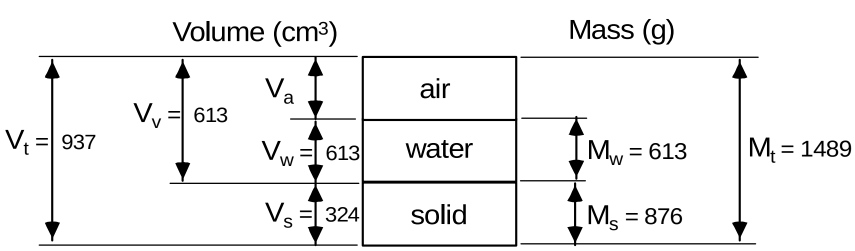
$$M_w = 1489 - 876 = 613\text{ g}; \quad M_t = 876 + 613 = 1489\text{ g}$$

$$w = \frac{M_w}{M_s} \times 100 = \frac{613}{876} \times 100 = \underline{\underline{70.0\%}}$$

$$V_w = \frac{M_w}{\rho_w} = \frac{613}{1.0\, \text{g/cm}^3} = 613\text{ cm}^3, \quad V_v = \frac{V_w}{S} = \frac{613}{1.0} = 613\text{ cm}^3$$

Assume $G_s = 2.70$

$$\text{thus; } V_s = \frac{M_s}{G_s \rho_w} = \frac{876}{(2.7)(1.0)} = 324.4\text{ cm}^3; \quad V_t = 613 + 324 = 937\text{ cm}^3$$



2.45. *For the soil in the previous problem, compute the void ratio and the porosity. Does your answer compare with what you would expect for a saturated cohesive soil?*

SOLUTION:

$$e = \frac{V_v}{V_s} = \frac{613}{324} = \underline{\underline{1.89}}; \quad n = \frac{V_v}{V_t} \times 100\% = \frac{613}{937} \times 100 = \underline{\underline{65.4\%}}$$

2.46. *For the soil in the previous two problems, compute (a) the total or wet density and (b) the dry density. Provide your answers in units of Mg/m^3, kN/m^3, and lbf/ft^3.*

SOLUTION:

(a) $\rho_t = \dfrac{M_t}{V_t} = \dfrac{1489}{937} = 1.5891\ ^{g}/_{cm^3} = \underline{\underline{1.59\ ^{Mg}/_{m^3}}}$

$\gamma_t = \underline{\underline{15.59\ ^{kN}/_{m^3}}} = \underline{\underline{99.2\ ^{lbf}/_{ft^3}}}$

(b) $\rho_{dry} = \dfrac{M_s}{V_t} = \dfrac{876}{937} = 0.9349\ ^{g}/_{cm^3} = \underline{\underline{0.93\ ^{Mg}/_{m^3}}}$

$\gamma_{dry} = \underline{\underline{9.17\ ^{kN}/_{m^3}}} = \underline{\underline{58.3\ ^{lbf}/_{ft^3}}}$

2.47. *A soil specimen had a buoyant density of 73 pounds per cubic foot. Calculate its wet density in kg/m^3.*

SOLUTION:

$\gamma' = 73\ ^{lb}/_{ft^3};\quad \gamma_t = \gamma_{sat} = \gamma' + \gamma_w = 73 + 62.4 = 135.4\ ^{lb}/_{ft^3}$

$\rho_t = (135.4)(16.018) = \underline{\underline{2169\ ^{kg}/_{m^3}}}$

2.53. *The "chunk density" method is often used to determine the unit weight of a specimen of irregular shape. A specimen of cemented silty sand is treated in this way to obtain the "chunk density." From the information given below, determine the (a) wet density, (b) dry density, (c) void ratio, and (d) degree of saturation of the sample.*

Mass of specimen at natural water content in air = 181.8 g
Mass of specimen + wax coating in air = 215.9 g
Mass of specimen + wax in water = 58.9 g
Soil solid density = 2650 kg/m^3
Natural water content = 2.5%
Wax solid density = 940 kg/m^3

SOLUTION:

$$w=\frac{M_t-M_s}{M_s}\quad\rightarrow\quad M_s=\frac{M_t}{1+w}=\frac{181.8}{1+.025}=177.366\text{ g}\quad\rightarrow\quad M_w=M_t-M_s=181.8-177.4=4.43\text{ g}$$

$$V_s=\frac{M_s}{\rho_s}=\frac{177.366\text{ g}}{2.650\ \text{g/cm}^3}=66.93\text{ cm}^3\quad\rightarrow\quad V_w=\frac{M_w}{\rho_w}=\frac{4.43\text{ g}}{1.0\ \text{g/cm}^3}=4.43\text{ cm}^3$$

$$M_{wax}=M_{t+wax}-M_t=215.9-181.8=34.1\text{ g}\quad\rightarrow\quad V_{wax}=\frac{M_{wax}}{\rho_{wax}}=\frac{34.1}{0.940}=36.28\text{ cm}^3$$

$$M_{\text{water displaced}}=M_{t+wax}-M_{\text{water displaced}}\quad\rightarrow\quad M_{\text{water displaced}}=215.9-58.9=157.0\text{ g}$$

$$V_{t+wax}=\frac{M_{\text{water displaced}}}{\rho_w}=\frac{157.0}{1.0}=157.0\text{ cm}^3$$

$$V_{air}=V_{t+wax}-V_{wax}-V_w-V_s=157.0-36.28-4.43-66.93=49.36\text{ cm}^3$$

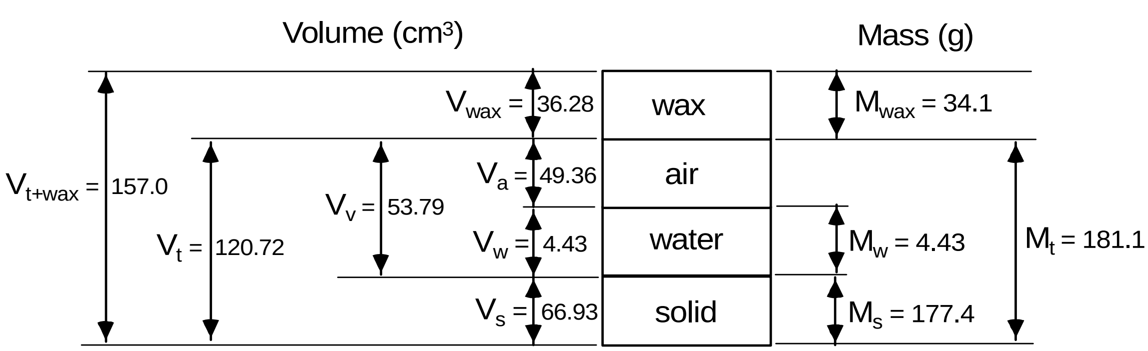
$$V_v=V_{air}+V_w=49.36+4.43=53.79\text{ cm}^3\quad\rightarrow\quad V_t=V_{t+wax}-V_{wax}=157.0-36.28=120.72\text{ cm}^3$$

(a) $\rho_t=\dfrac{M_t}{V_t}=\dfrac{181.8}{120.72}=\underline{\underline{1.50\ \text{Mg/m}^3}}$

(b) $\rho_{dry}=\dfrac{M_s}{V_t}=\dfrac{177.366}{120.72}=\underline{\underline{1.47\ \text{Mg/m}^3}}$

(c) $e=\dfrac{V_v}{V_s}=\dfrac{53.79}{66.93}=\underline{\underline{0.80}}$

(d) $S=\dfrac{V_w}{V_v}\times 100\%=\dfrac{4.43}{53.79}\times 100=\underline{\underline{8.2\ \%}}$



2.54. *A sensitive volcanic clay soil was tested in the laboratory and found to have the following properties:*

(a) ρ = 1.28 Mg/m^3, (b) e = 0.90, (c) S = 95%, (d) ρ_s = 2.75 Mg/m^3, (e) w = 311%.

In rechecking the above values, one was found to be inconsistent with the rest. Find the inconsistent value and report it correctly. Show all your computations and phase diagrams.

SOLUTION:

Assume $V_s = 1\,\text{cm}^3$

$M_s = \rho_s \times V_s = 2.75$ Mg; $\quad M_w = w \times M_s = (3.11)(2.75) = 8.55$ Mg

$M_t = M_s + M_w = 2.75 + 8.55 = 11.30$ Mg

(1) $V_t = \dfrac{M_t}{\rho_t} = \dfrac{11.30}{1.28} = \text{~~8.83 cm}^3\text{~~}$

$V_v = e \times V_s = (9)(1) = 9\text{ cm}^3; \quad V_w = \dfrac{M_w}{\rho_w} = \dfrac{8.55}{1.0} = 8.55\text{ cm}^3$

(2) $V_t = V_v + V_s = 9 + 1 = 10\text{ cm}^3 \neq 8.83\text{ cm}^3$

check : $G_s w = Se$

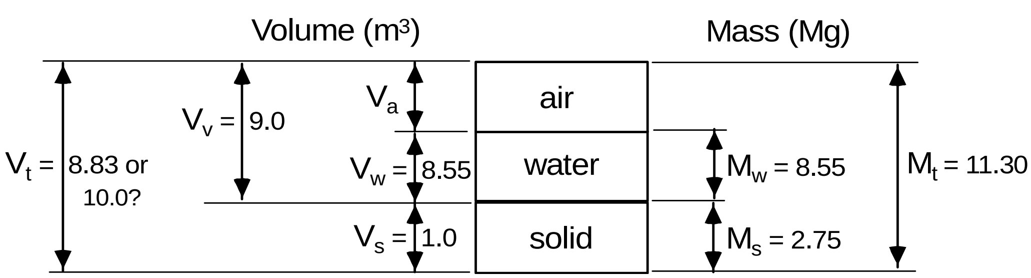
$(2.65)(3.11) = (0.95)(9.0)$

$8.55 = 8.55 \rightarrow$ These values are in correct proportion.

Consequently, the error must be in the ρ_t value.

Re-calculate $\rho_t = \dfrac{M_t}{V_t} = \dfrac{11.30}{10} = 1.13\ ^{Mg}/_{m^3}$

<u>Solution : $\rho_t = 1.13\ ^{Mg}/_{m^3}$</u>



2.55. *A cylinder contains 510 cm³ of loose dry sand which weighs 740 g. Under a static load of 200 kPa the volume is reduced 1%, and then by vibration it is reduced 10% of the original volume. Assume the solid density of the sand grains is 2.65 Mg/m³. Compute the void ratio, porosity, dry density, and total density corresponding to each of the following cases:*

(a) Loose sand. (b) Sand under static load. (c) Vibrated and loaded sand.

SOLUTION:

$$V_s = \frac{M_s}{\rho_s} = \frac{740}{2.65} = 279.24 \text{ cm}^3; \qquad M_t = 740 \text{ g};$$

dry sand: $M_w = V_w = 0.0$

(a) Loose sand - initial condition

$V_t = 510 \text{ cm}^3$

$V_v = V_t - V_s = 510 - 279.24 = 230.75 \text{ cm}^3$

$$e = \frac{V_v}{V_s} = \frac{230.75}{279.24} = \underline{\underline{0.83}}; \qquad n = \frac{V_v}{V_t} \times 100\% = \frac{230.75}{510} \times 100 = \underline{\underline{45.2\%}}$$

$$\rho_{dry} = \frac{M_s}{V_t} = \frac{740}{510} = 1.45 \text{ g/cm}^3 = \underline{\underline{1.45 \text{ Mg/m}^3}}$$

$\rho_{dry} = \rho_t = \underline{\underline{1.45 \text{ Mg/m}^3}}$

(b) Sand under static load

$$V_t = \frac{510}{1.01} = 504.95 \text{ cm}^3$$

$V_v = V_t - V_s = 504.95 - 279.24 = 225.71 \text{ cm}^3$

$$e = \frac{V_v}{V_s} = \frac{225.71}{279.24} = \underline{\underline{0.81}}; \qquad n = \frac{V_v}{V_t} \times 100\% = \frac{225.71}{510} \times 100 = \underline{\underline{44.2\%}}$$

$$\rho_{dry} = \frac{M_s}{V_t} = \frac{740}{504.95} = 1.47 \text{ g/cm}^3 = \underline{\underline{1.47 \text{ Mg/m}^3}}$$

$\rho_{dry} = \rho_t = \underline{\underline{1.47 \text{ Mg/m}^3}}$

(c) Vibrated and loaded sand

$$V_t = \frac{510}{1.10} = 454.54 \text{ cm}^3$$

$V_v = V_t - V_s = 454.54 - 279.24 = 175.30 \text{ cm}^3$

$$e = \frac{V_v}{V_s} = \frac{175.30}{279.24} = \underline{\underline{0.63}}; \qquad n = \frac{V_v}{V_t} \times 100\% = \frac{175.30}{454.54} \times 100 = \underline{\underline{38.6\%}}$$

$$\rho_{dry} = \frac{M_s}{V_t} = \frac{740}{454.54} = 1.63 \text{ g/cm}^3 = \underline{\underline{1.63 \text{ Mg/m}^3}}$$

$\rho_{dry} = \rho_t = \underline{\underline{1.63 \text{ Mg/m}^3}}$

2.56. *On five-cycle semilogarithmic paper, plot the grain-size distribution curves from the following mechanical analysis data on six soils, A through F. For each soil determine the effective size as well as the uniformity coefficient and the coefficient of curvature. Determine also the percentages of gravel, sand silt, and clay according to (a) ASTM, (b) AASHTO, (c) USCS, and (d) the British Standard.*

U.S. Standard Sieve No. or Particle Size	Percent Passing by Weight					
	Soil A	Soil B	Soil C	Soil D	Soil E	Soil F
75 mm (3 in.)	100		100			
38 (1-½)	70		—			
19 (¾)	49	100	91			
9.5 (⅜)	36	—	87			
No. 4	27	88	81		100	
No. 10	20	82	70	100	89	
No. 20	—	80	—	99	—	
No. 40	8	78	49	91	63	
No. 60	—	74	—	37	—	
No. 100	5	—	—	9	—	
No. 140	—	65	35	4	60	
No. 200	4	55	32	—	57	100
40 μm	3	31	27		41	99
20 μm	2	19	22		35	92
10 μm	1	13	18		20	82
5 μm	< 1	10	14		8	71
2 μm	—	—	11		—	52
1 μm	—	2	10		—	39

Note: Missing data is indicated by a dash in the column.

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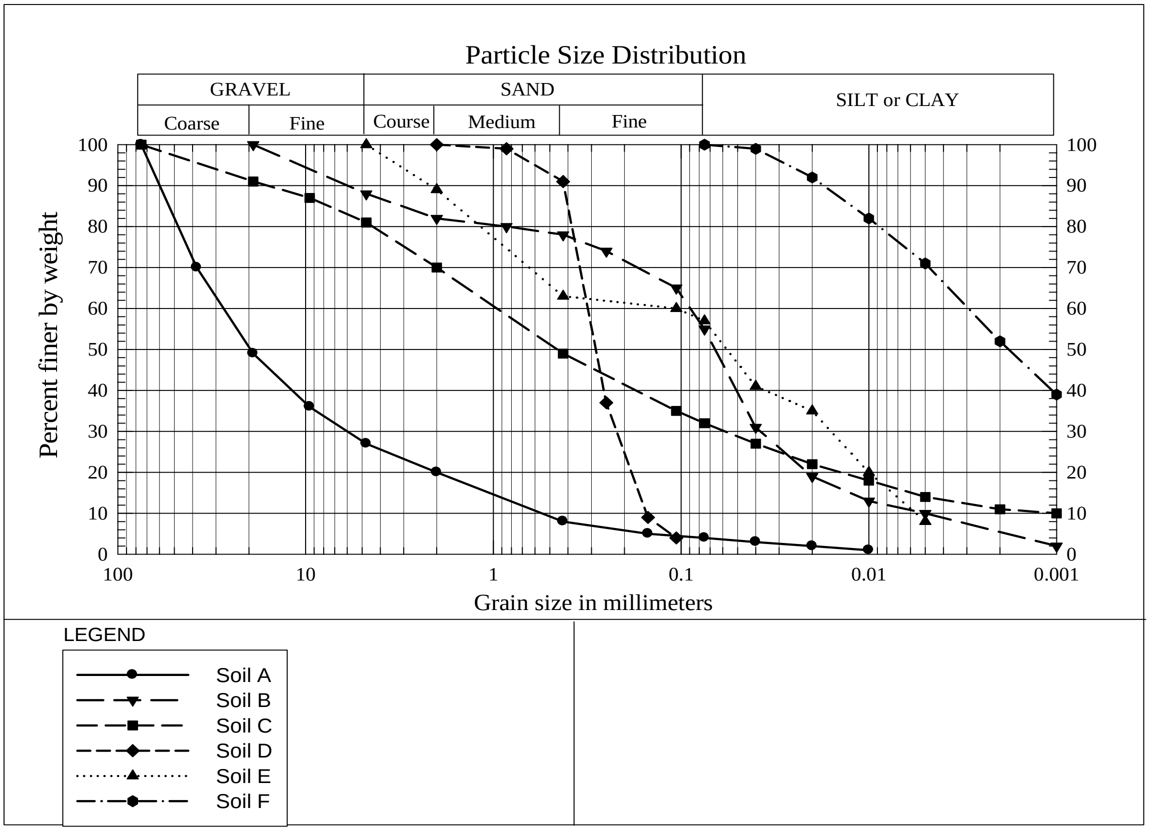
2.56. continued.

SOLUTION: $C_u = \dfrac{D_{60}}{D_{10}}$ $C_c = \dfrac{(D_{30})^2}{D_{60} \times D_{10}}$



Soil	Effective Size D_{10} (mm)	D_{30} (mm)	D_{60} (mm)	C_u	C_c
A	0.6	6	28	46.7	2.1
B	0.005	0.04	0.09	18.0	3.6
C	0.001	0.06	1	1000.0	3.6
D	0.16	0.22	0.3	1.9	1.0
E	0.006	0.015	0.1	16.7	0.4
F	N/D	N/D	0.003	N/D	N/D

continued next page

2.56. continued.

(a) Percentages according to ASTM.

Soil	Gravel (%)	Sand (%)	Fines (silt + clay)	Silt (%)	Clay (%)
A	73	23	4	4	0
B	12	33	55	45	10
C	19	49	32	18	14
D	0	100	0	0	0
E	0	43	57	49	8
F	0	0	100	29	71

(b) Percentages according to AASHTO.

Soil	Gravel (%)	Sand (%)	Fines (silt + clay)	Silt (%)	Clay (%)
A	80	16	4	4	0
B	18	27	55	45	10
C	30	38	32	18	14
D	0	100	0	0	0
E	11	32	57	49	8
F	0	0	100	29	71

(c) Percentages according to USCS.

Soil	Gravel (%)	Sand (%)	Fines (silt + clay)	Silt (%)	Clay (%)
A	73	23	4	--	--
B	12	33	55	--	--
C	19	49	32	--	--
D	0	100	0	--	--
E	0	43	57	--	--
F	0	0	100	--	--

(d) Percentages according to the British Standard.

Soil	Gravel (%)	Sand (%)	Fines (silt + clay)	Silt (%)	Clay (%)
A	80	16	4	4	0
B	18	27	55	53	2
C	30	38	32	21	11
D	0	100	0	0	0
E	11	32	57	57	0
F	0	0	100	48	52

2.58. *The soils in Problem 2.56 have the following Atterberg limits and natural water contents. Determine the PI and LI for each soil and comment on their general activity.*

SOLUTION: $PI = LL - PL;\qquad LI = \dfrac{w_n - PL}{PI}$

Property	Soil A	Soil B	Soil C	Soil D	Soil E	Soil F
w_n (%)	27	14	14	11	8	72
LL	13	35	35	--	28	60
PL	8	29	18	NP	NP	28
PI	5	6	17	0	0	32
LI	3.8	-2.5	-0.24	--	--	1.38

Soil A: very sensitive, highly active

Soil B: most likely a clay above the water table that has experienced a decrease in moisture

Soil C: most likely a clay above the water table that has experienced a decrease in moisture

Soil D: most likely a fine sand

Soil E: most likely a silt

Soil F: slightly sensitive and active

2.59. *Comment on the validity of the results of Atterberg limits on soils G and H.*

SOLUTION:

Based on Atterberg's definitions: LL > PL > SL. Soil G violates the definitions because the SL > PL (25 > 20). Soil H violates the definitions because the PL > LL (42 > 38).

2.60. *The following data were obtained from a liquid-limit test on a silty clay. Two plastic-limit determinations had water contents of 23.1% and 23.6%. Determine the LL, PI, the flow index, and the toughness index. The flow index is the slope of the water content versus log of number of blows in the liquid-limit test, and the toughness index is the PI divided by the flow index.*

SOLUTION:

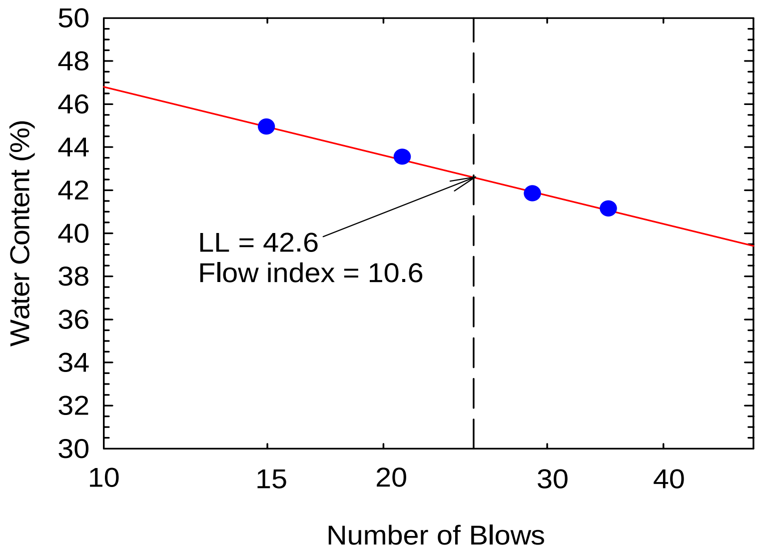
From the plot below, LL = 42.6

Flow Index = slope = $I_F = \dfrac{w_1 - w_2}{\log\left(\dfrac{N_2}{N_1}\right)} = 10.6$ (from linear regresion of best fit line)

use average value of PL tests; $PL = \dfrac{23.1 + 23.6}{2} = 23.4$

$PI = LL - PL = 42.6 - 23.4 = 19.2$

Toughness Index $= \dfrac{PI}{I_F} = \dfrac{19.2}{10.6} = 1.81$



2.61. *Classify the following soils according to the USCS:*

(a) A sample of well-graded gravel with sand has 73% fine to coarse subangular gravel, 25% fine to coarse subangular sand, and 2% fines. The maximum size of the particles is 75 mm. The coefficient of curvature is 2.7, while the uniformity coefficient is 12.4.

(b) A dark brown, wet, organic-odor soil has 100% passing the No. 200 sieve. The liquid limit is 32% (not dried, and is 21% when oven dried!) and the plastic index is 21% (not dried).

(c) This sand has 61% predominately fine sand, 23% silty fines, and 16% fine subrounded gravel size. The maximum size is 20 mm. The liquid limit is 33% and the plastic limit is 27%.

(d) This material has 74% fine to coarse subangular reddish sand and 26% organic and silty dark brown fines. The liquid limit (not dried) is 37% while it is 26% when oven dried. The plastic index (not dried) is 6.

(e) Although this soil has only 6% nonplastic silty fines, it has everything else! It has gravel content of 78% fine to coarse subrounded to subangular gravel, and 16% fine to coarse subrounded to subangular sand. The maximum size of the subrounded boulders is 500 mm. The uniformity coefficient is 40, while the coefficient of curvature is only 0.8. (After U.S. Dept. of the Interior, 1990.)

SOLUTION: Refer to Table 2.7 and corresponding footnotes

(a) GW – Well-graded gravel with sand.

(b) OL -- Organic clay

(c) SM – Silty sand with gravel

(d) SM – Silty sand with organic fines

(e) GP-GM – Poorly graded gravel with silt, sand, cobbles, and boulders (or GP-GC)

2.62. *Classify the five soils in the preceding question according to the AASHTO method of soil classification*.

SOLUTION:

(a) A-1-a

(b) A-8

(c) A-2-4

(d) A-2-4 or A-8

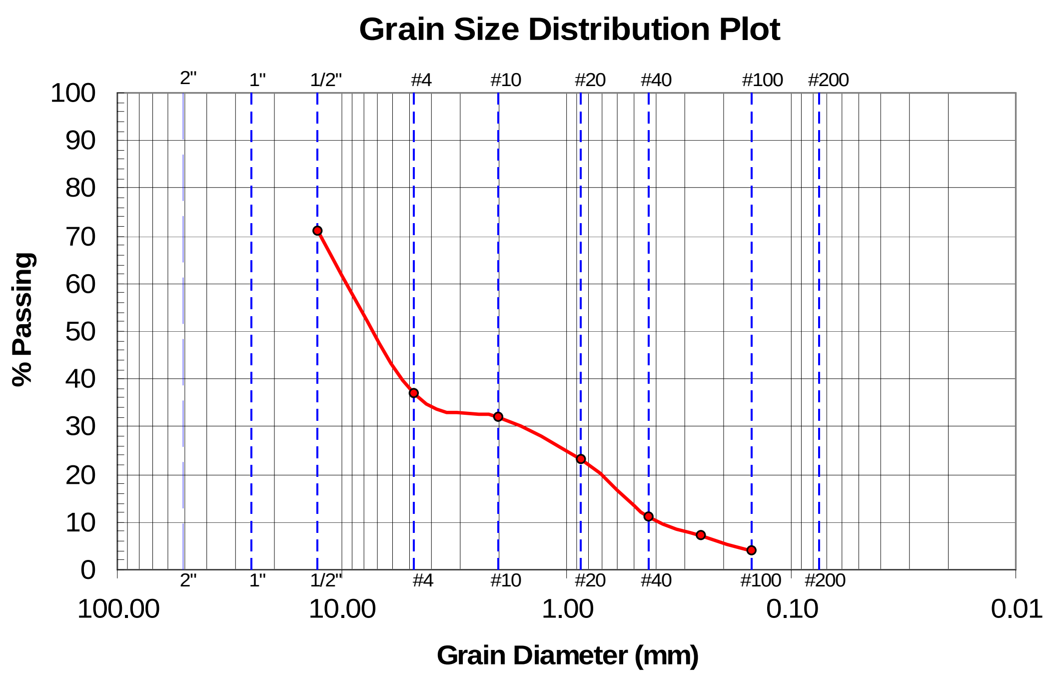
(e) A-1-a

2.63. *The results of a sieve test below give the percentage passing through the sieve.*
(a) Using a spreadsheet, plot the particle-size distribution.
(b) Calculate the uniformity coefficient.
(c) Calculate the coefficient of curvature.

Sieve	Percent Finer by Weight
½"	71
No. 4	37
No. 10	32
No. 20	23
No. 40	11
No. 60	7
No. 100	4

SOLUTION:

$$C_u = \frac{D_{60}}{D_{10}} = \frac{9.5}{0.39} = \underline{\underline{24}}; \qquad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(1.6)^2}{(9.5)(0.39)} = \underline{\underline{0.69}}$$



2.64. For the data given below, classify the soils according to the USCS. For each soil, give both the letter symbol and the narrative description.

(a) 65% material retained on No. 4 sieve, 32% retained on No. 200 sieve. $C_u = 3, C_c = 1.$
(b) 100% material passed No. 4 sieve, 90% passed No. 200 sieve. LL = 23, PL = 17.
(c) 70% material retained on No. 4 sieve, 27% retained on No. 200 sieve. $C_u = 5, C_c = 1.5.$

SOLUTION:

(a) GP – Poorly graded gravel with sand

(b) CL-ML – Silty clay

(c) GW – Well-graded gravel with sand

2.65. *A sample of soil was tested in the laboratory and the following grain size analysis results were obtained. Classify this soil according to the USCS, providing the group symbol for it.*

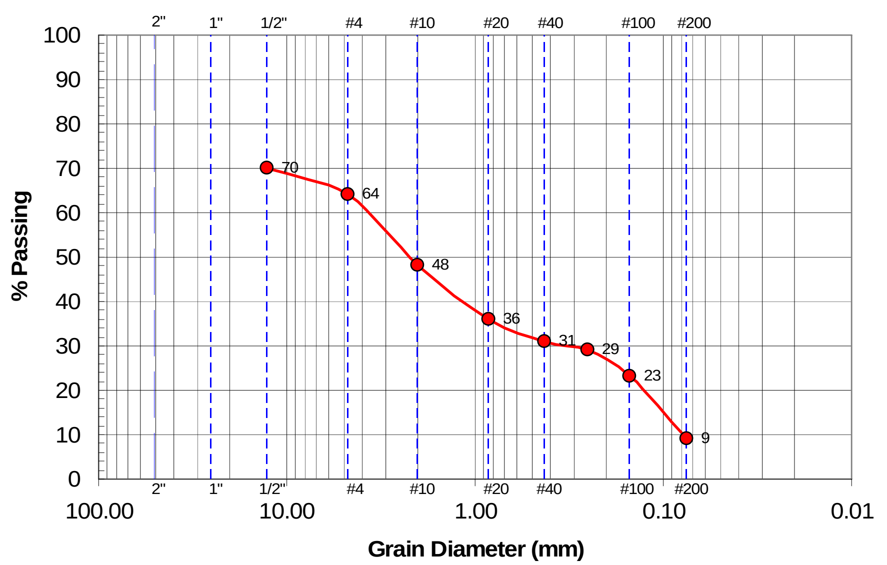
Sieve	Sieve Opening (mm)	Percent Coarser by Weight	Percent Finer by Weight
1/2"	12.7	30	70
4	4.75	36	64
10	2.00	52	48
20	0.85	64	36
40	0.425	69	31
60	0.25	71	29
100	0.15	77	23
200	0.075	91	9

SOLUTION:

(a) $PI = LL - PL = 26 - 23 = 3$

$$C_u = \frac{D_{60}}{D_{10}} = \frac{3.9}{0.08} = 49; \qquad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(0.4)^2}{(3.9)(0.08)} = 0.51 \rightarrow \therefore \text{ well graded}$$

SW-SM (Well-graded sand with silt)



2.66. *A minus No. 40 material had a liquidity index of 0.73, a natural water content of 44.5%, and a plasticity index of 24.7. Classify this soil according to the USCS, provide the group symbol.*

SOLUTION:

$$LI = \frac{w - PL}{PI} \qquad 0.73 = \frac{44.5 - PL}{24.7} \quad \rightarrow \quad PL = \underline{26.5}$$

$$PI = LL - PL; \qquad LL = PI + PL = 24.7 + 26.5 = \underline{51.2}$$

CH (Fat clay)

2.67. *A sample of soil was tested in the laboratory and the following grain size analysis results were obtained. Classify this soil according to the USCS, providing the group symbol for it.*

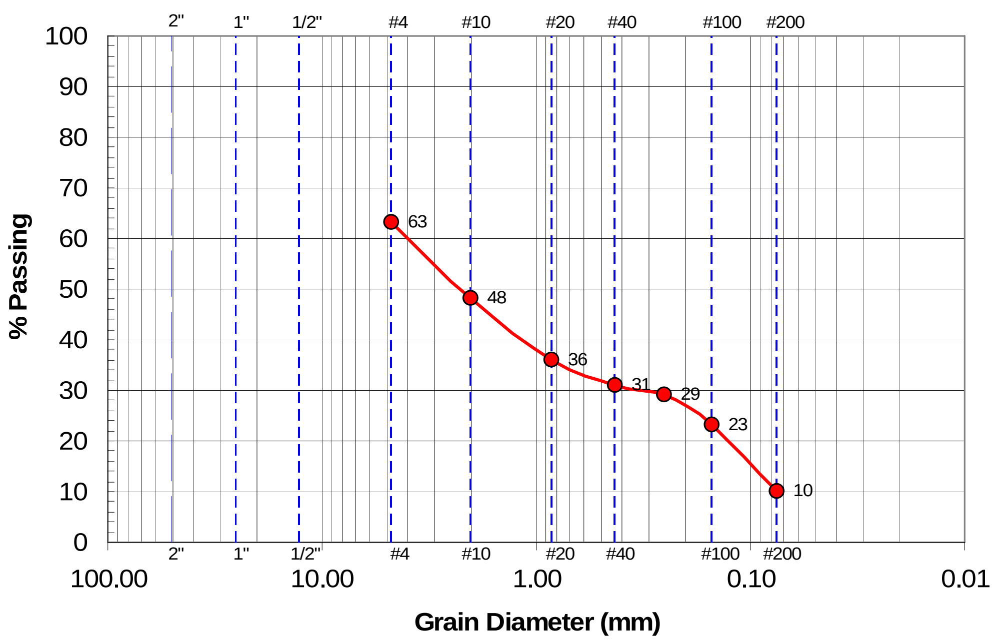
Sieve No.	Sieve Opening (mm)	Percent Coarser by Weight	Percent Finer by Weight
4	4.75	37	63
10	2.00	52	48
20	0.85	64	36
40	0.425	69	31
60	0.25	71	29
100	0.15	77	23
200	0.075	90	10

SOLUTION:

(a) $PI = LL - PL = 60 - 26 = 34$

$$C_u = \frac{D_{60}}{D_{10}} = \frac{4.75}{0.075} = 63; \qquad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(0.425)^2}{(4.75)(0.075)} = 0.51 \rightarrow \therefore \text{ well graded}$$

SW-SC (Well-graded sand with clay)



2.68. *A sample of soil was tested in the laboratory and the following grain size analysis results were obtained: Atterberg limits on minus No. 40 material were: LL = 36, PL = 14. Determine the USCS classification symbol for this soil. Extra credit - determine the full AASHTO classification for this soil (symbol plus group index).*

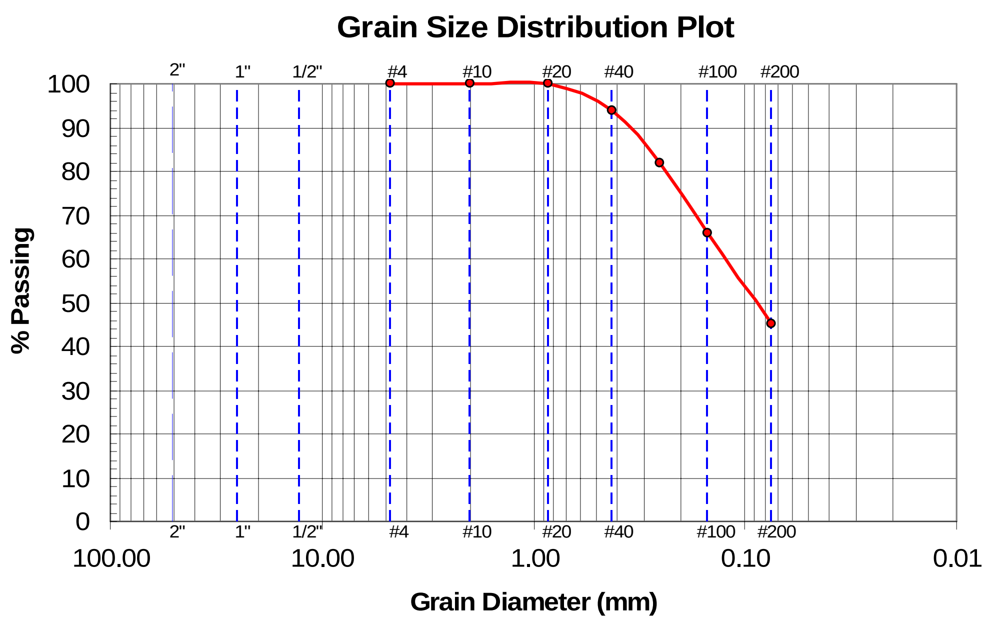
Sieve No.	Sieve Opening (mm)	Percent Finer by Weight
4	4.75	100
10	2.00	100
20	0.85	100
40	0.425	94
60	0.25	82
100	0.15	66
200	0.075	45
Pan	--	0

SOLUTION:

$$PI = LL - PL = 36 - 14 = \underline{22}$$
$$GI = (F - 35)[0.2 + 0.005(LL - 40)] + 0.01(F - 15)(PI - 10)$$
$$GI = (45 - 35)[0.2 + 0.005(36 - 40)] + 0.01(45 - 15)(22 - 10)$$
$$GI = \underline{5.4}$$

(a) USCS: SC (Clayey Sand)

(b) AASHTO: A-6 (5)



2.69. *Laboratory testing was performed on two soil samples (A and B).*
(a) Determine the USCS classification symbol for Sample A.
(b) Determine the AASHTO classification for Sample B.

Sieve No.	Sieve Opening (mm)	A - Percent Passing	B - Percent Passing
3 inch	76.2	100	
1.5 inch	38.1	98	
0.75 inch	19.1	96	
4	4.75	77	100
10	2.00	--	96
20	0.85	55	94
40	0.425	--	73
100	0.15	30	--
200	0.075	18	55
Liquid Limit		32	52
Plastic Limit		25	32

SOLUTION:

(a) $PI = LL - PL = 32 - 25 = 7$

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.3}{0.044} = 29.5; \qquad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(0.15)^2}{(1.3)(0.044)} = 0.393 \rightarrow \therefore \text{ poorly graded}$$

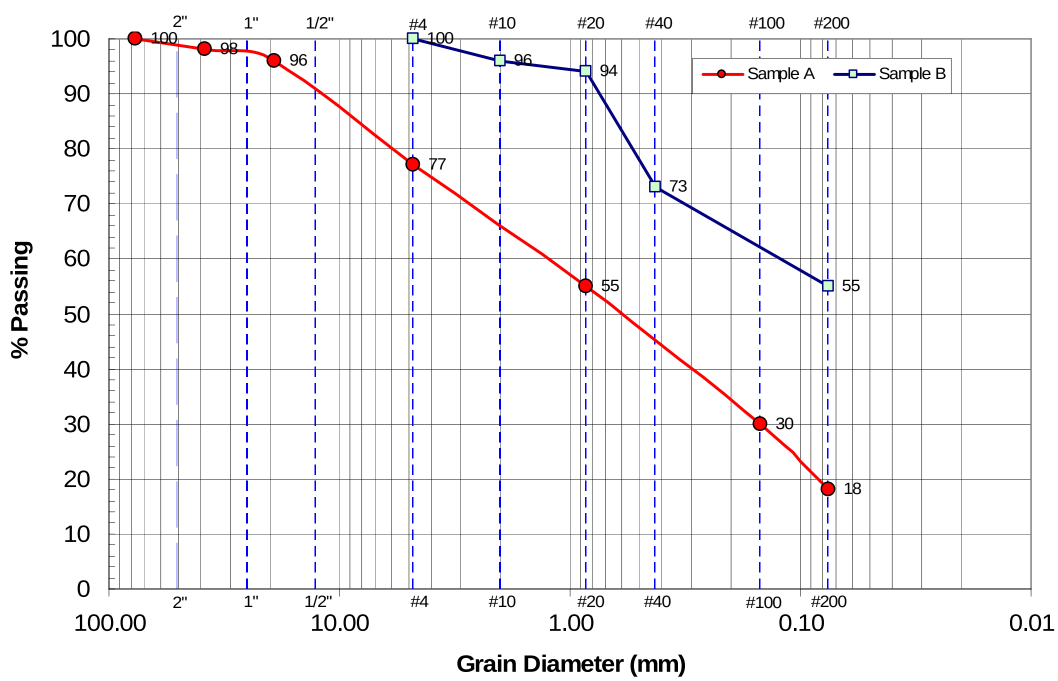
USCS → SM (Silty sand with gravel)

(b) $PI = LL - PL = 52 - 32 = 20; \qquad LL - 30 = 52 - 30 = 22 < 32; \quad \therefore A-7-5$

$GI = (F-35)[0.2 + 0.005(LL-40)] + 0.01(F-15)(PI-10)$

$GI = (55-35)[0.2 + 0.005(52-40)] + 0.01(55-15)(20-10) = 9.2$

AASHTO → A-7-5 (9)



2.70. *A sample of soil was tested in the laboratory and the following grain size analysis results were obtained: Atterberg limits on minus No. 40 material were: LL = 62, PL = 20. Determine the USCS letter symbol for this soil.*

Sieve No.	Sieve Opening (mm)	Percent Coarser by Weight	Percent Finer by Weight
4	4.75	0.0	100.0
10	2.00	5.1	94.9
20	0.85	10.0	90.0
40	0.425	40.7	59.3
60	0.25	70.2	29.8
100	0.15	84.8	15.2
200	0.075	90.5	9.5
Pan	--	100.0	0.0

SOLUTION:

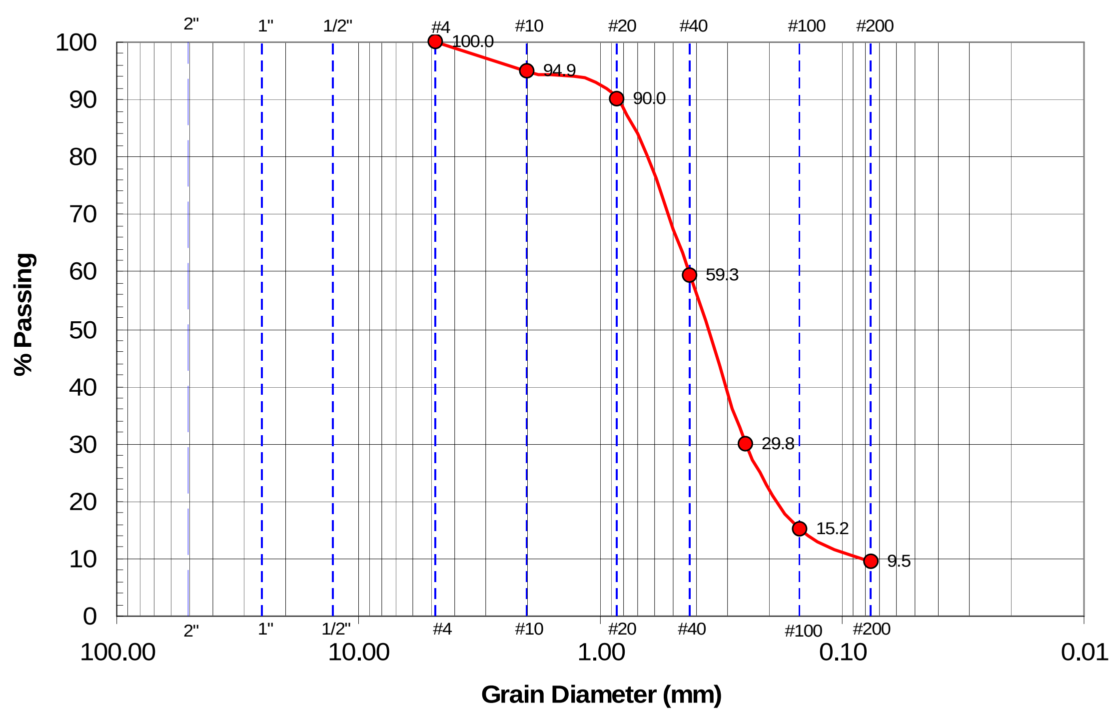
$PI = LL - PL = 62 - 20 = 42$

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.43}{0.075} = 5.7 \cong 6; \qquad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(0.25)^2}{(0.43)(0.075)} = 1.9$$

The C_u value is close to 6. Technically, this soil classifies as poorly graded; however, a well graded determination is not unreasonable.

USCS $\rightarrow$ $\underline{\underline{\text{SP} - \text{SC (Poorly graded sand with clay)}}}$

Grain Size Distribution Plot



2.71. *A sample of a brown sandy clay was obtained to determine its Atterberg limits and then classify its soil type according to the Unified Soil Classification System. For one of the PL determinations, the wet + dish = 11.53 g and the dry weight + dish = 10.49 g. The dish only weighed 4.15 g. Compute the plastic limit. Another plastic limit was 16.9%. Three determinations of the liquid limit were made. For 17 blows, the water content was 49.8%; for 26 blows, the water content was 47.5%; and for 36 blows, the water content was 46.3%. Evaluate the soil type, indicate the information on a plasticity chart, and give the Unified Soil Classification symbol.*

SOLUTION:

$M_w = 11.53 - 10.49 = 1.04 \text{ g}$

$M_s = 10.49 - 4.15 = 6.34 \text{ g}$

$w_{PL} = PL = \frac{M_w}{M_s} \times 100 = \frac{1.04}{6.34} \times 100 = \underline{16.4}$

$PL_{(avg)} = \frac{16.4 + 16.9}{2} = 16.65;$ From plot below: $LL = \underline{48}$

$PI = 48 - 16.7 = \underline{31}$

Based on Casagrande's plasticity chart (Fig. 2.13), the soil fines classify as CL

$\therefore$ USCS $\rightarrow$ $\underline{\underline{\text{CL (Sandy lean clay)}}}$