Solutions

1.1. The kinetic energy of the Boeing = 8×10^9 J. The mass of a mosquito is, say, 1 mg. The mosquito-antimosquito annihilation produces the energy $2\times10^{-6}\,(3\times10^8)^2$=$2\times10^{11}$ J.

1.2. $s=(3E)^2-0=9E^2=9(p^2+m^2)=88.9\ \text{GeV}^2$; $m=\sqrt{s}=9.43$ GeV.

1.3. $\Gamma_{\pi^\pm}=h/\tau_{\pi^\pm}=\left(6.6\times10^{-16}\ \text{eV s}\right)/\left(2.6\times10^{-8}\ \text{s}\right)=25\ \text{neV}$, $\Gamma_K=54$ neV, $\Gamma_\Lambda=2.5\ \mu$eV

1.4. $\tau_\rho=h/\Gamma_\rho=\left(6.6\times10^{-16}\ \text{eV s}\right)/\left(1.49\times10^{12}\ \text{eV}\right)=4.4\times10^{-24}\ \text{s}$, $\tau_\omega=8\times10^{-23}$ s; $\tau_\phi=1.6\times10^{-22}$ s; $\tau_{K^*}=1.3\times10^{-23}$ s; $\tau_{J/\psi}=7\times10^{-21}$ s; $\tau_\Lambda=5.5\times10^{-24}$ s.

1.5. Neglecting the recoil, the momentum transfer would be $q=E_e\sin\theta=2.1$ GeV, corresponding to the resolving power $D\approx 197\ (\text{MeV fm})/2100\ (\text{MeV})=0.1$ fm.

1.6. Our reaction is $p+p\to p+p+m$. In the CM frame the total momentum is zero. The lowest energy configuration of the system is when all particles in the final state are at rest.

a. Let us write down the equality between the expressions of s in the CM and L frames, i. e.

$$s=\left(E_p+m_p\right)^2-p_p^2=\left(2m_p+m\right)^2.$$

Recalling that $E_p^2=m_p^2+p_p^2$, we have $E_p=\dfrac{\left(2m_p+m\right)^2-2m_p^2}{2m_p}=m_p+2m+\dfrac{m^2}{2m_p}$.

b. The two momenta are equal and opposite because the two particles have the same mass, hence we are in the CM frame. The threshold energy E_p^* is given by $s=\left(2E_p^*\right)^2=\left(2m_p+m\right)^2$ which gives $E_p^*=m_p+m/2$.

c. $E_p=1.218$ GeV; $p_p=0.78$ GeV; $T_p=280$ MeV; $E_p^*=1.007$ GeV; $p_p^*=0.36$ GeV.

1.7. a. $s=\left(E_\gamma+m_p\right)^2-p_\gamma^2=\left(E_\gamma+m_p\right)^2-E_\gamma^2=\left(m_p+m_\pi\right)^2=1.16\ \text{GeV}^2$, hence we have $E_\gamma=149$ MeV

b. $s=\left(E_\gamma+E_p\right)^2-\left(\mathbf{p}_\gamma+\mathbf{p}_p\right)^2=m_p^2+2E_\gamma E_p-2\mathbf{p}_\gamma\cdot\mathbf{p}_p$. For a given proton energy, s reaches a maximum for a head-on collision. Consequently, $\mathbf{p}_\gamma\cdot\mathbf{p}_p=-E_\gamma p_p$ and, taking into account that the energies are very large, $s=m_p^2+2E_\gamma\left(E_p+p_p\right)\approx m_p^2+4E_\gamma E_p$. In conclusion

$$E_p=\frac{s-m_p^2}{4E_\gamma}=\frac{(1.16-0.88)\times10^{18}\ \text{eV}^2}{4\times10^{-3}\ \text{eV}}=7\times10^{19}\ \text{eV}=70\ \text{EeV}.$$

c. The attenuation length is $\lambda=1/(\sigma\rho)=1.5\times10^{22}\ \text{m}=5\ \text{Mpc}\ \left(1\ \text{Mpc}=3.1\times10^{22}\ \text{m}\right)$

This is a short distance on the cosmological scale. The cosmic ray spectrum (Fig. 1.10) should not go beyond the above computed energy. This is called the Greisen, Zatzepin and Kusmin (GZK) bound. The AUGER observatory is now exploring this extreme energy region.

1.8. We call E_i the incident gamma energy and E_f the background gamma energy. At threshold $s=(2m_e)^2$.

For a given E_i, s is a maximum for head-on collisions: $s=\left(E_i+E_f\right)^2-\left(E_i-E_f\right)^2=4E_iE_f$.

Hence at threshold: $E_i = m_e^2 / E_f$.

a. $E_f = \frac{1}{\lambda} = \left(10^6 \text{ m}^{-1}\right) \times \left(1.97 \times 10^{-7} \text{ eV/m}^{-1}\right) \simeq 0.2 \text{ eV}$ and $E_i = \frac{\left(5 \times 10^5\right)^2 \text{ eV}^2}{0.2 \text{ eV}} = 1.25 \text{ TeV}$.

b. $E_i = \frac{\left(5 \times 10^5\right)^2 \text{ eV}^2}{10^{-3} \text{ eV}} = 250 \text{ TeV}$.

1.9. $s = \left(E_p + m_p\right)^2 - p_p^2 = \left(4m_p\right)^2 \Rightarrow E_{p,\min} = 7m_p = 6.6 \text{ GeV}$.

1.10. Calling E_b the beam energy at fixed target and E_p the energies of the colliding beams, the condition is $2m_pE_b = 4E_p^2$, hence he have $E_b = 100 \text{ PeV}$. This value is well above the 'knee' of the cosmic ray spectrum, but it is much smaller than the GZK bound

1.11. We must consider the reaction

$$M \rightarrow m_1 + m_2 .$$

The figure defines the CM variables

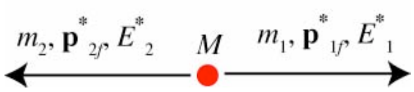

Fig. S.1

We can use equations (P1.5) and (P1.6) with $\sqrt{s}=M$, obtaining

$$E_{2f}^* = \frac{M^2 + m_2^2 - m_1^2}{2M};\ E_{1f}^* = \frac{M^2 + m_1^2 - m_2^2}{2M} .$$

The corresponding momenta are

$$\mathbf{p}_f^* \equiv \mathbf{p}_{1f}^* = -\mathbf{p}_{2f}^* = \sqrt{E_{1f}^{*2} - m_1^2} = \sqrt{E_{2f}^{*2} - m_2^2} .$$

1.12. In the Λ decay we have

$$E_\pi^* = \frac{m_\Lambda^2 - m_p^2 + m_\pi^2}{2m_\Lambda} = 0.17 \text{ GeV};\quad E_p^* = m_\Lambda - E_\pi^* = 0.94 \text{ GeV};\quad p^* = \sqrt{E_\pi^{*2} - m_\pi^2} = 0.1 \text{ GeV} .$$

And in the Ξ decay we have: $E_\pi^* = 0.20 \text{ GeV}$; $E_\Lambda^* = 1.12 \text{ GeV}$; $p^* = 0.14 \text{ GeV}$.

1.13. The expressions found in problem 1.11 become $E_2^* = \frac{M^2 - m_1^2}{2M}$ and $E_1^* = \frac{M^2 + m_1^2}{2M}$.

Since m_2=0, the CM momentum is $p^* = E_2^* = \frac{M^2 - m_1^2}{2M}$.

1.14. Let call x a coordinate along the beam. The velocity of the pions in L should not be larger than the velocity of the muon in the CM, i. e. $\beta_\pi \le \beta^*_{\mu x} \le \beta^*_\mu$. Let us use the formulae found in problem 1.14 to calculate the Lorentz parameters for the CM-L transformation

$$\beta_\mu^* = \frac{p^*}{E_\mu^*} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2};\quad \gamma_\mu^* = \frac{E_\mu^*}{m_\mu} = \frac{m_\pi^2 + m_\mu^2}{2m_\mu m_\pi} \quad \Rightarrow \beta_\mu^*\gamma_\mu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\mu m_\pi} .$$

The condition $\beta_\pi < \beta_\mu^*$ gives $p_\pi = \beta_\pi \gamma_\pi m_\pi < \beta_\mu^* \gamma_\pi^* m_\pi = \frac{m_\pi^2 - m_\mu^2}{2m_\mu} = 39.35 \text{ MeV}$.

1.15. When dealing with a Lorentz transformation problem, the first step is the accurate drawing of the momenta in the two frames and the definition of the kinematic variables.

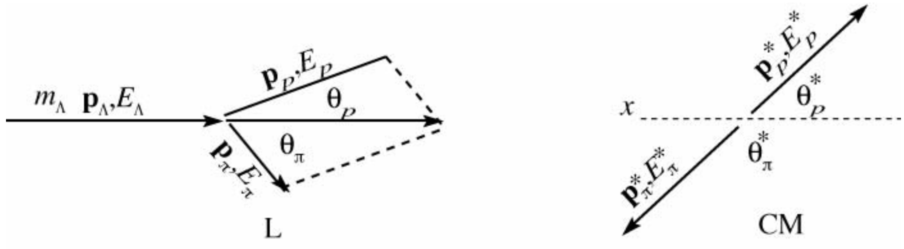

Fig. S.2

Using the expressions we found in the introduction we have:

a. $E_\pi^* = \dfrac{m_\Lambda^2 - m_p^2 + m_\pi^2}{2m_\Lambda} = 0.17$ GeV; $E_p^* = 0.95$ GeV; $p_\pi^* = p_p^* = \sqrt{E_\pi^{*2} - m_\pi^2} = 0.096$ GeV.

b. We calculate the Lorentz factors for the transformation:

$E_\Lambda = \sqrt{p_\Lambda^2 + m_\Lambda^2} = 2.29$ GeV; $\beta_\Lambda = \dfrac{p_\Lambda}{E_\Lambda} = 0.87$; $\gamma_\Lambda = \dfrac{E_\Lambda}{m_\Lambda} = 2.05$.

c. We do the transformation and calculate the requested quantities

$p_\pi \sin\theta_\pi = p_\pi^* \sin\theta_\pi^* = 0.096 \times \sin 210° = -0.048$ GeV

$p_\pi \cos\theta_\pi = \gamma_\Lambda \left(p_\pi^* \cos\theta_\pi^* + \beta_\Lambda E_\pi^*\right) = 2.05(0.096 \times \cos 210° + 0.87 \times 0.17) = 0.133$ GeV.

$\tan\theta_\pi = \dfrac{-0.048}{0.133} = -0.36$ $\theta_\pi = -20°$; $p_\pi = \sqrt{\left(p_\pi \sin\theta_\pi\right)^2 + \left(p_\pi \cos\theta_\pi\right)^2} = 0.141$ GeV.

$p_p \sin\theta_p = p_p^* \sin\theta_p^* = 0.048$ GeV

$p_p \cos\theta_p = \gamma_\Lambda \left(p_p^* \cos\theta_p^* + \beta_\Lambda E_p^*\right) = 2.05(0.096 \times \cos 30° + 0.87 \times 0.95) = 1.86$ GeV

$\tan\theta_p = \dfrac{0.048}{1.86} = 0.026$ $\theta_p = 1.5°$.

$p_p = \sqrt{\left(p_p \sin\theta_p\right)^2 + \left(p_p \cos\theta_p\right)^2} = 1.9$ GeV; $\theta = \theta_p - \theta_\pi = 21.5°$.

1.16. Remember to start by drawing the momentum vectors in the two reference frames, as in problem 1.15. We now have, being in non-relativistic conditions,

$$E_1 = E_3 + E_4 \quad \Rightarrow \quad \frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_4^2}{2m} \quad \Rightarrow \quad p_1^2 = p_3^2 + p_4^2.$$

$$\mathbf{p}_1 = \mathbf{p}_3 + \mathbf{p}_4 \quad \Rightarrow \quad p_1^2 = p_3^2 + p_4^2 + 2\mathbf{p}_3 \cdot \mathbf{p}_4 = p_3^2 + p_4^2 \quad \Rightarrow \quad \mathbf{p}_3 \cdot \mathbf{p}_4 = 0.$$

$\theta_{34} = \theta_{13} + \theta_{14} = \pi/2$: at non-relativistic speeds the angle between the final directions is 90°.

1.17. We continue to refer to the figure of problem 1.15. We shall solve our problem in two ways: by performing a Lorentz transformation and by using the Lorentz invariants.

We start with the first method. We calculate the Lorentz factors. The energy of the incident proton is $E_1 = \sqrt{p_1^2 + m_p^2} = 3.143$ GeV. Firstly, let us calculate the CM energy squared of the two-proton system (i. e. its mass squared).

$p_{pp} = p_1 = 3$ GeV; $E_{pp} = E_1 + m_p = 4.081$ GeV. Hence $s = 2m_p^2 + 2E_1 m_p = 7.656$ GeV^2.

The Lorentz factors are $\beta_{pp} = p_{pp}/E_{pp} = 0.735$ and $\gamma_{pp} = E_{pp}/\sqrt{s_{pp}} = 1.47$.

Since all the particles are equal, we have

$$E_1^* = E_2^* = E_3^* = E_4^* = \frac{\sqrt{s}}{2} = 1.385 \text{ GeV}; \; p_1^* = p_2^* = p_3^* = p_4^* = \sqrt{E_1^{*2} - m_p^2} = 1.019 \text{ GeV}.$$

We now perform the transformation. To calculate the angle we must calculate firstly the components of the momenta

$$p_3 \sin\theta_{13} = p_3^* \sin\theta_{13}^* = 1.019 \times \sin 10° = 0.177 \text{ GeV}.$$

$$p_3 \cos\theta_{13} = \gamma\left(p_3^* \cos\theta_{13}^* + \beta E_3^*\right) = 1.473 \times (1.019 \times \cos 10° + 0.735 \times 1.385) = 2.978 \text{ GeV}.$$

$$\tan\theta_{13} = \frac{0.177}{2.978} = 0.0594; \qquad \theta_{13} = 3°.$$

$$-p_4 \sin\theta_{14} = -p_4^* \sin\theta_{14}^* = -1.019 \times \sin 170° = -0.1769 \text{ GeV}.$$

$$p_4 \cos\theta_{14} = \gamma\left(p_4^* \cos\theta_{14}^* + \beta E_4^*\right) = 1.473 \times (1.019 \times \cos 170° + 0.735 \times 1.385) = 0.0213 \text{ GeV}.$$

$$\tan\theta_{14} = -0.1769 / 0.0213 = -8.305 \qquad \theta_{14} = -83° \qquad \Rightarrow \qquad \theta_{34} = \theta_{13} - \theta_{14} = 86°.$$

In relativistic conditions the angle between the final momenta in a collision between two equal particles is always, as in this example, smaller than 90°.

We now solve the problem using the invariants and the expressions in the introduction. We want the angle between the final particles in L. We then write down the expression of s in L in the initial state, which have already calculated, i. e.

$$s = (E_3 + E_4)^2 - (\mathbf{p}_3 + \mathbf{p}_4)^2 = m_3^2 + m_4^2 + 2E_3E_4 - 2\mathbf{p}_3 \cdot \mathbf{p}_4$$

that gives $\mathbf{p}_3 \cdot \mathbf{p}_4 = m_p^2 + E_3E_4 - s/2$ and hence $\cos\theta_{34} = \dfrac{m_p^2 + E_3E_4 - s/2}{p_3 p_4}$.

We need E_3 and E_4 (and their momenta); we can use (P.1.13) if we have t. With the data of the problem we can calculate t in the CM:

$$t = 2m_p^2 + 2p_i^{*2} \cos\theta_{13}^* - 2E_i^{*2} = 2p_i^{*2}\left(\cos\theta_{13}^* - 1\right) = 2 \times 1.019^2 \left(\cos 10° - 1\right) = -0.0316 \text{ GeV}^2.$$

We then obtain

$$E_3 = \frac{s + t - 2m_p^2}{2m_p} = \frac{7.656 - 0.0316 - 2 \times 0.938^2}{2 \times 0.938} = 3.126 \text{ GeV}; \qquad p_3 = 2.982 \text{ GeV}.$$

From energy conservation we have

$$E_4 = E_1 + m_p - E_3 = 3.143 + 0.938 - 3.126 = 0.955 \text{ GeV}; \qquad p_4 = 0.179 \text{ GeV}.$$

Finally we obtain

$$\cos\theta_{34} = \frac{0.938^2 + 3.126 \times 0.955 - 7.656/2}{2.982 \times 0.179} = 0.0696 \qquad \Rightarrow \theta_{34} = 86°.$$

1.18. We must take into account that β_D is close to 1. We write

$$\gamma_D = \frac{E}{m_D} = 16.1; \quad \beta_D = \sqrt{\frac{\gamma_D^2 - 1}{\gamma_D^2}} = \sqrt{1 - \gamma_D^{-2}} \approx 1 - \frac{\gamma_D^{-2}}{2} = 0.998.$$

In the L reference frame the D life was $t = \dfrac{d}{\beta c} = 10$ ps long. In its rest-frame was $t_0 = t/\gamma_D = 0.62$ ps.

From $p_K = p_\pi = p^*$; $E_K + E_\pi = m_D$; $m_D = \sqrt{p_K^2 + m_K^2} + \sqrt{p_\pi^2 + m_\pi^2}$ we obtain

$$p_\pi = \sqrt{\left(\frac{m_D^2 + m_\pi^2 - m_K^2}{2m_D}\right)^2 - m_\pi^2} = 860 \text{ MeV}.$$

1.19. The distance travelled by a pion in a lifetime in the L frame is $l_0 = \gamma\beta c\tau_\pi$. If the initial number of pions is N_0, their number at the distance l is $N(l) = N_0 \exp\left(-\frac{l}{\gamma\tau_\pi\beta c}\right)$. Hence

$$\gamma\beta = \frac{l}{\tau_\pi c \ln\frac{N_0}{N(l)}} = \frac{20}{2.6\times10^{-8}\times3\times10^{-8}\times\ln(1/0.9)} = 24.3$$

and $p = m\gamma\beta = 0.14\times24.3 = 3.4$ GeV; $E = \sqrt{p^2 + m_\pi^2} = \sqrt{3.4^2 + 0.14^2} = 3.42$ GeV.

1.20. In this case the reference frames L and CM coincide. We have

$\mathbf{p}_{\pi^0} + \mathbf{p}_n = 0 \quad \Rightarrow \quad p_{\pi^0} = p_n = p^*$.

The total energy is $E = E_{\pi^0} + E_n = m_{\pi^-} + m_n = 1079$ MeV.

Subtracting the members of the two relationships $E_n^2 = p^{*2} + m_n^2$ and $E_{\pi^0}^2 = p^{*2} + m_{\pi^0}^2$ we obtain

$E_n^2 - E_{\pi^0}^2 = m_n^2 - m_{\pi^0}^2$

From $E_n = E - E_{\pi^\circ}$ we have $E_n^2 = E^2 + E_{\pi^0}^2 - 2EE_{\pi^0}$; and finally

$$E_{\pi^0} = \frac{E^2 + E_{\pi^0}^2 - E_n^2}{2E} = \frac{E^2 + m_{\pi^0}^2 - m_n^2}{2E} = 138.8 \text{ MeV}; T_n = E - E_{\pi^0} - m_n = 0.6 \text{ MeV}.$$

The Lorentz factors are $\gamma_{\pi^0} = E_{\pi^0}/m_{\pi^0} = 1.028$ nd $\beta_{\pi^0} = \sqrt{1 - 1/\gamma_{\pi^0}^2} = 0.23$.

The distance travelled in a lifetime is then

$l = \gamma_{\pi^0}\tau_{\pi^0}\beta_{\pi^0}c = 1.028\times8.4\times10^{-17}\times0.23\times3\times10^8 = 6$ nm.

1.21. The maximum momentum transfer is at background scattering. Eq. (6.25) gives in these conditions $Q^2=4EE'$, where E' is the energy of the scattered electron. Using Eq. (6.11) we have

$$Q_{\max}^2 = \frac{4E^2M}{M+2E} = \frac{4\times4\times56}{56+4} = 15 \text{ GeV}^2.$$

1.22. Having the α particle charge z=2, the cross section is

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E_k^2\sin^4\frac{\theta}{2}} = \frac{Z^2\alpha^2}{E_k^2}\frac{1}{(1-\cos\theta)^2}.$$

Integrating on the angles we have

$$\int_0^{2\pi} d\phi \int_{\theta_1}^{\theta_2} d\cos\theta \frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{E^2}2\pi \int_{\theta_1}^{\theta_2} \frac{1}{(1-\cos\theta)^2} d\cos\theta = \left(\frac{Z^2\alpha^2}{E^2}2\pi\right) \left.\frac{1}{\cos\theta - 1}\right|_{\theta_1}^{\theta_2}.$$

Hence $\left(\frac{d\sigma}{d\Omega}\right)_{\theta>90^0} / \left(\frac{d\sigma}{d\Omega}\right)_{\theta>10^0} = 0.0074$.

1.23. The requested rate is given by $R_s = \frac{\sigma(\theta>0.1)R_i t\rho N_A}{197\times(10^{-3}\text{kg})}$. We calculate the cross section

$$\sigma(\theta > \theta_1) = \int_{\theta_1}^{\pi} d\cos\theta \frac{d\sigma}{d\cos\theta} = \left(\frac{Z^2\alpha^2}{E_k^2}2\pi\right)\left(\frac{1}{\cos\pi - 1} - \frac{1}{\cos\theta_1 - 1}\right) = \left(\frac{Z^2\alpha^2}{E_k^2}2\pi\right)\left(\frac{1}{1-\cos\theta_1} - \frac{1}{2}\right)$$

$=4.5\times10^3$ barn. The requested rate is

$$R_s = \frac{4.5\times10^{-25}(\mathrm{m}^2)\times10^3(\mathrm{s}^{-1})\times10^{-6}(\mathrm{m})\times1.93\times10^4(\mathrm{kg\ m}^{-3})\times6.02\times10^{23}}{197\times\left(10^{-3}\mathrm{kg}\right)} = 26\ \mathrm{s}^{-1}.$$

1.24. At any angle the scattered electron energy reaches its maximum if the scattering is elastic and we have $E' = \dfrac{E}{1+\dfrac{E}{M}(1-\cos\theta)} = \dfrac{10}{1+\dfrac{10}{1}(1-0.87)} = 4.3\ \mathrm{GeV}$.

1.25. $\cos\theta = 1 - \dfrac{E/E'-1}{E/M} = 1 - \dfrac{2.5-1}{20} = 0.925 \qquad \theta = 22^\circ$.

1.26. 0.5.

1.27. The equation of motion is $q\mathbf{v}\times\mathbf{B} = \dfrac{d\mathbf{p}}{dt}$. Since in this case the Lorentz factor γ is constant, we can write $q\mathbf{v}\times\mathbf{B} = \gamma m\dfrac{d\mathbf{v}}{dt}$. The centripetal acceleration is then: $\left|\dfrac{d\mathbf{v}}{dt}\right| = \dfrac{qvB}{\gamma m} = \dfrac{v^2}{\rho}$. Simplifying we obtain $p = qB\rho$. We now want pc in GeV, B in tesla and ρ in metres. Starting from $pc = qcB\rho$ we have

$$pc[\mathrm{GeV}]\times1.6\times10^{-10}[\mathrm{J/GeV}] = 1.6\times10^{-19}[\mathrm{C}]\times3\times10^8[\mathrm{m/s}]\times B[\mathrm{T}]\times\rho[\mathrm{m}].$$

Finally in N.U.: $p[\mathrm{GeV}] = 0.3\times B[\mathrm{T}]\times\rho[\mathrm{m}]$.

1.28. The number of protons in the unit volume of the target is $n_p = \dfrac{\rho\times N_A}{1\times10^{-3}} = 3.6\times10^{28}\ \mathrm{m}^{-3}$; N_H and N_0 are linked by the relationship $N_H = N_o e^{-n_p\sigma l}$. Consequently, we have

$$\sigma = \frac{1}{n_p l}\ln\frac{N_o}{N_H} = \frac{10^{-29}}{0.36}\ln\frac{7.5}{6.9} = 23.2\ \mathrm{mb}.$$

The statistical uncertainty about the incoming particles number is $\Delta N_i = \sqrt{N_i}$ and similarly for the outgoing number. The statistical error on the cross section is

$$\Delta\sigma = \left[\left(\frac{\partial\sigma}{\partial N_o}\right)^2 N_o + \left(\frac{\partial\sigma}{\partial N_H}\right)^2 N_H\right]^{\frac{1}{2}} = \frac{1}{n_p l}\left[\frac{1}{N_o} + \frac{1}{N_H}\right]^{\frac{1}{2}} = 0.6\ \mathrm{mb}.$$

The final result is $\sigma = 23.2 \pm 0.6$ mb.

1.29. The Lorentz factor of the antiproton is $\gamma = \sqrt{p^2+m^2}/m = 1.62$ and its velocity $\beta = \sqrt{1-\gamma^{-2}} = 0.787$. The condition in order to have the antiproton above the Cherenkov threshold is that the index is $n \geq 1/\beta = 1.27$.

If the index is n=1.5, the Cherenkov angle is given by $\cos\theta = 1/n\beta = 0.85$. Hence $\theta = 32^\circ$.

1.30. The speed of a particle of momentum $p=m\gamma\beta$ is $\beta = \left(1+\frac{m^2}{p^2}\right)^{-1/2} \approx 1-\frac{m^2}{2p^2}$, that is a good approximation for speeds close to c. The difference between the flight times is $\Delta t = L\frac{m_2^2 - m_1^2}{2p^2}$ in N.U. In order to have Δt>600 ps, we need a base-length L>26 m.

1.31. The threshold condition is $n>\beta^{-1}$. Consequently, the index must satisfy the condition $1-\beta_\pi < n-1 < 1-\beta_K$. Since the speeds are very near to 1, we calculate the differences $1-\beta$ directly. From $\beta^{-1} = \frac{E}{p} \simeq 1+\frac{m^2}{2p^2}$ we have $\beta - 1 \simeq \frac{m^2}{2p^2}$. Hence $1-\beta_\pi = 2.45\times10^{-5}$ and $1-\beta_K = 3.05\times10^{-4}$. Consequently the condition on the pressure is 8.2 kPa<Π<102 kPa.

1.32. Superman saw the light blue shifted due to Doppler effect. Taking for the wavelengths λ_R=650 nm and λ_G=520 nm, we have $\nu_G/\nu_R = 1.25$. Solving for β the Doppler shift expression $\nu_G = \nu_R\sqrt{\frac{1+\beta}{1-\beta}}$, we obtain β=0.22.

1.33

1. The minimum velocity is $\beta_{\min} = \frac{1}{n} = 0.75$; 2.The minimum kinetic energy for a proton is $E_{kin,\min}(p) = m_p\left(\frac{1}{\sqrt{1-\beta_{\min}^2}} - 1\right) = 938\times0.51 = 480\ \text{MeV}$ and of the pion: $E_{kin,\min}(\pi) = m_\pi\left(\frac{1}{\sqrt{1-\beta_{\min}^2}} - 1\right) = 139.6\times0.51 = 71.2\ \text{MeV}$; 3. the Lorentz factor is $\gamma = \frac{E_\pi}{m_\pi} = \frac{400}{139.6} = 2.87$ and $\beta = \sqrt{1-\gamma^{-2}} = 0.94$. The Cherenkov angle is then $\theta = \cos^{-1}\left(\frac{1}{\beta n}\right) = 36.9^0$

1.34.

a. The Cherenkov threshold is $\beta_{thr}^{-1} = n$. For a generic mass m

$$\beta^{-1} - 1 = \frac{E}{p} - 1 = \sqrt{\frac{p^2+m^2}{p^2}} - 1 \simeq \frac{1}{2}\frac{m^2}{p^2}$$

Threshold condition for pions is given by

$$\beta^{-1} - 1 = n - 1 = 3\times10^{-9}\Pi = 3\times10^{-9}\times5.2\times10^3 = 1.56\times10^{-5}$$

and $p = \frac{m_\pi}{\sqrt{2\times1.56\times10^{-5}}} = 25\ \text{GeV}$

b. $\Pi(K) = 5.2\times10^3\ \frac{m_K^2}{m_\pi^2} = 5.2\times10^3\ \left(\frac{0.494}{0.140}\right)^2 = 6.5\times10^4$ Pa=650 mbar

c. $\Pi(p) = 5.2\times10^3\ \frac{m_p^2}{m_\pi^2} = 5.2\times10^3\ \left(\frac{0.938}{0.140}\right)^2 = 2.33\times10^5$ Pa=2330 mbar

1.35.

1. $E = p = 0.3 \times B \times R = 0.3 \times 10^{-9} \times 10^{13} = 3$ TeV
2. $E = p = 0.3 \times B \times R = 0.3 \times 5 \times 10^{-11} \times 3 \times 10^{20} = 5 \times 10^{9}$ GeV

1.36

1. The total energy of the deuterons is $E_d = m_d + T_d = 1875.7$ MeV. The motion of the deuterons is not relativistic. Their momentum is
$p_d = \sqrt{2m_d T_d} = \sqrt{2 \times 1875.6 \times 0.13} = 61.25$ MeV. This is also the total momentum, which is so small that in this case the L frame is also in practice the CM frame.
The CM energy squared is $\sqrt{s} = \sqrt{(E_d + m_t)^2 - p_d^2} \simeq E_d + m_t = 4684.6$. The result could be obtained by simply summing the two masses and the deuteron kinetic energy. This because the situation is non relativistic. The total kinetic energy available after the reaction is $E_{kin,t} = E_d + m_t - m_\alpha - m_n = 17.6$ MeV, which is mainly taken by the lighter particle, the neutron. To be precise

$$T_n = \frac{s + m_n^2 - m_\alpha^2}{2\sqrt{s}} - m_n = \frac{4684.6^2 + 939.6^2 - 3727.4^2}{2 \times 4684.6} - 939.6 = 953.6 - 939.6 = 14.0 \text{ MeV}$$

and

$$T_\alpha = \frac{s + m_\alpha^2 - m_n^2}{2\sqrt{s}} - m_n = \frac{4684.6^2 + 3727.4^2 - 939.6^2}{2 \times 4684.6} - 3727.4 = 3.6 \text{ MeV}$$

2. The flux is $\Phi = \frac{I_n}{4\pi R^2} = \frac{3 \times 10^{10}}{4\pi \times 1^2} = 2.4 \times 10^9$ nutrons/(m^{2}s).

3. We can calculate the momentum of the neutron non relativistically
$p_n = \sqrt{2m_n T_n} = \sqrt{2 \times 939.6 \times 14} = 162.2$ MeV, and its velocity
$\beta_n = \frac{p_n}{E_n} = \frac{162.2}{953.6} = 0.17$ $\quad\upsilon_n = 5.1 \times 10^7$ m/s. We need 1 ns time resolution

1.37. The minimum momentum to resolve the structure is $p_{\min} = \frac{197 \text{ MeV fm}}{R_A} = 50$ MeV. The momentum of the neutron of (non relativistic) kinetic energy $E_{k,0}$ is $p_n = \sqrt{2m_n E_{k,0}}$.

The coherence condition is $\sqrt{2m_n E_{k,0}} < p_{\min}$ or $E_{k,0} < \frac{p_{\min}^2}{8m_n} = \frac{50^2}{4 \times 940} = 0.7$ MeV.

Call p_0 the initial neutron momentum, corresponding to the kinetic energy E_{k0}, p_1 and E_{k1} the momentum and kinetic energy of the final neutron, p_2 and E_{k2} those of the recoiling Ar nucleus. Momentum and kinetic energy conservation in the non-relativistic kinematics of the elastic background scattering give

$$\frac{p_0^2}{2m_n} = \frac{p_1^2}{2m_n} + \frac{p_2^2}{2m_{Ar}}$$
$$p_0 = p_2 - p_1$$

From the first equation we have $p_0^2 = p_1^2 + \frac{m_n}{m_{Ar}} p_2^2$

And from the second $p_0^2 = p_1^2 + p_2^2 - 2p_1p_2$

Equating the second members we obtain (provided $p_2 \neq 0$) $p_1 = \frac{1}{2} p_2 \left(1 - \frac{m_n}{m_{Ar}}\right)$.

We substitute this expression in the momentum conservation equation, obtaining $p_2 = \frac{2p_o}{1 + \frac{m_n}{m_{Ar}}} = \frac{2 \times 50}{1 + \frac{0.94}{37.2}} = \frac{100}{1.025} = 97.6 \text{ MeV}$.

The recoil kinetic energy is $E_{k2} = \frac{p_2^2}{2m_{Ar}} = \frac{97.6^2}{2 \times 37200} = 130 \text{ keV}$

1.38.

(a) $E_2 = \frac{h}{\lambda} = \frac{1240 \text{ eV nm}}{694 \text{ nm}} = 1.79 \text{ eV}$.

The CM energy for the head-on geometry is $s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = 2E_1E_2 + 2E_1E_2$.

At threshold $s = 4E_1E_2 = (2m_e)^2$, that is $E_1 = \frac{m_e^2}{E_2} = \frac{(0.5)^2}{1.79 \times 10^{-6}} = 140 \text{ GeV}$

(b) $1 - \beta = 1 - \frac{\mathbf{p}_1 + \mathbf{p}_2}{E_1 + E_2} = 1 - \frac{E_1 - E_2}{E_1 + E_2} = 1 - \frac{1 - E_2/E_1}{1 + E_2/E_1} \simeq 2\frac{E_2}{E_1} = 2.6 \times 10^{-11}$

(c) $s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = 2E_1E_2 - 2E_1E_2 = 0$. The mass is zero for any values of the two energies.

2.1. From the result of the Problem 11.13 we have $p^* = E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV}$. From this we obtain $E_\mu^* = m_\pi - p^* = 110 \text{ MeV}$.

2.2. a) $p_\nu^* = E_\nu^* = p_\mu^* = 236 \text{ MeV}$; $E_\mu^* = 259 \text{ MeV}$

b) p_K=5 GeV, hence $E_K = \sqrt{p_K^2 + m_K^2} = 5.02 \text{ GeV}$; $\gamma = \frac{E_K}{m_K} = 10.2$; $\gamma\beta = \frac{p_K}{m_K} = 10.1$.

The muons with maximum energy in L are those that are emitted backwards by the kaon. Their momentum is $p_\mu = \gamma p_\mu^* + \beta\gamma E_\mu^* = 10.2 \times 0.236 + 10.1 \times 0.259 = 5.02 \text{ GeV}$.

2.3. The second gamma moves backwards. The total energy is $E = E_1 + E_2$; the total momentum is $P = p_1 - p_2 = E_1 - E_2$. The square of the mass of the two-gamma system is equal to the square of the pion mass: $m_{\pi^0}^2 = (E_1+E_2)^2-(E_1-E_2)^2 = 4E_1E_2$, from which we obtain $E_2 = \frac{m_{\pi^0}^2}{4E_1} = \frac{135^2}{4\times 150} = 30.4\ \text{MeV}$. The speed of the π^0 is $\beta = \frac{P}{E} = \frac{E_1-E_2}{E_1+E_2} = 0.662$.

2.4. The Lorentz factor for E_μ=5 GeV is $\gamma = E_\mu / m_\mu = 47$. In its rest frame the distance of the Earth surface is $l_0 = l/\gamma = 630\ \text{m}$. For E_μ=5 TeV, the distance of the Earth is $l_0 = l/\gamma = 0.63\ \text{m}$. The first muon travels in a lifetime $\gamma\beta c\tau \simeq \gamma c\tau = 28\ \text{km}$, the second would travel 28 000 km if it did not hit the surface first.

2.5. The Lorentz factor for E_π=5 GeV is $\gamma = E_\pi / m_\pi = 36$. In its rest frame it sees the Earth's surface at the distance $l_0 = l/\gamma = 830\ \text{m}$. In a lifetime it travels $\gamma c\tau = 280\ \text{m}$. We see that only a few such pions survive. To find them we must go to high altitude.

2.6. The momenta of the electrons are $p = 0.3B\rho$=12 MeV. The gamma energy is E_γ=24 MeV.

2.8. Since the decay is isotropic, the probability of observing a photon is a constant $P(\cos\theta^*,\phi^*) = K$. We determine K by imposing that the probability of observing a photon at any angle is 2, i. e. the number of photons.

We have $2 = \int K\sin\theta^* d\theta^* d\phi = \int_0^{2\pi} d\phi \int_0^{\pi} Kd(\cos\theta^*) = K4\pi$. Hence $K = 1/2\pi$ and $P(\cos\theta^*,\phi^*) = 1/2\pi$.

The distribution is isotropic in azimuth in L too. To have the dependence of θ, that is given by $P(\cos\theta) \equiv \frac{dN}{d\cos\theta} = \frac{dN}{d\cos\theta^*}\frac{d\cos\theta^*}{d\cos\theta}$, we must calculate the 'Jacobian' $J = \frac{d\cos\theta^*}{d\cos\theta}$.

Calling β and γ the Lorentz factors of the transformation and taking into account that $p^* = E^*$, we have

$$p\cos\theta = \gamma\left(p^*\cos\theta^* + \beta E^*\right) = \gamma p^*\left(\cos\theta^* + \beta\right)$$
$$E = p = \gamma\left(E^* + \beta p^*\cos\theta^*\right) = \gamma p^*\left(1+\beta\cos\theta^*\right).$$

We differentiate the first and third members of these relationships, taking into account that p^* is a constant. We obtain

$$dp\times\cos\theta + p\times d(\cos\theta) = \gamma p^* d(\cos\theta^*) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*}\cos\theta + p\frac{d\cos\theta}{d\cos\theta^*} = \gamma p^*.$$

$$dp = \gamma\beta p^* d(\cos\theta^*) \quad \Rightarrow \quad \frac{dp}{d\cos\theta^*} = \gamma\beta p^*$$

and $J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma\frac{p^*}{p}(1-\beta\cos\theta)$.

The inverse transformation is $E^* = \gamma(E - \beta p\cos\theta)$, i. e. $p^* = \gamma p(1-\beta\cos\theta)$, giving

$$J^{-1} = \frac{d\cos\theta}{d\cos\theta^*} = \gamma^2(1-\beta\cos\theta)^2.$$

Finally we obtain $P(\cos\theta) \equiv \frac{dN}{d\cos\theta} = \frac{1}{2\pi}\gamma^{-2}(1-\beta\cos\theta)^{-2}$

2.10. $\mu_e / \mu_\mu = m_\mu / m_e = 207$; $\mu_e / \mu_\tau = m_\tau / m_e = 3477$.

2.11. The energy needed to produce an antiproton is minimum when the Fermi motion is opposite to the beam direction. If E_f is the total energy of the target proton and p_f its momentum, the threshold condition is $(E_p+E_f)^2-(p_p-p_f)^2=(4m_p)^2$. From this we have $E_pE_f + p_pp_f = 7m_p^2$. We simplify by setting $p_p \simeq E_p$ obtaining

$$E_p = \frac{7m_p^2}{E_f + p_f} \simeq \frac{7m_p^2}{m_p + p_f} \simeq 7m_p\left(1 - \frac{p_f}{m_p}\right) = 5.5 \text{ GeV}.$$

This value should be compared to E_p=6.6 GeV on free protons.

2.12. By differentiating (1.79) we obtain $\Delta\theta = 0.3BL\Delta p / p^2$. The slit of opening d at the distance l defines the angle within $\Delta\theta = d / l$. The requested distance is then $l = \dfrac{d \times p}{0.3BL\Delta p / p} = 3.3 \text{ m}$.

2.13. Considering the beam energy and the event topology, the event is probably an associate production of a K^0 and a Λ. Consequently the V^0 may be one of these two particles. The negative track is in both cases a π, while the positive track may be a π or a proton. We need to measure the mass of the V. With the given data we start by calculating the Cartesian components of the momenta

$p_x^- = 121 \times \sin(-18.2°)\cos 15° = -36.5$ MeV; $p_y^- = 121 \times \sin(-18.2°)\sin 15° = -9.8$ MeV;

$p_z^- = 121 \times \cos(-18.2°) = 115$ MeV.

$p_x^+ = 1900 \times \sin(20.2°)\cos(-15°) = 633.7$ MeV; $p_y^- = 1900 \times \sin(20.2°)\sin(-15°) = -169.8$ MeV;

$p_z^- = 1900 \times \cos(20.2°) = 1783.1$ MeV.

Summing the components, we obtain the momentum of the V, i. e. $p = 1998$ MeV.

The energy of the negative pion is $E^- = \sqrt{(p^-)^2 + m_\pi^2} = 185$ MeV. If the positive track is a π its energy is $E_\pi^+ = \sqrt{(p^+)^2 + m_\pi^2} = 1905$ MeV, while if it is a proton its energy is $E_p^+ = 2119$ MeV. The energy of the V is $E_\pi^V = 2090$ MeV in the first case, $E_p^V = 2304$ MeV in the second case. The mass of the V is consequently $m_\pi^V = \sqrt{E_\pi^{V2} - p^2} = 620$ MeV in the first hypothesis, $m_p^V = 1150$ MeV in the second. Within the ±4% uncertainty, the first hypothesis is incompatible with any known particle, while the second is compatible with the particle being a Λ.

2.14.

1. The CM energy squared is $s = (E_\nu + m_n)^2 - p_\nu^2 = m_n^2 + 2m_nE_\nu$. The threshold condition is

$$s = (m_e + m_p)^2 = m_p^2 + m_e^2 + 2m_em_p.$$

Hence, the threshold condition is $E_\nu = \dfrac{(m_e + m_p)^2 - m_n^2}{2m_n} < 0$, meaning that there is no threshold, the reaction proceeds also at zero neutrino energy.

2. The threshold condition is $s=\left(m_\mu+m_p\right)^2=m_p^2+m_\mu^2+2m_\mu m_p$. The threshold energy is

$$E_\nu=\frac{\left(m_\mu+m_p\right)^2-m_n^2}{2m_n}=\frac{(105.7+938.3)^2-939.6^2}{2\times 939.6}=110\text{ MeV}$$

3. The threshold energy is

$$E_\nu=\frac{\left(m_\tau+m_p\right)^2-m_n^2}{2m_n}=\frac{(1777+938.3)^2-939.6^2}{2\times 939.6}=3.45\text{ GeV}.$$

2.15. We first find an expression valid in both cases. Call $E_{\gamma1}$ and $p_{\gamma1}=E_{\gamma1}$ the energy and momentum of the initial photon and $E_{\gamma2}$ and $p_{\gamma2}=E_{\gamma2}$ those of the final one. Similarly E_{e1}, p_{p1} and E_{e2}, p_{p2} for the electron.

The initial values of energies and momenta are given; hence the total energy and momentum and CM energy squared

$E_T=E_{e1}+E_{\gamma1}$ $\quad p_T=p_{e1}-E_{\gamma1}$ $\quad s=E_T^2-p_T^2$.

Energy conservation gives $E_T=E_{e2}+E_{\gamma2}$ $\quad p_T=p_{e2}-E_{\gamma2}$.

We can eliminate the final energy and momentum of the electron by imposing $E_{e2}^2-p_{e2}^2=m_e^2$.

$E_{e2}=E_T-E_{\gamma2}$ $\quad p_{e2}=p_T+E_{\gamma2}$. Hence: $\left(E_T-E_{\gamma2}\right)^2-\left(p_T+E_{\gamma2}\right)^2=m_e^2$. Solving for $E_{\gamma2}$ we have $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}$.

1. We have, in MeV: $E_{\gamma1}=0.511$, E_{e1}=0.511, p_{e1}=0.

E_T=1.02, p_T=0.511, s=0.78 and $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{0.78-0.511^2}{2(1.02+0.511)}=0.170\text{ MeV}$.

2. We have $E_{\gamma1}=0.511$, E_{e1}=1.02, $p_{e1}=\sqrt{1.02^2-0.511^2}=0.88$.

E_T=1.53, p_T=0.511, s=2.08 and $E_{\gamma2}=\dfrac{s-m_e^2}{2\left(E_T+p_T\right)}=\dfrac{2.08-0.511^2}{2(1.53+0.511)}=0.446\text{ MeV}$.

2.16. The LASER photon energy is $E_{\gamma i}=\dfrac{h}{\lambda}=\dfrac{1240\text{ eV nm}}{694\text{ nm}}=1.79\text{ eV}$.

The electron initial momentum (we shall need its difference from energy) is

$$p_{ei}=\sqrt{E_{ei}^2-m_e^2}\simeq E_{ei}-\frac{m_e^2}{2E_{ei}}$$

The total energy and momentum are $E_T=E_{ei}+E_{\gamma i}$ $\quad p_T=p_{ei}-E_{\gamma i}$

Energy conservation gives $E_T=E_{ef}+E_{\gamma f}$ $\quad p_T=E_{\gamma f}-p_{ef}$

We can eliminate the final energy and momentum of the electron by imposing $E_{ef}^2-p_{ef}^2=m_e^2$.

$E_{ef}=E_T-E_{\gamma f}$ $\quad p_{ef}=E_{\gamma f}-p_T$. Hence: $\left(E_T-E_{\gamma f}\right)^2-\left(E_{\gamma f}-p_T\right)^2=m_e^2$. Solving for $E_{\gamma f}$ we have $E_{\gamma f}=\dfrac{s-m_e^2}{2\left(E_T-p_T\right)}$.

$$E_T - p_T = \left(E_{ei} + E_{\gamma i}\right) - \left(p_{ei} - E_{\gamma i}\right) \simeq \frac{m_e^2}{2E_{ei}} + 2E_{\gamma i} =$$

$$= \frac{0.5^2 \times 10^{-6}}{2 \times 20} + 2 \times 1.79 \times 10^{-9} = \left(6.25 + 3.58\right)10^{-9} \text{ GeV} = 9.83 \text{ eV}$$

$s = \left(E_{\gamma i} + E_{ei}\right)^2 - \left(E_{\gamma i} - p_{ei}\right)^2 = m_e^2 + 4E_{\gamma i}E_{ei}$. Hence

$s - m_e^2 = 4E_{\gamma i}E_{ei} = 4 \times 1.79 \times 20 \times 10^9 \text{ eV}^2 = 14.3 \times 10^{10} \text{ eV}^2$, and

$$E_{\gamma f} = \frac{s - m_e^2}{2\left(E_T - p_T\right)} = \frac{14.3 \times 10^{10}}{2 \times 9.83} = 7.3 \text{ GeV}$$

2.17. The kinetic energy is $T = \sqrt{p^2 + m^2} - m$

For a proton we have $T = \sqrt{23^2 + 938.3^2} - 938.3 = 280$ keV

For a positron we have $T = \sqrt{23^2 + 0.51^2} - 0.51 = 22.5$ MeV

2.18. $E \simeq p = 0.3BR = 0.3 \times 0.3 \times 0.14 = 12.6$ MeV.

2.19. In problem 2.1 we already calculated the CM momentum $p^* = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8$ MeV. The CM muon energy is $E_\mu^* = \sqrt{p^{*2} + m_\mu^2} = 110$ MeV. For the Lorentz transformation to the L frame we have $\beta \simeq 1$ and $\gamma = \dfrac{E_\pi}{m_\pi} = \dfrac{200}{0.14} == 1400$. The maximum and minimum muon energies are

$E_{\mu\,\min}^{\max} = \gamma\left(E_\mu^* \pm \beta p^*\right) = 1400\left(0.110 \pm 0.030\right) = 112 - 196$ GeV.

3.2. Strangeness conservation requires that a K^+ or a K^0 is produced together with the K^-. The third component of the isospin in the initial state is –1/2. Let us check if it is conserved in the two reactions. The answer is yes for $\pi^- + p \to K^- + K^+ + n$ because in the final state we have $I_z = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = +\frac{1}{2}$, and yes also for $\pi^- + p \to K^- + K^0 + p$ because in the final state we have $I_z = -\frac{1}{2} - \frac{1}{2} + \frac{1}{2} = -\frac{1}{2}$. The threshold of the first reaction is just a little smaller of that of the second reaction because $m_n + m_{K^+} < m_p + m_{K^0}$ (1433 MeV < 1436 MeV). For the former we have

$$E_\pi = \frac{\left(2m_K + m_n\right)^2 - m_\pi^2 - m_p^2}{2m_p} = 1.5 \text{ GeV}.$$

3.3. To conserve both strangeness and baryonic number a pair of $\Lambda\bar{\Lambda}$ must be produced. The reaction is $\pi^- + p \to \Lambda + \bar{\Lambda} + n$. On free protons, we obtain $E_\pi^0 = 4.9$ GeV. The threshold energy on bound protons, having Fermi momentum p_f, following Problem 2.11, is found to be $E_\pi = E_\pi^0\left(1 - p_f / m_p\right) = 4.1$ GeV.

In the first case pions in the beam have γ=35 and $\beta\approx 1$. The flux at the emulsion stack is N= 0.97 10^6 π/cm^2.

3.4. 1. OK, S; 2. OK, W; 3. Violates $\mathcal{L}_\mu$; 4. OK, EM; 5. Violates C; 6. Cannot conserve both energy and momentum; 7. violates $\mathcal{B}$ and S; 8. violates $\mathcal{B}$ and S; 9. violates J and $\mathcal{L}_e$; 10. violates energy conservation.

3.5. 1. Violates $\mathcal{L}_e$ and $\mathcal{L}_\mu$; 2. Violates charge conservation; 3. Violates $\mathcal{B}$, I and I_z; 4. Violates the charge; 5. Violates $\mathcal{B}$, I and I_z; 6. Violates charge and S; 7. Violates S and I_z; 8.Violates S and I_z; 9. Violates energy conservation; 10. Violates $\mathcal{L}_e$ and $\mathcal{L}$.

3.6. a) J and $\mathcal{L}$, b) $\mathcal{B}$ and $\mathcal{L}$, c) energy conservation ; d) electric charge.

3.7. a) $0+0 \to -1+0$, NO; b) $0+0 \to 1-1$, YES; c) $-1+0 \to 1-2+0$, YES; d) $1+0 \to -1-2+0$, NO; e) $-1+0 \to -3+1+1$, YES.

3.8. a) NO for J and $\mathcal{L}$; b) NO for J and $\mathcal{L}$; c) YES; d) NO for $\mathcal{L}$; e) YES; f) NO for $\mathcal{L}_e$ and $\mathcal{L}_\mu$; g) NO for $\mathcal{L}$; h) YES.

3.9. $\left|\pi^- p\right\rangle = |1,-1\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$

$$\left|\pi^+ p\right\rangle = |1,+1\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle$$

$$\left|\Sigma^0 K^0\right\rangle = |1,0\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\left|\Sigma^- K^+\right\rangle = |1,-1\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\left|\Sigma^+ K^+\right\rangle = |1,+1\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle$$

$\left\langle K^+\Sigma^+\middle|\pi^+ p\right\rangle = A_{3/2}$; $\left\langle \Sigma^- K^+\middle|\pi^- p\right\rangle = \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}A_{3/2} = \frac{1}{3}A_{3/2}$; $\left\langle \Sigma^0 K^0\middle|\pi^- p\right\rangle = \sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}A_{3/2} = \frac{\sqrt{2}}{3}A_{3/2}$

Hence: $\sigma\left(\pi^+ p \to \Sigma^+ K^+\right) : \sigma\left(\pi^- p \to \Sigma^- K^+\right) : \sigma\left(\pi^- p \to \Sigma^0 K^0\right) = 9:1:2$

3.10. From the expressions found in the solution of problem 3.9, we have
$\sigma(1):\sigma(2):\sigma(3) = 2\left|A_{3/2} - A_{1/2}\right|^2 : \left|A_{3/2} + 2A_{1/2}\right|^2 : 9\left|A_{3/2}\right|^2$

3.11. $\sigma(1)/\sigma(2) = 1$

3.12. We can proceed as in the previous solutions or also as follows.

$$|p,d\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle|0,0\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle$$

$$\left|\frac{1}{2},+\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}|1,0\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1,1\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}\left|\pi^0\right\rangle\left|\mathrm{He}^3\right\rangle + \sqrt{\frac{2}{3}}\left|\pi^+\right\rangle\left|\mathrm{H}^3\right\rangle$$

$$\sigma\left(p+d \to \mathrm{He}^3 + \pi^0\right)/\sigma\left(p+d \to \mathrm{H}^3 + \pi^+\right) = 1/2$$

3.13.From $|p,p\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = |1,+1\rangle$ and $\left|d,\pi^+\right\rangle = |0,0\rangle|1,+1\rangle = |1,+1\rangle$ we have $\left\langle d,\pi^+\middle|p,p\right\rangle = A_1$.

From $\left|p,n\right\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|0,0\right\rangle + \frac{1}{\sqrt{2}}\left|1,0\right\rangle$ and $\left|d,\pi^0\right\rangle = \left|0,0\right\rangle\left|1,0\right\rangle = \left|1,0\right\rangle$ we have $\left\langle d,\pi^0 \middle| p,n\right\rangle = \frac{1}{\sqrt{2}}A_1$. Finally we obtain $\sigma\left(pp \to d\pi^+\right)/\sigma\left(pn \to d\pi^0\right) = 2$.

3.14. $\left|K^-,\mathrm{He}^4\right\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|0,0\right\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle$.

$$\left|\Sigma^0,\mathrm{H}^3\right\rangle = \left|1,0\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \qquad \Rightarrow \left\langle K^-,\mathrm{He}^4 \middle| \Sigma^0,\mathrm{H}^3\right\rangle = \sqrt{\frac{1}{3}}A_{1/2}$$

$$\left|\Sigma^-,\mathrm{He}^3\right\rangle = \left|1,-1\right\rangle\left|\frac{1}{2},+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \qquad \Rightarrow \left\langle K^-,\mathrm{He}^4 \middle| \Sigma^-,\mathrm{He}^3\right\rangle = -\sqrt{\frac{2}{3}}A_{1/2}$$

$\sigma\left(K^- + \mathrm{He}^4 \to \Sigma^0\mathrm{H}^3\right)/\sigma\left(K^- + \mathrm{He}^4 \to \Sigma^-\mathrm{He}^3\right) = 1/2$.

3.15. $\sigma(1):\sigma(2):\sigma(3) = \left|-\frac{1}{\sqrt{6}}A_0 + \frac{1}{2}A_1\right|^2 : \left|\frac{1}{\sqrt{6}}A_0\right|^2 : \left|\frac{1}{\sqrt{6}}A_0 + \frac{1}{2}A_1\right|^2$.

3.16. $\sigma\left(\pi^- p \to \pi^- p\right)/\sigma\left(\pi^- p \to \pi^0 n\right) = \left|\frac{2}{3}A_{1/2} + \frac{1}{3}A_{3/2}\right|^2 / \left|-\frac{\sqrt{2}}{3}A_{1/2} + \frac{\sqrt{2}}{3}A_{3/2}\right|^2$.

3.17. a) The initial parity is $P_i = P(\pi)P(d)(-1)^{l_i} = (-)(+)(+) = -$ and the final one is $P_f = P(n)P(n)(-1)^{l_f} = (-1)^{l_f}$. Parity conservation requires l_f=1,3,5… Angular momentum conservation requires that l_f<3. Only l_f=1 remains. The two-neutron wave function must be completely antisymmetric. Since the spatial part is antisymmetric, the spin part must be symmetric. In conclusion the state is $^3\mathrm{S}_1$, with total spin S=1.

b) Since P_i=+, l_f is even. The spin function is antisymmetric. Hence the state is $^1\mathrm{S}_0$ and its total spin is S=0.

3.18. From (3.18) the initial charge conjugation is $C=(-1)^{l+s}$. The final one is $C(n\gamma)=(-1)^n$. The charge conjugation is conserved if $l+S+n$=even.

In the ortho-positronium $l+s$=1, consequently n= odd. The minimum number of photons is n=3 (n=1 forbidden by energy-momentum conservation).

In the para-positronium $l+s$=0, hence n= even and the minimum number of photons is n=2.

3.19.

1. $C(\bar{p}p)=(-1)^{l+s}=C(n\pi^0)=+$. Then $l+s$=even. The possible states are $^1\mathrm{S}_0$, $^3\mathrm{P}_1$, $^3\mathrm{P}_2$, $^3\mathrm{P}_3$, $^1\mathrm{D}_2$.

2. The orbital momentum is even, because the wave function of the $2\pi^0$ state must be symmetric. Since the total angular momentum is just orbital momentum, only the states $^1\mathrm{S}_0$, $^3\mathrm{P}_2$, $^1\mathrm{D}_2$ are left. Parity conservation gives $P(2\pi^0)=+ = P(\bar{p}p)=(-1)^{l+1}$. Hence, l=odd, leaving only $^3\mathrm{P}_2$.

3.20. The G-parity is positive, because it is conserved and is such in the final state. As $G=C(-1)^I$, if I=0 then C=+, i. e. $C=(-1)^l=+1$. Then l=even. If I=1, then C=–, i.e. $(-1)^l=-1$. We have l=odd.

3.21. It is convenient to prepare a table with the possible values of the initial J^{PC} and of the final

l^{CP} with l=J to satisfy angular momentum conservation. Only the cases with the same parity and charge conjugation are allowed. Recall that $P(\bar{p}p)=(-1)^{l+1}$ and $C(\bar{p}p)=(-1)^{l+s}$.

	$^1S_0$	$^3S_1$	$^1P_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3D_1$	$^3D_2$	$^3D_3$
J^{PC}	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}	2^{-+}	1^{--}	2^{--}	3^{--}
l^{PC}	0^{++}	1^{--}	1^{--}	0^{++}	1^{--}	2^{++}	2^{++}	1^{--}	2^{++}	3^{--}
		Y		Y		Y		Y		Y

In conclusion 1. $^1S_0$; 2. $^3S_1$, $^3D_1$ e 3. $^3P_2$.

3.22. Given the quark content, Λ_c has electric charge +1. Since the processes is strong, a) and c) are forbidden by charm conservation, while b) is allowed. d) is violates charge conservation.

3.23. Λ_b is neutral. a) violates charm and beauty, b) and c) are allowed, d) violates beauty, e) violates baryon number.

3.26.

1. The minimum velocity is $\beta_{\min}=\frac{1}{n}=0.75$.

2. The minimum kinetic energy for electrons is

$$E_{kin,\min}(e)=m_e\left(\frac{1}{\sqrt{1-\beta_{\min}^2}}-1\right)=0.511\times 0.51=0.26\ \text{MeV}$$

and for K^+: $E_{kin,\min}(K^+)=m_K\left(\frac{1}{\sqrt{1-\beta_{\min}^2}}-1\right)=497.6\times 0.51=254\ \text{MeV}$

3. In the decay $p\to e^+ +\pi^0$, the CM kinetic energy of the e^+ is

$$E_{kin}(e^+)=\frac{m_p^2+m_e^2-m_{\pi^0}^2}{2m_p}-m_e=\frac{938.3^2+0.51^2-0.135^2}{2\times 938.3}-0.51=469\ \text{MeV}$$, above threshold

In the decay $p\to K^+ +\nu$, the CM kinetic energy of the K is

$$E_{kin}(K^+)=\frac{m_p^2+m_K^2-m_\nu^2}{2m_p}-m_K=\frac{938.3^2+497.6^2}{2\times 938.3}-497.6=104\ \text{MeV}$$, below threshold

3.27. (a) forbidden by lepton number, (b) forbidden by angular momentum and lepton number, (c) forbidden by charge conjugation, (e) allowed, (f) forbidden by baryon and lepton numbers, (g) forbidden by angular momentum and lepton number.

3.28. (a) X must have charge Q=0 and strangeness S=+1; it is a K^0; (b) X must have charge Q=0 and lepton number $\mathcal{L}_e$ =–1, it is a $\bar{\nu}_e$; (c) X must have Q=0; the reaction being weak, strangeness does not need to be conserved; it is a π^0.

3.29. The third component of the initial isospin is $I_{z,initial}=1+\frac{1}{2}=\frac{3}{2}$, hence the total isospin must be $I=\frac{3}{2}$. For the K^+K^+ we have I=1, I_z=1. Hence for the Ξ^0 may have $I_\Xi=\frac{1}{2}$ or $I_\Xi=\frac{3}{2}$.

The third component is $I_{\Xi,z} = +\frac{1}{2}$.

3.30. The total energy and its square are: $\sqrt{s} = m_p + m_{\pi^-} = 0.9383 + 0.1396 = 1.08$ GeV and $s = 1.162$ GeV^2.

The energy of the photon is $E_\gamma = \frac{s - m_n^2}{2\sqrt{s}} = \frac{1.162 - 0.883}{2 \times 1.08} = 0.129$ GeV.

The kinetic energy of the neutron is $T_n = \sqrt{s} - E_\gamma - m_n = m_p - m_n + m_\pi - E_\gamma$, which is a very small quantity, expressed as a difference between large quantities. It is then convenient to consider the nonrelativistic expression of the kinetic energy:

$$T_n = \frac{p_n^2}{2m_n} = \frac{E_\gamma^2}{2m_n} = \frac{130^2}{2 \times 939.6} = 9 \text{ MeV}.$$

3.31. The beam energy is enough to produce strange particles but not for heavier flavours. In order to conserve strangeness the V^0s must be a K^0 and a Λ. The simplest reaction is $\pi^+ + p \to \pi^+ + \pi^+ + K^0 + \Lambda$.

We calculate the mass of each V^0 assuming in turn it to be the K^0 or the a Λ.

If 1 is a Λ,

$$M^2 = m_p^2 + m_\pi^2 + 2\sqrt{p_{1+}^2 + m_p^2}\sqrt{p_{1-}^2 + m_\pi^2} - 2p_{1+}p_{1-}\cos\theta_1 =$$
$$= 0.938^2 + 0.139^2 + 2 \times 1.02 \times 1.905 - 2 \times 0.4 \times 1.9 \times \cos 24.5° = 3.38 \text{ GeV}^2$$

or M=1.83 GeV not compatible with being a Λ.

If 1 is a K^0

$$M^2 = 2m_\pi^2 + 2\sqrt{p_{1+}^2 + m_\pi^2}\sqrt{p_{1-}^2 + m_\pi^2} - 2p_{1+}p_{1-}\cos\theta_1 =$$
$$= 0.04 + 2 \times 0.424 \times 1.905 - 2 \times 0.4 \times 1.9 \times \cos 24.5° = 0.246 \text{ GeV}^2$$

or M=0.495 GeV compatible, within the errors, with the mass of the K^0

If 2 is a Λ,

$$M^2 = m_p^2 + m_\pi^2 + 2\sqrt{p_{2+}^2 + m_p^2}\sqrt{p_{2-}^2 + m_\pi^2} - 2p_{2+}p_{2-}\cos\theta_2 =$$
$$= 0.938^2 + 0.139^2 + 2 \times 1.20 \times 0.29 - 2 \times 0.75 \times 0.25 \times \cos 22° = 1.59 - 0.35 = 1.24 \text{ GeV}^2$$

or M=1.11 GeV compatible, within the errors, with the mass of the Λ.

If 2 is a K^0

$$M^2 = 2m_\pi^2 + 2\sqrt{p_{2+}^2 + m_\pi^2}\sqrt{p_{2-}^2 + m_\pi^2} - 2p_{2+}p_{2-}\cos\theta_2 =$$
$$= 0.04 + 2 \times 0.76 \times 0.29 - 0.35 = 0.138 \text{ GeV}^2$$

or M=0.371 GeV incompatible, within the errors, with the mass of the K^0.

4.2. For the ω, $G = C(-1)^I=(-1)(-1)^0 =-1$. For the ϕ, $G = C(-1)^I=(-1)(-1)^0=-1$. The K is not an eigenstate of G. For the η, $G = C(-1)^I=(+1)(-1)^0=+1$.

4.3. The ρ decays strongly into 2π, hence G=+. The possible values of its isospin are 0, 1 and 2. In the three cases the Clebsch-Gordan coefficients are $\langle 1,0|1,0;1,0\rangle = 0$, $\langle 0,0|1,0;1,0\rangle \neq 0$ and $\langle 2,0|1,0;1,0\rangle \neq 0$. Hence I=1.

Since I=1, the isospin wave function is antisymmetric. The spatial wave function must consequently be antisymmetric, i. e. the orbital momentum of the two π must be l=odd. The ρ spin is equal to l. $C=(-1)^l=-1$. $P=(-1)^l=-1$.

4.4. $\tau_{K*} = 1.3\times10^{-23}$ s, $\gamma = E/m \simeq p/m = 10.1$, $d = \gamma c\tau = 39$ fm.

4.5. a. From the size of the resonance width Γ we infer that it decays by strong interaction. As a consequence, S, I, I_z and Y are conserved. Therefore $S(\Sigma)= S(\Lambda)+ S(\pi)=1+0=+1$ and $Y(\Sigma)= Y(\Lambda)+ Y(\pi)=0+0=0$. $\mathbf{I}(\Sigma)= \mathbf{I}(\Lambda)\otimes\mathbf{I}(\pi)=\mathbf{0}\otimes\mathbf{1}=\mathbf{1}$ and $I_z(\Sigma)= I_z(\Lambda)+ I_z(\pi)=0+0=0$.
b. $\mathbf{J}(\Sigma)= \mathbf{J}(\Lambda)\otimes\mathbf{J}(\pi)\otimes\mathbf{L}=\mathbf{1/2}\otimes\mathbf{0}\otimes\mathbf{1} \Rightarrow J=1/2$ or $J=3/2$. $P(\Sigma)= P(\Lambda)\, P(\pi)\, (-1)^L=(+)(-)(-)=+$.

4.6. a. The decay is strong b. The initial strangeness in the reaction $K^- + p \to \pi^- + \Sigma^+(1385)$ is $S=-1$. The strangeness of the Σ (1385) is $S=-1$. Since the isospin is conserved in the strong decay, the isospin of the Σ (1385) is equal to the isospin of the $\pi^+\Lambda$ system, i. e. is 1.

4.7. 1. Two equal bosons cannot be in an antisymmetric state; 2. $C(2\pi^0)=+1$; 3. the Clebsch Gordan coefficient $\langle 1,0;1,0|1,0\rangle = 0$.

4.8. The charge conjugation of the final state $\pi^+\pi^-$ is $C=(-1)^l=(-1)^J$, i.e. ρ^0 has $C=-$; the f^0 has $C=+$. The system $\pi^0\gamma$ has $C=(+)(-)=-$. Hence $f^0\to\pi^0\gamma$ is forbidden.

4.9. $R=\Gamma\left(K^{*+} \to K^0 + \pi^+\right)/\Gamma\left(K^{*+} \to K^+ + \pi^0\right) = 1/2$ if $I_{K*}=3/2$. $R=2$ if $I_{K*}=1/2$.

4.10. $\Gamma\left(K^-p\right)/\Gamma\left(\bar{K}^0 n\right)=1$. $\Gamma\left(\pi^-\pi^+\right)/\Gamma\left(\bar{K}^0 n\right)=0$, because the decay into $\pi^+\pi^-$ would violate baryon number and strangeness.

4.11. It is useful to prepare a table with the quantum numbers of the relevant states

	$\bar{p}p\,^3S_1$	$\bar{p}p\,^3S_1$	$\bar{p}p\,^1S_0$	$\bar{p}p\,^1S_0$	$\bar{p}n\,^3S_1$	$\bar{p}n\,^1S_0$
J^P	1^-	1^-	0^-	0^-	1^-	0^-
C	–	–	+	+	X	X
I	0	1	0	1	1	1
G	–	+	+	–	+	–

$\bar{p}n \to \pi^-\pi^-\pi^+$. Since $G=-1$ in the final state, there is only one possible initial state, i.e. $^1S_0$

$$|\bar{p},n\rangle = |1,-1\rangle = \frac{1}{\sqrt{2}}|1,0;1,-1\rangle - \frac{1}{\sqrt{2}}|1,-1;1,0\rangle = \frac{1}{\sqrt{2}}\left|\rho^0;\pi^-\right\rangle - \frac{1}{\sqrt{2}}\left|\rho^-;\pi^0\right\rangle$$

hence $R\left(\bar{p}n \to \rho^0\pi^-\right)/R\left(\bar{p}n \to \rho^-\pi^0\right)=1$.

$$|\bar{p},p\rangle = |1,0\rangle = \frac{1}{\sqrt{2}}\left|\rho^-;\pi^+\right\rangle + 0\frac{1}{\sqrt{2}}\left|\rho^0;\pi^0\right\rangle - \frac{1}{\sqrt{2}}\left|\rho^+;\pi^-\right\rangle$$

hence $R\left(\bar{p}p(I=1)\to\rho^+\pi^-\right):R\left(\bar{p}p(I=1)\to\rho^0\pi^0\right):R\left(\bar{p}p(I=1)\to\rho^-\pi^+\right)=1:0:1$.

$$|\bar{p},p\rangle = |0,0\rangle = \frac{1}{\sqrt{3}}\left|\rho^-;\pi^+\right\rangle - \frac{1}{\sqrt{3}}\left|\rho^0;\pi^0\right\rangle + \frac{1}{\sqrt{3}}\left|\rho^+;\pi^-\right\rangle$$

hence $R\left(\bar{p}p(I=0)\to\rho^+\pi^-\right):R\left(\bar{p}p(I=0)\to\rho^0\pi^0\right):R\left(\bar{p}p(I=0)\to\rho^-\pi^+\right)=1:1:1$.

4.12. The isospin wave function must be symmetrical, because the spatial wave function is such. Hence $I = 0$ or 2.

4.13. The matrix element $\mathcal{M}$ must be symmetric under the exchange of each pair of pions. Consequently:

1. if $J^P = 0^-$, $\mathcal{M}$ = constant. There are no zeros.
2. if $J^P = 1^-$, $\mathcal{M} \propto \mathbf{q}(E_1 - E_2)(E_2 - E_3)(E_3 - E_1)$; zeros on the diagonals and on the border.
3. if $J^P = 1^+, M \propto \mathbf{p}_1 E_1 + \mathbf{p}_2 E_2 + \mathbf{p}_3 E_3$; zero in the centre, where E_1=E_2=E_3; zero at T_3=0, where $\mathbf{p}_3$=0, $\mathbf{p}_2$=– $\mathbf{p}_1$; E_2=E_1.

4.14. Since the parity is positive, the orbital momentum l of the two nucleons must be even. The total angular momentum is J=1, the sum of the two spins can be s=0 or s=1. Hence we can have l=0 or l=2. The two possible states are $^3S_1$ and $^3D_1$.

4.15. A baryon can contain between 0 and 3 c valence quarks; therefore the charm of a baryon can be C=0, 1, 2, 3. Since the charge of c is equal to 2/3, the baryons with Q=+1 can have charm C=2 (ccd, ccs, ccb), C=1 (e.g. cud) or C=0 (e.g. uud). If Q=0, one c can be present, as in cdd, or none as in udd. Hence C=1 or C=0.

4.16. Since $\mathcal{B}$=1 the particle is a baryon. Therefore the valence quarks are three. Since the charge Q=+1, two quarks are up-type, one is down-type. Since C=1, one up-type quark is c. Since S=0, B=0, T=0, the other two quarks are u and d. The state is udc.

4.17. sss, uuc, usc, ssc, udb.

4.18. $c\bar{d}$, $\bar{c}u$, $u\bar{b}$, $c\bar{b}$.

4.22. $\tau_{J/\psi} = \frac{1}{\Gamma} == \frac{1}{0.091 \times 1.52 \times 10^{21}\text{s}^{-1}} = 7.2 \times 10^{-21}\text{s}$.

The distance travelled in a lifetime is $l_{\text{lab}} = \gamma\tau_{J/\psi}\beta c = \tau_{J/\psi}\frac{p}{M}c = 3.5 \times 10^{-12}\text{m}$.

Let E_e be the energy and $p_e \approx E_e$ the momentum of the electron. From $E_J = 2E_e$ and $p_J = 2p_e\cos\theta_e$, we have $E_e = \frac{E_J}{2} = \frac{1}{2}\sqrt{p_J^2 + m_J^2} = 2.94\ \text{GeV}$.

From $m_e^2 = \left(\frac{E_J}{2}\right)^2 - \left(\frac{p_J}{2\cos\theta_e}\right)^2$, we have $\cos\theta_e = \frac{p_J}{\sqrt{E_J^2 - 4m_e^2}} \approx \frac{p_J}{E_J} = 0.85$; i. e. $\theta_e = 31.8^\circ$.

For p_J=50 GeV, $\theta_e = 3.6^\circ$.

4.23. $\gamma = E/m = 20/1.86 = 10.7$. The condition $I = I_0 e^{-\frac{t}{\gamma\tau}} > 0.9 I_0$ gives $t < \gamma\tau\ln\left(\frac{1}{0.9}\right)$. We need to resolve distances $d = ct < 139\ \mu\text{m}$.

Possible instruments: bubble chambers, emulsions, Silicon microstrips.

4.24. We start from $\sigma(E) = \frac{3\pi}{E^2}\frac{\Gamma_e\Gamma_f}{(E - M_R)^2 + (\Gamma/2)^2} = \frac{12\pi\Gamma_e\Gamma_f}{\Gamma^2}\frac{1}{E^2}\frac{1}{\left[2(E - M_R)/\Gamma\right]^2 + 1}$.

In the neighbourhood of the resonance peak the factor $1/E^2$ varies only slowly, compared to the resonant factor, and we can approximate it with the constant $1/M_R^2$, i.e.