True–False. Mark as true any statement that is always true. Mark as false any statement that is never true or that is not necessarily true. Be able to justify your answers.

1. An "exercise" can be solved by simply applying a routine procedure, but a "problem" is not routine and requires a well-thought-out plan of attack.
2. If there are infinitely many numbers involved in a problem, the Guess and Test problem-solving strategy is indicated.
3. If 12 coins are arranged in the shape of an equilateral triangle, then there are 5 coins along each side.
4. If the fourth phase of problem solving, "Looking Back," we should look for other ways to solve the problem even if we got the 'right' answer.
5. When you are asked to make a generalization, you should use the Guess and Test problem-solving strategy.
6. If a problem involves a large array or diagram, a good strategy to try is "Use a Variable."
7. If a problem involves a physical situation, the "Draw a Picture" strategy may be appropriate.
8. If a problem involves a sequence of numbers or figures, the "Draw a Picture" strategy may be appropriate.
9. If a problem asks you to make a prediction or generalization, the "Look for a Pattern" strategy may be appropriate.
10. If a problem asks "in how many ways" the "Look for a Pattern" strategy may be appropriate.
11. The fourth term in the sequence 3, 5, 7, . . . must be 9.
12. If a large cube is made up of 64 smaller cubes, then it has 16 cubes showing on each face of the larger cube.
13. Patterns in data may be easier to see if we arrange the data in tables.

Multiple Choice. Mark the letter of the single BEST response. Be sure to read all the choices for each problem before deciding.

14. George Polya presented a four step process for problem solving which is still used extensively today. Which of the following is NOT a step in his basic process?
    (a) Devise a plan.
    (b) Understand the problem.
    (c) Carry out your plan.
    (d) Draw picture.
    (e) Look back to analyze your solution.
15. The Guess and Test problem-solving strategy can be useful when
    (a) there are only a few possible solutions to a problem.
    (b) the problem suggests an equation.
    (c) you are trying to develop a formula.
    (d) you need to find the measures of the angles in a triangle.
    (e) there is an unknown quantity related to known quantities

16. You may wish to use the "Solve a Simpler Problem" strategy if:
    (a) The problem involves very large or very small numbers.
    (b) There are a large number of cases.
    (c) A direct solution is too complex.
    (d) Both (a) and (b) are correct.
    (e) Both (a) and (c) are correct.

17. When using the Guess and Test Problem Solving Strategy you will probably be most successful if you
    (a) simply try possibilities at random.
    (b) use a means of organizing your guesses.
    (c) try to learn from each of the trials.
    (d) apply both parts (a) and (c).
    (e) apply both parts (b) and (c).

18. If a problem asks "how many cubes are in the nth figure," an approach to solving the problem might be:
    (a) Guess and Test.
    (b) Draw a Picture.
    (c) Make a Table.
    (d) Look for a Pattern.
    (e) Use a Variable.

19. If a problem asks you to "Find a formula . . . " you might begin by
    (a) drawing a sketch of the problem situation.
    (b) understanding the terms used in the problem.
    (c) looking for relationships between known quantities and the unknown quantity.
    (d) making a tables of values, using n as the unknown quantity.
    (e) trying any one of the above.

20. If you are given a sequence of numbers and asked to predict the next number in the sequence, a problem solving strategy you might reasonably try is
    (a) Look for a Pattern.
    (b) Draw a Picture.
    (c) Guess and Test.
    (d) Trial and Error.
    (e) None of these is a particularly good strategy.

21. If a problem begins with "There are 200 people at a dance . . .", an approach to solving the problem may be
    (a) Draw a Picture.
    (b) Guess and Test.
    (c) Solve a Simpler Problem.
    (d) Deductive reasoning.
    (e) None of these is a good approach.

22. When we look at specific examples, observe a pattern, and draw a general conclusion, we are using
    (a) the Draw a Picture problem-solving strategy.
    (b) Deductive reasoning.
    (c) the Use a Variable problem-solving strategy.
    (d) The Guess and Test problem-solving strategy.
    (e) Inductive reasoning.

23. Deductive reasoning requires us to
    (a) accept some initial conditions as true.
    (b) make a table.
    (c) look for a pattern.
    (d) solve a simpler problem.
    (e) draw a picture.

24. When using formal problem-solving strategies, we
    (a) must choose the right one for the problem.
    (b) should not mix them in one problem.
    (c) should always start by choosing a variable.
    (d) are using deductive reasoning.
    (e) None of these answers is always true.

Fill in the Blanks. Complete each statement with a word or phrase that makes it true.

25. When solving a problem with geometric figures involved, we should use the _____ problem-solving strategy.

26. When solving a problem in which you can easily present information, we should use the _____ problem solving strategy.

27. If you wish to gain a better understanding of a complex problem, you might use the _____ problem solving strategy.

28. The sum of the measures of the angles in a triangle is _____.

29. When a problem asks us to predict a sequence of numbers or figures, we should use the _____ problem-solving strategy.

30. If we start with some general statements accepted as true and then draw conclusions from them, we are using _____ reasoning.

31. If we are using _____ reasoning, our answer may not be the only correct one.

Writing. Write your answers concisely and completely. Feel free to use figures and/or tables to illustrate the points you are making.

32. List the four steps in the problem-solving process first introduced by George Polya and employed in all the problem-solving strategies.

33. Read and solve the following problem. Then write a paragraph describing the method you used to get your solution. Which problem-solving strategy(s) did you use and why?

> At a family dinner Grandmother, Aunt Suzie, Mary, Father and Richard are to sit along one side of the table. Grandmother must sit on one end. Aunt Suzie and Richard cannot be seated next to each other. Aunt Suzie wants to sit next to Mary. Mary must sit next to Grandmother. How should their seating arrangements be made?

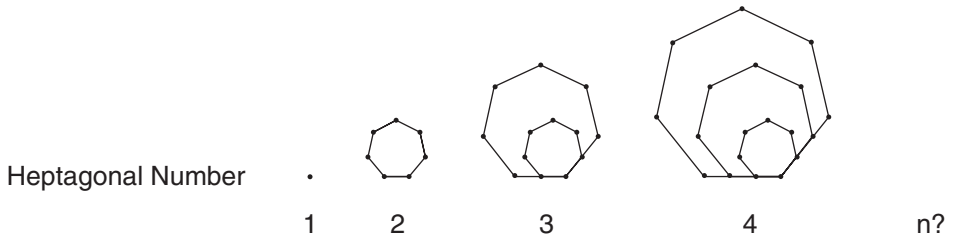
34. Read and solve the following problem. Then write a paragraph describing the method you used to get your solution. Which problem-solving strategy(s) did you use and why?

> Toothpicks can be arranged to form rows of rectangles as shown. How many toothpicks are required to form a row with 30 rectangles in it?

35. Compare and contrast inductive and deductive reasoning.

36. Read and solve the following problem. Then write a paragraph describing the method you used to get your solutions. Which problem solving strategies did you use and why?

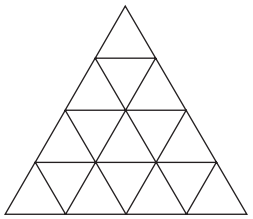
    The **heptagonal numbers** are the whole numbers that represent the following shapes.



    Find the number of dots in the nth heptagonal number.

37. Read and solve the following problem. Then write a paragraph describing the method you used to get your solution. Which problem-solving strategy(s) did you use and why?

    How many triangles of all sizes are there in the figure below?



Exercises and Problems.

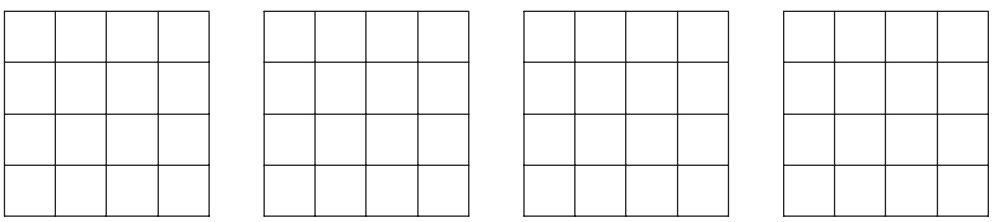
38. A large cube is formed by arranging 27 smaller white cubes of the same size into a cube shape. If three faces which share one corner of the larger cube are painted red, how many of the smaller cubes will have exactly two red faces?

39. If all the diagonals are drawn in a polygon with five sides (a pentagon), what is the maximum number of intersections of two diagonals?

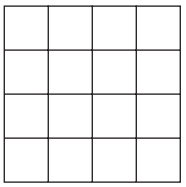
40. The measure of one angle of a triangle is twice the measure of a second angle. The measure of the third angle is twelve degrees more than the measure of the first angle. What is the measure of the largest angle of the triangle?

41. Samuel, Martha, James, Katie and Rafael all go to the movies. James insists on sitting on the aisle. Samuel and Martha must sit next to each other. Rafael sits next to Samuel and next to James. Who is sitting next to Katie?

42. How many squares of all sizes are there in a 4 by 4 square? Sketch all of the squares on the provided 4 by 4 squares and briefly explain your work.

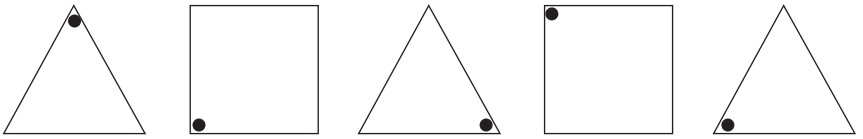


43. Shade exactly ten squares so that no three consecutive shaded squares line up vertically, horizontally or diagonally.



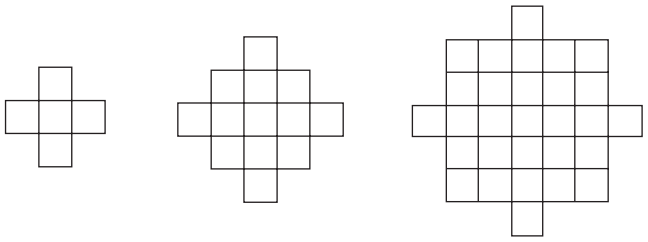
44. Toothpicks can be arranged to form triangles as shown below. How many toothpicks are required to form a triangle with ten toothpicks on a side?

45. Use inductive reasoning to determine the next figure in the sequence below.

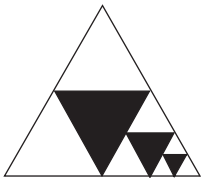


46. Use inductive reasoning to find the 6th, 10th and nth terms in the following sequence:
0, 3, 8, 15, 24, . . .

47. How many squares will be in the tenth figure in the following sequence?



48. What fraction of the large triangle is shaded?



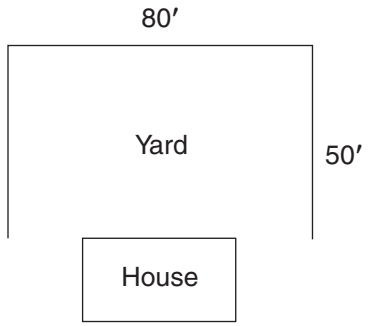
49. How can 24 cubes be stacked so that a minimum number of sides are exposed? How can 24 cubes be stacked so that a maximum number of sides are exposed? A side is one face of a smaller cube. Explain your thinking.

50. Determine the missing numbers in each of the following Fibonacci-type sequences
    (a) 1, 4, 5, 9, _____,, _____, _____
    (b) 2, _____, 6, _____, 16, _____
    (c) 3, _____, _____, _____, 27
    (d) 3, 7, 10, 17, 27

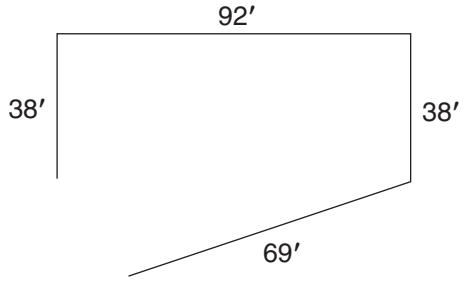
51. Use inductive reasoning to write the 6th, 7th and 12th terms of each sequence. Then write a formula for the nth term. Briefly explain your method.
    (a) $3, 6, 9, 12, 15, . .$
    (b) $4, 7, 11, 16, 22, \ldots$
    (c) $\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \frac{5}{243}, \ldots$

Applications.

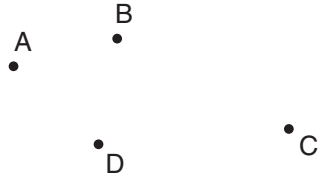
52. A landscape designer is working on the plans for a residential yard. Her clients want a privacy hedge planted on three sides of the rectangular backyard. (The side bordering the house will not be planted.) The plant selected for the hedge spans 5′ when mature. If the backyard is 50 feet deep and 80 feet long and there is to be a plant at the corners of the hedge, how many hedge plants are required? (See sketch.)



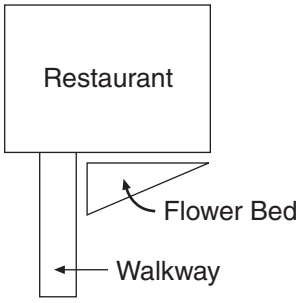
53. A water system must be installed in a field as shown below. If the pipe comes in both 8-foot and 15-foot lengths, and cannot be cut, how many pipes of each length will be required?



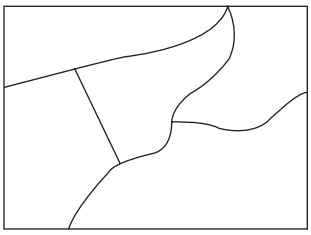
54. Four areas are newly designated as parks as shown below. Hiking trails are to be constructed so that each park is directly connected with each of the others. What is the least number of trails needed?



55. A triangular raised bed of annual flowers is to be built next to the door of a new restaurant, as shown. If the angle of the flower bed near the door is 90° and the other angle by the walkway is three times the third angle, what are the measures of the angles of this flower bed?



56. A state map has county lines as shown. If the counties are to be colored and no two counties which share a border can have the same color, what is the minimum number of colors required for this map? Explain your solution.



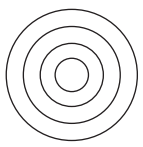
57. There is to be a board meeting in the Success Unlimited Inc. boardroom. The chair of the board always sits at the head of the long rectangular table. There are three seats along each side of the table and one seat at the far end which no one ever uses. Six people should attend the meeting in addition to the chair. However, two people are out of town on assignment and a third person is ill. In how many different ways can the remaining people sit at the table?

58. A rancher wishes to fence a rectangular area of 500 sq. ft. He wants to use only whole-number dimensions. What dimensions will require the least amount of fencing? [Remember the area of a rectangle is $A = l \times w$ and the perimeter is $P = 2(l + w)$.]

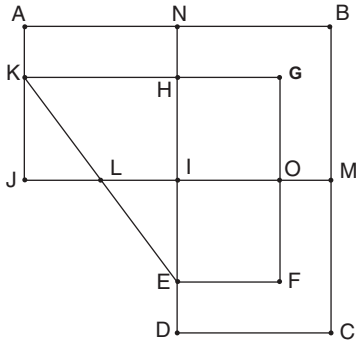
59. Ten people are meeting at a conference. Each person is instructed to introduce herself or himself to each of the other participants at the conference. If it is estimated that each introduction should take 1.5 minutes, how much time should be allowed in the schedule for this activity?

60. A waste water system is being installed in a small town. The large pipes required to reach from the town to the water treatment plant come in 20-foot and 24-foot lengths. The city planner finds that twelve fewer of the 24-foot pipes will be required than if the 20-foot pipes were used. What is the distance from the town to the water treatment plant?

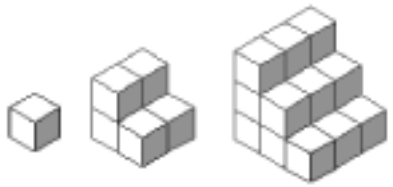
61. A school logo is made up of concentric circles as shown in the figure below. All the school colors, blue, green and yellow, are to be used. No two bordering regions can be the same color. If the center region must be yellow, how many different color schemes are possible?



62. A mail carrier must deliver mail to houses in a subdivision shown in the following map. Lines represent streets and letters represent houses. List the houses in the order the mail carrier will deliver mail as she travels each street exactly once and mark the map to show the route. Can she start and stop at the same house? Explain.

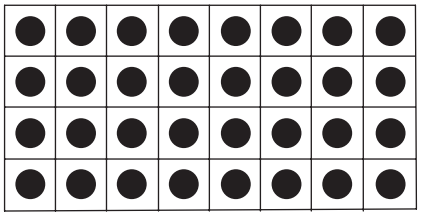


63. These three shapes represent the first three figures in a sequence of figures made from small cubes.



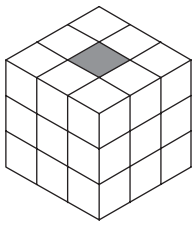
(a) How many cubes are in the 5th figure? Explain
(b) Describe the number of cubes in the nth figure. Don't simplify your answer, just list the pattern of the sum, for example, to sum 1 through n, you would write $1 + 2 + \ldots + (n - 1) + n$

64. The following is a 4×8 array of dots.

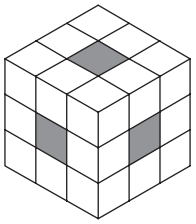


(a) If 10 dots are removed, how can you arrange the remaining dots so that each row and each column of the array contain an odd number of dots? Sketch the new arrangement or explain why it cannot be done.
(b) If 12 dots are removed, how can you arrange the remaining dots so that there is the same number of dots along each edge? Sketch the new arrangement or explain why it cannot be done.

65. Suppose three center cubes are removed from a large 3×3 cube and the resulting figure is dipped in red paint. How many cubes have red paint on 1 face? 2 faces? 3 faces? 4 faces? 5 faces? 6 faces? Explain.



66. Suppose all seven of the cubes from the center each face to the opposite face of a large 3×3 cube are removed and the resulting figure is dipped in red paint. How many cubes have red paint on 1 face? 2 faces? 3 faces? 4 faces? 5 faces? 6 faces? Explain.



67. Lynn purchased 24 decorative stone square pavers to create a border to enclose a rectangular brick patio. Each square paver measures 1 foot by 1 foot and the smaller rectangular bricks can be cut to fit any paver border configuration. What paver border configuration encloses the most brick patio area and what is that area? Explain. One border arrangement of pavers is shown here: