CHAPTER 2

Experimental Designs: An Oveview

2. d. (i) RB-3 design (ii) H_0: $\mu_{.1} = \mu_{.2} = \mu_{.3}$ (iii) $Y_{ij} = \mu + \alpha_j + \pi_i + \varepsilon_{ij}$ ($i = 1, \ldots, 14$; $j = 1, \ldots, 3$)

   e. (i) t test for dependent samples (ii) H_0: $\mu_{.1} \le \mu_{.2}$, where $\mu_{.1}$ and $\mu_{.2}$ denote the population means for English-Canadian and French-Canadian students, respectively. (iii) $Y_{ij} = \mu + \alpha_j + \pi_i + \varepsilon_{ij}$ ($i = 1, \ldots, 50$; $j = 1, 2$)

   f. (i) CRF-62 design (ii) H_0: $\mu_{1.} = \mu_{2.} = \cdots = \mu_{6.}$; H_0: $\mu_{.1} = \mu_{.2}$; H_0: $\mu_{jk} - \mu_{jk!} - \mu_{j!k} + \mu_{j!k!} = 0$ for all j and k (iii) $Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{i(jk)}$ ($i = 1, \ldots, 50$; $j = 1, \ldots, 6$; $k = 1, 2$)

   g. (i) CR-3 design (ii) H_0: $\mu_1 = \mu_2 = \mu_3$ (iii) $Y_{ij} = \mu + \alpha_j + \varepsilon_{i(j)}$ ($i = 1, \ldots, 30$; $j = 1, \ldots, 3$)

3. a. The grand mean is the average value around which the treatment means vary.

   b. A treatment effect is the deviation of the grand mean from a treatment mean.

   c. An error effect is all effects not attributable to a treatment level or treatment combination.

4. b. A completely randomized design is the simplest design to lay out and analyze. The randomization procedures for the randomized block and Latin square designs are more complex than those for the completely randomized design, but the latter designs enable a researcher to isolate the effects of one nuisance variable or, in the case of the Latin square design, two nuisance variables.

5. c. a_1b_1, a_1b_2, a_1b_3, a_2b_1, a_2b_2, a_2b_3, a_3b_1, a_3b_2, a_3b_3

   d. a_1b_1, a_1b_2, a_2b_1, a_2b_2, a_3b_1, a_3b_2, a_4b_1, a_4b_2

   e. $a_1b_1c_1$, $a_1b_1c_2$, $a_1b_2c_1$, $a_1b_2c_2$, $a_2b_1c_1$, $a_2b_1c_2$, $a_2b_2c_1$, $a_2b_2c_2$, $a_3b_1c_1$, $a_3b_1c_2$, $a_3b_2c_1$, $a_3b_2c_2$

6. d. CR-5 design with $n = 6$

		Treat. Level	Dep. Var.
	Subject$_1$	a_1	Y_{11}
Group$_1$	⋮	⋮	⋮
	Subject$_6$	a_1	Y_{61}
			$\bar{Y}_{.1}$
	Subject$_1$	a_2	Y_{12}
Group$_2$	⋮	⋮	⋮
	Subject$_6$	a_2	Y_{62}
			$\bar{Y}_{.2}$
	Subject$_1$	a_3	Y_{13}
Group$_3$	⋮	⋮	⋮
	Subject$_6$	a_3	Y_{63}
			$\bar{Y}_{.3}$
	Subject$_1$	a_4	Y_{14}
Group$_4$	⋮	⋮	⋮
	Subject$_6$	a_4	Y_{64}
			$\bar{Y}_{.4}$
	Subject$_1$	a_5	Y_{15}
Group$_5$	⋮	⋮	⋮
	Subject$_6$	a_5	Y_{65}
			$\bar{Y}_{.5}$

e. t test for dependent samples with $n = 7$

	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.
Block$_1$	a_1	Y_{11}	a_2	Y_{12}
Block$_2$	a_1	Y_{21}	a_2	Y_{22}
Block$_3$	a_1	Y_{31}	a_2	Y_{32}
⋮	⋮	⋮	⋮	⋮
Block$_7$	a_1	Y_{71}	a_2	Y_{72}
		$\bar{Y}_{.1}$		$\bar{Y}_{.2}$

f. RB-4 design with n = 6

	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.	
Block_1	a_1	Y_{11}	a_2	Y_{12}	a_3	Y_{13}	a_4	Y_{14}	$\bar{Y}_{1.}$
Block_2	a_1	Y_{21}	a_2	Y_{22}	a_3	Y_{23}	a_4	Y_{24}	$\bar{Y}_{2.}$
Block_3	a_1	Y_{31}	a_2	Y_{32}	a_3	Y_{33}	a_4	Y_{34}	$\bar{Y}_{3.}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Block_6	a_1	Y_{61}	a_2	Y_{62}	a_3	Y_{63}	a_4	Y_{64}	$\bar{Y}_{6.}$
		$\bar{Y}_{.1}$		$\bar{Y}_{.2}$		$\bar{Y}_{.3}$		$\bar{Y}_{.4}$	

g. CRF-222 design with n = 3

		Treat. Comb.	Dep. Var.
Group_1	Subject_1	$a_1b_1c_1$	Y_{1111}
	Subject_2	$a_1b_1c_1$	Y_{2111}
	Subject_3	$a_1b_1c_1$	Y_{3111}
			$\bar{Y}_{.111}$
Group_2	Subject_1	$a_1b_1c_2$	Y_{1112}
	Subject_2	$a_1b_1c_2$	Y_{2112}
	Subject_3	$a_1b_1c_2$	Y_{3112}
			$\bar{Y}_{.112}$
Group_3	Subject_1	$a_1b_2c_1$	Y_{1121}
	Subject_2	$a_1b_2c_1$	Y_{2121}
	Subject_3	$a_1b_2c_1$	Y_{3121}
			$\bar{Y}_{.121}$
	⋮	⋮	⋮
Group_8	Subject_1	$a_2b_2c_2$	Y_{1222}
	Subject_2	$a_2b_2c_2$	Y_{2222}
	Subject_3	$a_2b_2c_2$	Y_{3222}
			$\bar{Y}_{.222}$

h. LS-3 design with n = 3

Group	Subject	Treat. Comb.	Dep. Var.
$Group_1$	$Subject_1$	$a_1b_1c_1$	Y_{1111}
	$Subject_2$	$a_1b_1c_1$	Y_{2111}
	$Subject_3$	$a_1b_1c_1$	Y_{3111}
			$\bar{Y}_{.111}$
$Group_2$	$Subject_1$	$a_1b_2c_3$	Y_{1123}
	$Subject_2$	$a_1b_2c_3$	Y_{2123}
	$Subject_3$	$a_1b_2c_3$	Y_{3123}
			$\bar{Y}_{.123}$
$Group_3$	$Subject_1$	$a_1b_3c_2$	Y_{1132}
	$Subject_2$	$a_1b_3c_2$	Y_{2132}
	$Subject_3$	$a_1b_3c_2$	Y_{3132}
			$\bar{Y}_{.132}$
$Group_4$	$Subject_1$	$a_2b_1c_2$	Y_{1212}
	$Subject_2$	$a_2b_1c_2$	Y_{2212}
	$Subject_3$	$a_2b_1c_2$	Y_{3212}
			$\bar{Y}_{.212}$
	⋮	⋮	⋮
$Group_9$	$Subject_1$	$a_3b_3c_1$	Y_{1331}
	$Subject_2$	$a_3b_3c_1$	Y_{2331}
	$Subject_3$	$a_3b_3c_1$	Y_{3331}
			$\bar{Y}_{.331}$

8. d. RB-3 design with $n = 14$

	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.	
Block_1	a_1	Y_{11}	a_2	Y_{12}	a_3	Y_{13}	$\bar{Y}_{1\cdot}$
Block_2	a_1	Y_{21}	a_2	Y_{22}	a_3	Y_{23}	$\bar{Y}_{2\cdot}$
Block_3	a_1	Y_{31}	a_2	Y_{32}	a_3	Y_{33}	$\bar{Y}_{3\cdot}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Block_{14}	a_1	$Y_{14,1}$	a_2	$Y_{14,2}$	a_3	$Y_{14,3}$	$\bar{Y}_{14\cdot}$
		$\bar{Y}_{\cdot 1}$		$\bar{Y}_{\cdot 2}$		$\bar{Y}_{\cdot 3}$	

e. t test for dependent samples with n_1 and $n_2 = 50$

	Treat. Level	Dep. Var.	Treat. Level	Dep. Var.
Block_1	a_1	Y_{11}	a_2	Y_{12}
Block_2	a_1	Y_{21}	a_2	Y_{22}
Block_3	a_1	Y_{31}	a_2	Y_{32}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Block_{50}	a_1	$Y_{50,1}$	a_2	$Y_{50,2}$
		$\bar{Y}_{\cdot 1}$		$\bar{Y}_{\cdot 2}$

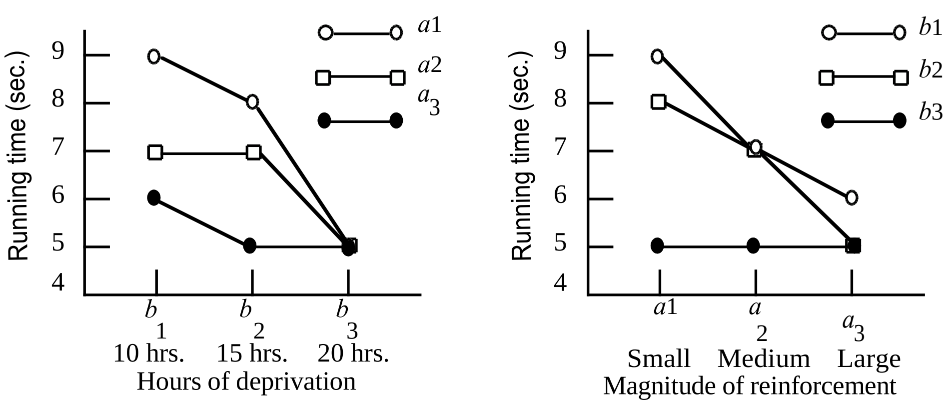
f. CRF-62 design with $n = 50$

		Treat. Comb.	Dep. Var.
Group$_1$	Subject$_1$	a_1b_1	Y_{111}
	⋮	⋮	⋮
	Subject$_{50}$	a_1b_1	$Y_{50,\,11}$
			$\bar{Y}_{.11}$
Group$_2$	Subject$_1$	a_1b_2	Y_{112}
	⋮	⋮	⋮
	Subject$_{50}$	a_1b_2	$Y_{50,\,12}$
			$\bar{Y}_{.12}$
Group$_3$	Subject$_1$	a_2b_1	Y_{121}
	⋮	⋮	⋮
	Subject$_{50}$	a_2b_1	$Y_{50,\,21}$
			$\bar{Y}_{.21}$
Group$_4$	Subject$_1$	a_2b_2	Y_{122}
	⋮	⋮	⋮
	Subject$_{50}$	a_2b_2	$Y_{50,\,22}$
			$\bar{Y}_{.22}$
	⋮	⋮	⋮
Group$_{12}$	Subject$_1$	a_6b_2	Y_{162}
	⋮	⋮	⋮
	Subject$_{50}$	a_6b_2	$Y_{50,\,62}$
			$\bar{Y}_{.62}$

g. CR-3 design with $n = 30$

		Treat. Level	Dep. Var.
Group_1	Animal_1	a_1	Y_{11}
	⋮	⋮	⋮
	Animal_{30}	a_1	$Y_{30,1}$
			$\bar{Y}_{.1}$
Group_2	Animal_1	a_2	Y_{12}
	⋮	⋮	⋮
	Animal_{30}	a_2	$Y_{30,2}$
			$\bar{Y}_{.2}$
Group_3	Animal_1	a_3	Y_{13}
	⋮	⋮	⋮
	Animal_{30}	a_3	$Y_{30,3}$
			$\bar{Y}_{.3}$

9. a.



b. As the number of hours of deprivation increases, the difference in running time among the three reinforcement conditions decreases.

12. a. A scientific hypothesis is a testable supposition that is tentatively adopted to account for certain facts and to guide in the investigation of others. A statistical hypothesis is a statement about one or more parameters of a population or the functional form of a population.

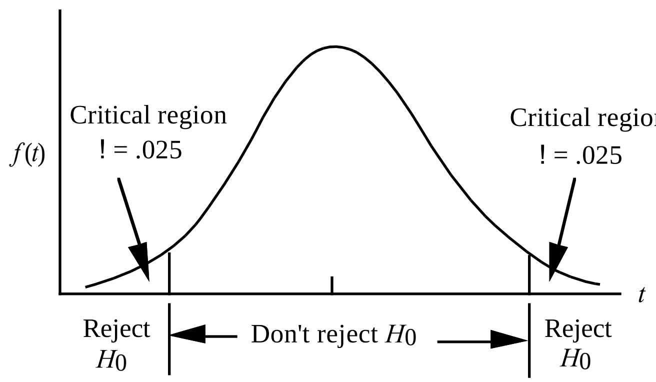
b. (i) Alternative hypothesis (ii) Null hypothesis

15. a. State the null and alternative hypotheses—H_0: $\mu = 45$, H_1: $\mu \neq 45$. Specify the test statistic—$t = (\bar{Y} - \mu_0) / (\hat{\sigma} / \sqrt{n})$. Specify the sample size—$n = 27$, and the sampling distribution—t distribution. Specify the level of significance—$\alpha = .05$. Obtain random

samples of size $n = 27$, compute t, and make a decision.

b. Reject the null hypothesis if t falls in either the lower or upper 2.5% of the sampling distribution of t; otherwise, do not reject the null hypothesis. If the null hypothesis is rejected, conclude that the mean for children in the experimental program is not equal to the mean for ninth-graders who have been observed during the past several years; if the null hypothesis is not rejected, do not draw this conclusion.

c.



d. $$t = \frac{\bar{Y} - \mu_0}{\sigma/\sqrt{n}} = \frac{52.5 - 45.0}{15/\sqrt{27}} = \frac{7.5}{2.89} = 2.60,\ p = .015.$$

The population mean for children in the experimental program was not equal to the mean for ninth-graders who have been observed during the past several years. The difference between children who did or did not participate in the experimental program, 52.5 versus 45.0, was statistically significant, $t(26) = 2.60$, $p = .015$.

e. $d = |52.5 - 45| / 15 = 0.5$; this is a medium size effect.

f. $$\bar{Y} - t_{.05/2,\,26}\frac{\sigma}{\sqrt{n}} < \mu < \bar{Y} + t_{.05/2,\,26}\frac{\sigma}{\sqrt{n}}$$

$$52.5 - \frac{2.056(15)}{\sqrt{27}} < \mu < 52.5 + \frac{2.056(15)}{\sqrt{27}}$$

$$46.6 < \mu < 58.4$$

The researcher can be 95% confident that μ is greater than 46.6 or less than 58.4. The null hypothesis is not tenable.

g. $$\bar{Y}_{.05} = \mu_0 + \frac{t_{.05/2,\,26}\,\sigma}{\sqrt{n}} = 45 + \frac{2.056(15)}{\sqrt{27}} = 50.935$$

$$t = \frac{\bar{Y}_{.05} - \mu}{\sigma/\sqrt{n}} = \frac{50.935 - 52.5}{15/\sqrt{27}} = -\frac{1.565}{2.887} = -0.54$$

TDIS(0.54,26,1) =.30 = $\hat{\beta}$; $1 - \hat{\beta}$ =.70

h.

		True Situation	
		$\mu = 45$	$\mu_1 = 52.5$
Researcher's Decision	$\mu = 45$	Correct acceptance $1 - \alpha = 1 - .05 = .95$	Type II error $\hat{\beta} = .30$
	$\mu \neq 45$	Type I error $\alpha = .05$	Correct rejection $1 - \hat{\beta} = 1 - .30 = .70$

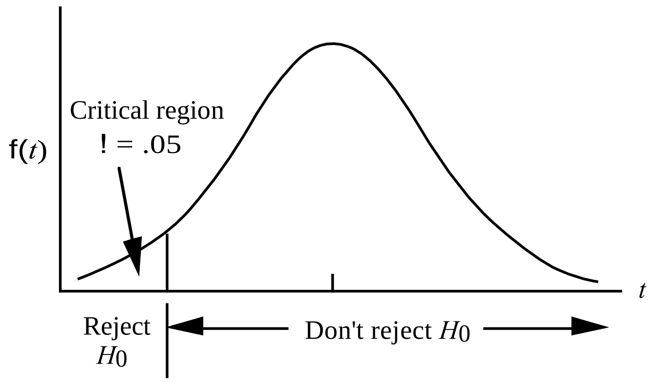
16. c. Correct rejection d. Correct acceptance
e. Correct rejection f. Type I error

17. The power of an experiment can be increased by (1) adopting a lower level of significance, (2) increasing the size of the sample, (3) refining the experimental methodology so as to decrease the size of the population standard deviation, and (4) increasing the magnitude of the treatment effects considered worth detecting. Increasing the sample size is often the simplest way to increase power. The other ways of increasing power may lead to problems or may not be feasible. For example, the adoption of $\alpha > .05$ may preclude the publication of the research. Refining the experimental methodology so as to decrease the size of the population standard deviation may be prohibitively expensive. Increasing the magnitude of the treatment effects considered worth detecting may not be appropriate.

18. a. State the null and alternative hypotheses—H_0: $\mu_1 - \mu_2 \leq 0$, H_1: $\mu_1 - \mu_2 > 0$. Specify the test statistic—$t = (\bar{Y}_{.1} - \bar{Y}_{.2}) / \sqrt{\hat{\sigma}^2_{\text{Pooled}}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$. Specify the sample size—$n_1 = 24$, $n_2 = 23$, and the sampling distribution—t distribution. Specify the level of significance—$\alpha = .05$. Randomly assign $N = 47$ subjects to the two game types, compute t, and make a decision.

b. Reject the null hypothesis if t falls in the upper 5% of the sampling distribution of t; otherwise, do not reject the null hypothesis. If the null hypothesis is rejected, conclude that the risk-related cognitions of men who play racing video games is higher than that for the men who play the neutral games; if the null hypothesis is not rejected, do not draw this conclusion.

c.



d. $\hat{\sigma}^2_{Pooled} = \dfrac{(n_1-1)\hat{\sigma}^2_1 + (n_2-1)\hat{\sigma}^2_2}{(n_1-1)+(n_2-1)} = \dfrac{(24-1)(1.3)^2 + (23-1)(1.2)^2}{(24-1)+(23-1)} = 1.568$

$t = (\bar{Y}_{.1} - \bar{Y}_{.2}) \Big/ \sqrt{\hat{\sigma}^2_{Pooled}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)} = (7.54 - 6.41) \Big/ \sqrt{1.568\left(\dfrac{1}{24} + \dfrac{1}{23}\right)}$

$= 1.13 / 0.365 = 3.09$. The p value is less than .002.

The mean risk-related cognitions for men who played the racing video games was higher than that for the men who played the neutral games. The difference between the means, 7.54 versus 6.41, was statistically significant, $t(45) = 3.09$, $p < .002$.

e. $g = \left|\bar{Y}_{.1} - \bar{Y}_{.2}\right| / \hat{\sigma}_{Pooled} = \left|7.54 - 6.41\right| / 1.25 = 0.90$; this is a large effect.

f. $(\bar{Y}_{.1} - \bar{Y}_{.2}) - t_{.05,45}\sqrt{\hat{\sigma}^2_{Pooled}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)} < \mu_1 - \mu_2$