Solutions Manual

Communication Systems

ANALYSIS AND DESIGN

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PEARSON
Prentice Hall

Upper Saddle River, New Jersey 07458

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Pearson Prentice Hall
Pearson Education, Inc.
Upper Saddle River, NJ 07458

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-091403-7

Pearson Education Ltd., *London*
Pearson Education Australia Pty. Ltd., *Sydney*
Pearson Education Singapore, Pte. Ltd.
Pearson Education North Asia Ltd., *Hong Kong*
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Solutions to Chapter 1 Problems

Problems 1.1 and 1.2 are designed to help the instructor determine the interests of the class. There are no strictly correct or strictly incorrect answers. These questions may help the instructor in developing examples for the lectures that will coincide with the class interests.

1.1 In one or two paragraphs, state why you want to study the design and analysis of communication systems.

1.2 What specific information would you like to learn in this class?

For Problems 1.3 – 1.5 provide enough explanation to justify each answer. Use your imagination.

The answers below are just some of the possible examples. The key to the solutions is to see that the student understands the various parameters involved in performance-versus-cost tradeoffs and that the student understands that different applications will produce different tradeoffs.

1.3 Give an example of a communication system where *accuracy of reception* is the most important parameter.

Examples include automated teller machines and medical imaging systems.

1.4 Give an example of a communication system where *reliability* is the most important parameter.

Consider a voice communication system on the space shuttle. If the voice message has slight static or buzzing, the user can still understand what is being said. If, however, the unit is unreliable, then the parties may not be able to communicate at all.

1.5 Give an example of a communication system where *equipment simplicity* (lack of complexity in hardware and software) is the most important parameter.

Consider an inexpensive AM/FM radio or a cheap wireline telephone handset. These products attempt to minimize consumer cost while still providing an "acceptable" level of service.

Solutions to Chapter 2 Problems

2.1 In your own words, define the following terms:

a. Time domain analysis

b. Frequency domain analysis

Answers should be consistent with the definitions given in Section 2.1 of the book: In time domain analysis, we examine the amplitude vs. time characteristics of the waveform. In frequency domain analysis, we replace the waveform with a summation of sinusoids which produce an equivalent waveform and we then examine the relative amplitudes, phases, and the frequencies of the sinusoids.

2.2 Give an example of a communication system application where bandwidth is very important.

Consider a cellular telephone system in a large city. The entire system has a certain amount of bandwith allocated to it by the Federal Communications Commission (say 600 kHz within each cell). Each cellular telephone call uses a certain amount of the 600 kHz bandwidth to communicate. If the cellular phones use an inefficient modulation technique which requires, say, 30 kHz of bandwidth for each phone call, then only 20 callers in each cell can use the system at any given time. If, on the other hand, the modulation technique used by the cellular telephone needs only 5 kHz of bandwidth, then 120 callers in each cell can simultaneously use the system. This six-fold increase in capacity will allow the cellular telephone service provider to generate six times as much revenue.

2.3 Why is it important to analyze the effects of the channel in a communication system?

As discussed in Chapter 1, all channels have physical limitations that will distort and attenuate the transmitted signal and that will add noise to the transmitted signal. Thus, the received signal will not be an exact duplicate of the transmitted signal. If we understand how a particular channel distorts, attenuates, and adds noise to a signal, then we can design our transmiter (and receiver) to use a modulation technique (and demodulation technique) which minimizes the channel's effects. By minimizing the channel's effects, we guarantee that the signal will be received more accurately.

2.4 Show that the set of harmonically related sines and cosines is orthogonal. In other words, show that for any positive integer values of m and n

a. $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)\,dt = 0$ if $m \neq n$ and $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)\,dt \neq 0$ if $m = n$

b. $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)\,dt = 0$ if $m \neq n$ and $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)\,dt \neq 0$ if $m = n$

c. $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\sin(2\pi m f_o t)\,dt = 0$ for all values of m and n

Solution

a. *Using trig identities (see the page opposite the inside front cover of the textbook),*

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt = \int_{t_o}^{t_o+T} 0.5\{\cos[2\pi(n+m)f_o t] + \cos[2\pi(n-m)f_o t]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt + 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt$$

Let's evaluate the terms one at a time.

Evaluating the first term:

$0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt = 0$ *for any positive values of m and n, since we are integrating a sinusoid over a whole number of periods.*

Evaluating the second term:

If $m \neq n$, $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = 0$ *because again we are integrating a sinusoid over a whole number of periods.*

If $m = n$, $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = 0.5\int_{t_o}^{t_o+T} 1dt = 0.5T$

Combining the results of the first and second terms,

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$$

b. *Using trig identities,*

$$\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt = \int_{t_o}^{t_o+T} 0.5\{\cos[2\pi(n-m)f_o t] - \cos[2\pi(n+m)f_o t]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt - 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt$$

As shown in Part a,

$0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt = 0$ *for any positive values of m and n*

and $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$

Thus, $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$

c. *Using trig identities,*

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\sin(2\pi m f_o t)dt = \int_{t_o}^{t_o+T} 0.5\{\sin[2\pi(n+m)f_o t] + \sin[2\pi(n-m)f_o t]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T} \sin[2\pi(n+m)f_o t]dt + 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt$$

Evaluating the first term,

$0.5\int_{t_o}^{t_o+T} \sin[2\pi(n+m)f_o t]dt = 0$ *for any positive values of m and n, since we are integrating a sinusoid over a whole number of periods.*

Evaluating the second term,

if $m \neq n,\ 0.5\int_{t_o}^{t_o+T} \sin[2\pi(n-m)f_o t]dt = 0$ *because again we are integrating a sinusoid over a whole number of periods.*

If $m = n,\ 0.5\int_{t_o}^{t_o+T} \sin[2\pi(n-m)f_o t]dt = 0.5\int_{t_o}^{t_o+T} 0dt = 0$

Thus, $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\sin(2\pi m f_o t)dt = 0$

2.5 Using the concept of orthogonality, derive the equations for the coefficients of the trigonometric form of the Fourier series (i.e., derive the expressions for a_0, a_n, and b_n). (Hint: to find the expression for a_n, start with the expression for the trigonometric form of the Fourier series, multiply both the left-hand and right-hand sides by $\cos 2\pi m f_o t$, and integrate over one period of the fundamental frequency. The a_0 and b_n expressions can be found in a similar manner.)

Starting with the trigonometric form of the Fourier series,

$$s(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t\right)$$

Multiplying both sides by $a_m \cos 2\pi m f_o t\ \ (m \neq 0)$ *and then integrating over one period of* $s(t)$*,*

$$\int_{t_0}^{t_o+T} s(t)\cos 2\pi m f_o t dt = \int_{t_0}^{t_o+T} \left\{a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t\right)\right\}\cos 2\pi m f_o t dt$$

$$= \int_{t_0}^{t_o+T} a_0 \cos 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \cos 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \cos 2\pi m f_o t dt$$

Examining the first term,

$\int_{t_0}^{t_o+T} a_0 \cos 2\pi m f_o t dt = 0$ *because we are integrating a sinusoid over a whole number of periods.*

Examining the second term,

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_o+T} \cos 2\pi n f_o t \cos 2\pi m f_o t dt$$

As we proved in Problem 2.4, due to orthogonality

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t) dt = \begin{cases} 0 & n \neq m \\ 0.5T & n = m \end{cases}$$

and so our second term reduces to

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_o+T} \cos 2\pi n f_o t \cos 2\pi m f_o t dt = a_m (0.5T)$$

Examining the third term,

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_o+T} \sin 2\pi n f_o t \cos 2\pi m f_o t dt$$

As we proved in Problem 2.4, due to orthogonality

$\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\cos(2\pi m f_o t) dt = 0$ *for all values of m and n, so*

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_o+T} \sin 2\pi n f_o t \cos 2\pi m f_o t dt = 0$$

Combining the three terms,

$$\int_{t_0}^{t_o+T} s(t) \cos 2\pi m f_o t dt = 0 + a_m (0.5T) + 0 = \frac{a_m T}{2}, \text{ or } a_m = \frac{2}{T} \int_{t_o}^{t_o+T} s(t) \cos(2\pi m f_o t) dt$$

Similarly, starting with the trigonometric form of the Fourier series, multiplying both sides by $\sin 2\pi m f_o t$*, and exploiting the orthogonality which we proved in Problem 2.4,*

$$\int_{t_0}^{t_o+T} s(t) \sin 2\pi m f_o t dt = \int_{t_0}^{t_o+T} \left\{ a_0 + \sum_{n=1}^{\infty} \left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t \right) \right\} \sin 2\pi m f_o t dt$$

$$= \int_{t_0}^{t_o+T} a_0 \sin 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \sin 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \sin 2\pi m f_o t dt$$

$$= 0 + 0 + b_m (0.5T) = \frac{b_m T}{2}$$

$$so\ b_m = \frac{2}{T}\int_{t_o}^{t_o+T} s(t)\sin(2\pi m f_o t)dt$$

Finally, starting with the trigonometric form of the Fourier series and integrating over one period,

$$\int_{t_0}^{t_o+T} s(t) = \int_{t_0}^{t_o+T} \left\{ a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t\right)\right\} dt$$

$$= \int_{t_0}^{t_o+T} a_0 dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_m \sin 2\pi n f_o t dt$$

$$= \int_{t_0}^{t_o+T} a_0 dt + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_o+T} \cos 2\pi n f_o t dt + \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_o+T} \sin 2\pi m f_o t dt$$

For all values of n, the integrals in the second and third terms are zero since we are integrating a sinusoid over a whole number of periods, and so

$$\int_{t_0}^{t_o+T} s(t) = \int_{t_0}^{t_o+T} a_0 dt = a_0 T\text{, or}$$

$$a_0 = \frac{1}{T}\int_{t_o}^{t_o+T} s(t)dt$$

2.6 Using the Fourier coefficients determined in Example 2.3 Part a and an appropriate software graphics package, reproduce the plots from Figures 2.8a – d.

The plots, of course, should look like Figures 2.8a – 2.8d.

2.7 Express the waveform given below in Figure P2-7 as a series of sinusoids. Your expression should use the actual Fourier coefficient values (i.e., the values of a_0, a_n, and b_n) for the dc term and at least the first four harmonics. Your expression should also use the actual value of the fundamental frequency (f_o). You may wish to set up a spreadsheet or software program to calculate your Fourier coefficients, since you will need to calculate the values for higher order harmonics in Problem 2.8.

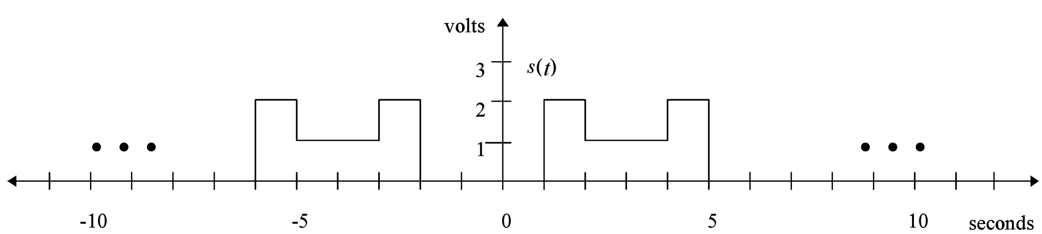


Figure P2-7

The waveform s(t) is periodic with finite energy per period, so the Fourier Series exists.

T = 7 seconds $\quad f_0 = \frac{1}{T} = \frac{1}{7}\ Hz$

$$a_0 = \frac{1}{T}\int_{t_o}^{t_o+T} s(t)dt = \frac{1}{7}\int_{t_o}^{t_o+T} s(t)dt = \frac{1}{7}\left[\int_0^1 0dt + \int_1^2 2dt + \int_2^4 1dt + \int_4^5 2dt + \int_5^7 0dt\right] = \frac{6}{7}\text{ volt}$$

$$a_n = \frac{2}{T}\int_{t_o}^{t_o+T} s(t)\cos(2\pi n f_o t)dt$$

$$= \frac{2}{7}\left[\int_0^1 0\cos(2\pi n f_o t)dt + \int_1^2 2\cos(2\pi n f_o t)dt + \int_2^4 1\cos(2\pi n f_o t)dt + \int_4^5 2\cos(2\pi n f_o t)dt + \int_5^7 0\cos(2\pi n f_o t)dt\right]$$

$$= \left(\frac{2}{7}\right)\left(\frac{7}{2\pi n}\right)\left[2\sin\left(\frac{2\pi n t}{7}\right)\Big|_1^2 + \sin\left(\frac{2\pi n t}{7}\right)\Big|_2^4 + 2\sin\left(\frac{2\pi n t}{7}\right)\Big|_4^5\right]$$

$$= \frac{1}{\pi n}\left[2\sin\left(\frac{10\pi n}{7}\right) - \sin\left(\frac{8\pi n}{7}\right) + \sin\left(\frac{4\pi n}{7}\right) - 2\sin\left(\frac{2\pi n}{7}\right)\right]$$

Similarly,

$$b_n = \frac{2}{T}\int_{t_o}^{t_o+T} s(t)\sin(2\pi n f_o t)dt$$

$$= \frac{-1}{\pi n}\left[2\cos\left(\frac{10\pi n}{7}\right) - \cos\left(\frac{8\pi n}{7}\right) + \cos\left(\frac{4\pi n}{7}\right) - 2\cos\left(\frac{2\pi n}{7}\right)\right]$$

Computing the a_n and b_n values,

n	*Terminology*	a_n	b_n
0	dc term	0.857	
1	first harmonic	-0.670	0.323
2	second harmonic	-0.366	0.459
3	third harmonic	0.0943	-0.413
4	fourth harmonic	-0.0707	-0.310
5	fifth harmonic	0.146	0.183
6	sixth harmonic	0.112	0.0538
7	seventh harmonic	0	0
8	eighth harmonic	-0.0837	0.0403
9	ninth harmonic	-0.0813	0.102
10	tenth harmonic	0.0283	-0.124

Thus,

$$
\begin{aligned}
s(t) &= a_0 + \sum_{n=1}^{\infty}(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t) \\
&= a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{2\pi n t}{7} + \quad b_n \sin\frac{2\pi n t}{7}) \\
&= a_0 + a_1\cos\left(\frac{2\pi t}{7}\right) + b_1\sin\left(\frac{2\pi t}{7}\right) + a_2\cos\left(\frac{4\pi t}{7}\right) + b_2\sin\left(\frac{4\pi t}{7}\right) + \ldots \\
&= 0.857 - 0.670\cos\left(\frac{2\pi t}{7}\right) + 0.323\sin\left(\frac{2\pi t}{7}\right) - 0.366\cos\left(\frac{4\pi t}{7}\right) + 0.459\sin\left(\frac{4\pi t}{7}\right) \\
&\quad + 0.0943\cos\left(\frac{6\pi t}{7}\right) - 0.413\sin\left(\frac{6\pi t}{7}\right) - 0.0707\cos\left(\frac{8\pi t}{7}\right) - 0.310\sin\left(\frac{8\pi t}{7}\right) + \ldots
\end{aligned}
$$

2.8 To see how many harmonics are needed for the Fourier series expression developed in Problem 2.7 to adequately replicate the original waveform.

a. Plot the waveform produced by summing the dc component and the sinusoids associated with the first five harmonics.

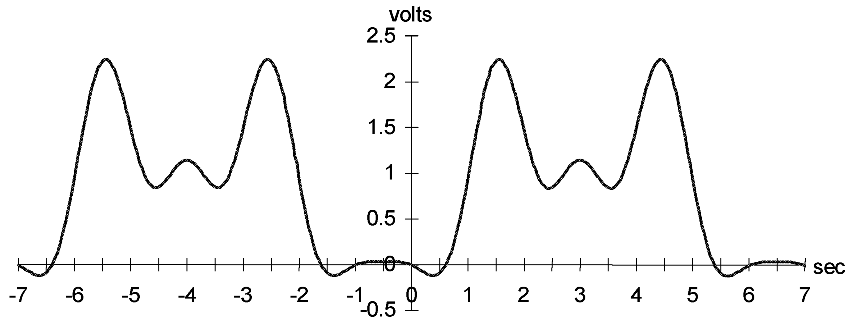
b. Plot the waveform produced by summing the dc component and the sinusoids associated with the first ten harmonics.

c. Plot the waveform produced by summing the dc component and the sinusoids associated with the first fifteen harmonics.

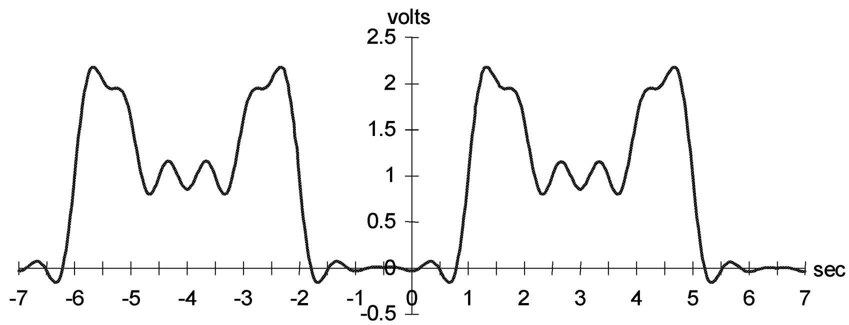
d. Plot the waveform produced by summing the dc component and the sinusoids associated with the first twenty-five harmonics.

Comment on how well the Fourier series is converging to produce the original waveform.

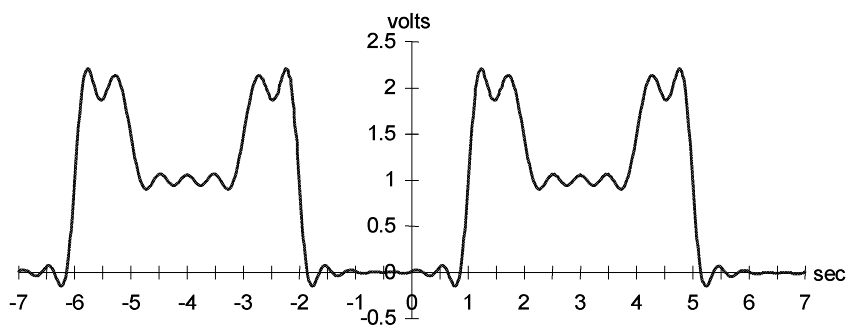
The plots are shown on the next page. With the dc term and the first five harmonics we can see a vague resemblence to the original function. With the dc term and ten harmonics, the vertical edges start to straighten up. With the dc term and 15 harmonics, the vertical edges are straighter and the horizontal lines start to flatten. The dc term plus the first twenty-five harmonics produces a pretty good replica of the original signal. The number of harmonics needed depends on how closely we need to reproduce the original waveform — as discussed in the textbook, accuracy requirements are application-dependent.



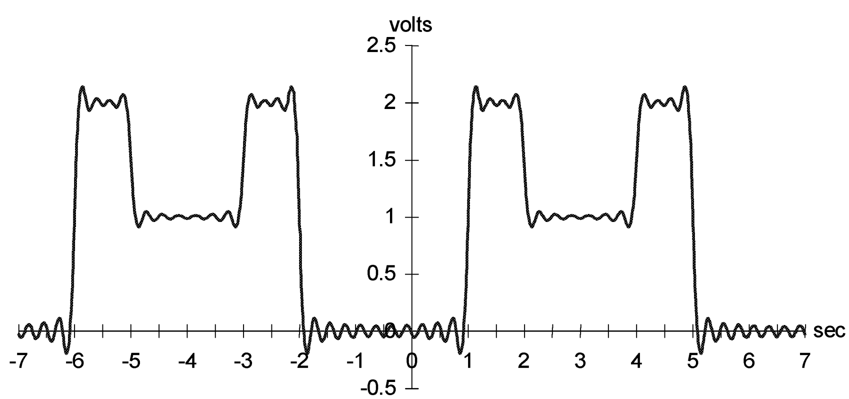
a. Waveform produced with dc and the first five harmonics



b. Waveform produced with dc and the first ten harmonics



c. Waveform produced with dc and the first 15 harmonics



d. Waveform produced with dc and the first 25 harmonics

2.9 Prove that the trigonometric form of the Fourier series and one-sided form of the Fourier series are equivalent. Think about the relationship between rectangular and polar coordinates and see how that concept relates to this proof.

We can prove equivalence by using the derivation in Section 2.2.2.1 of the textbook, which shows that the relationship between the trigonometric form and the one-sided form of the Fourier series is the same as the relationship between rectangular and polar coordinates, except that a positive b_n value corresponds to a negative value on the y-axis because sine lags cosine.

2.10 Express the waveform given in Figure P2-7 as a series of cosines using the one-sided Fourier series. Draw one-sided magnitude and phase spectra for the waveform.

In Problem 2.7 we calculated the trigonometric Fourier series coefficients for Figure P2-7. To convert to one-sided Fourier series coefficients,

$$\begin{aligned} s(t) &= a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t\right) \\ &= X_0 + \sum_{n=1}^{\infty} X_n \cos\left(2\pi n f_o t + \phi_n\right) \end{aligned} \tag{2.8}$$

where

$$X_0 = a_0 \tag{2.9}$$

$$X_n = \sqrt{a_n^2 + b_n^2} \tag{2.10}$$

$$\text{and}\,\phi_n = \arctan\left(\frac{-b_n}{a_n}\right) \tag{2.11}$$

[The equation numbers are from the textbook]. Computing the one-sided coefficients,

n	*Terminology*	a_n	b_n	X_n	ϕ_n *(radians)*
0	dc term	0.857		0.857	0 *
1	first harmonic	-0.670	0.323	0.744	-0.857π
2	second harmonic	-0.366	0.459	0.587	-0.714π
3	third harmonic	0.0943	-0.413	0.424	0.429π
4	fourth harmonic	-0.0707	-0.310	0.318	0.571π
5	fifth harmonic	0.146	0.183	0.235	-0.286π
6	sixth harmonic	0.112	0.0538	0.124	-0.143π
7	seventh harmonic	0	0	0	0 **
8	eighth harmonic	-0.0837	0.0403	0.0929	-0.857π
9	ninth harmonic	-0.0813	0.102	0.130	-0.714π
10	tenth harmonic	0.0283	-0.124	0.127	0.429π

* *Phase is meaningless for the dc term but is often shown as 0*

** *Phase is meaningless if X_n=0, but is often shown as 0*

Thus,

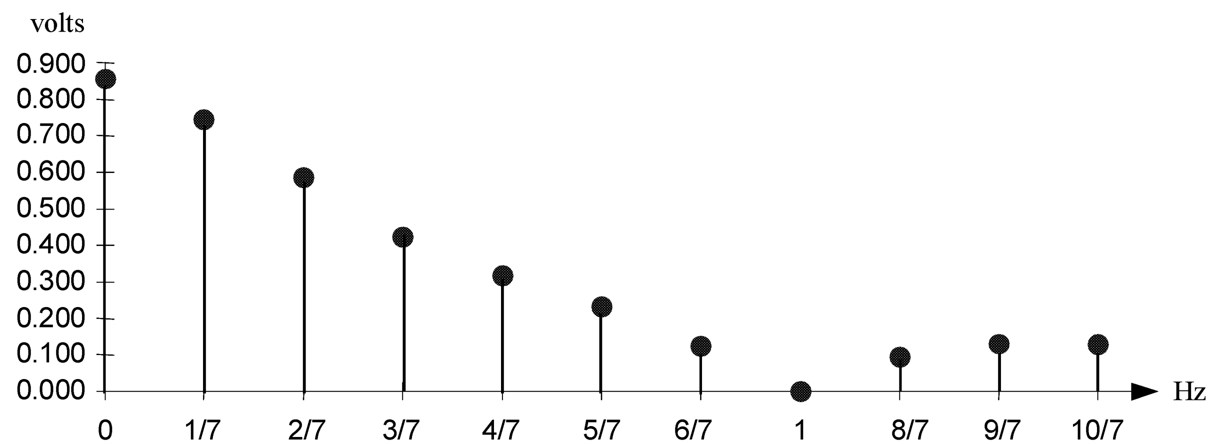
$$s(t) = X_0 + \sum_{n=1}^{\infty} X_n \cos(2\pi n f_o t + \phi_n)$$

$$= 0.857 + 0.744\cos\left(\frac{2\pi t}{7} + 0.857\pi\right) + 0.587\cos\left(\frac{4\pi t}{7} + 0.714\pi\right)$$

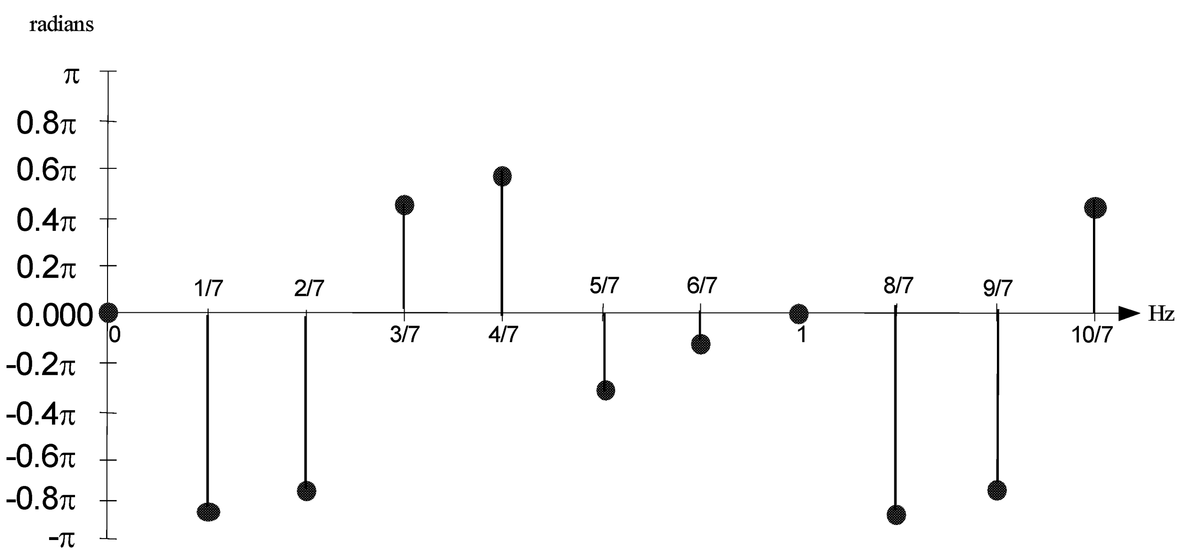
$$+ 0.424\cos\left(\frac{6\pi t}{7} - 0.429\pi\right) + 0.318\cos\left(\frac{8\pi t}{7} - 0.571\pi\right)$$

$$+ 0.235\cos\left(\frac{10\pi t}{7} + 0.286\pi\right) + \ldots$$

Magnitude Spectrum:



Phase Spectrum:



2.11 Prove Euler's identity (Hint: consider power series expansions of both the left-hand and right-hand sides of the identity).

The power series expansion for e^x *is* $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots$

$$\text{Therefore, } e^{j\omega t} = 1 + \frac{(j\omega t)}{1!} + \frac{(j\omega t)^2}{2!} + \frac{(j\omega t)^3}{3!} + \frac{(j\omega t)^4}{4!} + \frac{(j\omega t)^5}{5!} + \frac{(j\omega t)^6}{6!} + \ldots$$

$$= 1 + j\frac{\omega t}{1!} + j^2\frac{(\omega t)^2}{2!} + j^3\frac{(\omega t)^3}{3!} + j^4\frac{(\omega t)^4}{4!} + j^5\frac{(\omega t)^5}{5!} + j^6\frac{(\omega t)^6}{6!} + \ldots$$

$$= 1 + j\frac{\omega t}{1!} - \frac{(\omega t)^2}{2!} - j\frac{(\omega t)^3}{3!} + \frac{(\omega t)^4}{4!} + j\frac{(\omega t)^5}{5!} - \frac{(\omega t)^6}{6!} + \ldots$$

$$= \left[1 - \frac{(\omega t)^2}{2!} + \frac{(\omega t)^4}{4!} - \frac{(\omega t)^6}{6!} + \text{other even terms}\right]$$

Power series expansion for $\cos(\omega t)$

$$+ j\left[\frac{\omega t}{1!} - \frac{(\omega t)^3}{3!} + \frac{(\omega t)^5}{5!} + \text{other odd terms}\right]$$

Power series expansion for $\sin(\omega t)$

$$= \cos(\omega t) + j\sin(\omega t)$$

2.12 Prove that the trigonometric form of the Fourier series and complex exponential form of the Fourier series are equivalent.

This proof is found on pp. 23 - 25 of the textbook.

2.13 Express the waveform given in Figure P2-7 using the complex exponential Fourier series. Draw two-sided magnitude and phase spectra for the waveform.

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}$$

$$\text{where } c_n = \frac{1}{T}\int_{t_o}^{t_o+T} s(t)e^{-j2\pi n f_o t}\,dt = \frac{1}{7}\left[\int_0^1 0\,dt + \int_1^2 2e^{-j(2/7)\pi n t}\,dt + \int_2^4 e^{-j(2/7)\pi n t}\,dt + \int_4^5 2e^{-j(2/7)\pi n t}\,dt + \int_5^7 0\,dt\right]$$

$$= \frac{1}{7}\left[\left.\frac{-2e^{-j(2/7)\pi n t}}{j(2/7)\pi n}\right|_1^2 - \left.\frac{e^{-j(2/7)\pi n t}}{j(2/7)\pi n}\right|_2^4 - \left.\frac{2e^{-j(2/7)\pi n t}}{j(2/7)\pi n}\right|_4^5\right]$$

$$= \frac{1}{j2\pi n}\left[\left.-2e^{-j(2/7)\pi n t}\right|_1^2 - \left.e^{-j(2/7)\pi n t}\right|_2^4 - \left.2e^{-j(2/7)\pi n t}\right|_4^5\right]$$

$$= \frac{1}{j2\pi n}\left[2e^{-j(2/7)\pi n} - e^{-j(4/7)\pi n} + e^{-j(8/7)\pi n} - 2e^{-j(10/7)\pi n}\right]$$

Note that we could also calculate the two-sided Fourier series coefficients using the trigonometric Fourier series coefficients from the results of Problem 2.7 and remembering that

$$c_0 = a_0 \tag{2.15}$$

and for n > 0,

$$c_n = \frac{1}{2}(a_n - jb_n) \text{ and } c_{-n} = \frac{1}{2}(a_n + jb_n) \tag{2.21}$$

[*the equation numbers are from the textbook*].

As a quick check on our calculations, we can use the results of Problem 2.7 and note that for n > 0,

$$c_n = \frac{1}{2}(a_n - jb_n)$$

$$= \frac{1}{2}\left\{\begin{array}{l} \dfrac{1}{\pi n}\left[2\sin\left(\dfrac{10\pi n}{7}\right) - \sin\left(\dfrac{8\pi n}{7}\right) + \sin\left(\dfrac{4\pi n}{7}\right) - 2\sin\left(\dfrac{2\pi n}{7}\right)\right] \\ -j\dfrac{-1}{\pi n}\left[2\cos\left(\dfrac{10\pi n}{7}\right) - \cos\left(\dfrac{8\pi n}{7}\right) + \cos\left(\dfrac{4\pi n}{7}\right) - 2\cos\left(\dfrac{2\pi n}{7}\right)\right] \end{array}\right\}$$

$$= \frac{1}{2\pi n}\left\{\begin{array}{l} 2\left[\sin\left(\dfrac{10\pi n}{7}\right) + j\cos\left(\dfrac{10\pi n}{7}\right)\right] - \left[\sin\left(\dfrac{8\pi n}{7}\right) + j\cos\left(\dfrac{8\pi n}{7}\right)\right] \\ + \left[\sin\left(\dfrac{4\pi n}{7}\right) + j\cos\left(\dfrac{4\pi n}{7}\right)\right] - 2\left[\sin\left(\dfrac{2\pi n}{7}\right) + j\cos\left(\dfrac{2\pi n}{7}\right)\right] \end{array}\right\}$$

$$= \frac{1}{j2\pi n}\left\{\begin{array}{l} -2\left[\cos\left(\dfrac{10\pi n}{7}\right) - j\sin\left(\dfrac{10\pi n}{7}\right)\right] + \left[\cos\left(\dfrac{8\pi n}{7}\right) - j\sin\left(\dfrac{8\pi n}{7}\right)\right] \\ - \left[\cos\left(\dfrac{4\pi n}{7}\right) - j\sin\left(\dfrac{4\pi n}{7}\right)\right] + 2\left[\cos\left(\dfrac{2\pi n}{7}\right) - j\sin\left(\dfrac{2\pi n}{7}\right)\right] \end{array}\right\}$$

$$= \frac{1}{j2\pi n}\left[2e^{-j(2/7)\pi n} - e^{-j(4/7)\pi n} + e^{-j(8/7)\pi n} - 2e^{-j(10/7)\pi n}\right]$$

The two-sided magnitude and phase spectra are plotted on the next page. Note that we can either use the results from above to determine the phase and magnitude of each coefficient, or else we can use the one-sided Fourier Series coefficients calculated in Problem 2.10 and remember that

$$c_0 = X_0 \tag{2.23}$$

and for n > 0,

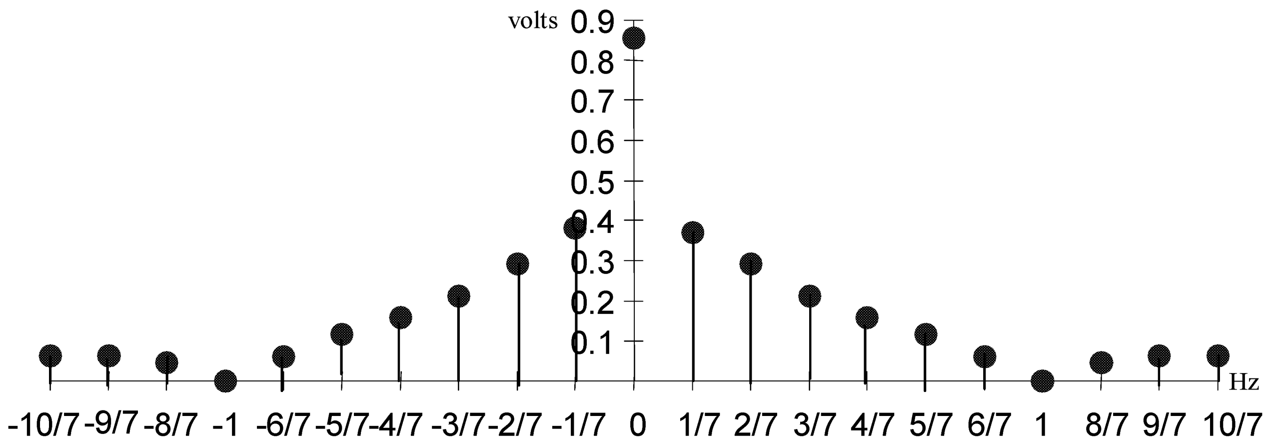
$$|c_n| = |c_{-n}| = \frac{X_n}{2} \tag{2.24}$$

$$\angle c_n = \phi_n \tag{2.25}$$

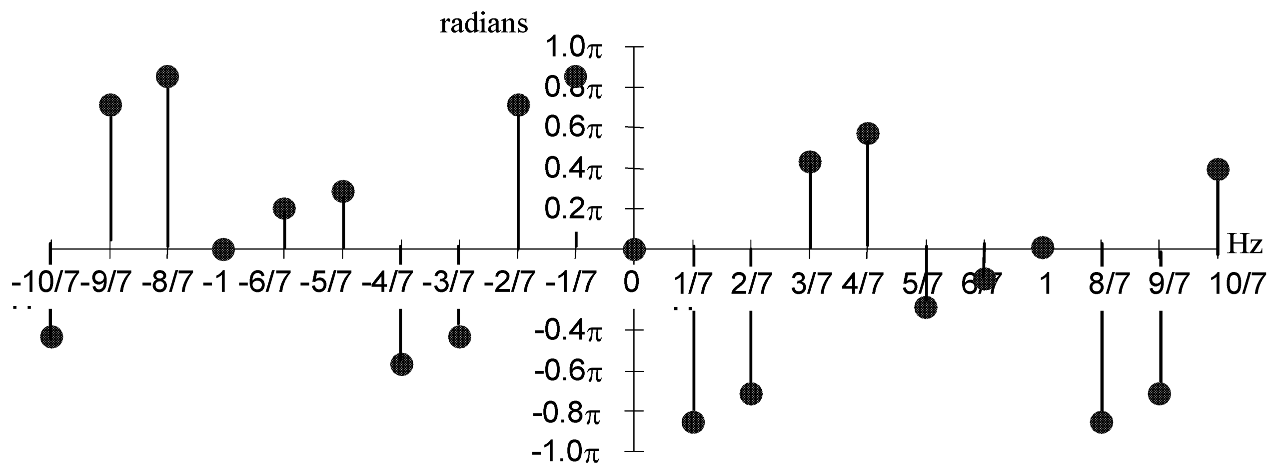
$$\angle c_{-n} = -\phi_n \tag{2.26}$$

[*the equation numbers are from the textbook*].

Magnitude Spectrum:



Phase Spectrum:



2.14 Discuss the relationship between the one-sided spectra (magnitude and phase) drawn in Problem 2.10 and the two-sided spectra drawn in Problem 2.13.

Let X_n and ϕ_n represent the magnitude and phase of the one-sided Fourier series coefficient relating to the nth harmonic. The frequency of the nth harmonic is nf_o. Let c_n represent the two-sided Fourier series coefficient whose magnitude and phase are plotted at nf_o.in the two-sided spectra. The one-sided and two-sided magnitude and phase spectra are related by

$$c_0 = X_0 \tag{2.23}$$

and for n > 0,

$$|c_n| = |c_{-n}| = \frac{X_n}{2} \tag{2.24}$$

$$\angle c_n = \phi_n \tag{2.25}$$

$$\angle c_{-n} = -\phi_n \tag{2.26}$$

[the equation numbers are from the textbook]. Graphically, the one-sided and two-sided magnitude spectra have the same dc component, the magnitude terms to the right of the dc component in the two-sided spectrum are only half as large as the corresponding terms in the one-sided magnitude spectrum, and the two-sided magnitude spectrum is symmetrical about the y-axis. Concerning the phase spectra, all terms to the right of the y-axis are the same for both the one-sided and two-sided spectra and the two-sided spectrum exhibits odd symmetry about the y axis. See page 29 in the textbook for an interpretation of the physical meaning of the terms to the left of the y-axis in the two-sided spectra.

2.15 Draw the two-sided amplitude spectrum for the waveform in Figure P2-15 for each of the following sets of parameters:

a. T = 50 milliseconds, τ = 10 milliseconds

b. T = 50 milliseconds, τ = 25 milliseconds

c. T = 50 milliseconds, τ = 5 milliseconds

d. T = 20 milliseconds, τ = 10 milliseconds

e. T = 100 milliseconds, τ = 10 milliseconds

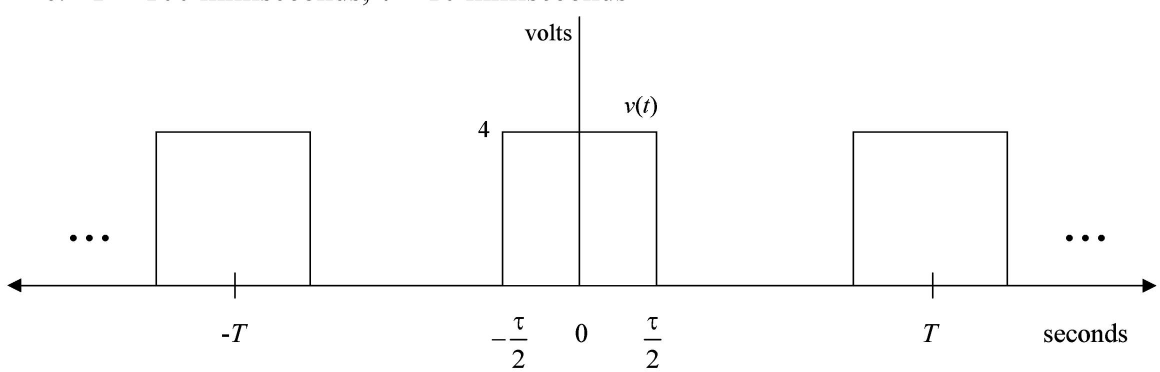


Figure P2-15

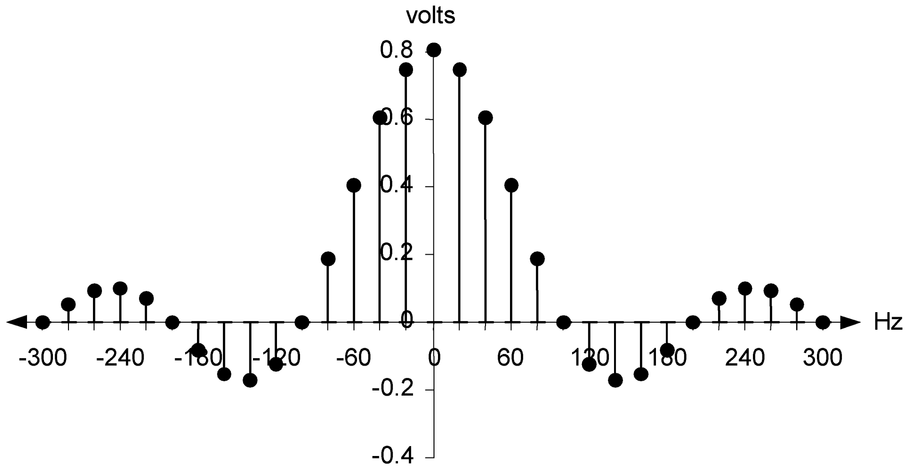
a. *As established in Example 2.6, the two-sided Fourier series representation of a rectangular pulse train with amplitude A, period T, and pulse width τ is*

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t} \quad \text{where } c_n = \left(\frac{A\tau}{T}\right)\text{sinc}\left(\frac{\pi n \tau}{T}\right)$$

Be sure you understand Example 2.6. For the pulse train in Part a, A = 4 volts, T = 50 milliseconds, and τ = 10 milliseconds. Substituting in,

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j40\pi nt} \quad \text{where} \quad c_n = 0.8\text{sinc}(0.2n\pi)$$

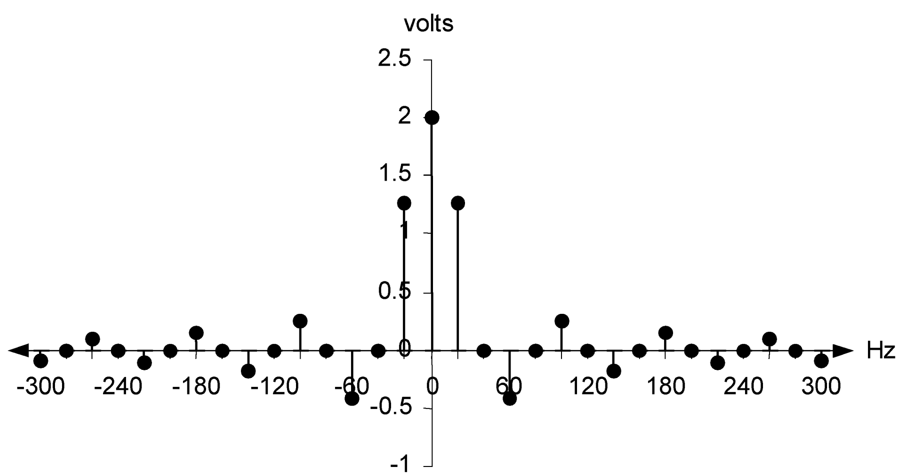
The amplitude spectrum is plotted below.



b. *A = 4 volts, T = 50 milliseconds, and τ = 25 milliseconds, so*

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j40\pi nt} \quad \text{where} \quad c_n = 2\text{sinc}(0.5n\pi)$$

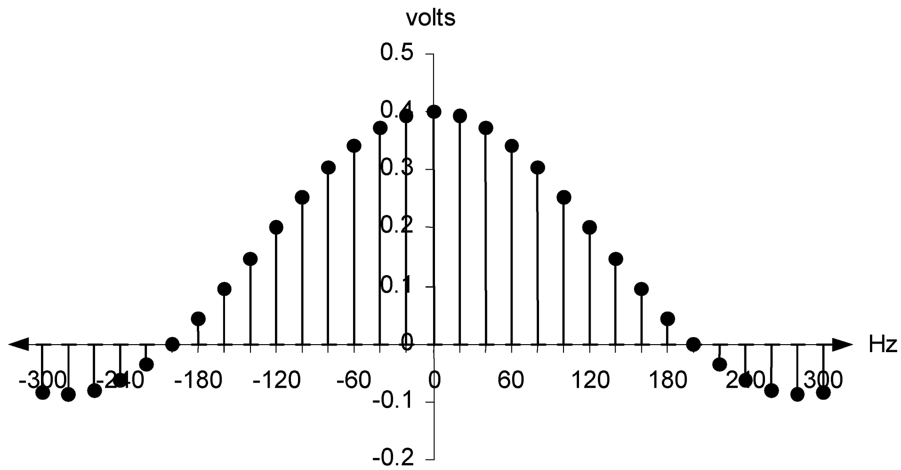
The amplitude spectrum is plotted below.



c. *A = 4 volts, T = 50 milliseconds, and τ = 5 milliseconds, so*

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j40\pi nt} \quad \text{where} \quad c_n = 0.4\text{sinc}(0.1n\pi)$$

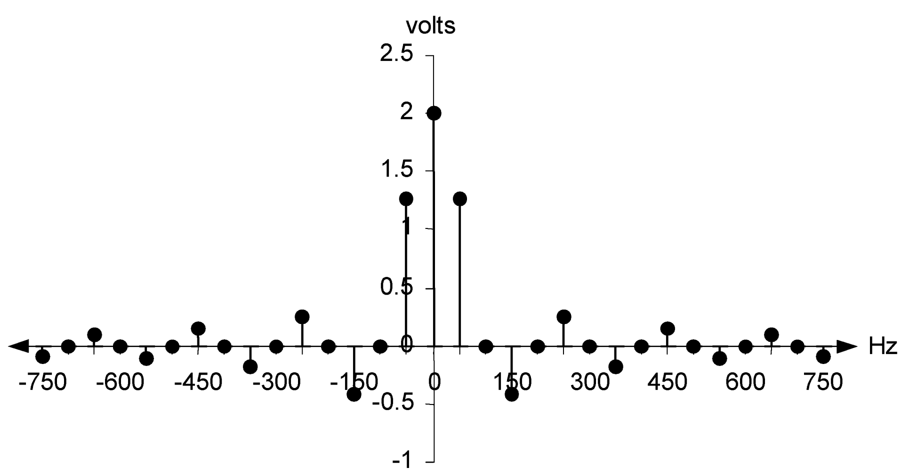
The amplitude spectrum is plotted below.



d. *A = 4 volts, T = 20 milliseconds, and τ = 10 milliseconds, so*

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j100\pi nt} \quad \text{where} \quad c_n = 2\text{sinc}(0.5n\pi)$$

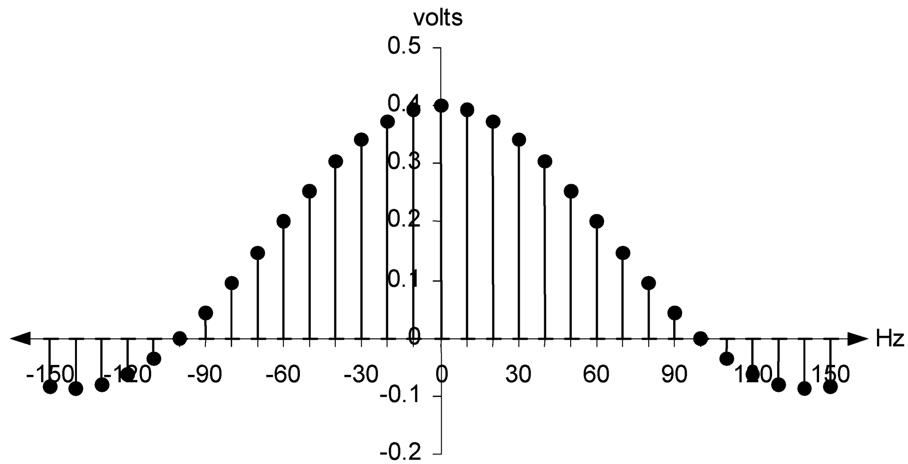
The amplitude spectrum is plotted below. Compare this spectrum to the spectrum in Part b and be sure you understand the reasons for the differences.



e. *A = 4 volts, T = 100 milliseconds, and τ = 10 milliseconds,*

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j20\pi nt} \quad \text{where} \quad c_n = 0.4\text{sinc}(0.1n\pi)$$

The amplitude spectrum is plotted below. Compare this spectrum to the spectrum in Part c and be sure you understand the reasons for the differences.



2.16 Discuss the advantages and disadvantages of each of the three forms of the Fourier series.

The trigonometric form is the most basic of the three forms and is very intuitive because we see that the series is a summation of sinusoids. Unfortunately, because there are two terms at each frequency, we cannot easily generate magnitude and phase spectra. The one-sided form still expresses the series as a summation of sinusoids, and thus retains intuition. Furthermore, since there is only one term at each frequency, we can now directly plot magnitude and phase spectra, which provide us with graphical tools that help us visualize the behavior of the signal. The disadvantage of the one-sided form is that the dc term must be calculated in a different way than the other terms. The major disadvantage of the two-sided form is that it is nonintuitive, both because it uses complex exponentials and because it introduces the abstract mathematical concept of "negative" frequency. The advantages of the two-sided form are that the dc component is calculated in the same way as all the other components, mathematical manipulation of exponentials is easier than mathematical manipulation of sines and cosines, and the two-sided form can be conceptually extended (the Fourier transform) to include nonperiodic waveforms.

2.17 Prove that average normalized power can also be expressed in the frequency domain as

$$P_s = \sum_{n=-\infty}^{\infty} \left| c_n \right|^2$$

Hint: First show that $P_s = \sum_{n=-\infty}^{\infty} c_n c_{-n}$ and then show that for all values of n, $c_n c_{-n} = \left| c_n \right|^2$

$$P_s = \frac{1}{T}\int_{t_o}^{t_o+T} s^2(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T}\left(\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}\right)^2 dt$$

$$= \frac{1}{T}\int_{t_o}^{t_o+T}\left[\left(\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}\right)\left(\sum_{m=-\infty}^{\infty} c_m e^{j2\pi m f_o t}\right)\right]dt$$

(Note that the second index has been changed from n to m in order to preserve the cross terms.)

$$= \frac{1}{T}\int_{t_o}^{t_o+T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_n e^{j2\pi n f_o t} c_m e^{j2\pi m f_o t} dt$$

$$= \frac{1}{T}\int_{t_o}^{t_o+T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_n c_m e^{j2\pi (n+m) f_o t} dt$$

$$= \frac{1}{T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_n c_m \int_{t_o}^{t_o+T} e^{j2\pi (n+m) f_o t} dt$$

$$= \frac{1}{T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_n c_m \left\{\int_{t_o}^{t_o+T}\cos 2\pi (n+m) f_o t dt + j\int_{t_o}^{t_o+T}\sin 2\pi (n+m) f_o t dt\right\}$$

Since we are integrating over a whole number of cycles of the sinusoid, the second integral is zero for all values of n and m and the first integral is zero for all values except m = –n. The equation thus reduces to

$$P_s = \frac{1}{T}\sum_{n=-\infty}^{\infty} c_n c_{-n}\int_{t_o}^{t_o+T}\cos(0)dt = \sum_{n=-\infty}^{\infty} c_n c_{-n} = \sum_{n=-\infty}^{\infty} c_n c_n^* = \sum_{n=-\infty}^{\infty}|c_n|^2$$

2.18 In Problem 2.15, you drew the two-sided amplitude spectrum for the Figure P2-15 waveform for each of the following sets of parameters:

a. T = 50 milliseconds, τ = 10 milliseconds

b. T = 50 milliseconds, τ = 25 milliseconds

c. T = 50 milliseconds, τ = 5 milliseconds

d. T = 20 milliseconds, τ = 10 milliseconds

e. T = 100 milliseconds, τ = 10 milliseconds

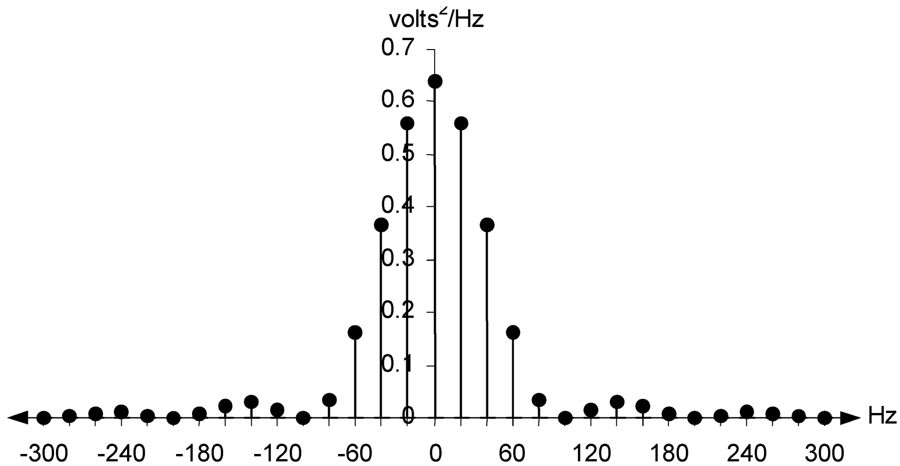
For each of the waveforms in Problem 2.15 a – e,

1. Calculate the average normalized power using the time domain.
2. Draw the two-sided average normalized power spectrum
3. Determine the bandwidth of the waveform (define bandwidth as the distance from dc (0 Hz) to the first zero crossing of the power spectrum).
4. Determine the percentage of each waveform's power which is contained within its bandwidth.

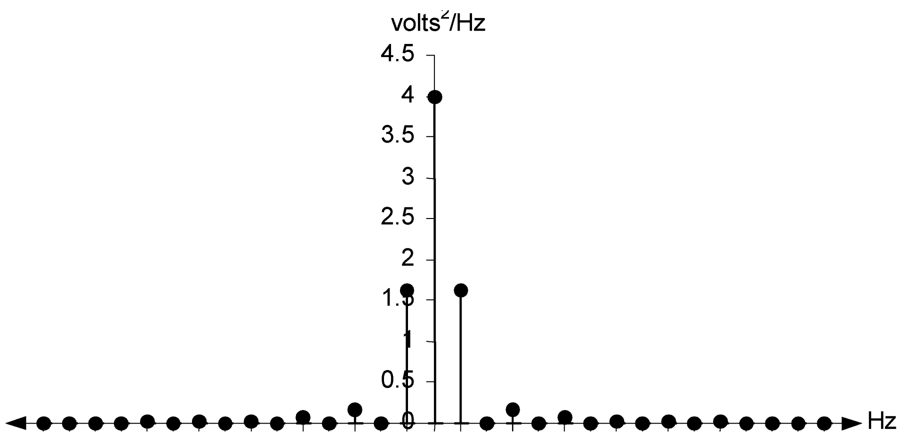
1. $P_v = \frac{1}{T}\int_{t_o}^{t_0+T} v^2(t)dt = \frac{1}{T}\int_{-\tau/2}^{\tau/2} 4^2\,dt = \frac{16\tau}{T}\ volts^2$

   For Part a, $P_v = 3.2\ volts^2$
   For Part b, $P_v = 8\ volts^2$
   For Part c, $P_v = 1.6\ volts^2$
   For Part d, $P_v = 8\ volts^2$
   For Part e, $P_v = 1.6\ volts^2$

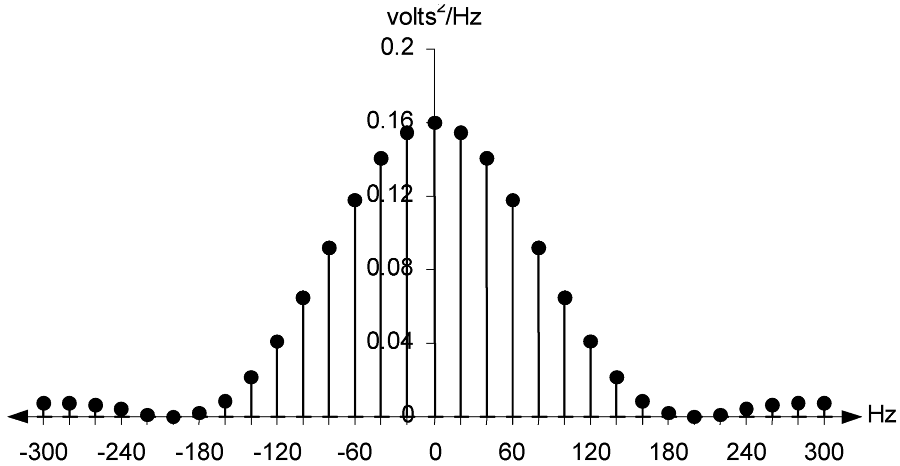
2. *Average normalized power spectrum for Part a:*



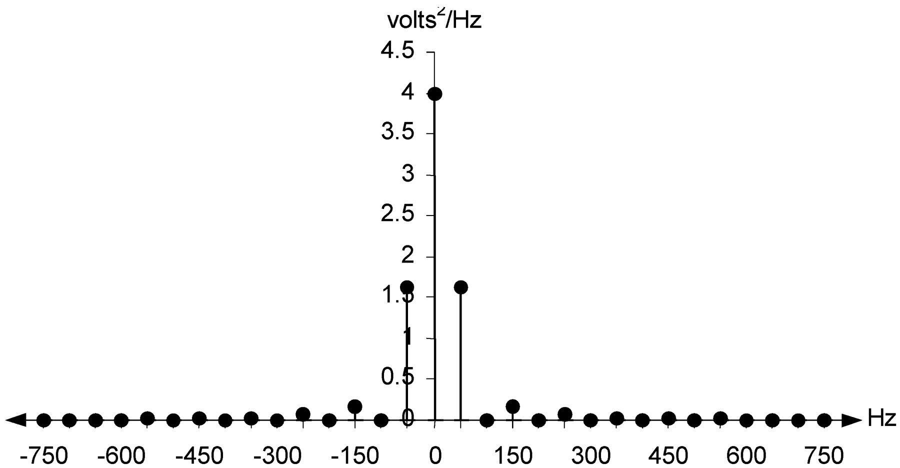
   Average normalized power spectrum for Part b:



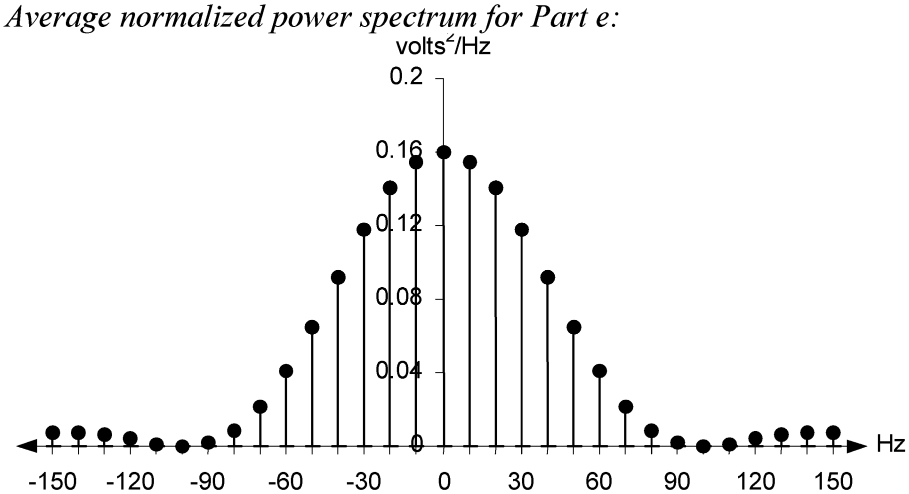
Average normalized power spectrum for Part c:



Average normalized power spectrum for Part d:



Average normalized power spectrum for Part e:



3. *For Part a, bandwidth = 100 Hz*
   For Part b, bandwidth = 40 Hz
   For Part c, bandwidth = 200 Hz
   For Part d, bandwidth = 100 Hz
   For Part e, bandwidth = 100 Hz

4. *For Part a, avg. normalized power within bandwidth =* $\sum_{n=-5}^{5} |c_n|^2 = 2.889\ volts^2$,

   so the percentage of power within the bandwidth = $\dfrac{2.889\ volts^2}{3.2\ volts^2} = 90.3\%$

   For Part b, avg. normalized power within bandwidth = $\sum_{n=-2}^{2} |c_n|^2 = 7.242\ volts^2$,

   so the percentage of power within the bandwidth = $\dfrac{7.242\ volts^2}{8\ volts^2} = 90.5\%$

   For Part c, avg. normalized power within bandwidth = $\sum_{n=-10}^{10} |c_n|^2 = 1.445\ volts^2$,

   so the percentage of power within the bandwidth = $\dfrac{1.445\ volts^2}{1.6\ volts^2} = 90.3\%$

   For Part d, the percentage of power within the bandwidth is the same as Part b (even though the spectrum has a much wider bandwidth than Part b). Be sure you see why. For Part e, the percentage of power within the bandwidth is the same as Part c (even though the bandwidth is different). Again, be sure you see why.

2.19 Explain why, when using the two-sided Fourier series, the "negative" frequency components must be included when performing power calculations but not when determining bandwidth.

These conventions (including the negative frequency components when performing power calculations but not when determining bandwidth) are necessary to make the results of power and bandwidth calculations using the two-sided Fourier series the same as the results of power and bandwidth calculations using the one-sided Fourier series. This equality is essential, since changing the mathematical representation of a signal (i.e., changing the form of the Fourier series) cannot change the physical characteristics of the signal. As discussed in the textbook, we have good, physical understanding and intuition concerning the bandwidth and power calculations performed using the one-sided Fourier Series. The two-sided form is used to derive the Fourier transform for nonperiodic, finite energy signals.

2.20 Why is orthogonality of the Fourier series components important when representing power in the frequency domain?

Orthogonality allows us to easily determine how average normalized power is distributed in the frequency domain because it eliminates the effects of the cross terms *when we square the frequency domain representation of the waveform (the squaring is necessary to calculate average normalized power). Thus, given an amplitude or magnitude spectrum for a waveform, we can easily draw the average normalized power spectrum for the waveform. As demonstrated in Problems 2.4 and 2.5, orthogonality lies at the very core of the Fourier series and the determination of the Fourier series coefficients.*

2.21 Draw the time domain waveform which corresponds to the amplitude spectrum given in Figure P2-21.

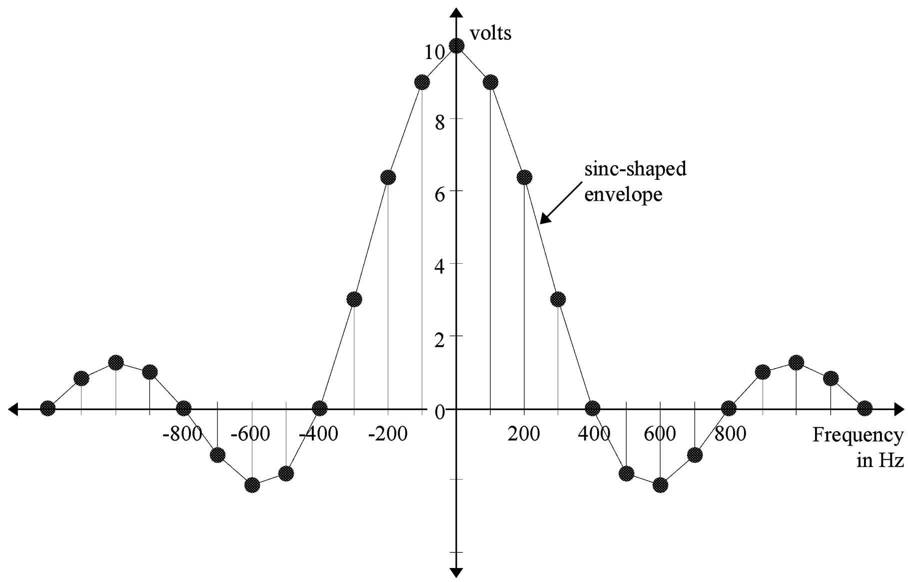


Figure P2-21

The time domain waveform is a rectangular pulse train whose period is $T = 1/f_o$.

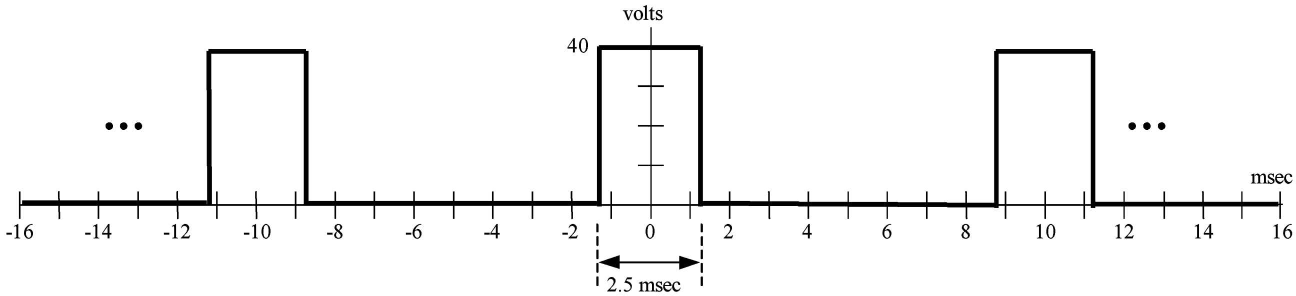
By inspection, $f_o = 100$ Hz, so $T = 10$ milliseconds.

The width of the rectangular pulses (τ) is the inverse of the first zero crossing of the amplitude spectrum, so

$$\tau = \frac{1}{400} = 2.5 \text{ milliseconds.}$$

Finally, the dc term of the amplitude spectrum is equal to $(A\tau)/T$, where A is the amplitude of the rectangular pulse. Solving, $A = \frac{10(.01)}{(.0025)} = 40$ volts.

Sketching the time domain waveform,



2.22 For the waveform in Figure P2-21, determine the bandwidth and the percentage of the waveform's power that lies within the bandwidth. Define the bandwidth of the waveform as the distance from dc (0 Hz) to the first zero crossing in the waveform's power spectrum.

By inspection, bandwidth = 400 Hz.

Average normalized power $= \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$.

Calculating average normalized power will be simpler using the time domain expression. In Problem 2.21 we established that the time domain representation of the Figure P2-21 amplitude spectrum is a rectangular pulse train centered at 0 with an amplitude of 40 volts, a pulse width of 2.5 msec, and a pulse period of 10 msec. Substituting in,

Average normalized power $= \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt = \frac{1}{.01}\int_{-.00125}^{.00125} 40^2\,dt = \frac{40^2}{.01}(.0025) = 400 \text{ volts}^2$

To calculate average normalized power within the 400 Hz bandwidth, we need to use the frequency domain expression for the waveform. The expression for the amplitude spectrum in Figure 2-21 is

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j200\pi nt} \quad \text{where} \quad c_n = 10\text{sinc}(0.25n\pi)$$

Substituting in, average normalized power within the bandwidth =

$$\sum_{n=-4}^{4} |c_n|^2 = 0^2 + 3.00^2 + 6.37^2 + 9.00^2 + 10^2 + 9.00^2 + 6.37^2 + 3.00^2 + 0^2 = 361.2 \text{ volts}^2$$

Percentage of power within the signal's bandwidth $= \frac{361.2}{400}(100\%) = 90.3\%$

2.23 Determine the frequency domain expression for each of the nonperiodic waveforms shown in Figure P2-23 and draw their magnitude and phase spectra.

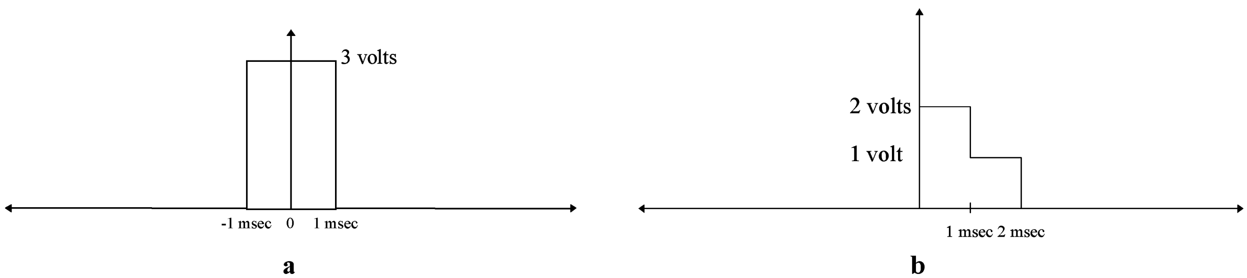
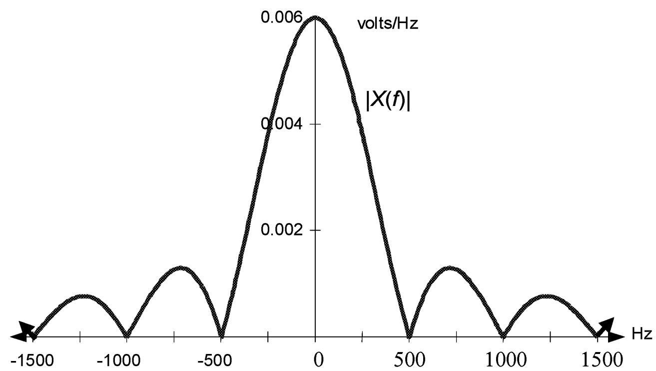


Figure P2-23

Figure P2-23a:

$$X(f) = A\tau \text{sinc}(\pi f\tau) = (3)(.002)\text{sinc}[\pi f(.002)] = 0.006\text{sinc}(.002\pi f) \text{ volts}$$

Magnitude Spectrum:



Phase spectrum:

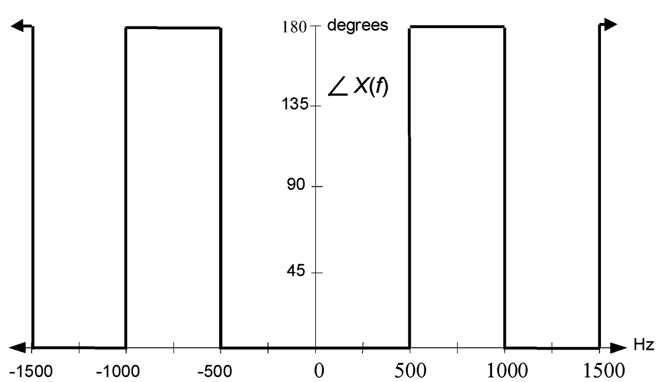


Figure P2-23b:

$$X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt=\int_{0}^{0.001}2e^{-j2\pi ft}dt+\int_{0.001}^{0.002}e^{-j2\pi ft}dt$$

$$=\frac{1}{j\pi f}\left(1-0.5e^{-j0.002\pi f}-0.5e^{-j0.004\pi f}\right)volts/Hz$$

To plot magnitude and phase spectra, let's convert to sines and cosines using Euler's identity.

$$X(f)=\frac{1}{j\pi f}\left(1-0.5e^{-j0.002\pi f}-0.5e^{-j0.004\pi f}\right)volts/Hz$$

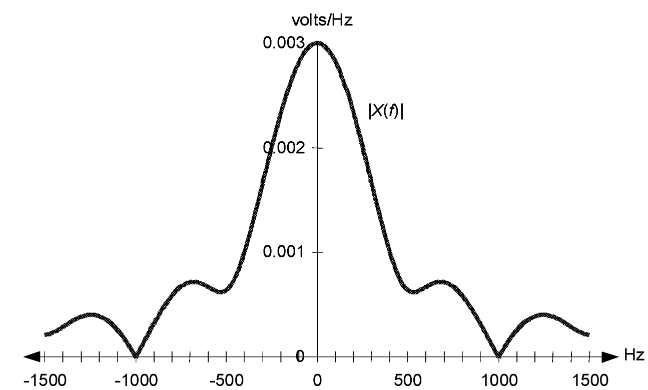
$$=\frac{1}{j\pi f}\{1-0.5[\cos(-.002\pi f)+j\sin(-.002\pi f)]-0.5[\cos(-.004\pi f)+j\sin(-.004\pi f)]\}$$

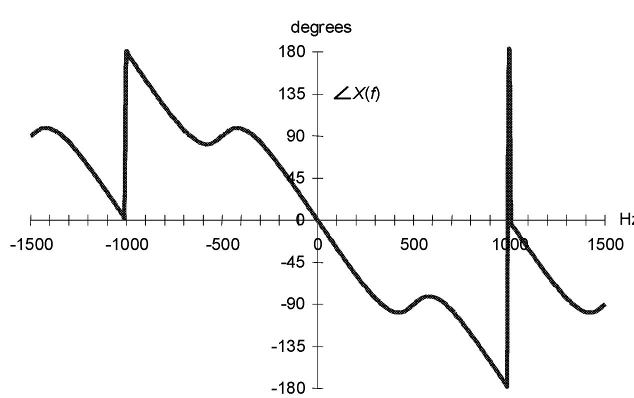
$$=\frac{1}{\pi f}\{0.5\sin(.002\pi f)+0.5\sin(.004\pi f)+j[-1+0.5\cos(.002\pi f)+0.5\cos(.004\pi f)]\}$$

$$|X(f)|=\left|\frac{1}{\pi f}\right|\sqrt{[0.5\sin(.002\pi f)+0.5\sin(.004\pi f)]^2+[-1+0.5\cos(.002\pi f)+0.5\cos(.004\pi f)]^2}$$

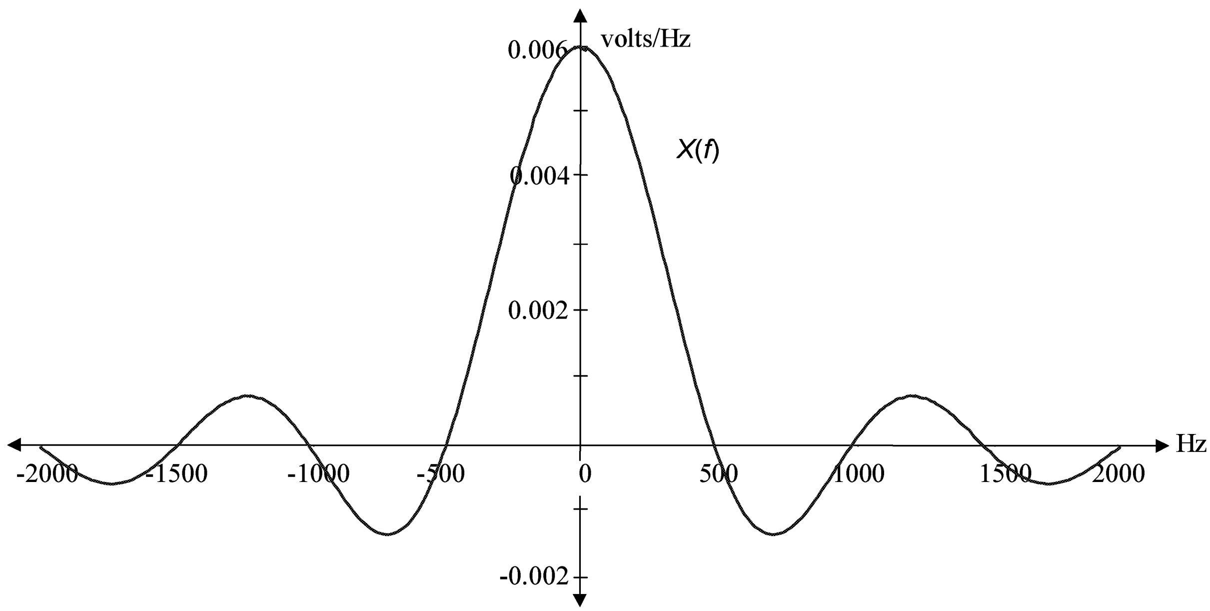
$$\angle X(f)=\arctan\left\{\frac{-1+0.5\cos(.002\pi f)+0.5\cos(.004\pi f)}{0.5\sin(.002\pi f)+0.5\sin(.004\pi f)}\right\}$$

Plotting magnitude and phase spectra:





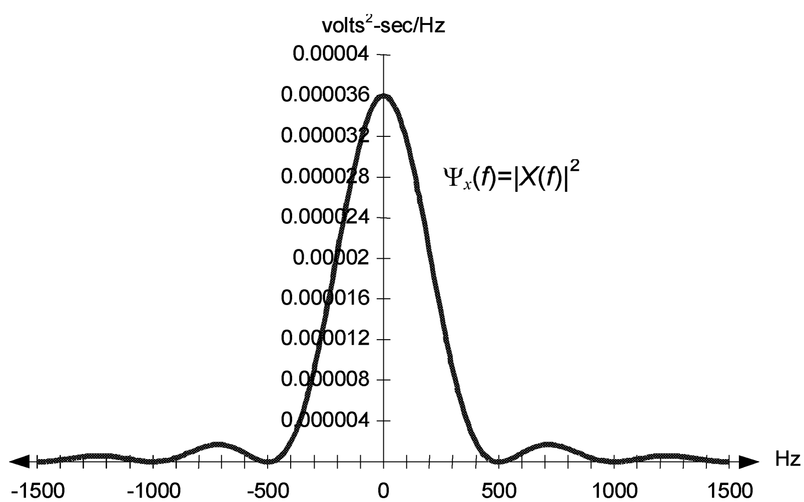
2.24 Draw an amplitude spectrum for the rectangular waveform shown above in Figure P2-23a. Why is this spectrum possibly more useful than phase and magnitude spectra? Are there any restrictions on what type of waveform can be represented using an amplitude spectrum?



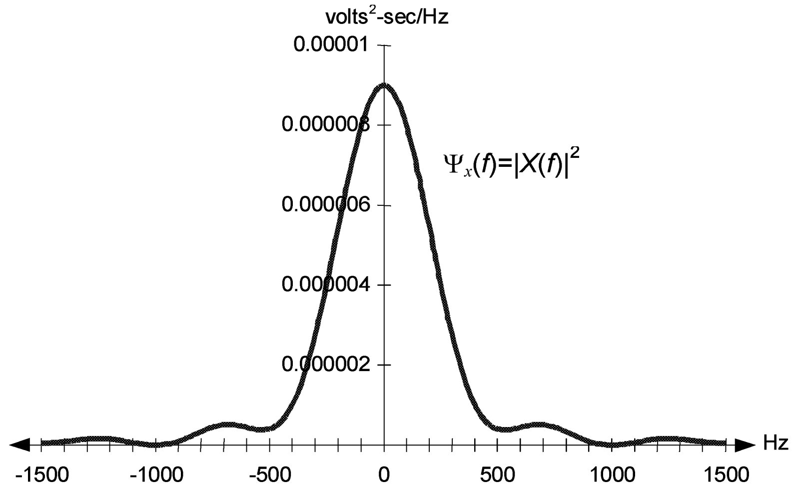
Amplitude spectra can only be drawn whenever all the Fourier series coefficients are real. When it can be drawn, the amplitude spectrum is often more useful than the magnitude and phase spectra because it contains both magnitude and phase information on the same plot.

2.25 Draw the normalized energy spectrum for the waveforms in Figures P2-23a and b.

Normalized energy spectrum for Figure 2-23a:



Normalized energy spectrum for Figure 2-23b:



2.26 Suppose the two waveforms in Figure P2-23a and b are passed through a filter with the transfer function shown in Figure P2-26. For each of the two waveforms, draw the magnitude spectrum and normalized energy spectrum at the filter's output. Additionally, calculate the normalized energy at the filter output for each of the two waveforms (you may leave your expression in integral form).

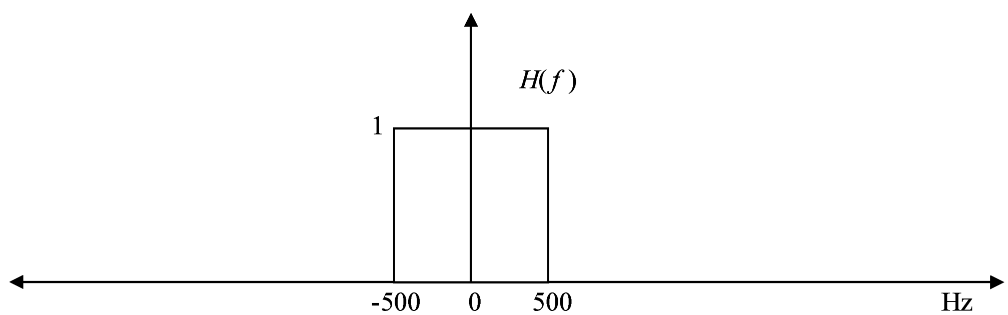
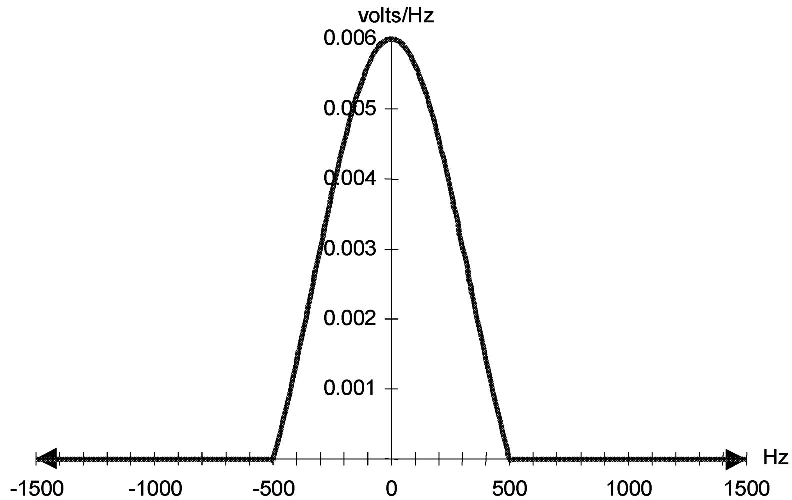


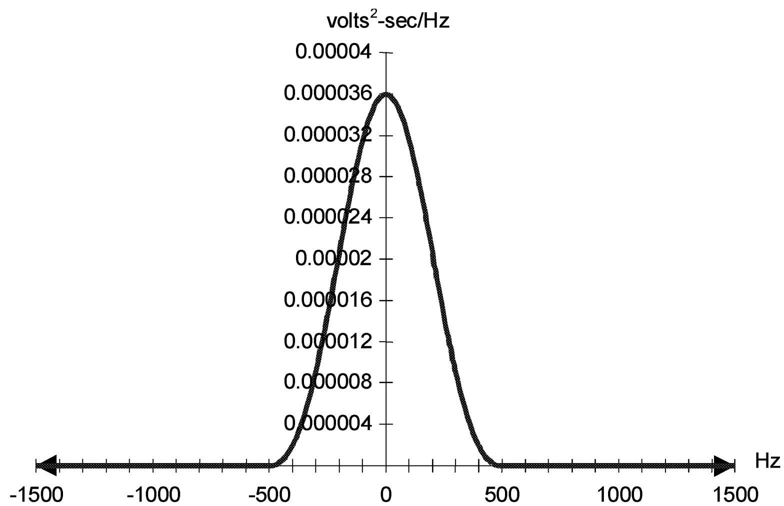
Figure P2-26

(continued on next page)

Magnitude spectrum forFigure P2-23a after filtering:

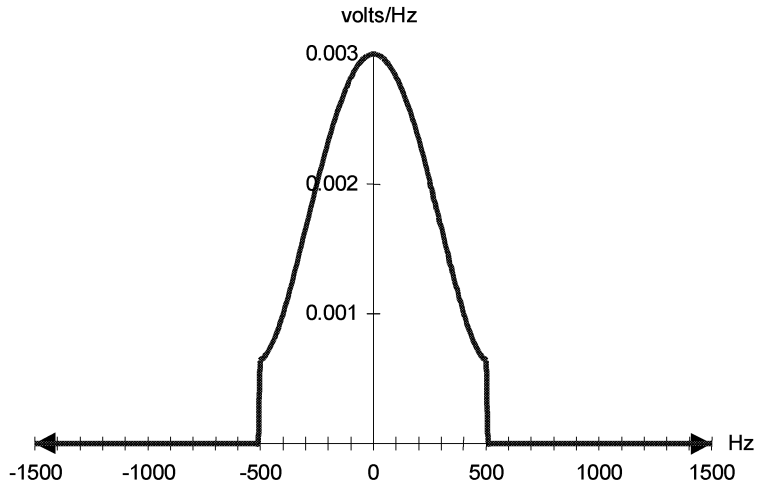


Normalized energy spectrum for Figure 2-23a after filtering:

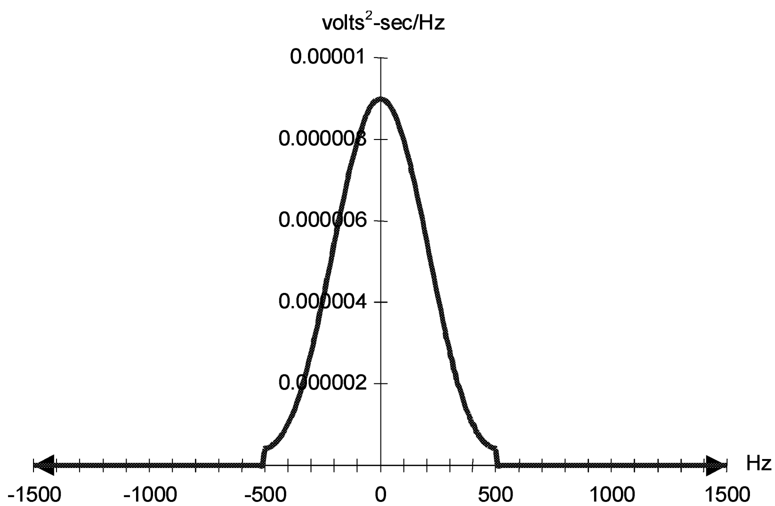


(continued on next page)

Magnitude spectrum forFigure P2-23b after filtering:



Normalized energy spectrum for Figure 2-23b after filtering:



Normalized energy in the Figure P2-23a waveform after filtering:

$$E_{after\ filtering} = \int_{-\infty}^{\infty} \Psi_x(f)\left|H(f)\right|^2 df = \int_{-500}^{500} \Psi_x(f)df = \int_{-500}^{500} 0.006^2 \operatorname{sinc}^2(.002\pi f)df$$

Normalized energy in the Figure P2-23b waveform after filtering:

$$E_{after\ filtering} = \int_{-\infty}^{\infty} \Psi_x(f)\left|H(f)\right|^2 = \int_{-500}^{500} \Psi_x(f)df$$

$$= \left(\frac{1}{\pi f}\right)^2 \int_{-500}^{500} \left[0.5\sin(.002\pi f) + 0.5\sin(.004\pi f)\right]^2 + \left[-1 + 0.5\cos(.002\pi f) + 0.5\cos(.004\pi f)\right]^2 df$$

2.27 Repeat Problem 2.26 using a filter with the transfer function shown in Figure P2-27.

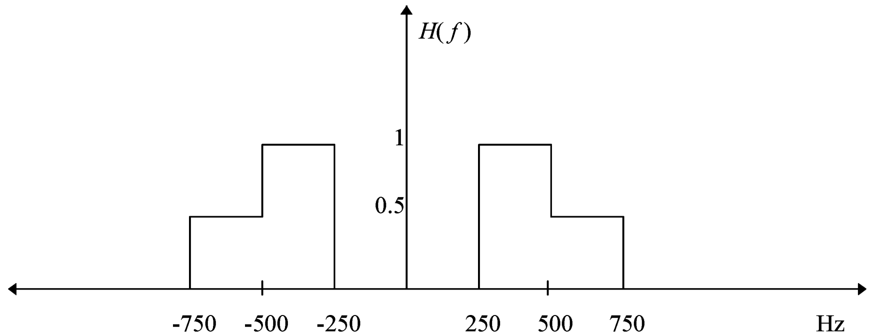
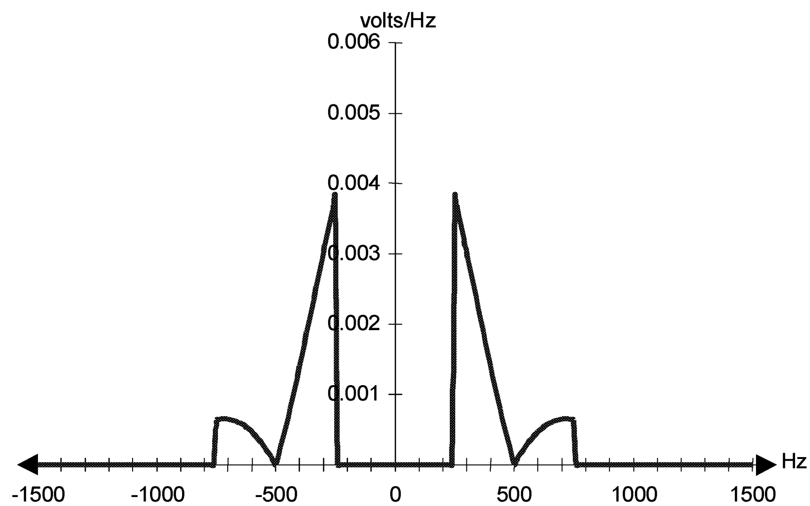
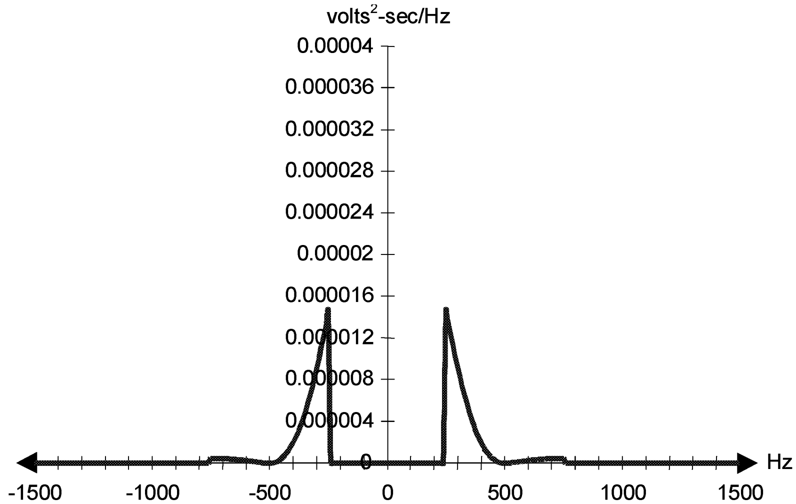


Figure P2-27

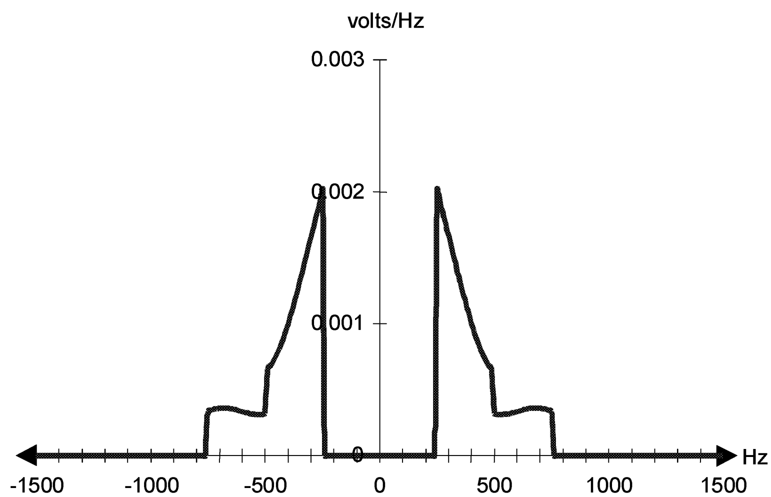
Magnitude spectrum forFigure P2-23a after filtering:



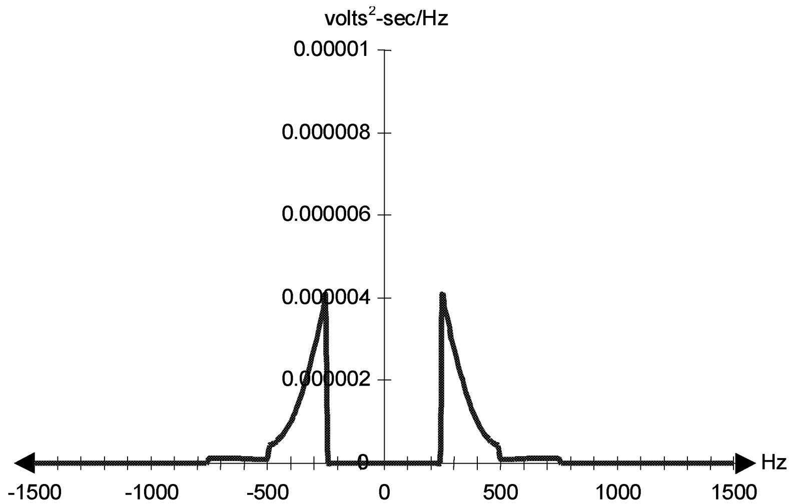
Normalized energy spectrum for Figure 2-23a after filtering (same vertical scale as used in Problems 2-23 and 2-26):



Magnitude spectrum forFigure P2-23b after filtering:



Normalized energy spectrum for Figure 2-23b after filtering (same vertical scale as used in Problems 2-23 and 2-26):



Normalized energy in the Figure P2-23a waveform after filtering:

$$E_{after\ filtering} = \int_{-\infty}^{\infty} \Psi_x(f)|H(f)|^2 df = \int_{-750}^{-500} 0.25\Psi_x(f)df + \int_{-500}^{-250} \Psi_x(f)df + \int_{250}^{500} \Psi_x(f)df + \int_{500}^{750} 0.25\Psi_x(f)df$$

$$= 2\left\{ \int_{-750}^{-500} 0.003^2 \operatorname{sinc}^2(.002\pi f)df + \int_{-500}^{-250} 0.006^2 \operatorname{sinc}^2(.002\pi f)df \right\}$$

Normalized energy in the Figure P2-23b waveform after filtering:

$$E_{after\ filtering} = \int_{-\infty}^{\infty} \Psi_x(f)|H(f)|^2 df = \int_{-750}^{-500} 0.25\Psi_x(f)df + \int_{-500}^{-250} \Psi_x(f)df + \int_{250}^{500} \Psi_x(f)df + \int_{500}^{750} 0.25\Psi_x(f)df$$

$$= 2\left(\frac{0.5}{\pi f}\right)^2 \int_{-750}^{-500} [0.5\sin(.002\pi f) + 0.5\sin(.004\pi f)]^2 + [-1 + 0.5\cos(.002\pi f) + 0.5\cos(.004\pi f)]^2 df$$

$$+ 2\left(\frac{1}{\pi f}\right)^2 \int_{-500}^{-250} [0.5\sin(.002\pi f) + 0.5\sin(.004\pi f)]^2 + [-1 + 0.5\cos(.002\pi f) + 0.5\cos(.004\pi f)]^2 df$$

2.28 Using the Fourier series as a starting point, derive the Fourier transform for a nonperiodic waveform with finite energy.

The derivation is performed in Section 2.4 of the textbook.

2.29 Prove the following property of the Fourier transform:

For any nonzero constant a,

$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

(Hint: perform separate proofs for $a > 0$ and $a < 0$.)

Consider $\alpha > 0$. By change of variable, let $\gamma = \alpha t$; $d\gamma = \alpha dt$

$$\begin{aligned}\Im\{x(\alpha t)\} &= \int_{-\infty}^{\infty} x(\alpha t) e^{-j2\pi f t} dt \\ &= \frac{1}{\alpha}\int_{-\infty}^{\infty} x(\gamma) e^{-j2\pi (f/\alpha)\gamma} d\gamma \\ &= \frac{1}{\alpha} X\left(\frac{f}{\alpha}\right)\end{aligned}$$

Now consider let $\alpha < 0$. By change of variable, let $\gamma = \alpha t$; $d\gamma = \alpha dt$

$$\Im\{x(\alpha t)\} = \int_{-\infty}^{\infty} x(\alpha t) e^{-j2\pi f t} dt$$

$$= \frac{1}{\alpha}\int_{\infty}^{-\infty} x(\gamma) e^{-j2\pi (f/\alpha)\gamma} d\gamma \quad (\textit{The integral limits flip because as } t \to -\infty, \gamma \to \infty \textit{ and vice-versa})$$

$$= -\frac{1}{\alpha}\int_{-\infty}^{\infty} x(\gamma) e^{-j2\pi (f/\alpha)\gamma} d\gamma$$

$$= -\frac{1}{\alpha} X\left(\frac{f}{\alpha}\right)$$

Thus,

$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

2.30 Determine the Fourier transform of the signal $s(t) = 3\cos(2000\pi t) + 5e^{-|t|}$ volts and draw its magnitude spectrum

solution on next page

$$S(f) = \Im\left\{3\cos(2000\pi t) + 5e^{-|t|}\right\} = \Im\left\{3\cos(2000\pi t)\right\} + \Im\left\{5e^{-|t|}\right\}$$

$$\Im\left\{3\cos(2000\pi t)\right\} = \frac{3}{2}\delta(f+1000) + \frac{3}{2}\delta(f-1000) \text{ volts/Hz}$$

$$\Im\left\{5e^{-|t|}\right\} = \int_{-\infty}^{\infty} 5e^{-|t|}e^{-j2\pi ft}dt = \int_{-\infty}^{0} 5e^{t}e^{-j2\pi ft}dt + \int_{0}^{\infty} 5e^{-t}e^{-j2\pi ft}dt$$

$$= \int_{-\infty}^{0} 5e^{(1-j2\pi f)t}dt + \int_{0}^{\infty} 5e^{(-1-j2\pi f)t}dt$$

$$= \frac{5}{1-j2\pi f}e^{(1-j2\pi f)t}\Big|_{-\infty}^{0} + \frac{5}{-1-j2\pi f}e^{(-1-j2\pi f)t}\Big|_{0}^{\infty}$$

$$= \frac{5}{1-j2\pi f} - \frac{5}{-1-j2\pi f}$$

$$= \frac{5(-1-j2\pi f) - 5(1-j2\pi f)}{-1-(2\pi f)^2}$$

$$= \frac{10}{1+(2\pi f)^2}$$

$$S(f) = \frac{3}{2}\delta(f+1000) + \frac{3}{2}\delta(f-1000) + \frac{10}{1+(2\pi f)^2} \text{ volts/Hz}$$

Magnitude spectrum of S(f):