Chapter 2

Fractions

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Chapter Notes, Teaching Tips and Lecture Launchers

- Remind students that the worked-out solutions, not just the answers, to all the Try-It Exercises are located after the Chapter Summary Chart.

- Encourage students to try the Business Decision exercise at the end of the Review Exercises. These are specifically designed to get students to apply the chapter material in a real world business situation, frequently with business decisions that go "beyond the math!"

- ***Spotlight***: Stress to students that business math is not just a math course, it's a business course. For maximum value, they should view the course as "applications" of math procedures, used to answer business questions and make profitable business decisions.

- As in Chapter 1, the Collaborative Learning Activity for this chapter, "Knowing Fractions is Half the Battle," on page 63, will get the students to think about and discuss the uses of math in business. It is also another good opportunity for them to get to know each other early in the term.

- Remind students about the MathCue.Business software and what a useful learning tool it can be, when used in the "tutorial" mode. Be sure they know by clicking "Solution Step 1" and the "…Next Step" buttons, they can view the step-by-step, worked-out solution to each exercise.

- To get reluctant students motivated to learn fractions, ask if they cook, do construction projects, or have pizza parties. Ask how an understanding of fractions might be helpful in those situations.

- Be sure to use visual representations of the problems. Drawing a pizza on the board is helpful, as it is something most everyone is familiar with and has favorable feelings about, and is very useful in showing the relationships between fractions. For example, a pizza can be cut into any number of pieces and still contain the same quantity of pizza. And when leftover pieces of pizza are put into the same box, they might add up to more than one pizza.

- Remind students that, although they may be able to do many problems involving fractions in their heads, it is better to practice using the formulas given, so that they will have a tool to use in all situations when the answer is not obvious.

- Point out that fractions represent a portion of a thing. For example, to find out how much rock remains after using ¾ of a half ton of rock, one cannot subtract ¼ from ½ because the ¼ is a portion, and the ½ ton is the weight of a pile of rocks. This problem also makes a good visual representation of fractions.

Section I Understanding and Working with Fractions

- Be sure students fully understand the types of fractions; proper, improper, and mixed numbers. Use Transparency 2-1 to illustrate the various types of fractions.

- Classroom Activity: Have students write an example of each type, and exchange theirs with those of a partner. Now ask them to resolve any differences.

- Classroom Activity: Ask students to fold a piece of paper in half and then open it. What fraction does each part represent? Ask them to continue folding the paper in halves, identifying the fractional part of the whole after each new fold.

- Make sure the students know the difference between the numerator and the denominator. Stress the fact that fractions are a way of expressing parts of a whole, whereby the denominator describes how many equal parts the whole is divided into and the numerator describes how many parts we are describing or referring to.

- Be sure students understand that the line between the numerator and the denominator is known as the "division line" or "fraction bar."

- Remind students that they should be careful when writing fractions. A horizontal fraction bar is better than a slanted one since a slanted bar can cause the fraction to be misread.
  For example: 5½ could be confused with 51/2.

- ***Spotlight***: Sometimes students have difficulty determining which of two fractions is the larger or smaller number. By having the students convert them to like fractions (same denominator), the answer will become evident.
  - For example: Which fraction is larger, $\frac{4}{5}$ or $\frac{5}{6}$?

$$\frac{4}{5} = \frac{24}{30}, \text{ while } \frac{5}{6} = \frac{25}{30}, \text{ the larger number!}$$

- Classroom Activity: Frequently fractions are used to express a given quantity as a part of a whole, such as 4 months equals $\frac{4}{12}$ or $\frac{1}{3}$ of a year. Ask students to write the following as a fractional part of the whole, reduced to lowest terms:
  - 6 hours is what part of a day? $\frac{6}{24} = \frac{1}{4}$
  - 9 inches is what part of a foot? $\frac{9}{12} = \frac{3}{4}$
  - 1 quart is what part of a gallon? $\frac{2}{4} = \frac{1}{2}$
  - 800 pounds is what part of a ton? $\frac{800}{2{,}000} = \frac{2}{5}$

    - 2 days is what part of a week? $\frac{2}{7}$

- When converting mixed numbers to improper fractions draw an arrow going in a clockwise direction starting at the denominator, around the whole number and ending at the numerator to help students remember the order of the steps.

- When reducing fractions, remind students they can use inspection, a trial and error process, or they may use the Rules of Divisibility, page 35. (Transparency 2-2)

- Show students that raising fractions is simply cutting the pie into smaller pieces; the amount of pie, or the value of the number, remains the same. Reducing fractions is just the opposite.

- Define greatest common denominator as the biggest number that can go into both the numerator and the denominator evenly.

- ***Spotlight***: In using the greatest common divisor method to reduce a fraction, be sure students realize that the first step, dividing the numerator into the denominator, is the "opposite" of how fractions actually work.
    - Four-fifths, $\frac{4}{5}$, for example, means four divided by five.
- Explain that, when converting a fraction into a decimal or percent where the denominator is itself a fraction, the fraction in the denominator must be converted first, with the result being added to the calculator's memory. Then enter the numerator divided by memory recall.

Section II Addition and Subtraction of Fractions

- Point out that a common denominator must be found before fractions can be added or subtracted. The "least" common denominator, LCM, is the most desirable to use since it simplifies the calculation.

- Review with students what is meant by a prime number, and how they are used to find the least common denominator of two or more fractions, page 40.

- Alert students that answers to fraction problems should be reduced to lowest terms.

- ***Spotlight***: A common error in addition of fractions is when students simply add the numerators and the denominators. For example: $\frac{2}{5} + \frac{4}{5} = \frac{6}{10}$.

- Be sure students understand that when they borrow 1 in subtraction, they are borrowing a whole unit, expressed in terms of the common denominator.

- Ask students to express a whole unit in terms of fourths, fifths, sixths, and twenty-fourths. ($\frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{24}{24}$)

Alert students, they must add the borrowed whole unit to the fractional quantity that was already part of the minuend.

- For example: $9\frac{1}{4} = 8\,(\frac{4}{4} + \frac{1}{4}) = 8\frac{5}{4}$

- Point out that students must always pay close attention to what the question is asking. Is it asking for the fraction of the whole being discussed in the question, or is it asking for the complement of that fraction, i.e. what we have or what we don't have, this or not this? Can the students use the fraction given, or must they subtract the fraction from 1?

- When raising fractions, show the fraction by which to multiply.
  - For example: $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$

- Collaborative Learning Activity: Have students work in pairs to create 4 word problem exercises involving addition and subtraction of fractions. Let them exchange their exercises with those of another group. After they have solved them, allow the groups to exchange answers and resolve their differences.

Section III Multiplication and Division of Fractions

- *Spotlight*: Show students that fractions raised to higher terms or reduced to lower terms are still the same number. This can be demonstrated by having the students multiply a number, say 60 for example, by $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{4}{16}$. All calculations yield the same answer, 15.
- Generally, students think multiplying and dividing fractions are easier than adding and subtracting.
  - Ask your students if they agree, and why? (Most should answer that multiplying and dividing fractions is easier because they don't have to find a common denominator)

- Point out to students that cancellation can be done in any order.

- Demonstrate that sometimes fractions can be solved mentally, without pencil and paper.
  - For example, $5\frac{1}{2} \div \frac{1}{4} = 5\frac{1}{2} \times 4$

    $= (5 + \frac{1}{2}) \times 4$

$= (5 \text{ x } 4) + (\frac{1}{2} \text{ x } 4)$

$= 20 + 2 = 22$

- ***Spotlight***: Ask students to divide a fraction, say $\frac{7}{8}$, by $\frac{5}{5}$. When the quotient turns out to be the original dividend, $\frac{7}{8}$, this will help them realize that any number divided by 1 is that number itself.
    - This illustration also works with multiplication, which proves that any number multiplied by 1 is that number itself.

- Classroom Activity: In division of fractions, be sure students invert the divisor, not the dividend. Point out that the divisor is the fraction after the term "divided by." Ask the students to identify the divisor and the dividend and then solve the following:
    - $\frac{3}{8}$ divided by $\frac{9}{16}$ (Answer: $\frac{2}{3}$)
    - How many times does $2\frac{3}{4}$ go into $14\frac{1}{2}$? (Answer: $5\frac{3}{11}$)
    - Chef Franco made 13 pounds of fresh pasta for a catered party. If there are 16 ounces in a pound, how many $6\frac{1}{2}$ ounce portions can be served? (Answer: 32)
    - How many pieces of wire, $12\frac{3}{4}$ feet long, can be cut from a roll measuring $956\frac{1}{4}$ feet? (Answer: 75)
    -

- Classroom Activity: To demonstrate that fractions raised to higher terms or reduced to lower terms are still the same number, have students multiply a number, for example 60, by $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{4}{16}$. All calculations will yield the same answer; 15.

Questions Students Always Ask

"When raising fractions to higher terms, why do I have to multiply the numerator?"

- Fractions are raised to higher numbers so that we can add and subtract like numbers, apples to apples, so to speak. However, we don't want to change the value of the number, just it's expression. For example, a pizza can be cut into quarters, eighths, maybe even sixteenths. Either way it's cut, it still is the same amount of pizza. If we had $\frac{1}{2}$ of a pepperoni pizza and $\frac{1}{8}$ of a veggie pizza left, how much pizza would we have? To find out, we would raise the fraction $\frac{1}{2}$ to $\frac{4}{8}$ by multiplying it by $\frac{4}{4}$, which, because it is the same value as 1, will not change the value of the fraction, then add $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$.

"To solve the equation 3 ½ - 1 ¾, why do I add 4/4 to the top fraction?

- First, you must raise all fractions to a common denominator. Then you will notice that the top fraction, $\frac{1}{2}$ raised to $\frac{2}{4}$, is less than the fraction you are subtracting from it. Thus, you must borrow a 1 from the 3 in the digits position and add it to the $\frac{2}{4}$. To add the 1, you must convert it to $\frac{4}{4}$, which has the same value as the number 1; it's just expressed differently: $\frac{2}{4} + \frac{4}{4} = \frac{6}{4}$. Then you can proceed with the subtraction. Be sure to note the borrowing by crossing out the 3 and writing 2 above it.

"When adding and subtracting mixed fractions, should I convert them into improper fractions?"

- No, just remember to note when you have carried a number over to the digits column when adding, or when you have borrowed from the digits column when subtracting.

"Do I have to cancel when multiplying fractions?"

- No, but cancellation is generally easier than reducing the product, and it often makes the multiplication easier.

"I don't understand the concept of dividing by a fraction."

- Demonstrate by drawing two pizzas on the board. Show how $2 \div \frac{1}{4} = 8$ is the same as dividing the two pizzas into fourths and ending up with 8 pieces.

"Does it matter which number I invert when dividing fractions?"

- Yes, it matters. Invert the divisor, that is, the second number in the equation, the number after the "divide by" sign.

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Level 2
Chapter 2 - Section II - Exercise 17

Chet Murray ran 3 1/2 miles on Monday, 2 4/5 miles on Tuesday, and 4 1/8 miles on Wednesday. What was Chet's total mileage for the 3 days?

Monday	3 1/2	
Tuesday	2 4/5	
Wednesday	4 1/8	
	10 17/40	Total miles

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Level 3

Chapter 2 - Section II - Exercise 19

At the Fresh Market, you buy 6 3/10 pounts of yams and 4 1/3 pounds of corn. What is the total weight of the purchase?

Yams	6 3/10	
Corn	4 1/3	
	10 19/30	Pounds

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Level 2
Chapter 2 - Section II - Exercise 29

Casey McKee sold 18 4/5 of his 54 2/3 acres of land. How many acres does Casey have lef

54 2/3	Original acres
18 4/5	Sold
35 13/15	Acres left

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Level 2

Chapter 2 - Section II - Exercise 32

Brady White weighed 196 1/2 pounds when he decided to join a gym to lose some weight.
At the end of the first month, he weighed 191 3/8 pounds.

a. How much did he lose that month?

196 1/2	
191 3/8	
5 1/8	Pounds

b. If his goal is 183 3/4 pounds, how much more does he have to lose?

191 3/8	
183 3/4	
7 5/8	Pounds

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Level 3
Chapter 2 - Section II - Exercise 34

Tim Kenney, a painter, used 6 4/5 gallons of paint on the exterior of a house and 9 3/4 gallons
on the interior.
a. What is the total amount of paint used on the house?

6 4/5
9 3/4
16 11/20

b. If an additional 8 3/5 gallons was used on the garage, what is the total amount of paint used on the house and garage?

8 3/5
16 11/20
25 3/20

c. Rounding you anser from part b up to the next whole gallon, calculate the total cost of the paint if you paid $23 for each gallon.

26	
$ 23	
$ 598	Total cost of paint

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Level 1

Chapter 2 - Section III - Exercise 12

Multiply the following fractions and reduce to lowest terms. Use cancellation whenever possible.

2/3 x 5 4/5 x 9

2/3	5 4/5	9	34 4/5

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Level 2
Chapter 2 - Section III - Exercise 18

Three partners share a business. Max owns 3/8, Sherry owns 2/5, and Duane owns the res
If the profits this year are $150,000, how much does each partner receive?

3/8	$150,000	$56,250	Max
2/5	$150,000	$60,000	Sherry
		$116,250	Total Max + Sherry
$150,000	$116,250	$33,750	Duane's share

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Level 1
Chapter 2 - Section III - Exercise 25

Divide the following fractions and reduce to lowest terms.

4 4/5 ÷ 7/8

4 4/5	7/8	5 17/35

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Level 2

Chapter 2 - Section III - Exercise 34

At the Cattleman's Market, 3 1/2 pounds of hamburger are to be divided into 7 eq
ages. How many pounds of meat will each package contain?

3 1/2	7	1/2	Pound

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Level 3
Chapter 2 - Section III - Exercise 36

The chef at the Sizzling Steakhouse has 140 pounds of sirloin steak on ha
night. If each portion is 10 1/2 ounces, how many sirloin steak dinners car
Round to the nearest whole dinner. (There are 16 ounces in a pound.)

140	16	10 1/2	213

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Level 2

Chapter 2 - Assessment Test - Exercise 22

Solve the following problems and reduce to lowest terms.

25 1/2 ÷ 1 2/3

25 1/2	1 2/3	15 3/10

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Chapter 2 - Assessment Test - Exercise 22

Ventura Coal mined 6 2/3 tons on Monday, 7 3/4 tons on Tuesday, and 4 1/2 tons on Wednesday. If the goal is to mine 25 tons this week, how many more tons must be mined?

6 2/3	Mon
7 3/4	Tue
4 1/2	Wed
18 11/12	First 3 days

25	Goal
18 11/12	First 3 days
6 1/12	Tons remaining

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Level 2

Chapter 2 - Assessment Test - Exercise 23

A blueprint of a house has a scale of 1 inch equals 4 1/2 feet. If the living room wall measur
5 1/4 inches on the drawing, what is the actual length of the wall?

4 1/2	5 1/4	23 5/8	Feet

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Level 3
Chapter 2 - Assessment Test - Exercise 29

A house has 4,400 square feet. The bedrooms occupy 2/5 of the space, the living and dining rooms occupy 1/4 of the space, the garage represents 1/10 of the space, and the balance is split evenly among three bathrooms and the kitchen.
a. How many square feet are in each bath and the kitchen?

Bedrooms	2/5			
Liv & Dine	1/4			
Garage	1/10			
	3/4	4,400	3,300	Sq ft

4,400			Whole house
3,300			Space for BRs, DR, LR & Gara
1,100	4	275	Sq ft in each Bath & Kitchen

b. If the owner wants to increase the size of the garage by 1/8, how many total square feet will the new garage have?

Old garage	4,400	1/10	440	Sq ft
Additional	440	1/8	55	
			495	Total sq ft in new garage