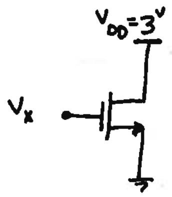
Chapter 2

2.1) a) NMOS :

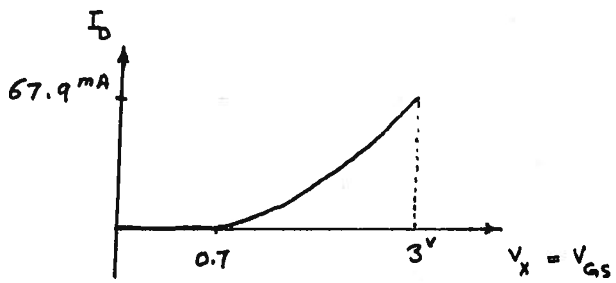


for $V_x < V_{TH} (= 0.7)$ device is off , $I_D \simeq 0$

for $V_x \geqslant 0.7$

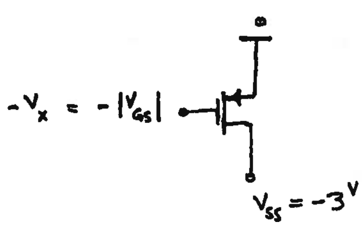
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_x - 0.7)^2 (1 + \lambda \cdot 3^V) \quad (L_{eff} = 0.5^{\mu} - 2L_D)$$

$$I_D = 12.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.7)^2$$



b) PMOS :

Solution is the same

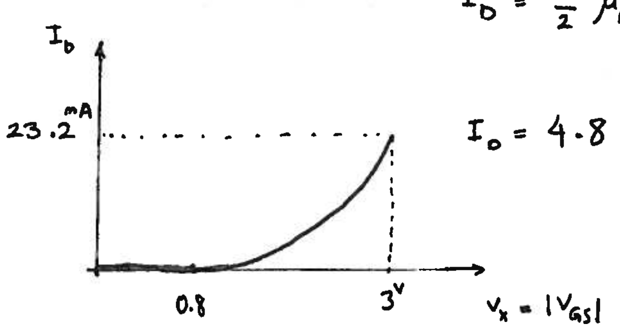


for $|V_{GS}| < V_{TH} (= 0.8)$ $\quad I_D \simeq 0$

for $|V_{GS}| \geqslant 0.8$

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L_{eff}} (V_x - 0.8)^2 (1 + \lambda \cdot 3^V)$$

$$I_D = 4.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.8)^2$$



2.2) a) Nmos

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \frac{mA}{V}$$ (Neglecting L_D)

$$r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$$

Intrinsic gain $= g_m r_o = 7\ 33 \frac{V}{V}$

b) PMOS

$$g_m = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} = 1.96 \frac{mA}{V}$$

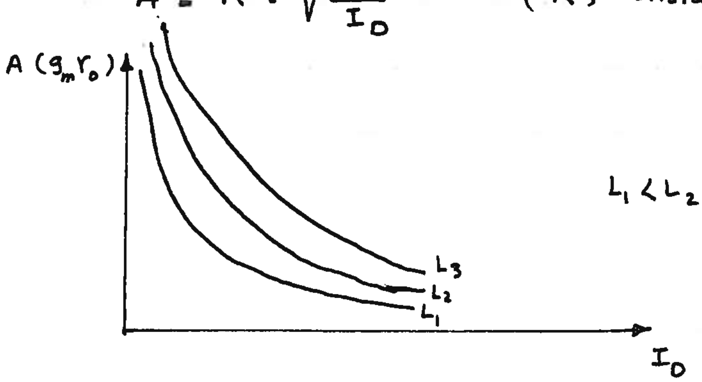
$$r_o = \frac{1}{\lambda I} = \frac{1}{0.2 \cdot 0.5^{mA}} = 10^{K\Omega}$$

$$g_m r_o = 19.6 \frac{V}{V}$$

2.3) $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $r_o = \frac{1}{\lambda I_D}$ Assume $\lambda = \frac{\alpha}{L}$

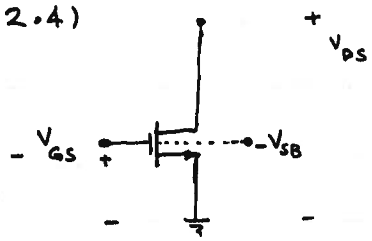
$$A = g_m r_o = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \cdot \frac{L}{\alpha I_D}$$

$$A = K \cdot \sqrt{\frac{WL}{I_D}}$$ (K ; Constant)



$L_1 < L_2 < L_3$

2.4)



I_D versus V_{GS} : (for NMOS)

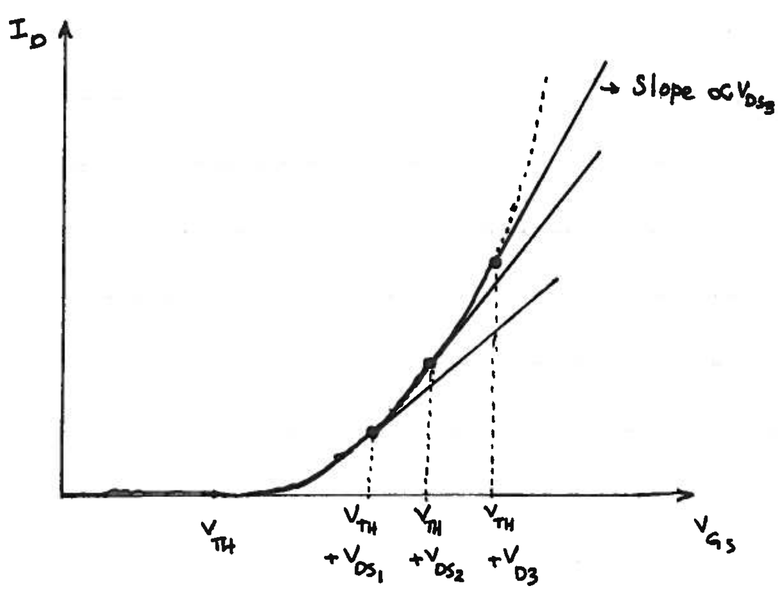
I) for $V_{GS} < V_{TH}$, $I_D \approx 0$

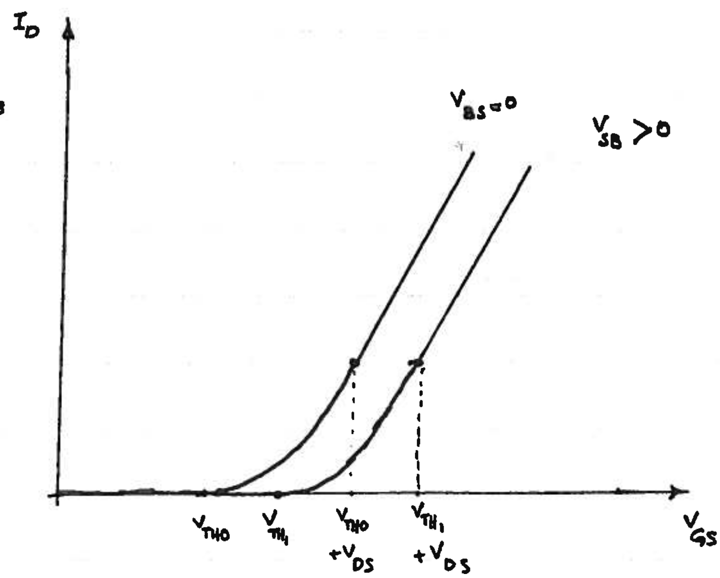
II) for $V_{TH} < V_{GS} < V_{TH} + V_{DS}$ $\Rightarrow$ Device is in the Saturation region

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

III) for $V_{GS} > V_{TH} + V_{DS}$ $\Rightarrow$ Device operates in the triode region

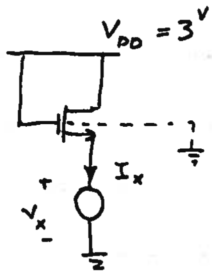
$$I_D \approx \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$





Changing V_{SB} just shifts the curve to the right for $V_{SB} > 0$ or to the left for $V_{SB} < 0$

2.5) a)



$\lambda = 0.1$, $\gamma = 0.45$, $2\phi_F = 0.9$, $V_{THO} = 0.7$

$V_{GS} = 3 - V_X$, $V_{DS} = 3 - V_X$, $V_{SB} = V_x$

$V_{TH} = V_{THO} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

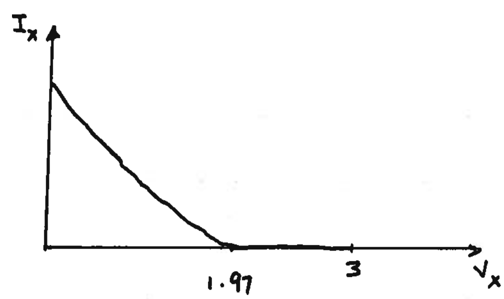
So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_X - 0.7 - 0.45\left(\sqrt{0.9 + V_X} - \sqrt{0.9}\right)\right)^2 (1 + \lambda(3 - V_x))$

The above equation is valid for

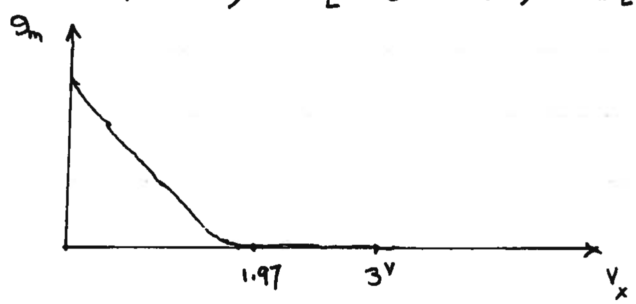
$3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right) > 0$, i.e. $V_x < 1.97^V$

So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2.727 - V_x - 0.45\sqrt{0.9 + V_x}\right)^2 (1.3 - 0.1 V_x)$

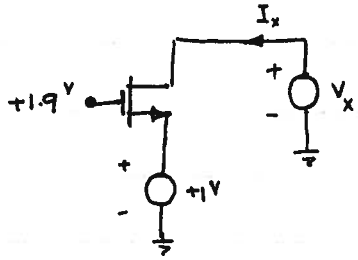
and $I_x = 0$ for $1.97 < V_x$



$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_x}$



2.5) b)



$\lambda = \gamma = 0 \qquad V_{TH} = 0.7$

for $0 < V_x < 1$, S and D exchange their roles.

$V_{GS} = 1.9 - V_X \qquad V_{DS} = 1 - V_X \quad , V_{OD} = 1.2 - V_X$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(1.2 - V_X) \times 2 \times (1 - V_X) - (1 - V_X)^2\right]$$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(1 - V_X)(1.4 - V_X)$$

$$g_m = \mu_n C_{ox}\frac{W}{L} V_{DS} = \mu_n C_{ox}\frac{W}{L}(1 - V_X) \quad \text{(absolute Value)}$$

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$V_{GS} = 1.9 - 1 = 0.9 \qquad V_{DS} = V_X - 1 \quad , V_{OD} = 0.9 - 0.7 = 0.2$

for $V_X < 1.2$; device operates in the triode region.

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2 \times 0.2 \times (V_X - 1) - (V_X - 1)^2\right]$$

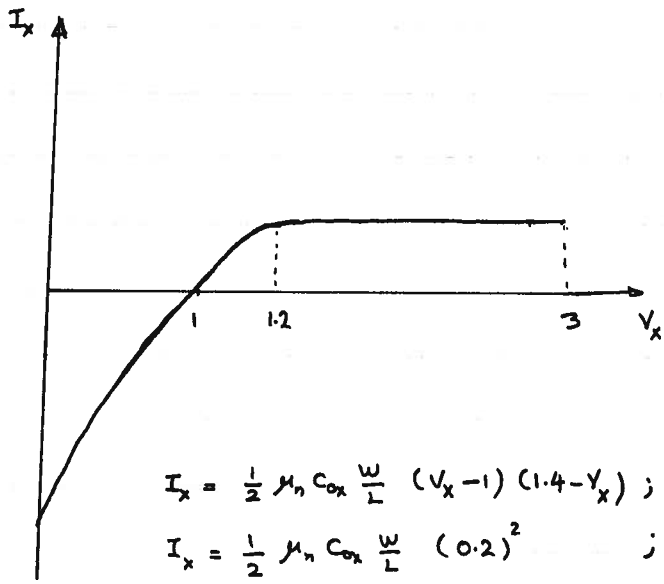
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_X - 1)$$

for $V_X > 1.2$, Device goes into saturation region

2.5) b Cont

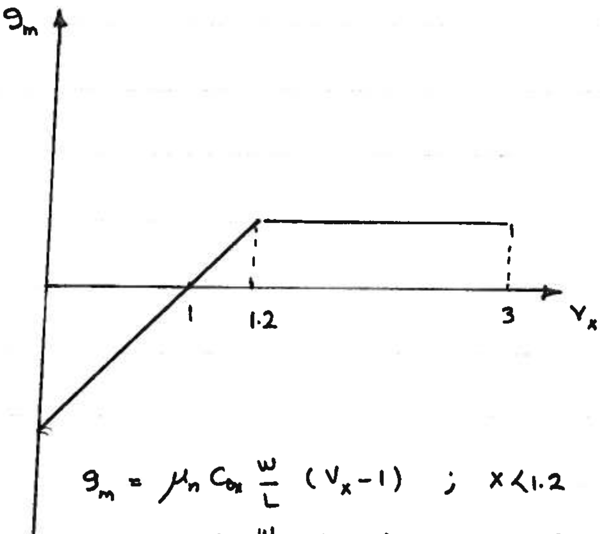
$$So, \quad I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2 ,$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$$



$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - 1)(1.4 - V_x)$; $x < 1.2$

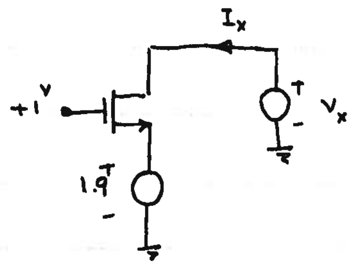
$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2$; $x > 1.2$



$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1)$; $x < 1.2$

$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$; $x > 1.2$

2.5) c



$\lambda = \gamma = 0$ $V_{TH} = 0.7$

S and D exchange their roles.

$V_{GS} = 1 - V_x$ $V_{DS} = 1.9 - V_x$ $V_{OD} = V_{GS} - V_{TH} = 0.3 - V_x$

Device is in Saturation region, So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$

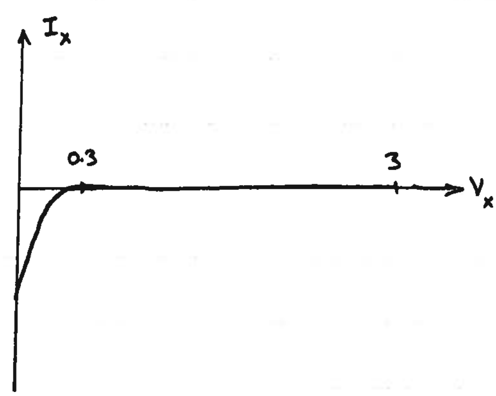
Device turns off when $V_x = 0.3$ and never turns on again.

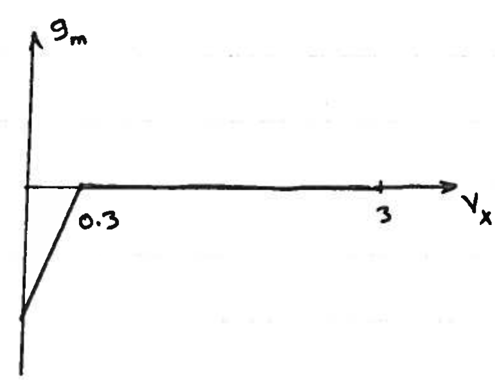
So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$; $x < 0.3$

$I_x = 0$; other wise

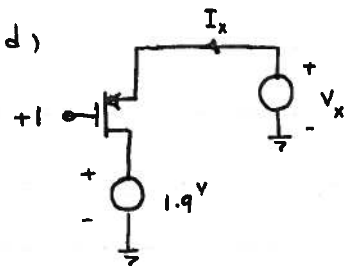
Then $g_m = -\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)$; $x < 0.3$

$g_m = 0$; other wise





2.5) d)



$V_{TH} = -0.8 \qquad \gamma = 0$

D and S exchang their roles.

$V_{GS} = -0.9 \qquad V_{DS} = V_x - 1.9$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L} (0.1)^2$$

$$g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox} \frac{W}{L} \left[(-0.1)(V_x - 1.9) - \frac{1}{2} (V_x - 1.9)^2 \right]$$

$$g_m = +\mu_p C_{ox} \frac{W}{L} (V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$V_{GS} = 1 - V_x \quad , \quad V_{DS} = 1.9 - V_x$

$$I_x = +\mu_p C_{ox} \frac{W}{L} \left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2} (1.9 - V_x)^2 \right]$$

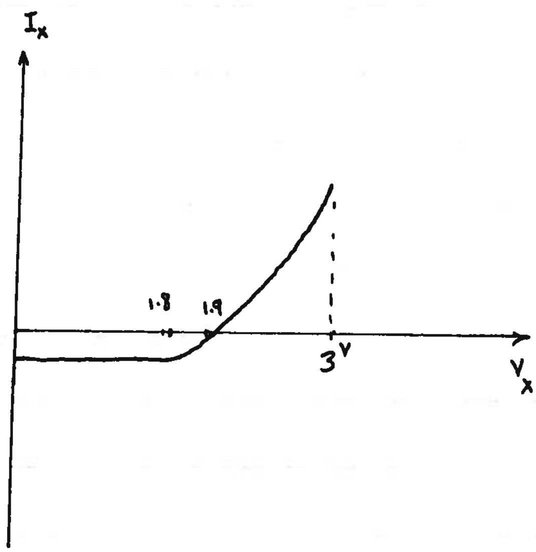
$$g_m = -\mu_p C_{ox} \frac{W}{L} (1.9 - V_x)$$

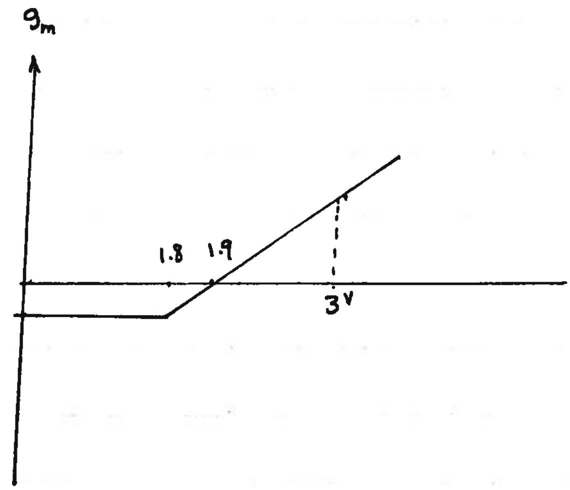
2.5)d So; $0 < V_x < 1.8$ $I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

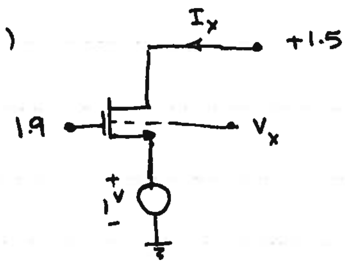
$1.8 < V_x < 3$ $I_x = +\mu_p C_{ox}\frac{W}{L} \times \frac{1}{2}(V_x - 1.9)(V_x - 1.7)$

$$g_m = \mu_p C_{ox}\frac{W}{L}(V_x - 1.9)$$





2.5) e)



$V_{TH0} = 0.7$ $\gamma = 0.45$ $2\phi_F = 0.9$, $\lambda = 0$

$V_{SB} = 1 - V_x$

$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + 1 - V_x} - \sqrt{0.9}\right)$

$V_{GS} = 0.9$ $V_{DS} = 0.5$

for $V_x = 0$, $V_{TH} = 0.893$ So device is in saturation region.

So $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)$$

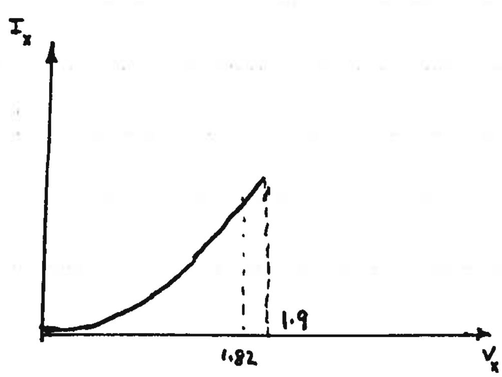
These equations are valid upto the edge of triode region, i.e.

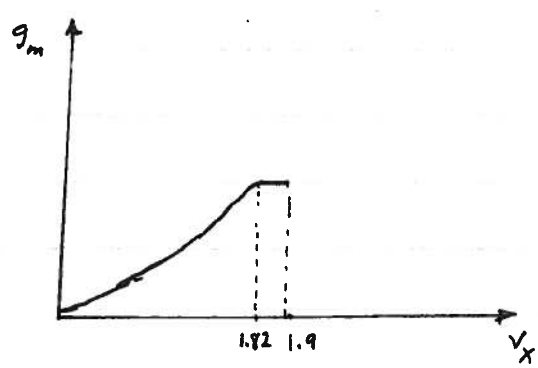
$0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right) = 0.5 \Rightarrow V_x = 1.82$

Above $V_x = 1.82$, device is in the triode region.

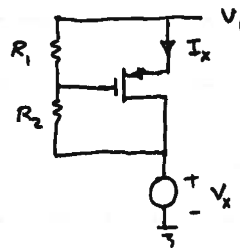
$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2 \times 0.5 \times \left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right) - 0.5^2\right]$$

$g_m = \mu_n C_{ox}\frac{W}{L}(0.5)$; This problem has been considered only for $0 < V_x < 1.9$ in which Schichman-Hodges Eq. is valid for V_{TH}.





2. (i) a)



$\gamma = 0$

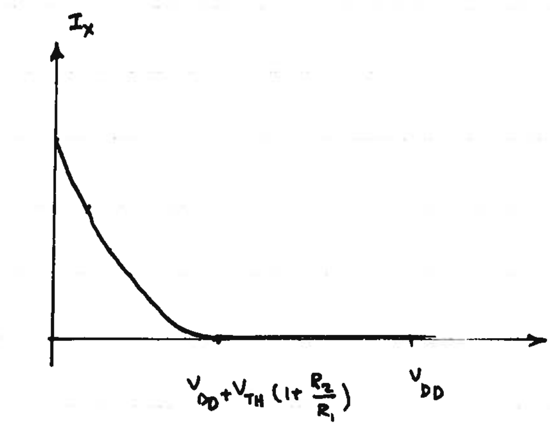
$$V_{SG} = (V_{DD} - V_X)\frac{R_1}{R_1+R_2} \qquad V_{SD} = V_{DD} - V_X$$

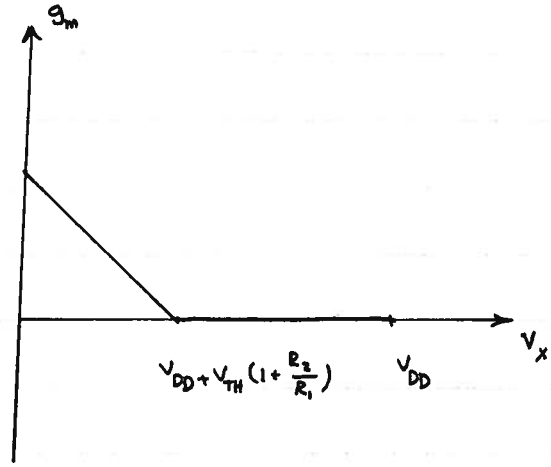
for $V_{SG} > |V_{TH}|$ Device is in the Saturation region (Device is off; otherwise)

$$(V_{DD} - V_X)\frac{R_1}{R_1+R_2} > -V_{TH}$$

$$V_X < V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow I_X = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{DD}-V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]^2$$

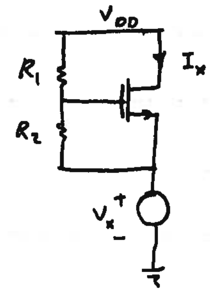
$$g_m = \mu_p C_{ox}\frac{W}{L}\left[(V_{DD}-V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]$$





If $V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ (e.g. for small value of R_1), device never turns on!

2.6) b)



$\gamma = 0$

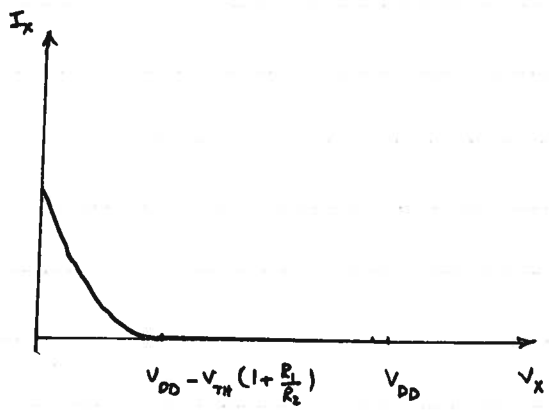
$$V_{GS} = (V_{DD} - V_X)\frac{R_2}{R_1 + R_2} \qquad V_{DS} = V_{DD} - V_X$$

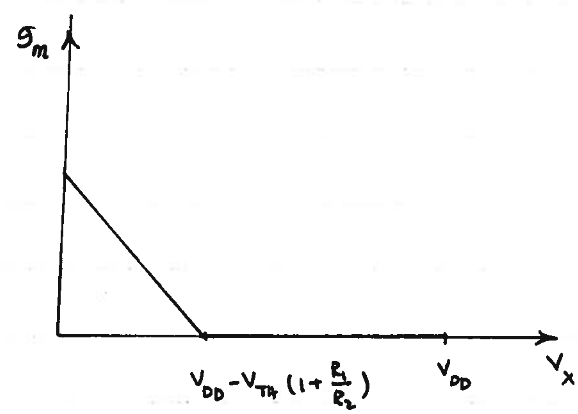
for $V_{GS} > V_{TH}$, Device is in the saturation region and

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_2}{R_1+R_2} - V_{TH}\right]^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_2}{R_1+R_2} - V_{TH}\right]$$

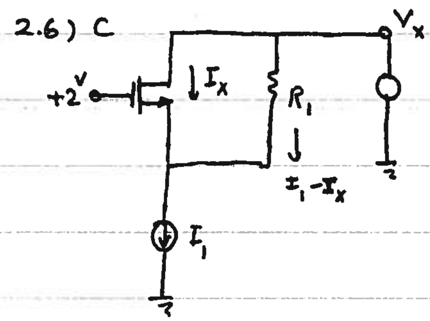
for $V_X < V_{DD} - V_{TH}\left(1 + \frac{R_1}{R_2}\right)$ (i.e. $V_{GS} > V_{TH}$)





If $V_{DD} - V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ device doesn't turn on.

2.6) C



I_x and $I_R = I_1 - I_x$ have the same polarity

So, $0 \leqslant I_x \leqslant I_1$

for $0 < V_x < 2 - V_{TH}$ (1.3) Device is in the triode.

$$V_{GS} = 2 - V_x + R_1 (I_1 - I_x) \quad , \quad V_{DS} = R_1 (I_1 - I_x)$$

$$I_x = I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{GS} - V_{TH}) - V_{DS} \right] V_{DS}$$

$$\Rightarrow (*) \; I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) + 2 (2 - V_{TH} - V_x) \right] \left\{ R_1 (I_1 - I_x) \right]$$

The above equation peresents $I_x - V_x$ characteristics in this region.

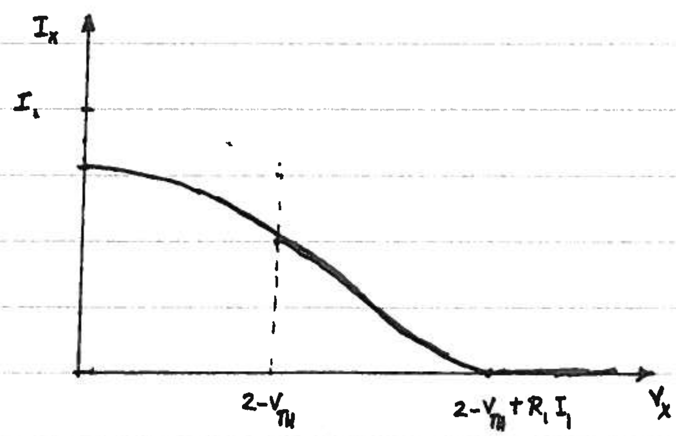
In this region $g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$

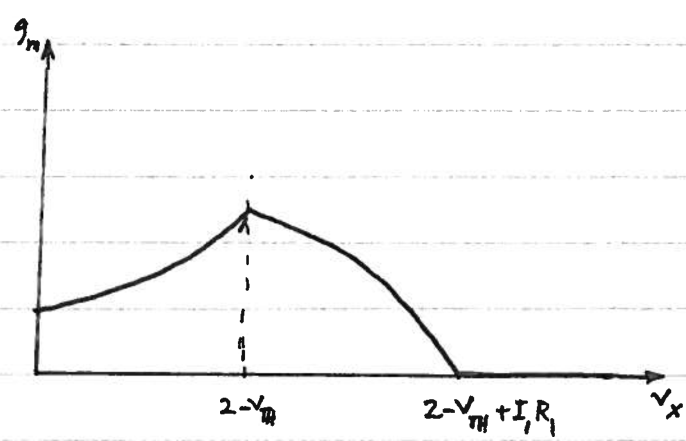
Then device enters the Saturation region; $V_{GS} = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]^2$$

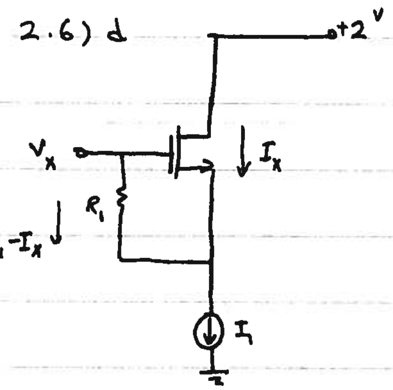
$$g_m = \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]$$

Then device turns off when $V_x = 2 - V_{TH} + R_1 I_1$





2.6) d



Assumption : $R_1 I_1 > V_{TH}$

for $0 < V_X < 2 + V_{TH}$: Device is in the saturation region

$$V_{GS} = R_1 (I_1 - I_X)$$

$$I_D = I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_X) - V_{TH} \right]^2$$

I_X is a constant that can be derived by solving the above equation.

Then device enters the triode region for $V_X > 2 + V_{TH}$

In this case $V_{GS} = R_1 (I_1 - I_X)$ $\quad V_{DS} = 2 - [V_X - R_1 (I_1 - I_X)] = 2 - V_X + R_1 (I_1 - I_X)$

$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Big(2 \left[R(I_1 - I_X) - V_{TH} \right] - 2 + V_X - R_1 (I_1 - I_X) \Big) \times (2 - V_X + R_1 (I_1 - I_X))$$

$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Big[\big(R_1 (I_1 - I_X) - V_{TH}\big) + (V_X - 2 - V_{TH}) \Big] \Big[(R_1 (I_1 - I_X) - V_{TH}) - (V_X - 2 - V_{TH}) \Big]$$

(✱) $$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\left(R_1 (I_1 - I_X) - V_{TH} \right)^2 - (V_X - 2 - V_{TH})^2 \right]$$

The second term shows that I_X decreases when we increase V_X

The polarity of I_X changes for higher V_X (Device still is in triode)

(✱) presents $I_X - V_X$ relationship in this region.