2.1

Situation: A system is separated from its surrounding by a

a. border

b. boundary

c. dashed line

d. dividing surface

SOLUTION

Answer is (b) boundary.  See definition in §2.1.

$\boxed{2.2}$

<u>Find</u>:

How are density and specific weight related?

$\boxed{\text{PLAN}}$

Consider their definitions (conceptual and mathematical)

$\boxed{\text{SOLUTION}}$

Density is a [mass]/[unit volume], and specific weight is a [weight]/[unit volume]. Therefore, they are related $\boxed{\text{by the equation } \gamma = \rho g}$, and density differs from specific weight by the factor $g$, the acceleration of gravity.

## 2.3

Density is (select all that apply)

a. weight/volume

b. mass/volume

c. volume/mass

d. mass/weight

## SOLUTION

Answer is (b) mass/volume.

Situation:

Which of these are units of density? (Select all that apply.)

a. $\text{kg/m}^3$

b. $\text{mg/cm}^3$

c. $\text{lbm/ft}^3$

d. $\text{slug/ft}^3$

SOLUTION

Correct answers are a, b, c, and d. Each of these is a mass/volume.

$\boxed{2.5}$

<u>Situation</u>:
Specific gravity (select all that apply)
a. can have units of $N/m^3$
b. is dimensionless
c. increases with temperature
d. decreases with temperature

$\boxed{\text{SOLUTION}}$

Correct answers are b and d.  Specific gravity is a ratio of the density of some liquid divided by the density of water at $4\,°C$. Therefore it is dimensionless.  As temperature goes up, the density of the numerator liquid decreases, but the denominator stays the same.  Therefore the SG decreases as temperature increases.  See Table 2.2 in §2.11 of EFM10e.

2.6

Situation:

If a liquid has a specific gravity of 1.7,
a) What is the density in slugs per cubic feet?
b) What is the specific weight in lbf per cubic feet?

SOLUTION

$$SG = 1.7$$
$$SG = \frac{\rho_l}{\rho_{water,4C}}$$

a)

$$\rho_l = 1.7 \left(1.94 \frac{\text{slug}}{\text{ft}^3}\right)$$
$$= \boxed{3.3 \frac{\text{slug}}{\text{ft}^3}}$$

b)

$$32.17\,\text{lbm} = 1\,\text{slug}$$
Therefore

$$\left(\frac{3.3\,\text{slug}}{\text{ft}^3}\right)\left(\frac{32.17\,\text{lbm}}{1\,\text{slug}}\right) = \boxed{106 \frac{\text{lbf}}{\text{ft}^3}}$$

6

<u>Situation</u>:

What are SG, $\gamma$, and $\rho$ for mercury?

State you answers in SI units and in traditional units.

SOLUTION

From table A.4 (EFM10e)

|     | SI | Traditional |
| --- | --- | --- |
| SG | 13.55 | 13.55 |
| $\gamma$ | 133,000 N/m$^3$ | 847 lbf/ft$^3$ |
| $\rho$ | 13,550 kg/m$^3$ | 26.3 slug/ft$^3$ |

Situation:

   If a gas has $\gamma = 15$ N/m$^3$ what is its density?

   State your answers in SI units and in traditional units.

SOLUTION

Density and specific seight are related according to

$$
\begin{aligned}
\gamma &= \frac{\rho}{g} \\
\text{So } \rho &= \frac{\gamma}{g} \\
\text{For } \gamma &= 15\frac{\text{N}}{\text{m}^3} \\
\text{In SI } \rho &= \left(\frac{15\,\text{N}}{\text{m}^3}\right)\left(\frac{1\,\text{s}^2}{9.81\,\text{m}}\right) \\
&= \boxed{1.53\frac{\text{kg}}{\text{m}^3}}
\end{aligned}
$$

Converting to traditional units

$$
\begin{aligned}
&= \left(\frac{1.53\,\text{kg}}{\text{m}^3}\right)\left(\frac{1\,\text{m}^3}{(3.281^3)\,\text{ft}^3}\right)\left(\frac{1\,\text{slug}}{14.59\,\text{kg}}\right) \\
&= \boxed{3.0 \times 10^{-3}\frac{\text{slug}}{\text{ft}^3}}
\end{aligned}
$$

2.9

If you have a bulk modulus of elasticity that is a very large number, then a small change in pressure would cause

a. a very large change in volume

b. a very small change in volume

SOLUTION

Examination of Eq. 2.6 in EFM10e shows that the answer is b.

Situation:

Dimensions of the bulk modulus of elasticity are

a. the same as the dimensions of pressure/density

b. the same as the dimensions of pressure/volume

c. the same as the dimensions of pressure

SOLUTION

Examination of Eq. 2.6 in EFM10e shows that the answer is c. The volume units in the denominator cancel, and the remaining units are pressure.

**2.11**

Situation:

   Elasticity of ethyl alcohol and water.

   $E_{ethyl} = 1.06 \times 10^9$ Pa.

   $E_{water} = 2.15 \times 10^9$ Pa.

Find:

   Which substance is easier to compress?

   a. ethyl alcohol

   b. water

**PLAN**

Use bulk density equation.

**SOLUTION**

The bulk modulus of elasticity is given by:

$$E = -\Delta p \frac{V}{\Delta V} = \frac{\Delta p}{d\rho/\rho}$$

This means that elasticity is inversely related to change in density, and to the negative change in volume.

Therefore, the liquid with the smaller elasticity is easier to compress.

Correct answer is a.  Ethyl alcohol is easier to compress because it has the smaller elasticity, because elasticity is inversely related to change in density.

Situation:

Pressure is applied to a mass of water.

$V = 2000\, \text{cm}^3$, $p = 2 \times 10^6\, \text{N}/\text{m}^2$.

Find:

Volume after pressure applied $(\text{cm}^3)$.

Properties:

From Table A.5, $E = 2.2 \times 10^9\, \text{Pa}$

PLAN

1. Use modulus of elasticity equation to calculate volume change resulting from pressure change.
2. Calculate final volume based on original volume and volume change.

SOLUTION

1. Elasticity equation

$$
\begin{aligned}
E &= -\Delta p \frac{V}{\Delta V} \\
\Delta V &= -\frac{\Delta p}{E} V \\
&= -\left[ \frac{(2 \times 10^6)\, \text{Pa}}{(2.2 \times 10^9)\, \text{Pa}} \right] 2000\, \text{cm}^3 \\
&= -1.82\, \text{cm}^3
\end{aligned}
$$

2. Final volume

$$
\begin{aligned}
V_{final} &= V + \Delta V \\
&= (2000 - 1.82)\, \text{cm}^3
\end{aligned}
$$

$$\boxed{V_{final} = 1998\, \text{cm}^3}$$

Situation:

Water is subjected to an increase in pressure.

Find:

Pressure increase needed to reduce volume by 2%.

Properties:

From Table A.5, $E = 2.2 \times 10^9$ Pa.

**PLAN**

Use modulus of elasticity equation to calculate pressure change required to achieve the desired volume change.

**SOLUTION** Modulus of elasticity equation

$$
\begin{aligned}
E &= -\Delta p \frac{\cancel{V}}{\Delta \cancel{V}} \\
\Delta p &= E \frac{\Delta \cancel{V}}{\cancel{V}} \\
&= -\left(2.2 \times 10^9 \,\text{Pa}\right) \left(\frac{-0.02 \times \cancel{V}}{\cancel{V}}\right) \\
&= \left(2.2 \times 10^9 \,\text{Pa}\right) (0.02) \\
&= 4.4 \times 10^7 \,\text{Pa}
\end{aligned}
$$

$$\boxed{\Delta p = 44 \,\text{MPa}}$$

Situation:

Open tank of water.

$T_{20} = 20\,°\mathrm{C}$, $T_{80} = 80\,°\mathrm{C}$.

$V = 400\,\mathrm{l}$, $d = 3\,\mathrm{m}$.

Hint: Volume change is due to temperature.

Find:

Percentage change in volume.

Water level rise for given diameter.

Properties:

From Table A.5: $\rho_{20} = 998\,\frac{\mathrm{kg}}{\mathrm{m}^3}$, and $\rho_{80} = 972\,\frac{\mathrm{kg}}{\mathrm{m}^3}$.

## PLAN

This problem is NOT solved using the elasticity equation, because the volume change results from a change in temperature causing a density change, NOT a change in pressure. The tank is open, so the pressure at the surface of the tank is always atmospheric.

## SOLUTION

a. Percentage change in volume must be calculated for this mass of water at two temperatures.

For the first temperature, the volume is given as $V_{20} = 400\ \mathrm{L} = 0.4\,\mathrm{m}^3$. Its density is $\rho_{20} = 998\,\frac{\mathrm{kg}}{\mathrm{m}^3}$. Therefore, the mass for both cases is given by.

$$
\begin{aligned}
m &= 998\frac{\mathrm{kg}}{\mathrm{m}^3} \times 0.4\,\mathrm{m}^3 \\
&= 399.2\,\mathrm{kg}
\end{aligned}
$$

For the second temperature, that mass takes up a larger volume:

$$
\begin{aligned}
V_{80} &= \frac{m}{\rho} = \frac{399.2\,\mathrm{kg}}{972\frac{\mathrm{kg}}{\mathrm{m}^3}} \\
&= 0.411\,\mathrm{m}^3
\end{aligned}
$$

Therefore, the percentage change in volume is

$$
\begin{aligned}
\frac{0.411\,\mathrm{m}^3 - 0.4\,\mathrm{m}^3}{0.4\,\mathrm{m}^3} &= 0.0275 \\
\text{volume \% change} &= \boxed{= 2.8\%}
\end{aligned}
$$

b. If the tank has $D = 3\,\mathrm{m}$, then $V = \pi r^2 h = 7.07h$. Therefore:

$$h_{20} \quad = \quad .057\,\mathrm{m}$$
$$h_{80} \quad = \quad .058\,\mathrm{m}$$

And water level rise is $0.0581 - 0.0566\,\mathrm{m} = 0.0015\,\mathrm{m} = 2\,\mathrm{mm}$.

$\boxed{\text{water level rise is} = 0.002\,\mathrm{m} = 2\,\mathrm{mm}}$

$\boxed{\text{REVIEW}}$

Density changes can result from temperature changes, as well as pressure changes.

Find:

Where in this text can you find:

a. density data for such liquids as oil and mercury?

b. specific weight data for air (at standard atmospheric pressure) at different temperatures?

c. specific gravity data for sea water and kerosene?

SOLUTION

a. Density data for liquids other than water can be found in Table A.4 in EFM10e. Temperatures are specified.

b. Data for several properties of air (at standard atmospheric pressure) at different temperatures are in Table A.3 in EFM10e.

c. Specific gravity and other data for liquids other than water can be found in Table A.4 in EFM10e. Temperatures are specified.

$\boxed{2.16}$

Regarding water and seawater:

    a. Which is more dense, seawater or freshwater?

    b. Find (SI units) the density of seawater (10°C, 3.3% salinity).

    c. Find the same in traditional units.

    d. What pressure is specified for the values in (b) and (c)?

$\boxed{\text{SOLUTION}}$

a. Seawater is more dense, because of the weight of the dissolved salt.

b. The density of seawater (10°C, 3.3% salinity) in SI units is 1026 kg/m$^3$, see Table A.3 in EFM10e.

c. The density of seawater (10°C, 3.3% salinity) in traditional units is 1.99 slugs/ft$^3$, see Table A.3 in EFM10e.

d. The specified pressure for the values in (b) and (c) is standard atmospheric pressure; as stated in the title of Table A.3 in EFM10e.

Situation:

If the density, $\rho$, of air (in an open system at atmospheric pressure) increases by a factor of 1.4x due to a temperature change,

a. specific weight increases by 1.4x

b. specific weight increases by 13.7x

c. specific weight remains the same

SOLUTION

Since specific weight is the product $\rho g$, if $\rho$ increases by a factor of 1.4, then specific weight increases by 1.4 times as well. The answer is (a).

2.18

Situation:

The following questions relate to viscosity.

Find:

(a) The primary dimensions of viscosity and five common units of viscosity.

(b) The viscosity of motor oil (in traditional units).

SOLUTION

a) Primary dimensions of viscosity are $\left[\frac{M}{LT}\right]$.

Five common units are:

i) $\frac{N \cdot s}{m^2}$; ii) $\frac{dyn \cdot s}{cm^2}$; iii) poise; iv) centipoise; and v) $\frac{lbf \cdot s}{ft^2}$

(b) To find the viscosity of SAE 10W-30 motor oil at 115 °F, there are no tabular data in the text. Therefore, one should use Figure A.2. For traditional units (because the temperature is given in Farenheit) one uses the left-hand axis to report that

$\mu = 1.2 \times 10^{-3} \frac{lbf \cdot s}{ft^2}$.

Note: one should be careful to identify the correct factor of 10 for the log cycle that contains the correct data point. For example, in this problem, the answer is between $1 \times 10^{-3}$ and $1 \times 10^{-2}$. Therefore the answer is $1.2 \times 10^{-3}$ and not $1 \times 10^{-2}$.

## 2.19

Situation:
Shear stress has dimensions of
a. force/area
b. dimensionless

## SOLUTION

The answer is (a).  See Eq. 2.10 in EFM10e, and discussion.

$\boxed{2.20}$

The term $dV/dy$, the velocity gradient
a. has dimensions of L/T, and represents shear strain
b. has dimensions of $T^{-1}$, and represents the rate of shear strain

$\boxed{\text{SOLUTION}}$

The answer is (b). See Eq. 2.14 in EFM10e, and discussion.

Situation:

For the velocity gradient $dV/dy$

a. The change in velocity $dV$ is in the direction of flow

b. The change in velocity $dV$ is perpendicular to flow

SOLUTION

The answer is (b). See Fig. 2.9 in EFM10e, and related preceding and following discussion.

## 2.22

<u>Situation:</u>

The no-slip condition

a. only applies to ideal flow

b. only applies to rough surfaces

c. means velocity, $V$, is zero at the wall

d. means velocity, $V$, is the velocity of the wall

## SOLUTION

The answer is (d); velocity, $V$, is the velocity of the wall.

Kinematic viscosity (select all that apply)

a. is another name for absolute viscosity

b. is viscosity/density

c. is dimensionless because forces are canceled out

d. has dimensions of $L^2/T$

SOLUTION

The answers are (b) and (d).

2.24

Situation:

Change in viscosity and density due to temperature.

$T_1 = 10\,°\mathrm{C}$, $T_2 = 70\,°\mathrm{C}$.

Find:

Change in viscosity and density of water.

Change in viscosity and density of air.

Properties:

$p = 101\,\mathrm{kN/m^2}$.

**PLAN**

For water, use data from Table A.5.   For air, use data from Table A.3

**SOLUTION**

Water

$$\mu_{70} = 4.04 \times 10^{-4}\ \mathrm{N \cdot s/m^2}$$
$$\mu_{10} = 1.31 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$$
$$\boxed{\Delta\mu = -9.06 \times 10^{-4}\ \mathrm{N \cdot s/m^2}}$$

$$\rho_{70} = 978\ \mathrm{kg/m^3}$$
$$\rho_{10} = 1000\ \mathrm{kg/m^3}$$
$$\boxed{\Delta\rho = -22\ \mathrm{kg/m^3}}$$

Air

$$\mu_{70} = 2.04 \times 10^{-5}\ \mathrm{N \cdot s/m^2}$$
$$\mu_{10} = 1.76 \times 10^{-5}\ \mathrm{N \cdot s/m^2}$$
$$\boxed{\Delta\mu = 2.8 \times 10^{-6}\ \mathrm{N \cdot s/m^2}}$$

$$\rho_{70} = 1.03\ \mathrm{kg/m^3}$$
$$\rho_{10} = 1.25\ \mathrm{kg/m^3}$$
$$\boxed{\Delta\rho = -0.22\ \mathrm{kg/m^3}}$$

Situation:

    Air at certain temperatures.

    $T_1 = 10\,°\mathrm{C}$, $T_2 = 70\,°\mathrm{C}$.

Find:

    Change in kinematic viscosity.

Properties:

    From Table A.3, $\nu_{70} = 1.99 \times 10^{-5}$ m$^2$/s, $\nu_{10} = 1.41 \times 10^{-5}$ m$^2$/s.

PLAN

Use properties found in Table A.3.

SOLUTION

$$
\Delta\nu_{\mathrm{air},10\to70} \quad = \quad (1.99 - 1.41) \times 10^{-5}
$$

$$
\boxed{\Delta\mathrm{v}_{\mathrm{air},10\to70} = 5.8 \times 10^{-6}\mathrm{m}^2/\mathrm{s}}
$$

REVIEW

Sutherland's equation could also be used to solve this problem.

Situation:

Viscosity of SAE 10W-30 oil, kerosene and water.

$T = 38\,^{\circ}\mathrm{C} = 100\,^{\circ}\mathrm{F}.$

Find:

Dynamic and kinematic viscosity of each fluid.

PLAN

Use property data found in Table A.4, Fig. A.2 and Table A.5.

SOLUTION

| | Oil (SAE 10W-30) | kerosene | | water |
|---|---|---|---|---|
| $\mu(\mathrm{N}\cdot\mathrm{s/m}^2)$ | $6.7\times10^{-2}$ | $1.4\times10^{-3}$ | (Fig. A-2) | $6.8\times10^{-4}$ |
| $\rho(\mathrm{kg/m}^3)$ | 880 | 814 | | 993 |
| $\nu(\mathrm{m}^2/\mathrm{s})$ | $7.6\times10^{-5}$ | $1.7\times10^{-6}$ | (Fig. A-2) | $6.8\times10^{-7}$ |

Situation:

Comparing properties of air and water.

Find:

Ratio of dynamic viscosity of air to that of water.

Ratio of kinematic viscosity of air to that of water.

Properties:

Air ($20\,^\circ$C, 1 atm), Table A.3, $\mu = 1.81 \times 10^{-5}$ N·s/m$^2$; $\nu = 1.51 \times 10^{-5}$ m$^2$/s

Water ($20\,^\circ$C, 1 atm), Table A.5, $\mu = 1.00 \times 10^{-3}$ N·s/m$^2$; $\nu = 1.00 \times 10^{-6}$ m$^2$/s

## SOLUTION

Dynamic viscosity

$$\frac{\mu_{\text{air}}}{\mu_{\text{water}}} = \frac{1.81 \times 10^{-5}\,\text{N} \cdot \text{s/m}^2}{1.00 \times 10^{-3}\,\text{N} \cdot \text{s/m}^2}$$

$$\boxed{\frac{\mu_{\text{air}}}{\mu_{\text{water}}} = 1.81 \times 10^{-2}}$$

Kinematic viscosity

$$\frac{\nu_{\text{air}}}{\nu_{\text{water}}} = \frac{1.51 \times 10^{-5}\,\text{m}^2/\text{s}}{1.00 \times 10^{-6}\,\text{m}^2/\text{s}}$$

$$\boxed{\frac{\nu_{\text{air}}}{\nu_{\text{water}}} = 15.1}$$

## REVIEW

1. Water at these conditions (liquid) is about 55 times more viscous than air (gas).

2. However, the corresponding kinematic viscosity of air is 15 times higher than the kinematic viscosity of water. The reason is that kinematic viscosity includes density and $\rho_{\text{air}} \ll \rho_{\text{water}}$.

3. Remember that

   (a) kinematic viscosity ($\nu$) is related to dynamic viscosity ($\mu$) by: $\nu = \mu/\rho$.

   (b) the labels "viscosity," "dynamics viscosity," and "absolute viscosity" are synonyms.

Situation:

At a point in a flowing fluid, the shear stress is $1 \times 10^{-4}$ psi, and the velocity gradient is $1 \text{ s}^{-1}$.

Find:

    a. What is the viscosity in traditional units?
    b. Convert this viscosity to SI units.
    c. Is this fluid more, or less, viscous than water?

**SOLUTION**

a.

$$\tau = \mu \frac{dV}{dy}$$

$$\mu = \frac{\tau}{dV/dy} = \left( \frac{1 \times 10^{-4} \, \text{lbf}}{\text{in}^2} \right) \left( \frac{\text{s}}{1} \right)$$

$$= \boxed{1 \times 10^{-4} \frac{\text{lbf} \cdot \text{s}}{\text{in}^2}}$$

b. Convert to SI units, using grid method

$$\mu = \left( \frac{1 \times 10^{-4} \, \text{lbf} \cdot \text{s}}{\text{in}^2} \right) \left( \frac{144 \, \text{in}^2}{1 \, \text{ft}^2} \right) \left( \frac{(3.28)^2 \, \text{ft}^2}{1 \, \text{m}^2} \right) \left( \frac{4.448 \, \text{N}}{1 \, \text{lbf}} \right)$$

$$= \boxed{0.689 \frac{\text{N} \cdot \text{s}}{\text{m}^2}}$$

c. The fluid is more viscous than water, based upon a comparison to tabular values for water.

Situation:

SAE 10W30 motor oil is used as a lubricant between two machine parts

$\mu = 1 \times 10^{-4} \, \text{lbf} \cdot \text{s/ft}^2$

$dV = 6 \, \text{ft/s}$

$\tau_{\text{max}} = 2 \, \text{lbf/ft}^2$

Find:

What is the required spacing, in inches?

SOLUTION

1. Use

$$\tau = \mu \frac{dV}{dy}$$

2. Find $dy$

$$
\begin{aligned}
dy &= \frac{\mu \cdot dV}{\tau} \\
&= \left( \frac{1 \times 10^{-4} \, \text{lbf} \cdot \text{s}}{\text{ft}^2} \right) \left( \frac{6 \, \text{ft}}{\text{s}} \right) \left( \frac{\text{ft}^2}{2 \, \text{lbf}} \right) \\
&= (3 \times 10^{-4} \, \text{ft}) \left( \frac{12 \, \text{in}}{1 \, \text{ft}} \right) \\
&= \boxed{3.6 \times 10^{-3} \, \text{in}}
\end{aligned}
$$

The spacing needs to be equal to, or wider than $3.6 \times 10^{-3}$ in in order for the shear stress to be less than 2 lbf/ ft$^2$.

Situation:

Water flows near a wall. The velocity distribution is

$$u(y) = a \left( \frac{y}{b} \right)^{1/6}$$

$a = 10 \, \text{m/s}$, $b = 2 \, \text{mm}$ and $y$ is the distance (mm) from the wall.

Find:

Shear stress in the water at $y = 1$ mm.

Properties:

Table A.5 (water at 20 °C): $\mu = 1.00 \times 10^{-3} \, \text{N} \cdot \text{s/m}^2$.

SOLUTION

Rate of strain (algebraic equation)

$$\begin{aligned}
\frac{du}{dy} &= \frac{d}{dy} \left[ a \left( \frac{y}{b} \right)^{1/6} \right] \\
&= \frac{a}{b^{1/6}} \frac{1}{6y^{5/6}} \\
&= \frac{a}{6b} \left( \frac{b}{y} \right)^{5/6}
\end{aligned}$$

Rate of strain (at $y = 1 \, \text{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \frac{a}{6b} \left( \frac{b}{y} \right)^{5/6} \\
&= \frac{10 \, \text{m/s}}{6 \times 0.002 \, \text{m}} \left( \frac{2 \, \text{mm}}{1 \, \text{mm}} \right)^{5/6} \\
&= 1485 \, \text{s}^{-1}
\end{aligned}$$

Shear Stress

$$\begin{aligned}
\tau_{y=1 \, \text{mm}} &= \mu \frac{du}{dy} \\
&= \left( 1.00 \times 10^{-3} \frac{\text{N} \cdot \text{s}}{\text{m}^2} \right) \left( 1485 \, \text{s}^{-1} \right) \\
&= 1.485 \, \text{Pa}
\end{aligned}$$

$$\boxed{\tau \, (y = 1 \, \text{mm}) = 1.49 \, \text{Pa}}$$

## 2.31

<u>Situation</u>:

Velocity distribution of crude oil between two walls.

$\mu = 8 \times 10^{-5}$ lbf s/ ft$^2$, $B = 0.1$ ft.

$u = 100y(0.1 - y)$ ft/ s, $T = 100\,°$F.

<u>Find</u>:

Shear stress at walls.

## SOLUTION

Velocity distribution

$$u = 100y(0.1 - y) = 10y - 100y^2$$

Rate of strain

$$
\begin{aligned}
du/dy &= 10 - 200y \\
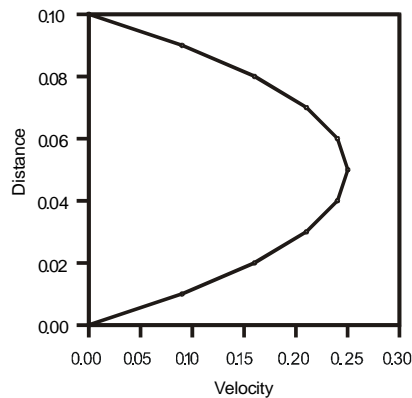(du/dy)_{y=0} &= 10 \text{ s}^{-1} \text{ and } (du/dy)_{y=0.1} = -10 \text{ s}^{-1}
\end{aligned}
$$

Shear stress

$$\tau_0 = \mu\frac{du}{dy} = (8 \times 10^{-5}) \times 10$$

$$\boxed{\tau_0 = 8\times10^{-4} \text{ lbf/ft}^2}$$

$$\boxed{\tau_{0.1} = 8\times10^{-4} \text{ lbf/ft}^2}$$

Plot, where distance is in ft, and velocity is in ft/s.

<u>Situation</u>:

A liquid flows between parallel boundaries.

$y_0 = 0.0\,\text{mm}$, $V_0 = 0.0\,\text{m/s}$.

$y_1 = 1.0\,\text{mm}$, $V_1 = 1.0\,\text{m/s}$.

$y_2 = 2.0\,\text{mm}$, $V_2 = 1.99\,\text{m/s}$.

$y_3 = 3.0\,\text{mm}$, $V_3 = 2.98\,\text{m/s}$.

<u>Find</u>:

(a) Maximum shear stress.

(b) Location where minimum shear stress occurs.

## SOLUTION

(a) Maximum shear stress

$$
\begin{aligned}
\tau &= \mu dV/dy \\
\tau_{\max} &\approx \mu(\Delta V/\Delta y) \text{ next to wall} \\
\tau_{\max} &= (10^{-3}\text{N} \cdot \text{s/m}^2)((1 \text{ m/s})/0.001 \text{ m}) \\
&\boxed{\tau_{\max} = 1.0 \text{ N/m}^2}
\end{aligned}
$$

(b)The $\boxed{\text{minimum shear stress will occur midway between the two walls}}$. Its magnitude will be zero because the velocity gradient is zero at the midpoint.

Situation:

Glycerin is flowing in between two stationary plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

$dp/dx = -1.6\,\mathrm{kPa/\,m}$, $B = 5\,\mathrm{cm}$.

Find:

Velocity and shear stress at a distance of 12 mm from wall (i.e. at $y = 12\,\mathrm{mm}$).
Velocity and shear stress at the wall (i.e. at $y = 0\,\mathrm{mm}$).

Properties:

Glycerin $(20\,^\circ\mathrm{C})$, Table A.4: $\mu = 1.41\,\mathrm{N\cdot s/\,m^2}$.

PLAN

Find velocity by direct substitution into the specified velocity distribution.
Find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative $du/dy$) is found by differentiating the velocity distribution.

SOLUTION

a.) Velocity (at $y = 12\,\mathrm{mm}$)

$$
\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\\
&= -\frac{1}{2\,(1.41\,\mathrm{N\cdot s/\,m^2})}\left(-1600\,\mathrm{N/\,m^3}\right)\left((0.05\,\mathrm{m})\,(0.012\,\mathrm{m}) - (0.012\,\mathrm{m})^2\right)\\
&= 0.258\,7\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

$$\boxed{u\,(y = 12\,\mathrm{mm}) = 0.259\,\mathrm{m/\,s}}$$

Rate of strain (general expression)

$$
\begin{aligned}
\frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right)\\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right)\\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)
\end{aligned}
$$

Rate of strain (at $y = 12\,\mathrm{mm}$)

$$
\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)\\
&= \left(-\frac{1}{2\,(1.41\,\mathrm{N\cdot s/\,m^2})}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m^3}}\right)(0.05\,\mathrm{m} - 2\times 0.012\,\mathrm{m})\\
&= 14.75\,\mathrm{s^{-1}}
\end{aligned}
$$

Definition of viscosity

$$\tau = \mu\frac{du}{dy}$$

$$= \left(1.41\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(14.75\,\text{s}^{-1}\right)$$

$$= 20.\,798\,\text{Pa}$$

$$\boxed{\tau\,(y = 12\,\text{mm}) = 20.8\,\text{Pa}}$$

b.) Velocity (at $y = 0\,\text{mm}$)

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

$$= -\frac{1}{2\,(1.41\,\text{N}\cdot\text{s}/\,\text{m}^2)}\left(-1600\,\text{N}/\,\text{m}^3\right)\left((0.05\,\text{m})\,(0\,\text{m}) - (0\,\text{m})^2\right)$$

$$= 0.00\frac{\text{m}}{\text{s}}$$

$$\boxed{u\,(y = 0\,\text{mm}) = 0\,\text{m}/\,\text{s}}$$

Rate of strain (at $y = 0\,\text{mm}$)

$$\frac{du}{dy} = \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)$$

$$= \left(-\frac{1}{2\,(1.41\,\text{N}\cdot\text{s}/\,\text{m}^2)}\right)\left(-1600\frac{\text{N}}{\text{m}^3}\right)(0.05\,\text{m} - 2\times 0\,\text{m})$$

$$= 28.37\,\text{s}^{-1}$$

Shear stress (at $y = 0\,\text{mm}$)

$$\tau = \mu\frac{du}{dy}$$

$$= \left(1.41\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(28.37\,\text{s}^{-1}\right)$$

$$= 40.00\,\text{Pa}$$

$$\boxed{\tau\,(y = 0\,\text{mm}) = 40.0\,\text{Pa}}$$

$\boxed{\text{REVIEW}}$

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.

2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

**Situation:**
   Oil (SAE 10W30) fills the space between two plates.
   $\Delta y = 1/8 = 0.125\,\text{in}$, $u = 25\,\text{ft}/\text{s}$.

   Lower plate is at rest.

**Find:**
   Shear stress in oil.

**Properties:**
   Oil (SAE 10W30 @ $150\,°\text{F}$) from Figure A.2: $\mu = 5.2 \times 10^{-4}$ lbf·s/ft$^2$.

**Assumptions:**
   1.) Assume oil is a Newtonian fluid.
   2.) Assume Couette flow (linear velocity profile).

**SOLUTION**

Rate of strain

$$
\begin{aligned}
\frac{du}{dy} &= \frac{\Delta u}{\Delta y} \\
&= \frac{25\,\text{ft}/\text{s}}{(0.125/12)\,\text{ft}} \\
\frac{du}{dy} &= 2400\,\text{s}^{-1}
\end{aligned}
$$

Newton's law of viscosity

$$
\begin{aligned}
\tau &= \mu\left(\frac{du}{dy}\right) \\
&= \left(5.2 \times 10^{-4}\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \times \left(2400\frac{1}{\text{s}}\right) \\
&= 1.248\frac{\text{lbf}}{\text{ft}^2}
\end{aligned}
$$

$$
\boxed{\tau = 1.25 \ \tfrac{\text{lbf}}{\text{ft}^2}}
$$

**Situation:**

Sliding plate viscometer is used to measure fluid viscosity.

$A = 50 \times 100 \,\mathrm{mm}$, $\Delta y = 1 \,\mathrm{mm}$.

$u = 10 \,\mathrm{m/s}$, $F = 3 \,\mathrm{N}$.

**Find:**

Viscosity of the fluid.

**Assumptions:**

Linear velocity distribution.

**PLAN**

1. The shear force $\tau$ is a force/area.
2. Use equation for viscosity to relate shear force to the velocity distribution.

**SOLUTION**

1. Calculate shear force

$$
\begin{aligned}
\tau &= \frac{Force}{Area} \\
\tau &= \frac{3 \,\mathrm{N}}{50 \,\mathrm{mm} \times 100 \,\mathrm{mm}} \\
\tau &= 600 \,\mathrm{N/m^2}
\end{aligned}
$$

2. Find viscosity

$$
\begin{aligned}
\mu &= \frac{\tau}{\left(\frac{du}{dy}\right)} \\
\mu &= \frac{600 \,\mathrm{N/m^2}}{[10 \,\mathrm{m/s}]/[1 \,\mathrm{mm}]} \times \frac{1 \,\mathrm{m}}{1000 \,\mathrm{mm}}
\end{aligned}
$$

$$
\boxed{\mu = 6 \times 10^{-2} \, \tfrac{\mathrm{N \cdot s}}{\mathrm{m^2}}}
$$

Situation:

Laminar flow occurs between two horizontal parallel plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{ds}\left(Hy - y^2\right) + u_t\frac{y}{H}$$

Pressure $p$ decreases with distance $s$, and the speed of the upper plate is $u_t$. Note that $u_t$ has a negative value to represent that the upper plate is moving to the left.

Moving plate: $y = H$.

Stationary plate: $y = 0$.

Find:

(a) Whether shear stress is greatest at the moving or stationary plate.

(b) Location of zero shear stress.

(c) Derive an expression for plate speed to make the shear stress zero at $y = 0$.

Sketch:
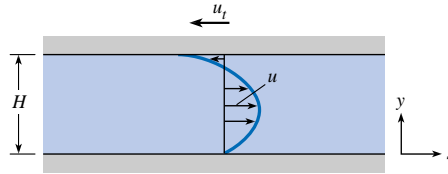


PLAN

By inspection, the rate of strain $(du/dy)$ or slope of the velocity profile is larger at the moving plate. Thus, we expect shear stress $\tau$ to be larger at $y = H$. To check this idea, find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$. Evaluate and compare the shear stress at the locations $y = H$ and $y = 0$.

SOLUTION

Part (a)

1. Shear stress, from definition of viscosity

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \mu\frac{d}{dy}\left[-\frac{1}{2\mu}\frac{dp}{ds}\left(Hy - y^2\right) + u_t\frac{y}{H}\right] \\
&= \mu\left[-\frac{H}{2\mu}\frac{dp}{ds} + \frac{y}{\mu}\frac{dp}{ds} + \frac{u_t}{H}\right] \\
\tau(y) &= -\frac{(H - 2y)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H}
\end{aligned}$$

Shear stress at $y = H$

$$
\begin{aligned}
\tau\,(y = H) &= -\frac{(H - 2H)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\
&= \frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H}
\end{aligned}
\tag{1}
$$

2. Shear stress at $y = 0$

$$
\begin{aligned}
\tau\,(y = 0) &= -\frac{(H - 0)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\
&= -\frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H}
\end{aligned}
\tag{2}
$$

Since pressure decreases with distance, the pressure gradient $dp/ds$ is negative. Since the upper wall moves to the left, $u_t$ is negative. Thus, maximum shear stress occurs at $y = H$ because both terms in Eq. (1) have the same sign (they are both negative.) In other words,

$$
|\tau\,(y = H)| > |\tau\,(y = 0)|
$$

.

Maximum shear stress occur at $y = H$ .

---

Part (b)
Use definition of viscosity to find the location ($y$) of zero shear stress

$$
\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= -\mu(1/2\mu)\frac{dp}{ds}(H - 2y) + \frac{u_t\mu}{H} \\
&= -(1/2)\frac{dp}{ds}(H - 2y) + \frac{u_t\mu}{H}
\end{aligned}
$$

Set $\tau = 0$ and solve for $y$

$$
0 = -(1/2)\frac{dp}{ds}(H - 2y) + \frac{u_t\mu}{H}
$$

$$
\boxed{y = \frac{H}{2} - \frac{\mu u_t}{H\,dp/ds}}
$$

---

Part (c)

$$\tau = \mu\frac{du}{dy} = 0 \text{ at } y = 0$$

$$\frac{du}{dy} = -(1/2\mu)\frac{dp}{ds}(H - 2y) + \frac{u_t}{H}$$

$$\text{Then, at } y = 0 : du/dy = 0 = -(1/2\mu)\frac{dp}{ds}H + \frac{u_t}{H}$$

$$\text{Solve for } u_t \quad : \quad \boxed{u_t = (1/2\mu)\frac{dp}{ds}H^2}$$

$$Note \quad : \quad \text{because } \frac{dp}{ds} < 0, u_t < 0.$$

2.37

Situation:

A cylinder falls inside a pipe filled with oil.

$d = 100\,\text{mm}$, $D = 100.5\,\text{mm}$.

$\ell = 200\,\text{mm}$, $W = 15\,\text{N}$.

Find:

Speed at which the cylinder slides down the pipe.

Properties:

SAE 20W oil (10°C) from Figure A.2: $\mu = 0.35$ N·s/m².

Assumptions:

Assume that buoyant forces can be neglected.

SOLUTION

$$\tau = \mu\frac{dV}{dy}$$

$$\frac{W}{\pi d\ell} = \frac{\mu V_{\text{fall}}}{(D-d)/2}$$

$$V_{\text{fall}} = \frac{W(D-d)}{2\pi d\ell\mu}$$

$$V_{\text{fall}} = \frac{15\,\text{N}(0.5\times10^{-3}\,\text{m})}{(2\pi\times0.1\,\text{m}\times0.2\,\text{m}\times3.5\times10^{-1}\,\text{N s/\,m}^2)}$$

$$\boxed{V_{\text{fall}} = 0.17\text{m/s}}$$

Situation:

A disk is rotated very close to a solid boundary with oil in between.

$\omega_a = 1\,\text{rad/s}$, $r_2 = 2\,\text{cm}$, $r_3 = 3\,\text{cm}$.

$\omega_b = 2\,\text{rad/s}$, $r_b = 3\,\text{cm}$.

$H = 2\,\text{mm}$, $\mu_c = 0.01\,\text{N s/m}^2$.

Find:

(a) Ratio of shear stress at 2 cm to shear stress at 3 cm.

(b) Speed of oil at contact with disk surface.

(c) Shear stress at disk surface.

Assumptions:

Linear velocity distribution: $dV/dy = V/y = \omega r/y$.

SOLUTION

(a) Ratio of shear stresses

$$\tau = \mu \frac{dV}{dy} = \frac{\mu \omega r}{y}$$

$$\frac{\tau_2}{\tau_3} = \frac{\mu \times 1 \times 2/y}{\mu \times 1 \times 3/y}$$

$$\boxed{\frac{\tau_2}{\tau_3} = \frac{2}{3}}$$

(b) Speed of oil

$$V = \omega r = 2 \times 0.03$$

$$\boxed{V = 0.06\text{m/s}}$$

(c) Shear stress at surface

$$\tau = \mu \frac{dV}{dy} = 0.01\,\text{N s/m}^2 \times \frac{0.06\,\text{m/s}}{0.002\,\text{m}}$$

$$\boxed{\tau = 0.30\ \text{N/m}^2}$$

Situation:

A disk is rotated in a container of oil to damp the motion of an instrument.

Find:

Derive an equation for damping torque as a function of $D, S, \omega$ and $\mu$.

PLAN

Apply the Newton's law of viscosity.

SOLUTION

Shear stress

$$
\begin{aligned}
\tau &= \mu \frac{dV}{dy} \\
&= \frac{\mu r \omega}{s}
\end{aligned}
$$

Find differential torque—on an elemental strip of area of radius $r$ the differential shear force will be $\tau dA$ or $\tau(2\pi r dr)$. The differential torque will be the product of the differential shear force and the radius $r$.

$$
\begin{aligned}
dT_{\text{one side}} &= r[\tau(2\pi r dr)] \\
&= r\left[\frac{\mu r \omega}{s}(2\pi r dr)\right] \\
&= \frac{2\pi \mu \omega}{s} r^3 dr \\
dT_{\text{both sides}} &= 4\left(\frac{r\pi \mu \omega}{s}\right) r^3 dr
\end{aligned}
$$

Integrate

$$
T = \int_0^{D/2} \frac{4\pi \mu \omega}{s} r^3 dr
$$

$$
T = \frac{1}{16}\frac{\pi \mu \omega D^4}{s}
$$

43

Situation:

One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder. $T_{\min} = 50\,°\text{F}$, $T_{\max} = 200\,°\text{F}$.

Find:

(a) Design a viscometer that can be used to measure the viscosity of motor oil.

Assumptions:

Motor oil is SAE 10W-30. Data from Fig A-2: $\mu$ will vary from about $2 \times 10^{-4}$ lbf-s/ft$^2$ to $8 \times 10^{-3}$ lbf-s/ft$^2$.

Assume the only significant shear stress is developed between the rotating cylinder and the fixed cylinder.

Assume we want the maximum rate of rotation ($\omega$) to be 3 rad/s.

Maximum spacing is 0.05 in.

SOLUTION

One possible design solution is given below.
Design decisions:

1. Let $h = 4.0$ in. $= 0.333$ ft

2. Let I.D. of fixed cylinder $= 9.00$ in. $= 0.7500$ ft.

3. Let O.D. of rotating cylinder $= 8.900$ in. $= 0.7417$ ft.

Let the applied torque, which drives the rotating cylinder, be produced by a force from a thread or small diameter monofilament line acting at a radial distance $r_s$. Here $r_s$ is the radius of a spool on which the thread of line is wound. The applied force is produced by a weight and pulley system shown in the sketch below.
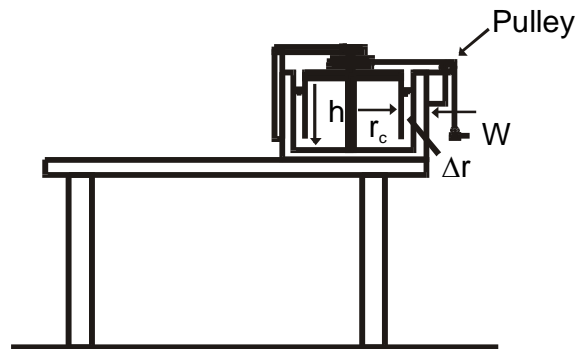


The relationship between $\mu, r_s, \omega, h$, and $W$ is now developed.

$$T = r_c F_s \tag{1}$$

where $T$ = applied torque

44

$r_c$ = outer radius of rotating cylinder
$F_s$ = shearing force developed at the outer radius of the rotating cylinder but $F_s = \tau A_s$ where $A_s$ = area in shear = $2\pi r_c h$

$$\tau = \mu dV/dy \approx \mu \Delta V/\Delta r \text{ where } \Delta V = r_c\omega \text{ and } \Delta r = \text{ spacing}$$

Then $T = r_c(\mu \Delta V/\Delta r)(2\pi r_c h)$

$$= r_c\mu(\frac{r_c\omega}{\Delta r})(2\pi r_c h) \tag{2}$$

But the applied torque $T = Wr_s$ so Eq. (2) become

$$Wr_s = r_c^3\mu\omega(2\pi)\frac{h}{\Delta r}$$

Or

$$\mu = \frac{Wr_s\Delta r}{2\pi\omega h r_c^3} \tag{3}$$

The weight $W$ will be arbitrarily chosen (say 2 or 3 oz.) and $\omega$ will be determined by measuring the time it takes the weight to travel a given distance. So $r_s\omega = V_{\text{fall}}$ or $\omega = V_{\text{fall}}/r_s$. Equation (3) then becomes

$$\mu = \left(\frac{W}{V_f}\right)\left(\frac{r_s^2}{r_c^3}\right)\left(\frac{\Delta r}{2\pi h}\right)$$

In our design let $r_s = 2$ in. $= 0.1667$ ft. Then

$$\mu = \left(\frac{W}{V_f}\right)\frac{(0.1667)^2}{(.3708)^3}\frac{0.004167}{(2\pi \times .3333)}$$

$$\mu = \left(\frac{W}{V_f}\right)\left(\frac{0.02779}{0.05098}\right)$$

$$\mu = \left(\frac{W}{V_f}\right)(1.085 \times 10^{-3})\text{ lbf}\cdot\text{s/ft}^2$$

Example: If $W = 2$oz. $= 0.125$lb. and $V_f$ is measured to be 0.24 ft/s then

$$\mu = \frac{0.125}{0.24}(1.085 \times 10^{-3})\text{ lbf s/ ft}^2$$

$$= 0.564 \times 10^{-4}\text{ lbf}\cdot\text{s/ ft}^2$$

### REVIEW

Other things that could be noted or considered in the design:

1. Specify dimensions of all parts of the instrument.

2. Neglect friction in bearings of pulley and on shaft of cylinder.

3. Neglect weight of thread or monofilament line.

4. Consider degree of accuracy.

5. Estimate cost of the instrument.

Situation:

Questions on the effect of temperature upon different types of fluids.

Find:

(a) If temperature increases, does the viscosity of water increase or decrease? Why?

(b) If temperature increases, does the viscosity of air increase or decrease? Why?

SOLUTION

(a) The viscosity of water decreases with increasing temperature . This is true for all liquids, and is because the loose molecular lattice within liquids, which provides a given resistance to shear at a relatively cool temperature, has smaller energy barriers resisting movement at higher temperatures.

---

(b) The viscosity of air increases with increasing temperature . This is true for all gases, and is because gases do not have a loose molecular lattice. The only resistance to shear provided in gases is due to random collision between different layers. As the temperature increases, there are more likely to be more collisions, and therefore a higher viscosity.

## 2.42

Sutherland's equation (select all that apply):

    a. relates temperature and viscosity

    b. must be calculated using Kelvin

    c. requires use of a single universal constant for all gases

    d. requires use of a different constant for each gas

## SOLUTION

Answers are (a) and (b). Answers c and d are relevant to the ideal gas law, not Sutherland's equation.

Situation:

When looking up values for density, absolute viscosity, and kinematic viscosity, which statement is true for BOTH liquids
and gases?

    a. all 3 of these properties vary with temperature

    b. all 3 of these properties vary with pressure

    c. all 3 of these properties vary with temperature and pressure

SOLUTION

Answer is (a). The absolute and kinematic viscosities of liquids do not vary with pressure.

Situation:

Common Newtonian fluids are:

    a. toothpaste, catsup, and paint

    b. water, oil and mercury

    c. all of the above

SOLUTION

The answer is (b). Toothpaste, catsup, and paint are not Newtonian, but are shear-thinning; see Fig. 2.14 in EFM10e.

<u>Situation</u>:

Which of these flows (deforms) with even a small shear stress applied?
    a. a Bingham plastic
    b. a Newtonian fluid

SOLUTION

The answer is (b); see Fig. 2.14 in EFM10e.

<u>Situation</u>:

Sutherland's equation and the ideal gas law describe behaviors of common gases.

<u>Find</u>:

Develop an expression for the kinematic viscosity ratio $\nu/\nu_o$, where $\nu$ is at temperature $T$ and pressure $p$.

<u>Assumptions</u>:

Assume a gas is at temperature $T_o$ and pressure $p_o$, where the subscript "o" defines the reference state.

**PLAN**

Combine the ideal gas law and Sutherland's equation.
**Note that $S =$ Sutherland's constant (and not specific gravity).

**SOLUTION**

The ratio of kinematic viscosities is

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} = \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o + S}{T + S}\frac{p_o}{p}\frac{T}{T_o}$$

$$\boxed{\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}}$$

Situation:

The dynamic viscosity of air.

$\mu_o = 1.78 \times 10^{-5}$ N·s/m$^2$.

$T_o = 15\,^\circ$C, $T = 100\,^\circ$C.

Find:

Dynamic viscosity $\mu$.

Properties:

From Table A.2, $S = 111K$.

**SOLUTION**

Sutherland's equation

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + S}{T + S}$$

$$= \left(\frac{373\,\mathrm{K}}{288\,\mathrm{K}}\right)^{3/2} \frac{288\,\mathrm{K} + 111\,\mathrm{K}}{373\,\mathrm{K} + 111\,\mathrm{K}}$$

$$\frac{\mu}{\mu_o} = 1.22$$

Thus

$$\mu = 1.22\mu_o$$

$$= 1.22 \times \left(1.78 \times 10^{-5}\,\mathrm{N \cdot s/\,m^2}\right)$$

$$\boxed{\mu = 2.17 \times 10^{-5}\ \text{N·s/m}^2}$$

**2.48**

<u>Situation</u>:

Methane gas.

$v_o = 1.59 \times 10^{-5}\, \mathrm{m^2/\,s}$.

$T_o = 15\,°\mathrm{C}$, $T = 200\,°\mathrm{C}$.

$p_o = 1\,\mathrm{atm}$, $p = 2\,\mathrm{atm}$.

<u>Find</u>:

Kinematic viscosity ($\mathrm{m^2/\,s}$).

<u>Properties</u>:

From Table A.2, $S = 198\,\mathrm{K}$.

**PLAN**

Apply the ideal gas law and Sutherland's equation.

**SOLUTION**

$$\nu = \frac{\mu}{\rho}$$

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho}$$

Ideal-gas law

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{p_o}{p}\frac{T}{T_o}$$

Sutherland's equation

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

so

$$\frac{\nu}{\nu_o} = \frac{1}{2}\left(\frac{473\,\mathrm{K}}{288\,\mathrm{K}}\right)^{5/2}\frac{288\,\mathrm{K} + 198\,\mathrm{K}}{473\,\mathrm{K} + 198\,\mathrm{K}}$$

$$= 1.252$$

and

$$\nu = 1.252 \times 1.59 \times 10^{-5}\ \mathrm{m^2/s}$$

$$\nu = \boxed{1.99 \times 10^{-5}\,\mathrm{m^2/\,s}}$$

**2.49**

Situation:

Nitrogen gas.

$\mu_o = 3.59 \times 10^{-7}\,\text{lbf} \cdot \text{s}/\,\text{ft}^2$.

$T_o = 59\,°\text{F}$, $T = 200\,°\text{F}$.

Find:

$\mu$ using Sutherland's equation.

Properties:

From Table A.2, $S = 192°\text{R}$.

**SOLUTION**

Sutherland's equation

$$
\begin{aligned}
\frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + S}{T + S} \\
&= \left(\frac{660°\text{R}}{519°\text{R}}\right)^{3/2} \frac{519°\text{R} + 192°\text{R}}{660°\text{R} + 192°\text{R}} \\
&= 1.197 \\
\mu &= 1.197 \times \left(3.59 \times 10^{-7} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \\
&= 4.297 \times 10^{-7}
\end{aligned}
$$

$$\boxed{\mu = 4.30 \times 10^{-7} \ \text{lbf-s/ft}^2}$$

Situation:

  Helium gas.
  $v_o = 1.22 \times 10^{-3}\,\mathrm{ft^2/s}$.
  $T_o = 59\,°\mathrm{F}$, $T = 30\,°\mathrm{F}$.
  $p_o = 1\,\mathrm{atm}$, $p = 1.5\,\mathrm{atm}$.

Find:

  Kinematic viscosity using Sutherland's equation.

Properties:

  From Table A.2, $S = 143°\mathrm{R}$.

PLAN

Combine the ideal gas law and Sutherland's equation.

SOLUTION

$$
\begin{aligned}
\frac{\nu}{\nu_o} &= \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S} \\
&= \frac{1.5}{1}\left(\frac{490°\mathrm{R}}{519°\mathrm{R}}\right)^{5/2}\frac{519°\mathrm{R} + 143°\mathrm{R}}{490°\mathrm{R} + 143°\mathrm{R}} \\
&= 1.359 \\
\nu &= 1.359 \times \left(1.22 \times 10^{-3}\frac{\mathrm{ft^2}}{\mathrm{s}}\right) \\
&= 1.658 \times 10^{-3}\frac{\mathrm{ft^2}}{\mathrm{s}}
\end{aligned}
$$

$$\boxed{\nu = 1.66 \times 10^{-3}\ \mathrm{ft^2/s}}$$

Situation:

Ammonia at room temperature.

$T_o = 68\,°\text{F}$, $\mu_o = 2.07 \times 10^{-7}\,\text{lbf s/ft}^2$.

$T = 392\,°\text{F}$, $\mu = 3.46 \times 10^{-7}\,\text{lbf s/ft}^2$.

Find:

Sutherland's constant.

SOLUTION

Sutherland's equation

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}} \tag{1}$$

Calculations

$$\frac{\mu}{\mu_o} = \frac{3.46 \times 10^{-7}\,\text{lbf s/ft}^2}{2.07 \times 10^{-7}\,\text{lbf s/ft}^2} = 1.671 \tag{a}$$

$$\frac{T_o}{T} = \frac{528\,°\text{R}}{852\,°\text{R}} = 0.6197 \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\frac{S}{T_o} = 1.71$$

$$\boxed{S = 903\ °\text{R}}$$

Situation:

SAE 10W30 motor oil.

$T_o = 38\,°\text{C}$, $\mu_o = 0.067\,\text{N s/ m}^2$.

$T = 99\,°\text{C}$, $\mu = 0.011\,\text{N s/ m}^2$.

Find:

The viscosity of motor oil, $\mu(60°\text{C})$, using the equation $\mu = Ce^{b/T}$.

PLAN

Use algebra and known values of viscosity ($\mu$) to solve for the constant b. Then, solve for the unknown value of viscosity.

SOLUTION

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$. Thus, evaluate $\mu$ at temperatures $T$ and $T_o$ and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[ b\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$$

Take the logarithm and solve for $b$.

$$b = \frac{\ln\left(\mu/\mu_o\right)}{\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

Data

$$\mu/\mu_o = \frac{0.011\,\text{N s/ m}^2}{0.067\,\text{N s/ m}^2} = 0.164$$
$$T = 372\,\text{K}$$
$$T_o = 311\,\text{K}$$

Solve for $b$

$$b = 3429 \ (\text{K})$$

Viscosity ratio at 60ºC

$$\frac{\mu}{\mu_o} = \exp\left[3429\left(\frac{1}{333\,\text{K}} - \frac{1}{311\,\text{K}}\right)\right]$$
$$= 0.4833$$
$$\mu = 0.4833 \times 0.067\,\text{N s/ m}^2$$
$$\mu = \boxed{0.032\ \text{N} \cdot \text{s/ m}^2}$$

Situation:
  Viscosity of grade 100 aviation oil.
  $T_o = 100\,°\text{F}$, $\mu_o = 4.43 \times 10^{-3}\,\text{lbf s}/\,\text{ft}^2$.
  $T = 210\,°\text{F}$, $\mu = 3.9 \times 10^{-4}\,\text{lbf s}/\,\text{ft}^2$.

Find:
  $\mu(150°\text{F})$, using the equation $\mu = Ce^{b/T}$.

PLAN

Use algebra and known values of viscosity ($\mu$) to solve for the constant b.  Then, solve for the unknown value of viscosity.

SOLUTION

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$.  Thus, evaluate $\mu$ at temperatures $T$ and $T_o$ and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Take the logarithm and solve for $b$

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Data

$$\begin{aligned}
\frac{\mu}{\mu_o} &= \frac{0.39 \times 10^{-3}\,\text{lbf s}/\,\text{ft}^2}{4.43 \times 10^{-3}\,\text{lbf s}/\,\text{ft}^2} = 0.08804 \\
T &= 670°\text{R} \\
T_o &= 560°\text{R}
\end{aligned}$$

Solve for $b$
$$b = 8288\ (°\text{R})$$

Viscosity ratio at 150°F

$$\begin{aligned}
\frac{\mu}{\mu_o} &= \exp\left[8288\left(\frac{1}{610°\text{R}} - \frac{1}{560°\text{R}}\right)\right] \\
&= 0.299 \\
\mu &= 0.299 \times \left(4.43 \times 10^{-3}\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \\
\end{aligned}$$

$$\boxed{\mu = 1.32 \times 10^{-3}\tfrac{\text{lbf} \cdot \text{s}}{\text{ft}^2}}$$

2.54

<u>Situation:</u>
Properties of air and water.
$T = 40\,°C$, $p = 170\,kPa$.

<u>Find:</u>
Kinematic and dynamic viscosities of air and water.

<u>Properties:</u>
Air data from Table A.3, $\mu_{air} = 1.91 \times 10^{-5}$ N·s/m$^2$
Water data from Table A.5, $\mu_{water} = 6.53 \times 10^{-4}$ N·s/m$^2$, $\rho_{water} = 992$ kg/m$^3$.

**PLAN**

Apply the ideal gas law to find density.    Find kinematic viscosity as the ratio of dynamic and absolute viscosity.

**SOLUTION**

A.) Air
Ideal gas law

$$
\begin{aligned}
\rho_{air} &= \frac{p}{RT} \\
&= \frac{170,000\,\text{Pa}}{(287\,\text{J/ kg K})\,(313.2\,\text{K})} \\
&= 1.89 \text{ kg/m}^3
\end{aligned}
$$

$$\boxed{\mu_{air} = 1.91 \times 10^{-5} \ \tfrac{\text{N·s}}{\text{m}^2}}$$

$$
\begin{aligned}
\nu &= \frac{\mu}{\rho} \\
&= \frac{1.91 \times 10^{-5}\,\text{N s/ m}^2}{1.89\,\text{kg/ m}^3}
\end{aligned}
$$

$$\boxed{\nu_{air} = 10.1 \times 10^{-5}\,\text{m}^2/\text{s}}$$

B.) water

$$\boxed{\mu_{water} = 6.53 \times 10^{-5} \ \text{N·s/m}^2}$$

$$
\begin{aligned}
\nu &= \frac{\mu}{\rho} \\
\nu &= \frac{6.53 \times 10^{-4}\,\text{N s/ m}^2}{992\,\text{kg/ m}^3}
\end{aligned}
$$

$$\boxed{\nu_{water} = 6.58 \times 10^{-7}\,\text{m}^2/\text{s}}$$

## 2.55

Situation:
Oxygen at $50\,°F$ and $100\,°F$.

Find:
Ratio of viscosities: $\frac{\mu_{100}}{\mu_{50}}$.

## SOLUTION

Because the viscosity of gases increases with temperature $\mu_{100}/\mu_{50} > 1$. Correct choice is (c).

<u>Situation</u>:

Surface tension: (select all that apply)

    a. only occurs at an interface, or surface

    b. has dimensions of energy/area

    c. has dimensions of force/area

    d. has dimensions of force/length

    e. depends on adhesion and cohesion

    f. varies as a function of temperature

**SOLUTION**

Answers are a, b, d, e, and f.

Situation:

Very small spherical droplet of water.

Find:

Pressure inside.

SOLUTION

Refer to Fig. 2-6(a). The surface tension force, $2\pi r\sigma$, will be resisted by the pressure force acting on the cut section of the spherical droplet or

$$p(\pi r^2) = 2\pi r\sigma$$
$$p = \frac{2\sigma}{r}$$
$$\boxed{p = \frac{4\sigma}{d}}$$

2.58

<u>Situation</u>:
A spherical soap bubble.
Inside radius $R$, wall-thickness $t$, surface tension $\sigma$.
Special case:  $R = 4\,\text{mm}$.

<u>Find</u>:
Derive a formula for the pressure difference across the bubble
Pressure difference for bubble with $R = 4\,\text{mm}$.
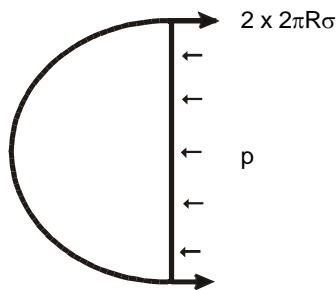
<u>Assumptions</u>:
The effect of thickness is negligible, and the surface tension is that of pure water.

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION

Force balance



2 x 2πRσ

p

Surface tension force

$$\sum F = 0$$
$$\Delta p \pi R^2 - 2(2\pi R \sigma) = 0$$

Formula for pressure difference

$$\Delta p = \frac{4\sigma}{R}$$

Pressure difference

$$\Delta p_{4\text{mm rad.}} = \frac{4 \times 7.3 \times 10^{-2}\ \text{N/m}}{0.004\ \text{m}}$$

$$\boxed{\Delta p_{4\text{mm rad.}} = 73.0\ \text{N/m}^2}$$

$\boxed{2.59}$

<u>Situation</u>:

A water bug is balanced on the surface of a water pond.

$n = 6$ legs, $\ell = 5\,\text{mm/leg}$.

<u>Find</u>:

Maximum mass of bug to avoid sinking.

<u>Properties</u>:

Surface tension of water, from Table A.4, $\sigma = 0.073\ \text{N/m}$.

$\boxed{\text{PLAN}}$

Apply equilibrium, then the surface tension force equation.

$\boxed{\text{SOLUTION}}$

Force equilibrium

$$\text{Upward force due to surface tension} \quad = \quad \text{Weight of Bug}$$
$$F_T \quad = \quad mg$$

To find the force of surface tension $(F_T)$, consider the cross section of one leg of the bug:



Cross section
of bug leg

Surface tension
force on one
side of leg

F  F

Assume θ is small
Then cos θ =1; F cos θ= F
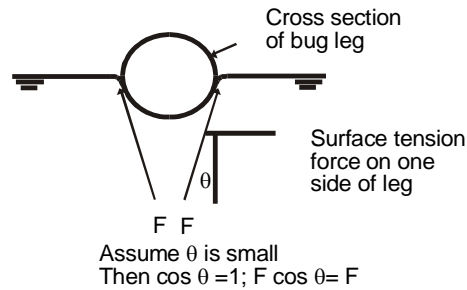
Surface tension force

$$
\begin{aligned}
F_T \quad &= \quad (2/\text{leg})(6\ \text{legs})\sigma\ell \\
&= \quad 12\sigma\ell \\
&= \quad 12(0.073\ \text{N/m})(0.005\ \text{m}) \\
&= \quad 0.00438\,\text{N}
\end{aligned}
$$

Apply equilibrium

$$
\begin{aligned}
F_T - mg \quad &= \quad 0 \\
m \quad &= \quad \frac{F_T}{g} = \frac{0.00438\,\text{N}}{9.81\,\text{m}^2/\text{s}} \\
&= \quad 0.4465 \times 10^{-3}\,\text{kg}
\end{aligned}
$$

$$\boxed{m = 0.447\,\text{g}}$$

Situation:

A water column in a glass tube is used to measure pressure.

$d_1 = 0.25\,\text{in}$, $d_2 = 1/8\,\text{in}$, $d_3 = 1/32\,\text{in}$.

Find:

Height of water column due to surface tension effects for all diameters.

Assumptions:

Assume that $\theta = 0$.

Properties:

From Table A.4: surface tension of water is 0.005 lbf/ft.

SOLUTION

Surface tension force

$$\Delta h = \frac{4\sigma}{\gamma d} = \frac{4 \times 0.005\,\text{lbf}/\text{ft}}{62.4\,\text{lbf}/\text{ft}^3 \times d\,\text{ft}} = \frac{3.21 \times 10^{-4}\,\text{ft}^2}{d\,\text{ft}}$$

$$d = \frac{1}{4}\,\text{in.} = \frac{1}{48}\,\text{ft.}; \ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}^2}{1/48\,\text{ft}} = 0.0154 \text{ ft.} = \boxed{0.185 \text{ in.}}$$

$$d = \frac{1}{8}\,\text{in.} = \frac{1}{96}\,\text{ft.}; \ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}^2}{1/96\,\text{ft}} = 0.0308 \text{ ft.} = \boxed{0.369 \text{ in.}}$$

$$d = \frac{1}{32}\,\text{in.} = \frac{1}{384}\,\text{ft.}; \ \Delta h = \frac{3.21 \times 10^{-4}\,\text{ft}^2}{1/384\,\text{ft}} = 0.123 \text{ ft.} = \boxed{1.48 \text{ in.}}$$

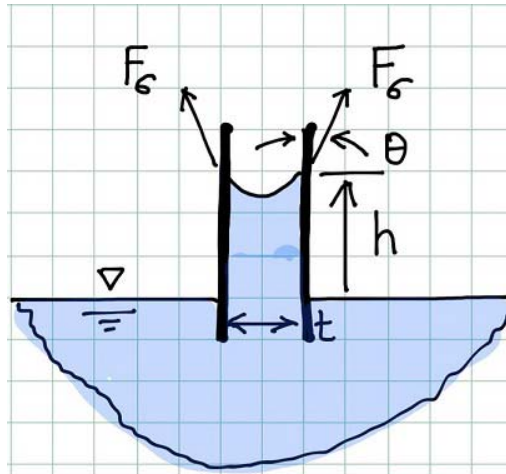Situation:

Two vertical glass plates

$t = 1\,\text{mm}$

Find:

Capillary rise $(h)$ between the plates.

Properties:

From Table A.4, surface tension of water is $7.3 \times 10^{-2}$ N/m.

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION



Equilibrium

$$\sum F_y = 0$$

$$\text{Force due to surface tension} = \text{Weight of fluid that has been pulled upward}$$

$$(2\ell)\,\sigma = (h\ell t)\,\gamma$$

Solve for capillary rise $(h)$

$$
\begin{aligned}
2\sigma\ell - h\ell t\gamma &= 0 \\
h &= \frac{2\sigma}{\gamma t} \\
h &= \frac{2 \times (7.3 \times 10^{-2}\,\mathrm{N/m})}{9810\,\mathrm{N/m^3} \times 0.001\,\mathrm{m}} \\
&= 0.0149 \text{ m} \\
h &= \boxed{14.9 \text{ mm}}
\end{aligned}
$$

Situation:

A spherical water drop.

$d = 1\,\text{mm}$

Find:

Pressure inside the droplet (N/m$^2$)

Properties:

From Table A.4, surface tension of water is $7.3 \times 10^{-2}$ N/m

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION

Equilibrium (half the water droplet)

$$
\begin{aligned}
\text{Force due to pressure} \;&=\; \text{Force due to surface tension} \\
pA \;&=\; \sigma L \\
\Delta p \pi R^2 \;&=\; 2\pi R \sigma
\end{aligned}
$$

Solve for pressure

$$
\begin{aligned}
\Delta p \;&=\; \frac{2\sigma}{R} \\
\Delta p \;&=\; \frac{2 \times 7.3 \times 10^{-2}\,\text{N/m}}{(0.5 \times 10^{-3}\,\text{m})} \\
\boxed{\Delta p = 292\ \text{N/m}^2}
\end{aligned}
$$

Situation:

A tube employing capillary rise is used to measure temperature of water

$T_0 = 0\,°C$, $T_{100} = 100\,°C$

$\sigma_0 = 0.0756\,N/m$, $\sigma_{100} = 0.0589\,N/m$

Find:

Size the tube (this means specify diameter and length).

Assumptions:

Assume for this problem that differences in $\gamma$, due to temperature change, are negligible.

PLAN

Apply equilibrium and the surface tension force equation.

SOLUTION

The elevation in a column due to surface tension is

$$\Delta h = \frac{4\sigma}{\gamma d}$$

where $\gamma$ is the specific weight and $d$ is the tube diameter. For the change in surface tension due to temperature, the change in column elevation would be

$$\Delta h = \frac{4\Delta\sigma}{\gamma d} = \frac{4 \times 0.0167\,N/m}{9810\,N/m^3 \times d} = \frac{6.8 \times 10^{-6}}{d}\,{}^2 \frac{m^2}{m} = \frac{6.8\,mm^2}{d\,mm}$$

The change in column elevation for a $\boxed{\text{1-mm diameter}}$ tube would be $\boxed{6.8\,mm}$. Special equipment, such the optical system from a microscope, would have to be used to measure such a small change in deflection  It is unlikely that smaller tubes made of transparent material can be purchased to provide larger deflections.

Situation:
Capillary rise is the distance water will rise above a water table, because the interconnected pores in the soil act like capillary tubes. This means that deep-rooted plants in the desert need only grow to the top of the "capillary fringe" in order to get water; they do not have to extend all the way down to the water table.

a. Assuming that interconnected pores can be represented as a continuous capillary tube, how high is the capillary rise in a soil consisting of a silty soil, with pore diameter of 10 $\mu$m?

b. Is the capillary rise higher in a soil with fine sand (pore $d$ approx. 0.1 mm), or in fine gravel (pore $d$ approx. 3 mm)?

c. Root cells extract water from soil using capillarity. For root cells to extract water from the capillary zone, do the pores in a root need to be smaller than, or greater than, the pores in the soil?

SOLUTION

a. Apply principals of surface tension, using Eq. 2.26 from EFM10e:

$$\Delta h = \frac{4\sigma}{\gamma d}$$

From Table A.5 in EFM10e, bottom line, $\sigma_{air/water} = 7.3 \times 10^{-2}\,\text{N}/\text{m}$

$$\Delta h = \left(\frac{4(7.3 \times 10^{-2})\,\text{N}}{\text{m}}\right)\left(\frac{\text{m}^3}{9810\,\text{N}}\right)\left(\frac{1}{10 \times 10^{-6}\,\text{m}}\right)$$
$$\boxed{= 3.0\,\text{m}}$$

b. By inspection of Eq. 2.26 of EFM10e, the pore diameter, $d$, is in the denominator, so as $d$ gets smaller, $\Delta h$ increases. Therefore, capillary rise is higher in a clay than in a gravel, because the pores are smaller.

c. In order to "wick" water from the soil, the pores in the roots need to be smaller than the pores in the soil.

$\boxed{2.65}$

A soap bubble and a droplet of water of equal diameter falling in air

$d = 2\,\text{mm}$, $\sigma_{bubble} = \sigma_{droplet}$

Find:

Which has the greater pressure inside.

$\boxed{\text{SOLUTION}}$

The soap bubble will have the greatest pressure because there are two surfaces (two surface tension forces) creating the pressure within the bubble. The correct choice is $\boxed{\text{a)}}$

Situation:
   A hemispherical drop of water is suspended under a surface

Find:
   Diameter of droplet just before separation

Properties:
   Table A.5 (20 °C): $\gamma = 9790 \, \text{N}/\, \text{m}^3, \sigma = 0.073 \, \text{N}/\, \text{m}$.

SOLUTION

Equilibrium

$$\text{Weight of droplet} = \text{Force due to surface tension}$$
$$\left(\frac{\pi D^3}{12}\right)\gamma = (\pi D)\,\sigma$$

Solve for $D$

$$
\begin{aligned}
D^2 &= \frac{12\sigma}{\gamma} \\
&= \frac{12 \times (0.073 \text{ N/m})}{9790 \text{ N/m}^3} = 8.\,948 \times 10^{-5} \, \text{m}^2 \\
D &= 9.\,459 \times 10^{-3} \, \text{m}
\end{aligned}
$$

$$\boxed{D = 9.46 \, \text{mm}}$$

Situation:

Surface tension is being measured by suspending liquid from a ring

$D_i = 10\,\text{cm}$, $D_o = 9.5\,\text{cm}$

$m = 10\,\text{g}$, $F = 16\,\text{g} \times g$, where $g$ is the acceleration of gravity

Find:

Surface tension ($\text{N}/\text{m}$)

PLAN

1.   Force equilibrium on the fluid suspended in the ring.   For force due to surface tension, use the form of the equation provided in the text for the special case of a ring being pulled out of a liquid.
2.  Solve for surface tension - all the other forces are known.

SOLUTION

1.  Force equilibrium

$$(\text{Upward force}) = (\text{Weight of fluid}) + (\text{Force due to surface tension})$$
$$F = W + \sigma(\pi D_i + \pi D_o)$$

2.  Solve for surface tension

$$\sigma = \frac{F - W}{\pi(D_i + D_o)}$$
$$\sigma = \frac{(0.016 - 0.010)\,\text{kg} \times 9.81\,\text{m}/\text{s}^2}{\pi(0.1 + 0.095)\,\text{m}}$$
$$\sigma = 9.61 \times 10^{-2}\frac{\text{kg}}{\text{s}^2}$$

$$\boxed{\sigma = 0.0961\ \text{N}/\text{m}}$$

## 2.68

<u>Situation:</u>

If liquid water at 30°C is flowing in a pipe and the pressure drops to the vapor pressure, what happens in the water?

    a. the water begins condensing on the walls of the pipe

    b. the water boils

    c. the water flashes to vapor

## SOLUTION

The answer is (b), it boils. Answer (c) is not correct. Flash vaporization is an industrial process used to separate volatile hydrocarbons; see the internet.

Find:

How does vapor pressure change with increasing temperature?
a. it increases
b. it decreases
c. it stays the same

SOLUTION

The answer is (a).

REVIEW    Vapor pressure increases with increasing temperature. To get an everyday feel for this, note from the Appendix that the vapor pressure of water at 212 °F (100 °C) is 101 kPa (14.7 psia). To get water to boil at a lower temperature, you would have to exert a vacuum on the water. To keep it from boiling until a higher temperature, you would have to pressurize it.

Situation:

$T = 20\,^{\circ}\mathrm{C}$,fluid is water.

Find:

The pressure that must be imposed to cause boiling

Properties:

Water $(60\,^{\circ}\mathrm{F})$, Table A.5: $P_v = 2340$ Pa abs

SOLUTION

Bubbles will be noticed to be forming when $P = P_v$.

$$\boxed{P = 2340 \text{ Pa abs}}$$

Water in a closed tank
$T = 20\,°C$, $p = 10400\,Pa$

Find:
Whether water will bubble into the vapor phase (boil).

Properties:
From Table A.5, at $T = 20\,°C$, $P_v = 2340$ Pa abs

SOLUTION

The tank pressure is 10,400 Pa abs, and $P_v = 2340$ Pa abs. So the tank pressure is higher than the $P_v$. Therefore the $\boxed{\text{water will not boil}}$.

REVIEW

The water can be made to boil at this temperature only if the pressure is reduced to 2340 Pa abs. Or, the water can be made to boil at this pressure only if the temperature is raised to approximately $50\,°C$.

2.72

Situation:

The boiling temperature of water decreases with increasing elevation
$\frac{\Delta p}{\Delta T} = \frac{-3.1\,\text{kPa}}{{}^\circ C}$.

Find:

Boiling temperature at an altitude of 3000 m

Properties:

$T = 100^o$C, $p = 101\,\text{kN}/\text{m}^2$.

$z_{3000} = 3000\,\text{m}$, $p_{3000} = 69\,\text{kN}/\text{m}^2$.

Assumptions:

Assume that vapor pressure versus boiling temperature is a linear relationship.

PLAN

Develop a linear equation for boiling temperature as a function of elevation.

SOLUTION

Let $BT =$ "Boiling Temperature." Then, $BT$ as a function of elevation is

$$BT\ (3000\ \text{m}) = \text{BT}\ (0\ \text{m}) + \left(\frac{\Delta BT}{\Delta p}\right)\Delta p$$

Thus,

$$\begin{aligned} BT\ (3000\ \text{m}) &= 100\,^\circ\text{C} + \left(\frac{-1.0\,^\circ\text{C}}{3.1\,\text{kPa}}\right)(101-69)\ \text{kPa} \\ &= 89.\,677\,^\circ\text{C} \end{aligned}$$

$$\boxed{\text{Boiling Temperature } (3000\ \text{m}) = 89.7\,^\circ\text{C}}$$