CHAPTER 1
Fundamentals of Algebra

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Section 1.1 The Real Number System

1. $\left\{-6, -\sqrt{6}, -\frac{4}{3}, 0, \frac{5}{8}, 1, \sqrt{2}, 2, \pi, 6\right\}$

(a) Natural numbers: $\{1, 2, 6\}$

(b) Integers: $\{-6, 0, 1, 2, 6\}$

(c) Rational numbers: $\left\{-6, -\frac{4}{3}, 0, \frac{5}{8}, 1, 2, 6\right\}$

(d) Irrational numbers: $\left\{-\sqrt{6}, \sqrt{2}, \pi\right\}$

2. $\left\{-\frac{10}{3}, -\pi, -\sqrt{3}, -1, 0, \frac{2}{5}, \sqrt{3}, \frac{5}{2}, 5, 101\right\}$

(a) Natural numbers: $\{5, 101\}$

(b) Integers: $\{-1, 0, 5, 101\}$

(c) Rational numbers: $\left\{-\frac{10}{3}, -1, 0, \frac{2}{5}, \frac{5}{2}, 5, 101\right\}$

(d) Irrational numbers: $\left\{-\pi, -\sqrt{3}, \sqrt{3}\right\}$

3. $\left\{-4.2, \sqrt{4}, -\frac{1}{9}, 0, \frac{3}{11}, \sqrt{11}, 5.\overline{5}, 5.543\right\}$

(a) Natural numbers: $\left\{\sqrt{4}\right\}$

(b) Integers: $\left\{\sqrt{4}, 0\right\}$

(c) Rational numbers: $\left\{4.2, \sqrt{4}, -\frac{1}{9}, 0, \frac{3}{11}, 5.\overline{5}, 5.543\right\}$

(d) Irrational numbers: $\left\{\sqrt{11}\right\}$

4. $\left\{-\sqrt{25}, -\sqrt{6}, -0.\overline{1}, -\frac{5}{3}, 0, 0.85, 3, 110\right\}$

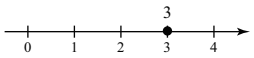
(a) Natural numbers: $\{3, 110\}$

(b) Integers: $\left\{-\sqrt{25}, 0, 3, 110\right\}$

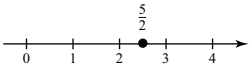
(c) Rational numbers: $\left\{-\sqrt{25}, -0.\overline{1}, -\frac{5}{3}, 0, 0.85, 3, 110\right\}$

(d) Irrational numbers: $\left\{-\sqrt{6}\right\}$

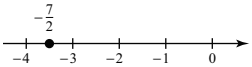
5. (a) The point representing the real number 3 lies between 2 and 4.



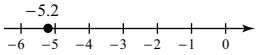
(b) The point representing the real number $\frac{5}{2}$ lies between 2 and 3.



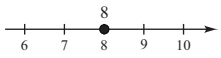
(c) The point representing the real number $-\frac{7}{2}$ lies between -4 and -3.



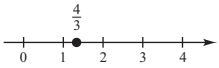
(d) The point representing the real number -5.2 lies between -6 and -5, but closer to -5.



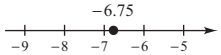
6. (a) The point representing the real number 8 lies between 7 and 9.



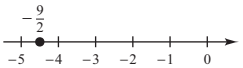
(b) The point representing the real number $\frac{4}{3}$ lies between 1 and 2, but closer to 1.



(c) The point representing the real number -6.75 lies between -7 and -6, but closer to -7.



(d) The point representing the real number $-\frac{9}{2}$ lies between -5 and -4.



7. $\frac{4}{5} < 1$ because $\frac{4}{5}$ is to the left of 1 on the real number line.

8. $2 > \frac{5}{3}$ because 2 is to the right of $\frac{5}{3}$ on the real number line.

9. $-5 < 2$ because -5 is to the left of 2 on the real number line.

10. $9 > -1$ because 9 is to the right of -1 on the real number line.

11. $-5 < -2$ because -5 is to the left of -2 on the real number line.

12. $-8 < -3$ because -8 is to the left of -3 on the real number line.

13. $\frac{5}{8} > \frac{1}{2}$ because $\frac{5}{8}$ is to the right of $\frac{1}{2}$ on the real number line.

14. $\frac{3}{2} < \frac{5}{2}$ because $\frac{3}{2}$ is to the left of $\frac{5}{2}$ on the real number line.

15. $-\frac{2}{3} > -\frac{10}{3}$ because $-\frac{2}{3}$ is to the right of $-\frac{10}{3}$ on the real number line.

16. $-\frac{5}{3} < -\frac{3}{2}$ because $-\frac{5}{3}$ lies to the left of $-\frac{3}{2}$ on the real number line.

17. Distance $= 10 - 4 = 6$

18. Distance $= 75 - 20 = 55$

19. Distance $= 7 - (-12) = 7 + 12 = 19$

20. Distance $= 32 - (-54) = 86$

21. Distance $= 18 - (-32) = 18 + 32 = 50$

22. Distance $= 14 - (-6) = 14 + 6 = 20$

23. Distance $= 0 - (-8) = 0 + 8 = 8$

24. Distance $= 125 - 0 = 125$

25. Distance $= 35 - 0 = 35$

26. Distance $= 0 - (-35) = 0 + 35 = 35$

27. Distance $= (-6) - (-9) = (-6) + 9 = 3$

28. Distance $= -7 - (-12) = -7 + 12 = 5$

29. $|10| = 10$

30. $|62| = 62$

31. $|-225| = 225$

32. $|-14| = 14$

33. $-\left|-\frac{3}{4}\right| = -\frac{3}{4}$

34. $-\left|\frac{3}{8}\right| = -\frac{3}{8}$

35. $|-6| > |2|$ because $|-6| = 6$ and $|2| = 2$, and 6 is greater than 2.

36. $|-2| = |2|$ because $|-2| = 2$ and $|2| = 2$.

37. $|47| > |-27|$ because $|47| = 47$ and $|-27| = 27$, and 47 is greater than 27.

38. $|150| < |-310|$ because $|150| = 150$ and $|-310| = 310$, and 150 is less than 310.

39. *Label:* The weight on the elevator $= x$
Inequality: $x \leq 2500$

40. *Label:* The speed of a car $= x$
Inequality: $x \leq 65$

41. *Label:* Contestant's weight $= x$
Inequality: $x > 200$

42. *Label:* Money saved $= x$
Inequality: $x \leq 2$

43. *Label:* Person's height $= x$
Inequality: $x \geq 52$

44. *Label:* Time to run a mile $= x$
Inequality: $8 \leq x \leq 10$

45. *Label:* Balance of checking account $= x$
Inequality: $200 \leq x \leq 700$

46. *Label:* The number of pages read $= x$
Inequality: $40 \leq x \leq 70$

47. The number line shows $-2.5 < 2$ because -2.5 is to the left of -2.

48. The number on the right is greater than the number on the left.

49. The fractions are converted to decimals and plotted on a number line to determine the order.

50. Rewrite the fractions with the same denominator 6, then plot each number on a number line or compare the numerators. Because the fractions are negative, the greater fraction has the lesser numerator.

51. $\{-5, -4, -3, -2, -1, 0, 1, 2, 3\}$

52. $\{-2, 0, 2, 4, 6, 8, 10\}$

53. $\{5, 7, 9\}$

54. $\{5, 7, 11, 13, 17, 19, 23\}$

55. $a = -1, b = \frac{1}{2}$

$-1 < \frac{1}{2}$

56. $a = -\frac{3}{2}, b = \frac{7}{2}$

$-\frac{3}{2} < \frac{7}{2}$

57. $a = -\frac{9}{2}, b = -2,$

$-\frac{9}{2} < -2$

58. $a = 61.2, b = 65$

$61.2 < 65$

59. $-|-85| = -85$

60. $-|-36.5| = -36.5$

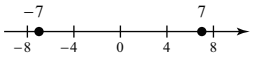
61. $-|3.5| = -3.5$

62. $|-1.4| = 1.4$

63. $|-\pi| = \pi$

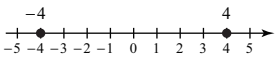
64. $-|\pi| = -\pi$

65. The opposite of -7 is 7.



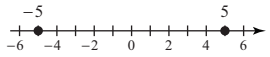
The distance of both -7 and 7 from 0 is 7.

66. The opposite of -4 is 4.



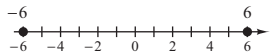
The distance of both -4 and 4 from 0 is 4.

67. The opposite of 5 is -5.



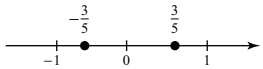
The distance of both -5 and 5 from 0 is 5.

68. The opposite of 6 is -6.



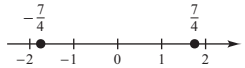
The distance of both 6 and -6 from 0 is 6.

69. The opposite of $-\frac{3}{5}$ is $\frac{3}{5}$.



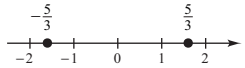
The distance of both $-\frac{3}{5}$ and $\frac{3}{5}$ from 0 is $\frac{3}{5}$.

70. The opposite of $\frac{7}{4}$ is $-\frac{7}{4}$.



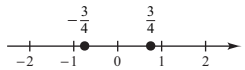
The distance of both $\frac{7}{4}$ and $-\frac{7}{4}$ from 0 is $\frac{7}{4}$.

71. The opposite of $\frac{5}{3}$ is $-\frac{5}{3}$.



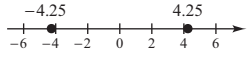
The distance of both $\frac{5}{3}$ and $-\frac{5}{3}$ from 0 is $\frac{5}{3}$.

72. The opposite of $-\frac{3}{4}$ is $\frac{3}{4}$.



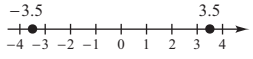
The distance of both $-\frac{3}{4}$ and $\frac{3}{4}$ from 0 is $\frac{3}{4}$.

73. The opposite of -4.25 is 4.25.



The distance of both -4.25 and 4.25 from 0 is 4.25.

74. The opposite of 3.5 is -3.5.



The distance of both 3.5 and -3.5 from 0 is 3.5.

75. $x < 0$

76. $y > 25$

77. $u \geq 16$

78. $x \geq 0$

79. You have more than 30 coins and fewer than 50 coins in a jar.

80. A basketball player scores no more than 310 points and no less than 280 points this season.

81. Because $|-4| = 4$ and $|4| = 4$, the two possible values of a are -4 and 4.

82. Because $-|-7| = -7$ and $-|7| = -7$, the two possible values of a are -7 and 7.

83. Because $|-2 - 3| = |-5| = 5$ and $|8 - 3| = |5| = 5$, the two possible values of a are -2 and 8.

84. Because $|-7 - (-1)| = |-6| = 6$ and $|5 - (-1)| = |6| = 6$, the two possible values of a are -7 and 5.

85. Sample answers: $-3, -100, -\frac{4}{1}$

86. Sample answers: 7, 1032, 15

87. Sample answers: $\sqrt{2}, \pi, -3\sqrt{3}$

88. Sample answers: $\frac{2}{3}, 201, 3.\overline{3}$

89. Sample answers: $\frac{3}{4}, 1\frac{1}{2}, 0.1\overline{6}$

90. Sample answers: $\frac{1}{2}, 10, 20\frac{1}{5}$

91. Sample answers: $-\frac{1}{2}, \pi, -\sqrt{2}$

92. Sample answers: $-1, -10, -100$

93. True. If a number can be written as ratio of two integers, it is rational. If not, the number is irrational.

94. True. The distance between 0 and the number b is the same as the distance between 0 and the opposite of b.

95. $0.15 = \frac{15}{100}$ and $0.\overline{15} = 0.151515\ldots = \frac{15}{99}$

96. Yes, the nonnegative real numbers include 0.

Section 1.2 Operations with Real Numbers

1. $-8 + 12 = +(12 - 8) = 4$

2. $-5 + 9 = +(9 - 5) = 4$

3. $13 + (-6) = +(13 - 6) = 7$

4. $12 + (-10) = +(12 - 10) = 2$

5. $-17 + (-6) = -(17 + 6)$
$= -23$

6. $-6.4 + (-3.7) = -(6.4 + 3.7)$
$= -10.1$

7. $-8 - 12 = -8 + (-12) = -(8 + 12) = -20$

8. $-3 - 17 = -3 + (-17) = -(3 + 17) = -20$

9. $13 - (-9) = 13 + 9$
$= 22$

10. $4 - (-11) = 4 + 11$
$= 15$

11. $-15 - (-18) = -15 + 18$
$= +(18 - 15)$
$= 3$

12. $-21.5 - (-6.3) = -21.5 + 6.3 = -(21.5 - 6.3) = -15.2$

13. $\frac{3}{8} + \frac{7}{8} = \frac{3 + 7}{8} = \frac{10}{8} = \frac{5}{4}$

14. $\frac{5}{6} + \frac{7}{6} = \frac{5 + 7}{6} = \frac{12}{6} = 2$

15. $\frac{3}{4} - \frac{1}{4} = \frac{3 - 1}{4} = \frac{2}{4} = \frac{1}{2}$

16. $\frac{5}{9} - \frac{1}{9} = \frac{5 - 1}{9} = \frac{4}{9}$

17. $\frac{3}{5} + \left(-\frac{1}{2}\right) = \frac{3(2)}{5(2)} - \frac{1(5)}{2(5)}$
$= \frac{6}{10} - \frac{5}{10}$
$= \frac{6 - 5}{10}$
$= \frac{1}{10}$

18. $\frac{6}{7} + \left(-\frac{3}{7}\right) = \frac{6}{7} - \frac{3}{7} = \frac{6 - 3}{7} = \frac{3}{7}$

19. $\frac{5}{8} - \frac{1}{8} = \frac{5-1}{8}$

$= \frac{4}{8}$

$= \frac{4}{2 \cdot 4}$

$= \frac{1}{2}$

20. $\frac{3}{10} - \frac{5}{2} = \frac{3}{10} - \frac{5(5)}{2(5)}$

$= \frac{3}{10} - \frac{25}{10}$

$= \frac{3-25}{10}$

$= \frac{-22}{10} = -\frac{11}{5}$

21. $3\frac{1}{2} + 4\frac{3}{8} = \frac{7}{2} + \frac{35}{8}$

$= \frac{7(4)}{2(4)} + \frac{35}{8}$

$= \frac{28}{8} + \frac{35}{8}$

$= \frac{28+35}{8} = \frac{63}{8}$

22. $5\frac{3}{4} + 7\frac{3}{8} = \frac{23}{4} + \frac{59}{8}$

$= \frac{23(2)}{4(2)} + \frac{59(1)}{8(1)}$

$= \frac{46+59}{8} = \frac{105}{8}$

23. $10\frac{5}{8} - 6\frac{1}{4} = \frac{85}{8} - \frac{25}{4}$

$= \frac{85}{8} - \frac{25(2)}{4(2)}$

$= \frac{85}{8} - \frac{50}{8}$

$= \frac{85-50}{8} = \frac{35}{8}$

24. $8\frac{1}{2} - 4\frac{2}{3} = \frac{17}{2} - \frac{14}{3}$

$= \frac{17(3)}{2(3)} - \frac{14(2)}{3(2)}$

$= \frac{51}{6} - \frac{28}{6}$

$= \frac{51-28}{6} = \frac{23}{6}$

25. $5(-6) = -30$

26. $3(-9) = -27$

27. $(-8)(-6) = 48$

28. $(-4)(-7) = 28$

29. $2(4)(-5) = 8(-5) = -40$

30. $3(-7)(10) = (-21)(10) = -210$

31. $(-1)(12)(-3) = (-12)(-3) = 36$

32. $(-2)(-6)(4) = (12)(4) = 48$

33. $\frac{1}{2}\left(\frac{1}{6}\right) = \frac{1}{12}$

34. $\frac{1}{3}\left(\frac{2}{3}\right) = \frac{2}{9}$

35. $-\frac{3}{2}\left(\frac{8}{5}\right) = -\frac{24}{10} = -\frac{12}{5}$

36. $\left(\frac{10}{13}\right)\left(-\frac{3}{5}\right) = -\frac{6}{13}$

37. $\left(-\frac{5}{8}\right)\left(-\frac{4}{5}\right) = \frac{1}{2}$

38. $\left(-\frac{4}{7}\right)\left(-\frac{4}{5}\right) = \frac{16}{35}$

39. $\frac{-18}{-3} = \frac{-6 \cdot -3}{-3} = 6$

40. $-\frac{30}{-15} = \frac{-2 \cdot -15}{-15} = 2$

41. $\frac{-48}{16} = \frac{-3 \cdot 16}{16} = -3$

42. $63 \div (-7) = \frac{63}{-7} = \frac{9 \cdot 7}{-7} = -9$

43. $-10 \div 0$ is undefined.

Division by zero is undefined.

44. $-125 \div 0$ is undefined.

Division by zero is undefined.

45. $-\frac{4}{5} \div \frac{8}{25} = -\frac{4}{5} \cdot \frac{25}{8} = \frac{(-4)(25)}{(5)(8)} = -\frac{5}{2}$

46. $-\frac{11}{12} \div \frac{5}{24} = -\frac{11}{12} \cdot \frac{24}{5} = -\frac{(11)(24)}{(12)(5)} = -\frac{22}{5}$

47. $\left(-\frac{1}{3}\right) \div \left(-\frac{5}{6}\right) = \left(-\frac{1}{3} \div -\frac{5}{6}\right)$
$= \left(\frac{-1}{3} \cdot \frac{-6}{5}\right) = \frac{(-1)(-6)}{(3)(5)} = \frac{2}{5}$

48. $\left(-\frac{3}{8}\right) \div \left(-\frac{4}{3}\right) = \left(-\frac{3}{8}\right) \cdot \left(-\frac{3}{4}\right) = \frac{3(3)}{8(4)} = \frac{9}{32}$

49. $4\frac{1}{8} \div 4\frac{1}{2} = \frac{33}{8} \div \frac{9}{2} = \frac{33}{8} \cdot \frac{2}{9} = \frac{(33)(2)}{(8)(9)} = \frac{11}{12}$

50. $26\frac{2}{3} \div 10\frac{5}{6} = \frac{80}{3} \div \frac{65}{6} = \frac{80}{3} \cdot \frac{6}{65} = \frac{(80)(6)}{(3)(65)} = \frac{32}{13}$

51. $-4\frac{1}{4} \div \left(-5\frac{5}{8}\right) = -\frac{17}{4} \div \left(-\frac{45}{8}\right)$
$= -\frac{17}{4} \cdot \left(-\frac{8}{45}\right) = \frac{17(8)}{4(45)} = \frac{34}{45}$

52. $-3\frac{5}{6} \div -2\frac{2}{3} = -\frac{23}{6} \div \frac{-8}{3}$
$= \frac{-23}{6} \cdot \frac{3}{-8} = \frac{(-23)(3)}{(6)(-8)} = \frac{23}{16}$

53. $(-7) \cdot (-7) \cdot (-7) = (-7)^3$

54. $(-4)(-4)(-4)(-4)(-4)(-4) = (-4)^6$

55. $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^4$

56. $\left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right) = \left(\frac{5}{8}\right)^4$

57. $-(7 \cdot 7 \cdot 7) = -7^3$

58. $-(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) = -5^6$

59. $2^5 = (2)(2)(2)(2)(2) = 32$

60. $5^3 = (5)(5)(5) = 125$

61. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

62. $(-3)^3 = (-3)(-3)(-3) = -27$

63. $-4^3 = -(4)(4)(4) = -64$

64. $-6^4 = -(6)(6)(6)(6) = -1296$

65. $\left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{64}{125}$

66. $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{81}$

67. $\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$
$= \frac{1}{4}$

68. $\left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) = -\frac{27}{64}$

69. $-\left(-\frac{1}{2}\right)^5 = -\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\left(-\frac{1}{32}\right) = \frac{1}{32}$

70. $-\left(-\frac{1}{4}\right)^3 = -\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)$
$= -\left(-\frac{1}{64}\right)$
$= \frac{1}{64}$

71. $(0.3)^3 = (0.3)(0.3)(0.3) = 0.027$

72. $(0.2)^4 = (0.2)(0.2)(0.2)(0.2) = 0.0016$

73. $5(-0.4)^3 = 5(-0.4)(-0.4)(-0.4) = 5(-0.064) = -0.32$

74. $-3(0.8)^2 = -3(0.8)(0.8) = -3(0.64) = -1.92$

75. $16 - 6 - 10 = (16 - 6) - 10 = 10 - 10 = 0$

76. $18 - 12 + 4 = (18 - 12) + 4 = 6 + 4 = 10$

77. $24 - 5 \cdot 2^2 = 24 - 5 \cdot 4$
$= 24 - (5 \cdot 4) = 24 - 20 = 4$

78. $18 + 3^2 - 12 = 18 + 9 - 12$
$= (18 + 9) - 12$
$= 27 - 12$
$= 15$

79. $28 \div 4 + 3 \cdot 5 = (28 \div 4) + (3 \cdot 5)$
$= 7 + 15$
$= 22$

80. $6 \cdot 7 - 6^2 \div 4 = 6 \cdot 7 - 36 \div 4$
$= (6 \cdot 7) - (36 \div 4)$
$= 42 - 9$
$= 33$

81. $14 - 2(8 - 4) = 14 - 2(4)$
$= 14 - 8$
$= 6$

82. $21 - 5(7 - 5) = 21 - 5(2)$
$= 21 - 10$
$= 11$

83. $17 - 5(16 \div 4^2) = 17 - 5(16 \div 16)$
$= 17 - 5(1)$
$= 17 - 5$
$= 12$

84. $72 - 8(6^2 \div 9) = 72 - 8(36 \div 9)$
$= 72 - 8(4)$
$= 72 - 32$
$= 40$

85. $5^2 - 2[9 - (18 - 8)] = 25 - 2[9 - 10]$
$= 25 - 2[-1]$
$= 25 + 2$
$= 27$

86. $8 \cdot 3^2 - 4(12 + 3) = 8 \cdot 9 - 4(15)$
$= 72 - 60$
$= 12$

87. $5^3 + |-14 + 4| = 125 + |-10|$
$= 125 + 10$
$= 135$

88. $|(-2)^5| - (25 + 7) = |-32| - 32$
$= 32 - 32$
$= 0$

89. $\dfrac{6 + 8(3)}{7 - 12} = [6 + 8(3)] \div (7 - 12)$
$= (6 + 24) \div (7 - 12)$
$= 30 \div (-5)$
$= -6$

90. $\dfrac{9 + 6(2)}{3 + 4} = [9 + 6(2)] \div (3 + 4)$
$= (9 + 12) \div (3 + 4)$
$= 21 \div 7$
$= 3$

91. Apply the order of operations as follows: Parentheses, Exponents, Multiplication and Division, Addition and Subtraction.

92. The first step in evaluating the expression $6^2 - 8(3 + 4)$ is to evaluate $3 + 4$ because this expression is inside parentheses.

93. To subtract the real number b from the real number a, add the opposite of b to a.

94. If $a > 0$, then $(-a)^n = -a^n$ when n is odd.

95. $85 - |-25| = 85 - 25 = 60$

96. $-36 + |-8| = -36 + 8 = -(36 - 8) = -28$

97. $-(-11.325) + |34.625| = 11.325 + 34.625 = 45.95$

98. $|-16.25| - 54.78 = 16.25 + (-54.78)$
$= -(54.78 - 16.25)$
$= -38.53$

99. $-\left|-6\frac{7}{8}\right| - 8\frac{1}{4} = -6\frac{7}{8} - 8\frac{1}{4}$
$= -\frac{55}{8} - \frac{33(2)}{4(2)}$
$= -\frac{55}{8} - \frac{66}{8}$
$= \frac{-55 - 66}{8}$
$= -\frac{121}{8}$

100. $-\left|-15\frac{2}{3}\right| - 12\frac{1}{3} = -15\frac{2}{3} - 12\frac{1}{3}$
$= -\frac{47}{3} - \frac{37}{3}$
$= -\frac{47 + 37}{3}$
$= -\frac{84}{3}$
$= -28$

101. $\dfrac{4^2 - 5}{11} - 7 = [(4^2 - 5) \div 11] - 7$
$= [(16 - 5) \div 11] - 7$
$= (11 \div 11) - 7$
$= 1 - 7$
$= -6$

102. $\dfrac{5^3 - 50}{-15} + 27 = \left[\left(5^3 - 50\right) \div -15\right] + 27$

$= \left[(125 - 50) \div -15\right] + 27$

$= (75 \div -15) + 27$

$= -5 + 27$

$= 22$

103. $\dfrac{6 \cdot 2^2 - 12}{3^2 + 3} = \left[\left(6 \cdot 2^2\right) - 12\right] \div \left(3^2 + 3\right)$

$= (24 - 12) \div (9 + 3)$

$= 12 \div 12$

$= 1$

104. $\dfrac{7^2 - 2(11)}{5^2 + 8(-2)} = \left[7^2 - 2(11)\right] \div \left[5^2 + 8(-2)\right]$

$= (49 - 22) \div (25 - 16)$

$= 27 \div 9$

$= 3$

105. $\dfrac{3 + \frac{3}{4}}{\frac{1}{8}} = \left(3 + \dfrac{3}{4}\right) \div \dfrac{1}{8}$

$= \left(\dfrac{12}{4} + \dfrac{3}{4}\right) \div \dfrac{1}{8}$

$= \dfrac{15}{4} \div \dfrac{1}{8}$

$= \dfrac{15}{4} \cdot \dfrac{8}{1}$

$= \dfrac{15(8)}{4} = 30$

106. $\dfrac{6 - \frac{2}{3}}{\frac{4}{9}} = \left(6 - \dfrac{2}{3}\right) \div \dfrac{4}{9}$

$= \left(\dfrac{18}{3} - \dfrac{2}{3}\right) \div \dfrac{4}{9}$

$= \dfrac{16}{3} \div \dfrac{4}{9}$

$= \dfrac{16}{3} \cdot \dfrac{9}{4}$

$= \dfrac{16(9)}{3(4)}$

$= 12$

107. $\dfrac{1}{4} + \dfrac{2}{9} + \dfrac{1}{10} + x + \dfrac{1}{3} = 1$

So,

$x = 1 - \left(\dfrac{1}{4} + \dfrac{2}{9} + \dfrac{1}{10} + \dfrac{1}{3}\right)$

$= 1 - \left(\dfrac{45}{180} + \dfrac{40}{180} + \dfrac{18}{180} + \dfrac{60}{180}\right)$

$= 1 - \left(\dfrac{45 + 40 + 18 + 60}{180}\right)$

$= 1 - \dfrac{163}{180} = \dfrac{180}{180} - \dfrac{163}{180} = \dfrac{17}{180}.$

108. $\dfrac{1}{7} + \dfrac{1}{6} + \dfrac{1}{5} + x + \dfrac{1}{3} = 1$

So,

$x = 1 - \left(\dfrac{1}{7} + \dfrac{1}{6} + \dfrac{1}{5} + \dfrac{1}{3}\right)$

$= 1 - \left(\dfrac{30}{210} + \dfrac{35}{210} + \dfrac{42}{210} + \dfrac{70}{210}\right)$

$= 1 - \left(\dfrac{30 + 35 + 42 + 70}{210}\right)$

$= 1 - \dfrac{177}{210}$

$= \dfrac{210}{210} - \dfrac{177}{210}$

$= \dfrac{33}{210} = \dfrac{11}{70}.$

109. \$2618.68 + \$1236.45 − \$25.62 − \$455.00 − \$125.00 − \$715.95 = \$2533.56

The balance at the end of the month was \$2533.56.

110. $l = 5$ meters, $w = 3$ meters

$A = lw$

$A = 5 \cdot 3 = 15$ square meters

111. $l = 14$ centimeters, $w = 8$ centimeters

$A = lw$

$A = 14 \cdot 8 = 112$ square centimeters

112. $b = 8$ inches, $h = 5$ inches

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(8)(5) = 20$ square inches

113. $b = 10$ feet, $h = 7$ feet

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot 10 \cdot 7 = 35$ square feet

114. (a) $\$50(12)(18) = \$10{,}800$

(b) The account would have \$15,832.22.

(c) $\$15{,}832.22 - \$10{,}800 = \$5032.22$

\$5032.22 is earnings from interest.

115. True. A nonzero rational number is an integer divided by an integer. The reciprocal of such a number is still an integer divided by an integer, and so it is a rational number.

116. False. The product of two fractions is the product of the numerators over the product of the denominators.

117. True. Any negative real number raised to an even numbered power will be a positive real number.

118. False. If a negative number is raised to an odd power, the result will be negative.

119. False. Division is not commutative.

120. No, the expressions are not equal.

$(2^2)^3 = 4^3 = 64$ and $2^{(2^3)} = 2^8 = 256$. Since

$64 \neq 256$, $(2^2)^3 \neq 2^{(2^3)}$.

121. If the numbers have like signs, the product or quotient is positive. If the numbers have unlike signs, the product or quotient is negative.

122. (a) $40 - 10 + 3 = 30 + 3 = 33 \neq 27$

Insert parentheses: $40 - (10 + 3) = 40 - 13 = 27$

(b) $5^2 + \frac{1}{2} \cdot 4 = 25 + \frac{1}{2} \cdot 4 = 25 + 2 = 27$

(c) $8 \cdot 3 + 30 \div 2 = 24 + 15 = 39 + 27$

Insert parentheses:

$(8 \cdot 3 + 30) \div 2 = (24 + 30) \div 2$

$= 54 \div 2 = 27$

(d) $75 \div 2 + 1 + 2 = 37.5 + 1 + 2$

$= 38.5 + 2 = 40.5 \neq 27$

Insert parentheses:

$75 \div (2 + 1) + 2 = 75 \div 3 + 2 = 25 + 2 = 27$

123. To add fractions with unlike denominators, you first find the least common denominator.

$$\frac{2}{3} + \frac{3}{2} = \frac{2(2)}{3(2)} + \frac{3(3)}{2(3)} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6}$$

124. Only common factors (not terms) of the numerator and denominator can be divided out.

$$\frac{5 + 12}{5} = \frac{17}{5}$$

Section 1.3 Properties of Real Numbers

1. $18 - 18 = 0$

Additive Inverse Property

2. $5 + 0 = 5$

Additive Identity Property

3. $\frac{1}{12} \cdot 12 = 1$

Multiplicative Inverse Property

4. $52 \cdot 1 = 52$

Multiplicative Identity Property

5. $(8 - 5)(10) = 8 \cdot 10 - 5 \cdot 10$

Distributive Property

6. $7(9 + 15) = 7 \cdot 9 + 7 \cdot 15$

Distributive Property

7. $15(-3) = (-3)15$

8. $6 + (5 + y) = (6 + 5) + y$

9. $5(6 + z) = 5 \cdot 6 + 5 \cdot z$

10. $(8 - y)(4) = 8(4) - y(4)$

11. $x + 4 = 5$

$(x + 4) - 4 = 5 - 4$

Addition Property of Equality

12. $7x = 14$

$\frac{1}{7}(7x) = \frac{1}{7}(14)$

Multiplication Property of Equality

13. $20(2 + 5) = 20 \cdot 2 + 20 \cdot 5$

14. $-3(4 - 8) = -3(4) + (-3)(-8)$

15. $(x + 6)(-2) = x \cdot (-2) + 6 \cdot (-2)$ or $-2x - 12$

16. $(z - 10)(12) = z(12) - 10(12)$ or $12z - 120$

17. $-6(2y - 5) = -6(2y) + (-6)(-5)$ or $-12y + 30$

18. $-4(10 - b) = -4(10) + (-4)(-b)$ or $-40 + 4b$

19. $7x + 2x = (7 + 2)x = 9x$

20. $8x - 6x = (8 - 6)x = 2x$

21. $\frac{7x}{8} - \frac{5x}{8} = (7 - 5)\left(\frac{x}{8}\right) = \frac{2x}{8} = \frac{x}{4}$

22. $\frac{3x}{5} + \frac{x}{5} = (3 + 1)\left(\frac{x}{5}\right) = \frac{4x}{5}$

23.

$ac = bc,\ c \neq 0$	Write original equation.
$\frac{1}{c}(ac) = \frac{1}{c}(bc)$	Multiplication Property of Equality
$\frac{1}{c}(ca) = \frac{1}{c}(cb)$	Commutative Property of Multiplication
$\left(\frac{1}{c} \cdot c\right)a = \left(\frac{1}{c} \cdot c\right)b$	Associative Property of Multiplication
$1 \cdot a = 1 \cdot b$	Multiplicative Inverse Property
$a = b$	Multiplicative Identity Property

24.

$a \cdot 1 = 1 \cdot a$	Write original equation.
$a \cdot 1 = a \cdot 1$	Commutative Property of Multiplication
$a = a$	Multiplicative Identity Property

25.

$a = (a + b) + (-b)$	Write original equation.
$a = a + [b + (-b)]$	Associative Property of Addition
$a = a + 0$	Additive Inverse Property
$a = a$	Additive Identity Property

26.

$a + (-a) = 0$	Write original equation.
$0 = 0$	Additive Inverse Property

27. $13 + 12 = 12 + 13$

Commutative Property of Addition

28. $(5 + 10)(8) = 8(5 + 10)$

Commutative Property of Multiplication

29. $(-4 \cdot 10) \cdot 8 = -4(10 \cdot 8)$

Associative Property of Multiplication

30. $3 + (12 - 9) = (3 + 12) - 9$

Associative Property of Addition

31. $10(2x) = (10 \cdot 2)x$

Associative Property of Multiplication

32. $1 \cdot 9k = 9k$

Multiplicative Identity Property

33. $10x \cdot \frac{1}{10x} = 1$

Multiplicative Inverse Property

34. $0 + 4x = 4x$

Additive Identity Property

35. $2x - 2x = 0$

Additive Inverse Property

36. $4 + (3 - x) = (4 + 3) - x$

Associative Property of Addition

37. $3(2 + x) = 3 \cdot 2 + 3x$

Distributive Property

38. $3(6 + b) = 3 \cdot 6 + 3 \cdot b$

Distributive Property

39. $(x + 1) - (x + 1) = 0$

Additive Inverse Property

40. $6(x + 3) = 6 \cdot x + 6 \cdot 3$

Distributive Property

41.

$x + 5 = 3$	Write original equation.
$(x + 5) + (-5) = 3 + (-5)$	Addition Property of Equality
$x + (5 + (-5)) = 3 - 5$	Associative Property of Addition
$x + 0 = -2$	Additive Inverse Property
$x = -2$	Additive Identity Property

42.

$x - 8 = 20$	Write original equation.
$(x - 8) + 8 = 20 + 8$	Addition Property of Equality
$x + (-8 + 8) = 28$	Associative Property of Addition
$x + 0 = 28$	Additive Inverse Property
$x = 28$	Additive Identity Property

43.

$2x - 5 = 6$	Write original equation.
$(2x - 5) + 5 = 6 + 5$	Addition Property of Equality
$2x + (-5 + 5) = 11$	Associative Property of Addition
$2x + 0 = 11$	Additive Inverse Property
$2x = 11$	Additive Identity Property
$\frac{1}{2}(2x) = \frac{1}{2}(11)$	Multiplication Property of Equality
$\left(\frac{1}{2} \cdot 2\right)x = \frac{11}{2}$	Associative Property of Multiplication
$1 \cdot x = \frac{11}{2}$	Multiplicative Inverse Property
$x = \frac{11}{2}$	Multiplicative Identity Property

44.

$3x + 4 = 10$	Write original equation.
$(3x + 4) + (-4) = 10 + (-4)$	Addition Property of Equality
$3x + [4 + (-4)] = 6$	Associative Property of Addition
$3x + 0 = 6$	Additive Inverse Property
$3x = 6$	Additive Identity Property
$\frac{1}{3}3x = \frac{1}{3}(6)$	Multiplication Property of Equality
$\left(\frac{1}{3} \cdot 3\right)x = 2$	Associative Property of Multiplication
$1 \cdot x = 2$	Multiplicative Inverse Property
$x = 2$	Multiplicative Identity Property

45.

$-4x - 4 = 0$	Write original equation.
$-4x - 4 + 4 = 0 + 4$	Addition Property of Equality
$-4x + (-4 + 4) = 4$	Associative Property of Addition
$-4x + 0 = 4$	Additive Inverse Property
$-4x = 4$	Additive Identity Property
$-\frac{1}{4}(-4x) = -\frac{1}{4}(4)$	Multiplication Property of Equality
$\left[-\frac{1}{4} \cdot (-4)\right]x = -1$	Associative Property of Multiplication
$1 \cdot x = -1$	Multiplicative Inverse Property
$x = -1$	Multiplicative Identity Property

46.

$-5x + 25 = 5$	Write original equation.
$(-5x + 25) + (-25) = 5 + (-25)$	Addition Property of Equality
$-5x + [25 + (-25)] = -20$	Associative Property of Addition
$-5x + 0 = -20$	Additive Inverse Property
$-5x = -20$	Additive Identity Property
$-\frac{1}{5}(-5x) = -\frac{1}{5}(-20)$	Multiplication Property of Equality
$\left[-\frac{1}{5} \cdot (-5)\right]x = 4$	Associative Property of Multiplication
$1 \cdot x = 4$	Multiplicative Inverse Property
$x = 4$	Multiplicative Identity Property

47. Every real number except zero has an additive inverse. The additive inverse (or opposite) of a number is the same distance from zero as that number. Because there is no distance from zero to zero, zero does not have an additive inverse.

48. Every real number except zero has a multiplicative inverse. The multiplicative inverse of every number is 1 over that number. Because zero cannot be in the denominator, zero does not have a multiplicative inverse.

49. No.

Subtraction: $8 - 2 = 6 \neq -6 = 2 - 8$

Division: $21 \div 7 = 3 \neq \frac{1}{3} = 7 \div 21$

50. No.

Subtraction: $(12 - 4) - 5 = 3 \neq 13 = 12 - (4 - 5)$

Division: $(48 \div 8) \div 2 = 3 \neq 12 = 48 \div (8 \div 2)$

51. $32 + (4 + y) = (32 + 4) + y$

52. $15 + (3 - x) = (15 + 3) - x$

53. $9(6M) = (9 \cdot 6)M$

54. $11(4n) = (11 \cdot 4)n$

55. $3(x + 5) = 3x + 15$

56. $4(x + 2) = 4x + 8$

57. $-2(x + 8) = -2x - 16$

58. $-9(x + 4) = -9x - 36$

59. $16(1.75) = 16\left(2 - \frac{1}{4}\right) = 16(2) - 16\left(\frac{1}{4}\right) = 32 - 4 = 28$

60. $15\left(1\frac{2}{3}\right) = 15\left(2 - \frac{1}{3}\right) = 15(2) - 15\left(\frac{1}{3}\right) = 30 - 5 = 25$

61. $7(62) = 7(60 + 2) = 7(60) + 7(2) = 420 + 14 = 434$

62. $5(51) = 5(50 + 1) = 5(50) + 5(1) = 250 + 5 = 255$

63.
$$\begin{aligned} 9(6.98) &= 9(7 - 0.02) \\ &= 9(7) - 9(0.02) \\ &= 63 - 0.18 \\ &= 62.82 \end{aligned}$$

64.
$$\begin{aligned} 12(19.95) &= 12(20 - 0.05) \\ &= 12(20) - 12(0.05) \\ &= 240 - 0.6 \\ &= 239.4 \end{aligned}$$

65. $a(b + c) = ab + ac$

66. $a(b - c) = ab - ac$

67.
$$\begin{aligned} 4 + (x + 5) + (3x + 2) &= 4 + (5 + x) + (3x + 2) \\ &= (4 + 5) + x + (3x + 2) \\ &= 9 + (x + 3x) + 2 \\ &= 9 + 4x + 2 \\ &= 4x + 9 + 2 \\ &= 4x + 11 \end{aligned}$$

68.
$$\begin{aligned} 2x + (3x - 4) + (2x + 4) &= (2x + 3x) - 4 + (2x + 4) \\ &= 5x - 4 + (4 + 2x) \\ &= 5x + (-4 + 4) + 2x \\ &= 5x + 0 + 2x \\ &= 7x \end{aligned}$$

69. (a)
$$\begin{aligned} 2(x + 6) + 2(2x) &= 2x + 12 + 4x \\ &= 2x + 4x + 12 \\ &= 6x + 12 \end{aligned}$$

(b) $(x + 6)(2x) = x(2x) + 6(2x) = 2x^2 + 12x$

70. (a)
$$\begin{aligned} 2(5x) + 2(2x - 1) &= 10x + 4x - 2 \\ &= 14x - 2 \end{aligned}$$

(b) $5x(2x - 1) = 5x(2x) + (-1)(5x)$

$= 10x^2 - 5x$

71. The additive inverse of a real number a is the number $-a$. The sum of a number and its additive inverse is the additive identity 0. For example, $8 + (-8) = 0$.

72. The multiplicative inverse of a real number a $(a \neq 0)$ is the number $\frac{1}{a}$. The product of a number and its multiplicative inverse is the multiplicative identity 1. For example, $8 \cdot \frac{1}{8} = 1$.

73. Given two real numbers a and b, the sum a plus b is the same as the sum b plus a.

74. To subtract the number a from both sides of an equation, use the Addition Property of Equality to add $(-a)$ to both sides.

75. Sample answer: $4 \odot 7 = 2 \cdot 4 + 7 = 8 + 7 = 15$

$7 \odot 4 = 2 \cdot 7 + 4 = 14 + 4 = 18$

Because $15 \neq 18$, $4 \odot 7 \neq 7 \odot 4$. So, the operation is not commutative.

$$\begin{aligned} 3 \odot (4 \odot 7) &= 3 \odot (2 \cdot 4 + 7) \\ &= 3 \odot 15 \\ &= 2 \cdot 3 + 15 \\ &= 6 + 15 \\ &= 21 \end{aligned}$$

$$\begin{aligned} (3 \odot 4) \odot 7 &= (2 \cdot 3 + 4) \odot 7 \\ &= 10 \odot 7 \\ &= 2 \cdot 10 + 7 \\ &= 20 + 7 \\ &= 27 \end{aligned}$$

Because $21 \neq 27$, $3 \odot (4 \odot 7) \neq (3 \odot 4) \odot 7$. So, the operation is not associative.

76. Sample answer: $9 \updownarrow 6 = 9 - (6 + 1) = 9 - 7 = 2$

$6 \updownarrow 9 = 6 - (9 + 1) = 6 - (10) = -4$

Because $2 \neq -4$, $9 \updownarrow 6 \neq 6 \updownarrow 9$. So, the operation is not commutative.

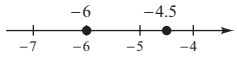
$$\begin{aligned} (10 \updownarrow 2) \updownarrow 7 &= [10 - (2 + 1)] \updownarrow 7 \\ &= (10 - 3) \updownarrow 7 \\ &= 7 \updownarrow 7 \\ &= 7 - (7 + 1) \\ &= 7 - 8 \\ &= -1 \end{aligned}$$

$$\begin{aligned} 10 \updownarrow (2 \updownarrow 7) &= 10 \updownarrow [2 - (7 + 1)] \\ &= 10 \updownarrow (2 - 8) \\ &= 10 \updownarrow (-6) \\ &= 10 - (-6 + 1) \\ &= 10 - (-5) \\ &= 15 \end{aligned}$$

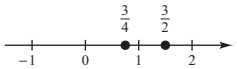
Because $-1 \neq 15$, $(10 \updownarrow 2) \updownarrow 7 \neq 10 \updownarrow (2 \updownarrow 7)$. So, the operation is not associative.

Mid-Chapter Quiz: Sections 1.1-1.3

1. $-4.5 > -6$



2. $\frac{3}{4} < \frac{3}{2}$



3. $|-15 - 7| = |-22| = 22$

4. $|-8.75 - (-2.25)| = |-8.75 + 2.25| = |-6.5| = 6.5$

5. $|-7.6| = 7.6$

6. $-|9.8| = -9.8$

7. $32 + (-18) = 14$

8. $-12 - (-17) = -12 + 17 = 5$

9. $\frac{3}{4} + \frac{7}{4} = \frac{3 + 7}{4} = \frac{10}{4} = \frac{5}{2}$

10. $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{4 - 1}{6} = \frac{3}{6} = \frac{1}{2}$

11. $(-3)(2)(-10) = (-6)(-10) = 60$

12. $\left(-\frac{4}{5}\right)\left(\frac{15}{32}\right) = \frac{(-4)(15)}{(5)(32)} = -\frac{3}{8}$

13. $\frac{7}{12} \div \frac{5}{6} = \frac{7}{12} \cdot \frac{6}{5} = \frac{(7)(6)}{(12)(5)} = \frac{7}{10}$

14. $\left(-\frac{3}{2}\right)^3 = \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right) = -\frac{27}{8}$

15. $3 - 2^2 + 25 \div 5 = 3 - 4 + 25 \div 5$
$= 3 - 4 + 5 = -1 + 5 = 4$

16. $\frac{18 - 2(3 + 4)}{6^2 - (12 \cdot 2 + 10)} = \left[18 - 2(3 + 4)\right] \div \left[6^2 - (12 \cdot 2 + 10)\right] = (18 - 14) \div (36 - 34) = 4 \div 2 = 2$

17. (a) $8(u - 5) = 8 \cdot u - 8 \cdot 5$ Distributive Property

(b) $10x - 10x = 0$ Additive Inverse Property

18. (a) $(7 + y) - z = 7 + (y - z)$ Associative Property of Addition

(b) $2x \cdot 1 = 2x$ Multiplicative Identity Property

19. $\$1406.98 - \$375.03 - \$59.20 - \$225.00 + \$320.45 = \$1068.20$

20. $\$45(2)(12)(8) = \$8640$

21.
$$1 = \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{8} + x$$
$$1 - \tfrac{1}{3} - \tfrac{1}{4} - \tfrac{1}{8} = x$$
$$\tfrac{24}{24} - \tfrac{8}{24} - \tfrac{6}{24} - \tfrac{3}{24} = x$$
$$\tfrac{7}{24} = x$$

The sum of the parts of a circle is equal to 1.

Section 1.4 Algebraic Expressions

1. Terms: $10x$, 5

Coefficients: 10, 5

2. Terms: $17y$, 4

Coefficients: 17, 4

3. Terms: $-6x^2$, 12

Coefficients: -6, 12

4. Terms: $-16t^2$, 48

Coefficients: -16, 48

5. Terms: $-3y^2$, $2y$, -8

Coefficients: -3, 2, -8

6. Terms: $9t^2$, $2t$, 10

Coefficients: 9, 2, 10

7. Terms: $-4a^3$, $1.2a$

Coefficients: -4, 1.2

8. Terms: $25z^3$, $-4.8z^2$

Coefficients: 25, -4.8

9. Terms: $4x^2$, $-3y^2$, $-5x$, 21

Coefficients: 4, -3, -5, 21

10. Terms: $7a^2$, $-b^2$, $4a$, 19

Coefficients: 7, -1, 4, 19

11. Terms: $-5x^2y$, $2y^2$, xy

Coefficients: -5, 2, 1

12. Terms: $14u^2$, $25uv$, $-3v^2$

Coefficients: 14, 25, -3

13. Terms: $\frac{1}{4}x^2$, $-\frac{3}{8}x$, 5

Coefficients: $\frac{1}{4}$, $-\frac{3}{8}$, 5

14. Terms: $\frac{2}{3}y$, $8z$, $\frac{5}{6}$

Coefficients: $\frac{2}{3}$, 8, $\frac{5}{6}$

15. $3x + 4x = (3 + 4)x = 7x$

16. $18z + 14z = (18 + 14)z = 32z$

17. $-2x^2 + 4x^2 = (-2 + 4)x^2 = 2x^2$

18. $20a^2 - 5a^2 = (20 - 5)a^2 = 15a^2$

19. $7x - 11x = (7 - 11)x = -4x$

20. $-23t + 11t = (-23 + 11)t = -12t$

21. $9y - 5y + 4y = (9 - 5 + 4)y = 8y$

22. $8y + 7y - y = (8 + 7 - 1)y = 14y$

23.
$$\begin{aligned} 3x - 2y + 5x + 20y &= (3x + 5x) + (-2y + 20y) \\ &= (3 + 5)x + (-2 + 20)y \\ &= 8x + 18y \end{aligned}$$

24.
$$\begin{aligned} -2a + 4b - 7a - b &= (-2a - 7a) + (4b - b) \\ &= -9a + 3b \end{aligned}$$

25.
$$\begin{aligned} 7x^2 - 2x - x^2 &= 7x^2 - x^2 - 2x \\ &= (7 - 1)x^2 - 2x \\ &= 6x^2 - 2x \end{aligned}$$

26. $9y + y^2 - 6y = y^2 + (9y - 6y) = y^2 + 3y$

27. $-3z^4 + 6z - z + 8 + z^4 - 4z^2 = (-3z^4 + z^4) - 4z^2 + (6z - z) + 8 = -2z^4 - 4z^2 + 5z + 8$

28. $-5y^3 + 3y - 6y^2 + 8y^3 + y - 4 = (-5y^3 + 8y^3) - 6y^2 + (3y + y) - 4 = 3y^3 - 6y^2 + 4y - 4$

29. $x^2 + 2xy - 2x^2 + xy + y = x^2 - 2x^2 + 2xy + xy + y = (1 - 2)x^2 + (2 + 1)xy + y = -x^2 + 3xy + y$

30. $3a - 5ab + 9a^2 + 4ab - a = 9a^2 + (-5 + 4)ab + (3 - 1)a = 9a^2 - ab + 2a$

31.
$$\begin{aligned} 10(x - 3) + 2x - 5 &= 10x - 30 + 2x - 5 \\ &= (10x + 2x) + (-30 - 5) \\ &= (10 + 2)x + (-30 - 5) \\ &= 12x - 35 \end{aligned}$$

32.
$$\begin{aligned} 3(x + 1) + x - 6 &= 3x + 3 + x - 6 \\ &= (3x + x) + (3 - 6) \\ &= 4x - 3 \end{aligned}$$

33. $x - (5x + 9) = x - 5x - 9 = (1 - 5)x - 9 = -4x - 9$

34. $y - (3y - 1) = y - 3y + 1 = -2y + 1$

35.
$$\begin{aligned} 5a - (4a - 3) &= 5a - 4a + 3 \\ &= (5 - 4)a + 3 \\ &= a + 3 \end{aligned}$$

36. $7x - (2x + 5) = 7x - 2x - 5 = 5x - 5$

37.
$$\begin{aligned} -3(3y - 1) + 2(y - 5) &= -9y + 3 + 2y - 10 \\ &= -9y + 2y + 3 - 10 \\ &= (-9 + 2)y - 7 \\ &= -7y - 7 \end{aligned}$$

38.
$$\begin{aligned} 5(a + 6) - 4(2a - 1) &= 5a + 30 - 8a + 4 \\ &= (5a - 8a) + (30 + 4) \\ &= -3a + 34 \end{aligned}$$

39.
$$\begin{aligned} -3(y^2 - 2) + y^2(y + 3) &= -3y^2 + 6 + y^3 + 3y^2 \\ &= (-3 + 3)y^2 + 6 + y^3 \\ &= 6 + y^3 \end{aligned}$$

40.
$$\begin{aligned} x(x^2 - 5) - 4(4 - x) &= x^3 - 5x - 16 + 4x \\ &= x^3 + (-5x + 4x) - 16 \\ &= x^3 - x - 16 \end{aligned}$$

41.
$$\begin{aligned} x(x^2 + 3) - 3(x + 4) &= x^3 + 3x - 3x - 12 \\ &= x^3 + (3 - 3)x - 12 \\ &= x^3 - 12 \end{aligned}$$

42.
$$\begin{aligned} 5(x + 1) - x(2x + 6) &= 5x + 5 - 2x^2 - 6x \\ &= -2x^2 + (-6 + 5)x + 5 \\ &= -2x^2 - x + 5 \end{aligned}$$

43.
$$\begin{aligned} 9a - [7 - 5(7a - 3)] &= 9a - [7 - 35a + 15] \\ &= 9a - [-35a + 22] \\ &= 9a + 35a - 22 \\ &= (9 + 35)a - 22 \\ &= 44a - 22 \end{aligned}$$