Instructor's Manual

Essential Mathematics for Economic Analysis

Fifth edition

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Preface

This Instructor's Manual accompanies *Essential Mathematics for Economic Analysis*, 5th Edition. Its main purpose is to provide instructors with a collection of problems that might be used for tutorials and exams. It supplements the problems in MyMathLab.

Most of the problems are taken from previous exams and problem sets at the Department of Economics, University of Oslo, and at Stanford University. We have endeavoured to select problems of varying difficulty, including some problems that might challenge even the best students. The number in parentheses after each problem indicates the appropriate section of the text that should be covered before attempting the (whole) problem.

For each chapter we offer some comments on the text. Sometimes we explain why certain topics are included and others are excluded. There are also occasional hints based on our experience of teaching the material. In some cases, we also comment on alternative approaches, sometimes with mild criticism of other ways of dealing with the material that we believe to be less suitable.

Chapters 2 and 3 in the main text review elementary algebra. This manual includes a Test 1 (page 213), designed for the students themselves to see if they need to review particular sections of Chapters 2 and 3. Many students using our text will probably have some background in calculus. The accompanying Test 2 (page 216) is designed to give information to both the students and the instructors about what students actually know about single variable calculus, and about what needs to be studied more closely, perhaps in Chapters 6–9 of the text.

For instructors who are unwilling to spend more than 5–10 minutes for a test of essentials, we have made Test 0 (page 211). Based on our experience, some instructors might be in for a shock if this test is given to students who have been away from mathematics for some time, even if they have an acceptable mathematical background.

As with the main text, we would like to acknowledge the extensive help from Cristina Maria Igreja in converting the original plain TEX files for this manual into LATEX.

Oslo and Coventry, April 2016

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CHAPTER 1

Essentials of Logic and Set Theory

Section 1.2 offers a very brief introduction to some key concepts in logic, and Section 1.3 attempts to give ambitious students a short discussion of proofs. Set theory, treated in Section 1.1, is in our opinion, not crucial for economics students, except when the need for it arises in their statistics courses.

Problem 1-01 (1.1)

In a group of 100 students, 25 study economics, 30 study political science, and 5 study both subjects. How many students study neither economics nor political science?

Problem 1-02 (1.1)

Given the sets $A = \{2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 7\}$, and $C = \{1, 3, 6, 7\}$, which of the following statements are true?

(a) $2 \in A \cap B$ (b) $(A \cup B) \cap C = \{1, 3, 7\}$ (c) $(A \setminus B) \cap C = \{2\}$ (d) $A \cap C \subseteq B$

Problem 1-03 (1.1)

Let A, B, and C be three sets. Which of the following statements are true? (Use Venn diagrams.)

(a) $A \cap B = A \cap C$ and $A \neq \emptyset \Rightarrow B = C$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

(d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B$

Problem 1-04 (1.2)

Which of the following statements are true?

(a) $x^3 + y^3 = 0 \Leftrightarrow x = y = 0$

(b) $x^2(1 + x) > 0 \Leftrightarrow x > -1$ and $x \neq 0$

(c) $x = \sqrt{16} \Rightarrow x^2 = 16$

(d) $x = 3$ and $y = 5 \Rightarrow 2x + 4y = 26$

Problem 1-05 (1.2)

Which of the following implications can be reversed?

(a) $x = 3 \Rightarrow x^3 = 27$

(b) $x = 0 \Rightarrow x(x^4 + 1) = 0$

(c) $x \geq 3 \Rightarrow (x + 2)^2(x - 3) \geq 0$

(d) $x = 3 \Rightarrow \sqrt{1 + x} = 5 - x$

Problem 1-06 (1.2)

Consider the statement: "A matrix can have an inverse only if its determinant is not 0." Which of the following statements express the same? (You do not need to know the meaning of the concepts.)

(a) A sufficient condition for a matrix to have an inverse is that its determinant is not 0.

(b) A matrix with determinant equal to 0 has no inverse.

(c) A necessary condition for a matrix to have an inverse is that its determinant is not 0.

Problem 1-07 (1.3)

Prove that $\sqrt{2}+3$ is irrational.

Problem 1-08 (Harder problem.) (1.3)

Let a and b be positive rational numbers. Prove that if $\sqrt{a}+\sqrt{b}$ is rational, so is $\sqrt{a}$.

Problem 1-09 (1.4)

Prove by induction that for all natural numbers $n \geq 3$,

$$2n+1 < 2^n \tag{$*$}$$

Problem 1-10 (1.4)

Prove by induction that the following equations hold for all natural numbers n.

(a) $1\cdot 2+2\cdot 3+\cdots+n(n+1)=\dfrac{1}{3}n(n+1)(n+2)$

(b) $1^3+2^3+\cdots+n^3=\dfrac{1}{4}n^2(n+1)^2$

Problem 1-11

Let n be a positive integer and consider the expression $s_n = n^2 - n + 41$. Verify that s_n is a prime number (and so has no factor except 1 and itself) for n = 1, 2, 3, 4, and 5. With some effort, one can prove that s_n is a prime number for n = 6, 7, …, 40 as well. Is s_n a prime for all n? (This problem was first suggested by the Swiss mathematician L. Euler.)