

Chapter 2 Problems

(2-1) Use the conversion formula $\bar{V} = V\angle\phi$

(a) $\bar{V}_1 = 100\angle -30^\circ \text{ V}$

(b) $\bar{V}_2 = 100\angle 30^\circ \text{ V}$

(2-2) We have,

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C} \int i(t)dt = \sqrt{2}V\cos(\omega t).$$

Taking Laplace Transform we get,

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = \sqrt{2}V \times \frac{s}{s^2 + \omega^2}$$

$$I(s) \left[R + Ls + \frac{1}{Cs} \right] = \sqrt{2}V \times \frac{s}{s^2 + \omega^2}$$

$$\begin{aligned} \therefore I(s) &= \frac{\sqrt{2}V \times Cs^2}{(s^2LC + RCs + 1)(s^2 + \omega^2)} \\ &= \frac{\sqrt{2}\frac{V}{L}s^2}{(s^2 + (\frac{R}{L})s + \frac{1}{LC})(s^2 + \omega^2)} \end{aligned}$$

Solving the above form by partial fractions and then taking the inverse Laplace transform we get

$$i(t) = 0.0368\cos(\omega t) + 0.466\sin(\omega t) = 0.468\cos(\omega t - 85.48^\circ) \text{ A}$$

(2-3)

$$\begin{aligned}Z_L &= |Z_L| \angle \phi \\|Z_L| &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\ \phi &= \tan^{-1} \left[\frac{\omega L - \frac{1}{\omega C}}{R} \right] \\ &= 85.48^\circ\end{aligned}$$

$$\begin{aligned}\bar{I} &= \frac{\bar{V}}{|Z_L|} \angle -\phi \\ &= \frac{120 \angle 0}{1.5^2 + \left(377 \times 20 \times 10^{-3} - \frac{1}{377 \times 100 \times 10^{-6}}\right)^2} \angle -85.48^\circ \\ &= 0.33 \angle -85.48^\circ \\ \therefore i(t) &= 0.33\sqrt{2}\cos(\omega t - 85.48) A \\ &= 0.467\cos(\omega t - 85.48) A\end{aligned}$$

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(2-4)

$$\begin{aligned}\bar{V} &= 90 \angle 30^\circ \\ \bar{I} &= 5 \angle 15^\circ \\ \text{If } \bar{V}_{new} &= 100 \angle 0^\circ\end{aligned}$$

$$\bar{I}_{new} = \left(\frac{\bar{V}_{new}}{\bar{V}}\right) \bar{I} = 5.56 \angle -15^\circ$$

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(2-5)

$$\begin{aligned}\bar{V} &= 100\angle 0^\circ V \\ Z &= j0.1 + \frac{(2)(-j5)}{2-j5} \\ &= 1.82\angle -18.9^\circ A \\ \bar{I} &= \frac{100\angle 0^\circ}{1.82\angle -18.9^\circ} \\ &= 54.95\angle 18.9^\circ A\end{aligned}$$

$$\begin{aligned}P &= VI\cos\phi = 5198.74W \\ Q &= VI\sin\phi = 1779.93VAR\end{aligned}$$

As the current leads the voltage, so the net load impedance is capacitive hence it draws negative reactive power
So net $Q = -1779.93VAR$.

$$p.f. = \cos\phi = 0.946(\text{Leading})$$

Let $X_1 = 0.1$ and $X_2 = 5$. Then,

$$\begin{aligned}Q_{X_1} &= I^2 X_1 = 301.95VAR \\ Q_{X_2} &= I_2^2 X_2\end{aligned}$$

where I_2 is the current through the capacitance.

$$\begin{aligned}\bar{I}_2 &= \bar{I} \times \frac{2}{2-j5} = 20.41\angle 87.1^\circ A \\ Q_{X_2} &= 5 \times (20.41)^2 = 2082.84VAR\end{aligned}$$

We further find that,

$$Q_{X_1} + (-Q_{X_2}) = -1780.57VAR \approx Q$$

Q_{X_2} takes a negative sign as the capacitor supplies reactive power.

$$\therefore Q = \sum_k I_k^2 X_k$$

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(2-6)

$$|I_s| = \frac{V}{\sqrt{R_s^2 + X_s^2}}$$

For the series circuit, the real power and the reactive power supplied to the load are

$$P_s = |I_s|^2 R_s = \frac{V^2}{R_s^2 + X_s^2} R_s$$

$$Q_s = |I_s|^2 X_s = \frac{V^2}{R_s^2 + X_s^2} X_s$$

For the parallel circuit, since the real and reactive powers of both circuits will be the same, we have

$$R_p = \frac{V^2}{P} = \frac{R_s^2 + X_s^2}{R_s}$$

$$X_p = \frac{V^2}{Q} = \frac{R_s^2 + X_s^2}{X_s}$$

(2-7) As calculated in Example 2-2,

$$\begin{aligned} Z_{in} &= 6.775 \angle 29.03^\circ \\ &= 5.924 + j3.289 \end{aligned}$$

Then, we will have,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{5.924^2 + 3.289^2}{5.924} = 7.75 \Omega$$

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{5.924^2 + 3.289^2}{3.289} = 13.96 \Omega$$

(2-8) In Example 2-2

$$\bar{I}_1 = 17.172 \angle -29.03^\circ \text{ A}$$

$$\bar{I}_{Rp} = \frac{jX_p}{R_p + jX_p} \bar{I}_1 = \frac{j13.96}{7.75 + j13.96} \times 17.172 \angle -29.03^\circ = 15.483 \angle 0^\circ \text{ A}$$

$$\bar{I}_{Xp} = \frac{R_p}{R_p + jX_p} \bar{I}_1 = \frac{7.75}{7.75 + j13.96} \times 17.172 \angle -29.03^\circ = 8.595 \angle 89.99^\circ \text{ A}$$

$$\therefore P = I_{Rp}^2 R_p = 15.483^2 \times 7.75 = 1857.86 \text{ W}$$

$$Q = I_{Xp}^2 X_p = 8.595^2 \times 13.96 = 1031.28 \text{ VAR}$$

Thus, P and Q are the same as calculated in Ex. 2-3. Hence the R_p and X_p calculations are correct.

(2-9) Referring to Figure 1,

$$\bar{V} = V\angle 0^\circ, I_{net} = I\angle \phi,$$

$$Z_{net} = \frac{\bar{V}}{I} = |Z|\angle -\phi, \cos\phi = 0.9$$

$$\sin(\phi) = \sqrt{1 - \cos^2\phi} = 0.436$$

$$\therefore Q_{net} = |P_L \tan\phi| = 900.29 \text{ VAR}$$

$$Q_C = Q_{net} + Q_L = 1931.59 \text{ VAR}$$

$$X_C = \frac{V^2}{Q_C}$$

$$\therefore \frac{1}{(2\pi f)C} = \frac{V^2}{Q_C}$$

$$C = \frac{Q_C}{(2\pi f)V^2} = 356 \mu\text{F}$$

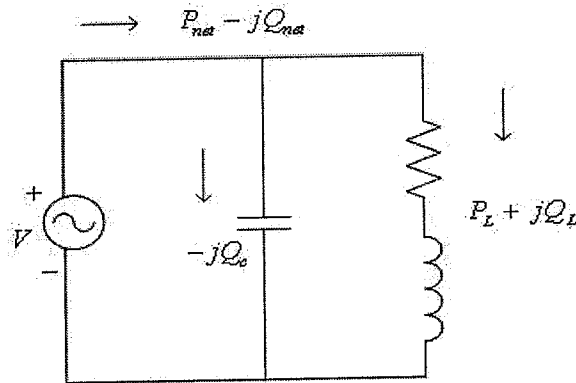


Figure 1:

(2-10) As shown in Figure 2,

$$Q_{net} = 5000 \tan[\cos^{-1}(0.95)] = 1643.42 \text{ VAR}$$

$$Q_{net} = Q_L - Q_C$$

$$Q_C = Q_L - Q_{net} = 2106.58 \text{ VAR}$$

$$X_C = \frac{V^2}{Q_C} = 6.835 \Omega$$

$$\therefore C = \frac{1}{(2\pi f)X_C} = 388 \mu\text{F}$$

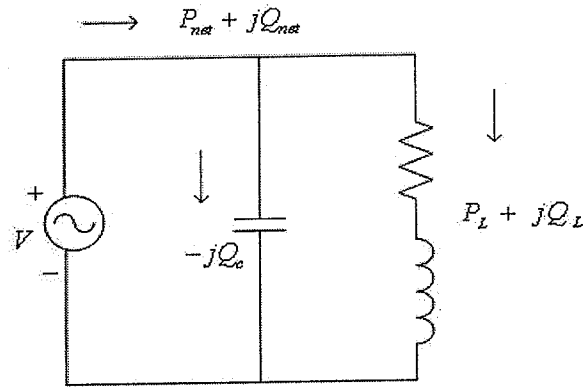


Figure 2:

(2-11) Note that

$$\bar{V}_a = 100\angle 30^\circ V$$

Thus,

$$\bar{V}_b = 100\angle -90^\circ V,$$

$$\bar{V}_c = 100\angle 150^\circ V$$

$$v_a(t) = \sqrt{2} \times 100 \cos(\omega t + 30) = 141.4 \cos(\omega t + 30)$$

$$v_b(t) = \sqrt{2} \times 100 \cos(\omega t + 30 - 120) = 141.4 \cos(\omega t - 90)$$

$$v_c(t) = \sqrt{2} \times 100 \cos(\omega t + 30 + 120) = 141.4 \cos(\omega t + 150)$$

$$\bar{V}_{ab} = \bar{V}_a - \bar{V}_b = 173.2\angle 60^\circ$$

$$v_{ab}(t) = \sqrt{2} \times 173.2 \cos(\omega t + 60) = 244.9 \cos(\omega t + 60)$$

(2-12) Given a Y-connected inductive load,

$$3V_a I_a \cos \phi = 10000$$

$$\begin{aligned} \therefore I_a &= \frac{10000}{3 \times 120 \times 0.9} \\ &= 30.86 A \end{aligned}$$

Assuming $V_a = 120\angle 0^\circ$

$$Z = |Z|\angle \cos^{-1}(0.9) = |Z|\angle 25.84^\circ$$

$$\therefore |Z| = \frac{V_a}{I_a} = 3.89\Omega$$

$$\bar{I}_a = \frac{\bar{V}_a}{Z} = \frac{120\angle -25.84^\circ}{3.89} = 30.86\angle -25.84^\circ A$$

$$\bar{I}_b = 3.86\angle -25.84 - 120^\circ 3.89 = 30.86\angle -145.84^\circ A$$

$$\bar{I}_c = 3.86\angle -25.84 + 120^\circ 3.89 = 30.86\angle 94.16^\circ A$$

(2-13) Same as above , only magnitude of load impedance changes

$$\therefore |Z| = 3.89 \times 3 = 11.67\Omega$$

(2-14)

$$\begin{aligned}\bar{V}_{aA} &= (Z_{self} - Z_{mutual})\bar{I}_a = (0.3 + j1) \times 10\angle -30^\circ \\ &= 10.44\angle 43.3^\circ V\end{aligned}$$

$$\begin{aligned}\bar{V}_{aA} &= \bar{V}_a - \bar{V}_A \\ &= 1000\angle 0^\circ - \bar{V}_A \\ \therefore \bar{V}_A &= 1000\angle 0^\circ - 10.44\angle 43.3^\circ \\ &= 992.43\angle -0.413^\circ V\end{aligned}$$

(2-15) In a balanced circuit,

$$\begin{aligned}\bar{V}_a + \bar{V}_b + \bar{V}_c &= 0 \\ \bar{V}_b + \bar{V}_c &= -\bar{V}_a \\ \bar{I}_a &= \frac{\bar{V}_a}{-jX_{c2}} + \frac{\bar{V}_a - \bar{V}_b}{-jX_{c1}} + \frac{\bar{V}_a - \bar{V}_c}{-jX_{c1}} \\ &= \frac{3\bar{V}_a}{-jX_{c1}} + \frac{\bar{V}_a}{-jX_{c2}} \\ &= \frac{\bar{V}_a}{Z_{eq}} \text{ where } Z_{eq} = \frac{1}{\frac{3}{-jX_{c1}} + \frac{1}{-jX_{c2}}}\end{aligned}$$

(2-16)

$$\begin{aligned}P_R &= \frac{V_S V_R}{X} \sin \delta \\ \sin \delta &= \frac{1000 \times 1.5}{100 \times 95} = 0.158 \\ \therefore \delta &= 9.091^\circ \\ \bar{I} &= \frac{\bar{V}_S - \bar{V}_R}{jX} = \frac{100 \angle \delta - 95 \angle 0}{j1.5} \\ &= \frac{100 \angle 9.091 - 95 \angle 0}{1.5 \angle 90} \\ &= 10.83 \angle -13.39^\circ\end{aligned}$$

From Eq 2-46

$$Q_R = \frac{V_S V_R}{X} \cos \delta - \frac{V_R^2}{X} = 237.11 \text{ VAR}$$

As a check

$$\begin{aligned}P_R + jQ_R &= \bar{V}_R \bar{I}^* = 1028.85 \angle 13.39^\circ \\ \therefore Q_R &= 238.3 \text{ VAR}\end{aligned}$$

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(2-17)

$$P = |V_S| |I| \cos \phi = 100 |I| \cos \phi$$

$$\overline{V_R} = \overline{V_S} - jX\overline{I}$$

Solving the above two equations we get an expression for $\frac{|V_S|}{|V_R|}$ and a corresponding plot in Matlab as shown above

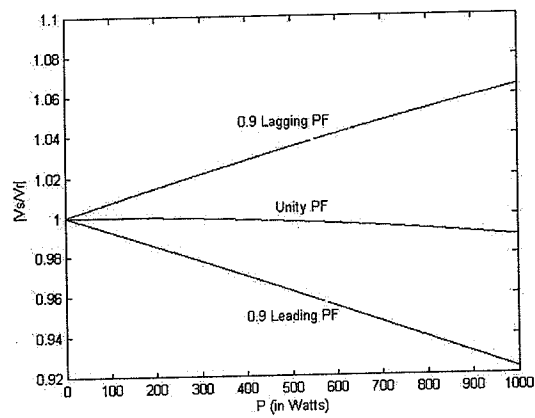


Figure 3:

(2-18) $V_{LL,base} = 208 \text{ V}$, $V_{ph,base} = \frac{208}{\sqrt{3}} = 120 \text{ V}$, $P_{ph,base} = 1200 \text{ W}$

$$I_{base} = \frac{P_{ph,base}}{V_{ph,base}} = 10 \text{ A}$$

$$Z_{base} = \frac{V_{ph,base}}{I_{base}} = 12 \Omega$$

With these base values, we calculate the per unit values as,

$$\begin{aligned}\overline{V}_{ph} &= \frac{120\angle 0}{120} = 1.0\angle 0^\circ pu \\ \overline{I}_L &= \frac{12\angle -36.87^\circ}{10} = 1.2\angle -36.87^\circ pu \\ \overline{Z}_L &= \frac{10\angle 36.87^\circ}{12} = 0.83\angle 36.87^\circ pu \\ P_L &= \frac{1152}{1200} = 0.96 pu \\ Q_L &= \frac{864}{1200} = 0.72 pu\end{aligned}$$

(2-19) $V_{LL,base} = 240 \text{ V}$, $V_{ph,base} = 138.57 \text{ V}$, $P_{ph,base} = 1800 \text{ W}$

$$\begin{aligned}I_{base} &= \frac{P_{ph,base}}{V_{ph,base}} = 12.99 \text{ A} \\ Z_{base} &= \frac{V_{ph,base}}{I_{base}} = 10.67 \Omega\end{aligned}$$

With these base values, we calculate the per unit values as,

$$\begin{aligned}\overline{V}_{ph} &= 0.867\angle 0^\circ pu \\ \overline{I}_L &= \frac{138.57}{|Z_L|} \angle -36.87^\circ = 13.86\angle -36.87^\circ \\ \overline{I} &= \frac{13.86\angle 36.87^\circ}{12.99} = 1.067\angle -36.87^\circ pu \\ \overline{Z}_L &= \frac{10\angle 36.87^\circ}{10.67} = 0.937\angle 36.87^\circ pu \\ P_L &= \frac{V^2}{|Z_L|} \cos \phi = 1536.13 \text{ W} = 0.853 pu \\ Q_L &= \frac{V^2}{|Z_L|} \sin \phi = 1152.1 \text{ VAR} = 0.64 pu\end{aligned}$$

(2-20) $V_{LL,base} = 240 \text{ V}$, $V_{ph,base} = 138.57 \text{ V}$, $P_{ph,base} = 1200 \text{ W}$

$$\begin{aligned}I_{base} &= \frac{P_{ph,base}}{V_{ph,base}} = 8.66 \text{ A} \\ Z_{base} &= \frac{V_{ph,base}}{I_{base}} = 16 \Omega\end{aligned}$$

With these base values, we calculate the per unit values as,

$$\begin{aligned}\overline{V}_{ph} &= 0.867 \angle 0^\circ pu \\ \overline{I}_L &= \frac{138.57}{|Z_L|} \angle -36.87^\circ = 13.86 \angle -36.87^\circ \\ \overline{I} &= \frac{13.86 \angle 36.87^\circ}{8.66} = 1.6 \angle -36.87^\circ pu \\ \overline{Z}_L &= \frac{10 \angle 36.87^\circ}{16} = 0.625 \angle 36.87^\circ pu \\ P_L &= \frac{V^2}{|Z_L|} \cos \phi = 1536.13 W = 1.28 pu \\ Q_L &= \frac{V^2}{|Z_L|} \sin \phi = 1152.1 VAR = 0.96 pu\end{aligned}$$

(2-21)(a)

$$\begin{aligned}l_m &= 2\pi r_m \\ H_{ID} &= \frac{Ni}{l_m} = 477.71 A/m\end{aligned}$$

(b)

$$\begin{aligned}l_m &= \pi OD \\ H_{OD} &= \frac{Ni}{l_m} = 433.53 A/m\end{aligned}$$

(c)

$$\begin{aligned}Error_{ID} &= \frac{H_{ID} - H_m}{H_m} \times 100 = 5.107\% \\ Error_{OD} &= \frac{H_{OD} - H_m}{H_m} \times 100 = 4.614\%\end{aligned}$$

(2-22)

$$\begin{aligned}\mathcal{R}_m &= \frac{l_m}{\mu_0 \mu_r A_m} \\ &= \frac{0.165}{4\pi \times 10^{-7} \times 2000 \times \frac{\pi}{4} \left(\frac{OD-ID}{2} \right)^2} \\ &= 1.337 \times 10^7 A/Wb\end{aligned}$$

(2-23) Using the \mathcal{R}_m from Prob 2-22,

$$L = \frac{N^2}{\mathcal{R}_m}$$

$$\therefore N = \sqrt{L\mathcal{R}_m} = 19$$

$$B_m = \mu_0 \mu_r \frac{NI}{l_m}$$

$$\therefore \mu_r = \frac{B_m l_m}{\mu_0 NI} = \frac{1.3 \times 0.165}{4\pi \times 10^{-7} \times 19 \times 3} = 2994.63$$

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(2-24)

$$\mathcal{R} = \mathcal{R}_m + \mathcal{R}_{gap} \approx \mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} \text{ [Using } A_{gap} \text{ from Prob 2-22]}$$

$$= \frac{l_{gap}}{4\pi \times 10^{-7} \times 4.91 \times 10^{-6}} = 16.2 \times 10^{10} l_{gap}$$

$$\lambda = N\phi = LI$$

$$\therefore NB_{max}A_m = LI$$

$$N = \frac{LI}{B_{max}A_m}$$

$$= \frac{25 \times 10^{-6} \times 3}{1.3 \times 4.91 \times 10^{-6}} = 12$$

$$\mathcal{R} = \frac{N^2}{L}$$

$$\therefore 16.2 \times 10^{10} l_{gap} = \frac{N^2}{25 \times 10^{-6}}$$

$$l_{gap} = \frac{N^2}{25 \times 10^{-6} \times 16.2 \times 10^{10}} = 0.00356 \text{ cm}$$

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