Calculus with Applications 11th Edition Lial Solutions Manual
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Chapter 3

THE DERIVATIVE

3.1 Limits

Your Turn 1

$f(x) = x^2 + 2$

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
$f(x)$	2.81	2.9801	2.998001	2.99980001	3	3.00020001	3.002001	3.0201	3.21

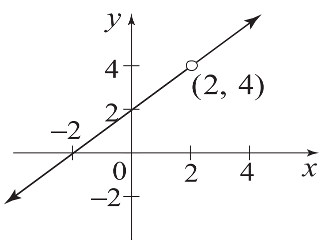
The table suggests that, as x get closer and closer to 1 from either side, $f(x)$ gets closer and closer to 3.

So, $\lim\limits_{x\to 1}(x^2 + 2) = 3.$

Your Turn 2

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x + 2)\cancel{(x - 2)}}{\cancel{(x - 2)}}$$

$$= x + 2, \text{ provided } x \neq 2$$



The graph of $y = \frac{x^2-4}{x-2}$ is the graph of $y = x + 2$, except there is a hole at (2, 4).

Looking at the graph, we see that as x is close to, but not equal to 2, $f(x)$ approaches 4.

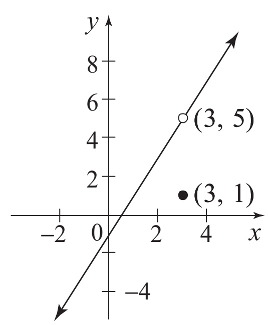
$$\lim_{x\to 2}\frac{x^2 - 4}{x - 2} = 4$$

Your Turn 3

Find $\lim\limits_{x\to 3} f(x)$ if $f(x) = \begin{cases} 2x - 1 & \text{if } x \neq 3 \\ 1 & \text{if } x = 3 \end{cases}$

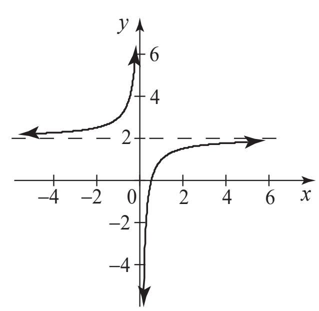
The graph of f is shown in the next column.

$$\lim_{x\to 3} f(x) = 5$$



Your Turn 4

Find $\lim\limits_{x\to 0}\dfrac{2x - 1}{x}.$



$$\lim_{x\to 0^-} f(x) = \infty$$

$$\lim_{x\to 0^+} f(x) = -\infty$$

Since there is no real number that $f(x)$ approaches as x approaches 0 from either side, nor does $f(x)$ approach either ∞ or $-\infty$, $\lim\limits_{x\to 0}\frac{2x-1}{x}$ does not exist.

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Your Turn 5

Let $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = 4$.

$$\begin{aligned}\lim_{x\to 2}\left[f(x)+g(x)\right]^2 &= \left[\lim_{x\to 2}\left[f(x)+g(x)\right]\right]^2\\ &= \left[\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)\right]^2\\ &= \left[3+4\right]^2\\ &= 7^2\\ &= 49\end{aligned}$$

Your Turn 6

$$\begin{aligned}\lim_{x\to -3}\frac{x^2-x-12}{x+3} &= \lim_{x\to -3}\frac{(x-4)\cancel{(x+3)}}{\cancel{x+3}} \quad (x \neq 3)\\ &= \lim_{x\to -3} x-4\\ &= (-3)-4\\ &= -7\end{aligned}$$

Your Turn 7

$$\begin{aligned}\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} &= \lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\\ &= \lim_{x\to 1}\frac{(\sqrt{x})^2-1}{(x-1)(\sqrt{x}+1)}\\ &= \lim_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}\\ &= \lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{1}{1+1} = \frac{1}{2}\end{aligned}$$

Your Turn 8

$$\begin{aligned}\lim_{x\to\infty}\frac{2x^2+3x-4}{6x^2-5x+7} &= \lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{3x}{x^2}-\frac{4}{x^2}}{\frac{6x^2}{x^2}-\frac{5x}{x^2}+\frac{7}{x^2}}\\ &= \lim_{x\to\infty}\frac{2+\frac{3}{x}-\frac{4}{x^2}}{6-\frac{5}{x}+\frac{7}{x^2}}\\ &= \frac{2+0-0}{6-0+0} = \frac{2}{6} = \frac{1}{3}\end{aligned}$$

3.1 Warmup Exercises

W1. Some trial with the factors of 8 and 15 shows that $8x^2 + 22x + 15 = (2x + 3)(4x + 5)$.

W2. Some trial with the factors of 12 shows that $12x^2 - 7x - 12 = (3x - 4)(4x + 3)$.

W3. $\dfrac{3x^2+x-14}{x^2-4} = \dfrac{(3x+7)(x-2)}{(x+2)(x-2)} = \dfrac{3x+7}{x+2}$, provided that x is not equal to 2 or -2.

W4. $\dfrac{2x^2+x-15}{x^2-9} = \dfrac{(2x-5)(x+3)}{(x+3)(x-3)} = \dfrac{2x-5}{x-3}$, provided that x is not equal to 3 or -3.

3.1 Exercises

1. Since $\lim_{x\to 2^-} f(x)$ does not equal $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2} f(x)$ does not exist. The answer is c.

2. Since $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -1$, $\lim_{x\to 2} f(x) = -1$. The answer is a.

3. Since $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 6$, $\lim_{x\to 4} f(x) = 6$. The answer is b.

4. Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = -\infty$, $\lim_{x\to 1} f(x) = -\infty$. The answer is b.

5. **(a)** By reading the graph, as x gets closer to 3 from the left or right, $f(x)$ gets closer to 3.

$$\lim_{x\to 3} f(x) = 3$$

(b) By reading the graph, as x gets closer to 0 from the left or right, $f(x)$ gets closer to 1.

$$\lim_{x\to 0} f(x) = 1.$$

6. **(a)** By reading the graph, as x gets closer to 2 from the left or right, $F(x)$ gets closer to 4.

$$\lim_{x\to 2} F(x) = 4$$

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(b) By reading the graph, as x gets closer to -1 from left or right, $F(x)$ gets closer to 4.

$$\lim_{x \to -1} F(x) = 4$$

7. **(a)** By reading the graph, as x gets closer to 0 from the left or right, $f(x)$ gets closer to 0.

$$\lim_{x \to 0} f(x) = 0$$

(b) By reading the graph, as x gets closer to 2 from the left, $f(x)$ gets closer to -2, but as x gets closer to 2 from the right, $f(x)$ gets closer to 1.

$\lim_{x \to 2} f(x)$ does not exist.

8. **(a)** By reading the graph, as x gets closer to 3 from the left or right, $g(x)$ gets closer to 2.

$$\lim_{x \to 3} g(x) = 2$$

(b) By reading the graph, as x gets closer to 5 from the left, $g(x)$ gets closer to -2, but as x gets closer to 5 from the right, $g(x)$ gets closer to 1.

$\lim_{x \to 5} g(x)$ does not exist.

9. **(a)** (i) By reading the graph, as x gets closer to -2 from the left, $f(x)$ gets closer to -1.

$$\lim_{x \to -2^-} f(x) = -1$$

(ii) By reading the graph, as x gets closer to -2 from the right, $f(x)$ gets closer to $-\frac{1}{2}$.

$$\lim_{x \to -2^+} f(x) = -\frac{1}{2}$$

(iii) Since $\lim_{x \to -2^-} f(x) = -1$ and $\lim_{x \to -2^+} f(x) = -\frac{1}{2}$, $\lim_{x \to -2} f(x)$ does not exist.

(iv) $f(-2)$ does not exist since there is no point on the graph with an x-coordinate of -2.

(b) (i) By reading the graph, as x gets closer to -1 from the left, $f(x)$ gets closer to $-\frac{1}{2}$.

$$\lim_{x \to -1^-} f(x) = -\frac{1}{2}$$

(ii) By reading the graph, as x gets closer to -1 from the right, $f(x)$ gets closer to $-\frac{1}{2}$.

$$\lim_{x \to -1^+} f(x) = -\frac{1}{2}$$

(iii) Since $\lim_{x \to -1^-} f(x) = -\frac{1}{2}$ and $\lim_{x \to -1^+} f(x) = -\frac{1}{2}$, $\lim_{x \to -1} f(x) = -\frac{1}{2}$.

(iv) $f(-1) = -\frac{1}{2}$ since $\left(-1, -\frac{1}{2}\right)$ is a point of the graph.

10. **(a)** (i) By reading the graph, as x gets closer to 1 from the left, $f(x)$ gets closer to 1.

$$\lim_{x \to 1^-} f(x) = 1$$

(ii) By reading the graph, as x gets closer to 1 from the right, $f(x)$ gets closer to 1.

$$\lim_{x \to 1^+} f(x) = 1$$

(iii) Since $\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = 1$, $\lim_{x \to 1} f(x) = 1$.

(iv) $f(1) = 2$ since (1, 2) is part of the graph.

(b) (i) By reading the graph, as x gets closer to 2 from the left, $f(x)$ gets closer to 0.

$$\lim_{x \to 2^-} f(x) = 0$$

(ii) By reading the graph, as x gets closer to 2 from the right, $f(x)$ gets closer to 0.

$$\lim_{x \to 2^+} f(x) = 0$$

(iii) Since $\lim_{x \to 2^-} f(x) = 0$ and $\lim_{x \to 2^+} f(x) = 0$, $\lim_{x \to 2} f(x) = 0$.

(iv) $f(2) = 0$ since (2, 0) is point of the graph.

11. By reading the graph, as x moves further to the right, $f(x)$ gets closer to 3. Therefore, $\lim_{x \to \infty} f(x) = 3$.

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12. By reading the graph, as x moves further to the left, $g(x)$ gets larger and larger. Therefore, $\lim_{x \to -\infty} g(x) = \infty$ (does not exist).

13. $\lim_{x \to 2} F(x)$ in Exercise 6 exists because $\lim_{x \to 2^-} F(x) = 4$ and $\lim_{x \to 2^+} F(x) = 4$.
$\lim_{x \to -2} f(x)$ in Exercise 9 does not exist since $\lim_{x \to -2^-} f(x) = -1$, but $\lim_{x \to -2^+} f(x) = -\frac{1}{2}$.

15. From the table, as x approaches 1 from the left or the right, $f(x)$ approaches 4.

$$\lim_{x \to 1} f(x) = 4$$

16. $f(x) = 2x^2 - 4x + 7$; find $\lim_{x \to 1} f(x)$.

Substitute 0.9 for x in the expression at the right to get $f(0.9) = 5.02$.

Continue substituting to complete the table.

x	0.9	0.99	0.999
$f(x)$	5.02	5.0002	5.000002

x	1.001	1.01	1.1
$f(x)$	5.000002	5.0002	5.02

As x approaches 1 from the left or the right, $f(x)$ approaches 5.

$$\lim_{x \to 1} f(x) = 5$$

17. $k(x) = \dfrac{x^3 - 2x - 4}{x - 2}$; find $\lim_{x \to 2} k(x)$.

x	1.9	1.99	1.999
$k(x)$	9.41	9.9401	9.9941

x	2.001	2.01	2.1
$k(x)$	10.006	10.0601	10.61

As x approaches 2 from the left or the right, $k(x)$ approaches 10.

$$\lim_{x \to 2} k(x) = 10$$

18. $f(x) = \dfrac{2x^3 + 3x^2 - 4x - 5}{x + 1}$; find $\lim_{x \to -1} f(x)$.

x	−1.1	−1.01	−1.001
$f(x)$	−3.68	−3.969	−3.996

x	−0.999	−0.99	−0.9
$f(x)$	−4.002	−4.02	−4.28

As x approaches -1 from the left or the right, $f(x)$ approaches -4.

$$\lim_{x \to -1} f(x) = -4$$

19. $h(x) = \dfrac{\sqrt{x} - 2}{x - 1}$; find $\lim_{x \to 1} h(x)$.

x	0.9	0.99	0.999
$h(x)$	10.51317	100.50126	1000.50013

x	1.001	1.01	1.1
$h(x)$	−999.50012	−99.50124	−9.51191

$$\lim_{x \to 1^-} = \infty$$
$$\lim_{x \to 1^+} = -\infty$$

Thus, $\lim_{x \to 1} h(x)$ does not exist.

20. $f(x) = \dfrac{\sqrt{x} - 3}{x - 3}$; find $\lim_{x \to 3} f(x)$.

x	2.9	2.99	2.999
$f(x)$	12.9706	127.0838	1268.237

x	3.001	3.01	3.1
$f(x)$	−1267.66	−125.506	−12.6795

$$\lim_{x \to 3^-} f(x) = \infty$$
$$\lim_{x \to 3^+} f(x) = -\infty$$

Thus, $\lim_{x \to 3} f(x)$ does not exist.

21. $\lim_{x \to 4} [f(x) - g(x)] = \lim_{x \to 4} f(x) - \lim_{x \to 4} g(x)$
$= 9 - 27 = -18$

22. $\lim_{x \to 4} [(g(x) \cdot f(x)]$
$= \left[\lim_{x \to 4} g(x)\right] \cdot \left[\lim_{x \to 4} f(x)\right]$
$= 27 \cdot 9 = 243$

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23. $\lim_{x\to 4} \frac{f(x)}{g(x)} = \frac{\lim_{x\to 4} f(x)}{\lim_{x\to 4} g(x)} = \frac{9}{27} = \frac{1}{3}$

24. $\lim_{x\to 4} \log_3 f(x) = \log_3 \lim_{x\to 4} f(x)$
$= \log_3 9 = 2$

25. $\lim_{x\to 4} \sqrt{f(x)} = \lim_{x\to 4} [f(x)^{1/2}]$
$= \left[\lim_{x\to 4} f(x)\right]^{1/2}$
$= 9^{1/2} = 3$

26. $\lim_{x\to 4} \sqrt[3]{g(x)} = \lim_{x\to 4} [g(x)]^{1/3}$
$= \left[\lim_{x\to 4} g(x)\right]^{1/3}$
$= 27^{1/3} = 3$

27. $\lim_{x\to 4} 2^{f(x)} = 2^{\lim_{x\to 4} f(x)}$
$= 2^9$
$= 512$

28. $\lim_{x\to 4} [1 + f(x)]^2 = \left[\lim_{x\to 4} (1 + f(x))\right]^2$
$= \left[\lim_{x\to 4} 1 + \lim_{x\to 4} f(x)\right]^2$
$= (1 + 9)^2 = 10^2$
$= 100$

29. $\lim_{x\to 4} \frac{f(x) + g(x)}{2g(x)}$
$= \frac{\lim_{x\to 4} [f(x) + g(x)]}{\lim_{x\to 4} 2g(x)}$
$= \frac{\lim_{x\to 4} f(x) + \lim_{x\to 4} g(x)}{2 \lim_{x\to 4} g(x)}$
$= \frac{9 + 27}{2(27)} = \frac{36}{54} = \frac{2}{3}$

30. $\lim_{x\to 4} \frac{5g(x) + 2}{1 - f(x)} = \frac{\lim_{x\to 4} [5g(x) + 2]}{\lim_{x\to 4} [1 - f(x)]}$
$= \frac{5 \lim_{x\to 4} g(x) + \lim_{x\to 4} 2}{\lim_{x\to 4} 1 - \lim_{x\to 4} f(x)}$
$= \frac{5.27 + 2}{1 - 9} = -\frac{137}{8}$

31. $\lim_{x\to 3} \frac{x^2 - 9}{x - 3} = \lim_{x\to 3} \frac{(x - 3)(x + 3)}{x - 3}$
$= \lim_{x\to 3} (x + 3)$
$= \lim_{x\to 3} x + \lim_{x\to 3} 3$
$= 3 + 3$
$= 6$

32. $\lim_{x\to -2} \frac{x^2 - 4}{x + 2} = \lim_{x\to -2} \frac{(x + 2)(x - 2)}{(x + 2)}$
$= \lim_{x\to -2} (x - 2)$
$= -2 - 2 = -4$

33. $\lim_{x\to 1} \frac{5x^2 - 7x + 2}{x^2 - 1} = \lim_{x\to 1} \frac{(5x - 2)(x - 1)}{(x + 1)(x - 1)}$
$= \lim_{x\to 1} \frac{5x - 2}{x + 1}$
$= \frac{5 - 2}{2}$
$= \frac{3}{2}$

34. $\lim_{x\to -3} \frac{x^2 - 9}{x^2 + x - 6} = \lim_{x\to -3} \frac{(x - 3)(x + 3)}{(x - 2)(x + 3)}$
$= \lim_{x\to -3} \frac{x - 3}{x - 2}$
$= \frac{-3 - 3}{-3 - 2}$
$= \frac{-6}{-5} = \frac{6}{5}$

35. $\lim_{x\to -2} \frac{x^2 - x - 6}{x + 2} = \lim_{x\to -2} \frac{(x - 3)(x + 2)}{x + 2}$
$= \lim_{x\to -2} (x - 3)$
$= \lim_{x\to -2} x + \lim_{x\to -2} (-3)$
$= -2 - 3$
$= -5$

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36. $$\lim_{x\to 5}\frac{x^2-3x-10}{x-5}=\lim_{x\to 5}\frac{(x-5)(x+2)}{(x-5)}$$
$$=\lim_{x\to 5}(x+2)$$
$$=5+2=7$$

37. $$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$$
$$=\lim_{x\to 0}\left(\frac{1}{x+3}-\frac{1}{3}\right)\left(\frac{1}{x}\right)$$
$$=\lim_{x\to 0}\left[\frac{3}{3(x+3)}-\frac{x+3}{3(x+3)}\right]\left(\frac{1}{x}\right)$$
$$=\lim_{x\to 0}\frac{3-x-3}{3(x+3)(x)}$$
$$=\lim_{x\to 0}\frac{-x}{3(x+3)x}$$
$$=\lim_{x\to 0}\frac{-1}{3(x+3)}$$
$$=\frac{-1}{3(0+3)}$$
$$=-\frac{1}{9}$$

38. $$\lim_{x\to 0}\frac{\frac{-1}{x+2}+\frac{1}{2}}{x}$$
$$=\lim_{x\to 0}\left(\frac{-1}{x+2}+\frac{1}{2}\right)\left(\frac{1}{x}\right)$$
$$=\lim_{x\to 0}\left[\frac{-2}{2(x+2)}+\frac{x+2}{2(x+2)}\right]\left(\frac{1}{x}\right)$$
$$=\lim_{x\to 0}\frac{-2+x+2}{2(x+2)(x)}$$
$$=\lim_{x\to 0}\frac{x}{2x(x+2)}$$
$$=\lim_{x\to 0}\frac{1}{2(x+2)}$$
$$=\frac{1}{2(0+2)}=\frac{1}{4}$$

39. $$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$$
$$=\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}\cdot\frac{\sqrt{x}+5}{\sqrt{x}+5}$$
$$=\lim_{x\to 25}\frac{x-25}{(x-25)(\sqrt{x}+5)}$$
$$=\lim_{x\to 25}\frac{1}{\sqrt{x}+5}$$
$$=\frac{1}{\sqrt{25}+5}$$
$$=\frac{1}{10}$$

40. $$\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}$$
$$=\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}\cdot\frac{\sqrt{x}+6}{\sqrt{x}+6}$$
$$=\lim_{x\to 36}\frac{(x-36)}{(x-36)(\sqrt{x}+6)}$$
$$=\lim_{x\to 36}\frac{1}{\sqrt{x}+6}$$
$$=\frac{1}{\sqrt{36}+6}$$
$$=\frac{1}{6+6}$$
$$=\frac{1}{12}$$

41. $$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$
$$=\lim_{h\to 0}\frac{x^2+2hx+h^2-x^2}{h}$$
$$=\lim_{h\to 0}\frac{2hx+h^2}{h}$$
$$=\lim_{h\to 0}\frac{h(2x+h)}{h}$$
$$=\lim_{h\to 0}(2x+h)$$
$$=2x+0=2x$$

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42. $$\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$
$$=\lim_{h\to 0}\frac{(x^3+3x^2+3xh^2+h^3)-x^3}{h}$$
$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$
$$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$$
$$=\lim_{h\to 0}(3x^2+3xh+h^2)$$
$$=3x^2+3x(0)+(0)^2=3x^2$$

43. $$\lim_{x\to\infty}\frac{3x}{7x-1}=\lim_{x\to\infty}\frac{\frac{3x}{x}}{\frac{7x}{x}-\frac{1}{x}}$$
$$=\lim_{x\to\infty}\frac{3}{7-\frac{1}{x}}$$
$$=\frac{3}{7-0}=\frac{3}{7}$$

44. $$\lim_{x\to-\infty}\frac{8x+2}{4x-5}=\lim_{x\to-\infty}\frac{\frac{8x}{x}+\frac{2}{x}}{\frac{4x}{x}-\frac{5}{x}}$$
$$=\lim_{x\to-\infty}\frac{8+\frac{2}{x}}{4-\frac{5}{x}}$$
$$=\frac{8+0}{4-0}=2$$

45. $$\lim_{x\to-\infty}\frac{-3x^2+2x}{2x^2-2x+1}$$
$$=\lim_{x\to-\infty}\frac{\frac{3x^2}{x^2}+\frac{2x}{x^2}}{\frac{2x^2}{x^2}-\frac{2x}{x^2}+\frac{1}{x^2}}$$
$$=\lim_{x\to-\infty}\frac{3+\frac{2}{x}}{2-\frac{2}{x}+\frac{1}{x^2}}$$
$$=\frac{3-0}{2+0+0}=\frac{3}{2}$$

46. $$\lim_{x\to\infty}\frac{x^2+2x-5}{3x^2+2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}$$
$$=\lim_{x\to\infty}\frac{1+\frac{2}{x}-\frac{5}{x^2}}{3+\frac{2}{x^2}}$$
$$=\frac{1+0-0}{3+0}=\frac{1}{3}$$

47. $$\lim_{x\to\infty}\frac{3x^3+2x-1}{2x^4-3x^3-2}$$
$$=\lim_{x\to\infty}\frac{\frac{3x^3}{x^4}+\frac{2x}{x^4}-\frac{1}{x^4}}{\frac{2x^4}{x^4}-\frac{3x^3}{x^4}-\frac{2}{x^4}}$$
$$=\lim_{x\to\infty}\frac{\frac{3}{x}+\frac{2}{x^3}-\frac{1}{x^4}}{2-\frac{3}{x}-\frac{2}{x^4}}$$
$$=\frac{0+0-0}{2-0-0}=0$$

48. $$\lim_{x\to\infty}\frac{2x^2-1}{3x^4+2}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^4}-\frac{1}{x^4}}{\frac{3x^4}{x^4}+\frac{2}{x^4}}$$
$$=\lim_{x\to\infty}\frac{\frac{2}{x^2}-\frac{1}{x^4}}{3+\frac{2}{x^4}}$$
$$=\frac{0-0}{3+0}=\frac{0}{3}=0$$

49. $$\lim_{x\to\infty}\frac{2x^3-x-3}{6x^2-x-1}$$
$$=\lim_{x\to\infty}\frac{\frac{2x^3}{x^2}-\frac{x}{x^2}-\frac{3}{x^2}}{\frac{6x^2}{x^2}-\frac{x}{x^2}-\frac{1}{x^2}}$$
$$=\lim_{x\to\infty}\frac{2x-\frac{1}{x}-\frac{3}{x^2}}{6-\frac{1}{x}-\frac{1}{x^2}}=\infty$$

The limit does not exist.

50. $$\lim_{x\to\infty}\frac{x^4-x^3-3x}{7x^2+9}=\lim_{x\to\infty}\frac{\frac{x^4}{x^2}-\frac{x^3}{x^2}-\frac{3x}{x^2}}{\frac{7x^2}{x^2}+\frac{9}{x^2}}$$
$$=\lim_{x\to\infty}\frac{x^2-x-\frac{3}{x}}{7+\frac{9}{x^2}}$$
$$=\infty$$

The limit does not exist.

51. $$\lim_{x\to\infty}\frac{2x^2-7x^4}{9x^2+5x-6}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}-\frac{7x^4}{x^2}}{\frac{9x^2}{x^2}+\frac{5x}{x^2}-\frac{6}{x^2}}$$
$$=\lim_{x\to\infty}\frac{2-7x^2}{9+\frac{5}{x}-\frac{6}{x^2}}$$

The denominator approaches 9, while the numerator becomes a negative number that is larger and larger in magnitude, so

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$$\lim_{x\to\infty} \frac{2x^2 - 7x^4}{9x^2 + 5x - 6} = -\infty \text{ (does not exist).}$$

52. $$\lim_{x\to\infty} \frac{-5x^3 - 4x^2 + 8}{6x^2 + 3x + 2} = \lim_{x\to\infty} \frac{\frac{-5x^3}{x^2} - \frac{4x^2}{x^2} + \frac{8}{x^2}}{\frac{6x^2}{x^2} + \frac{3x}{x^2} + \frac{2}{x^2}} = \lim_{x\to\infty} \frac{-5x - 4 + \frac{8}{x^2}}{6 + \frac{3}{x} + \frac{2}{x^2}}$$

The denominator approaches 6, while the numerator becomes a negative number that is larger and larger in magnitude, so

$$\lim_{x\to\infty} \frac{-5x^3 - 4x^2 + 8}{6x^2 + 3x + 2} = -\infty \text{ (does not exist).}$$

53. $\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1$.

Therefore $\lim_{x\to -1} f(x) = 1$.

54. $\lim_{x\to -2^-} g(x) = -1$ and $\lim_{x\to -2^+} g(x) = -1$.

Therefore $\lim_{x\to -2} g(x) = -1$.

55. **(a)** $\lim_{x\to 3} f(x) = 2$.

(b) $\lim_{x\to 5} f(x)$ does not exist since $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5^+} f(x) = 8$.

56. **(a)** $\lim_{x\to 0} g(x)$ does not exist since $\lim_{x\to 0^-} g(x) = 5$ and $\lim_{x\to 0^+} g(x) = -2$.

(b) $\lim_{x\to 3} g(x) = 7$.

57. The denominator

$x^2 - 3x + 2 = (x - 1)(x - 2)$. Thus

$\lim_{x\to 2} \frac{3x^2 + kx - 2}{x^2 - 3x + 2}$ will exist when the numerator contains a factor of $x - 2$. This occurs when $3(2)^2 + k(2) - 2 = 0$, which requires $k = -5$. The fraction is then

$\frac{3x^2 - 5x - 2}{(x - 1)(x - 2)} = \frac{(3x + 1)(x - 2)}{(x - 1)(x - 2)} = \frac{3x + 1}{x - 1}$ provided $x \neq 2$. The limit of the fraction as $x \to 2$ is $\frac{3(2) + 1}{2 - 1} = 7$.

58. The denominator

$x^2 - 4x + 3 = (x - 1)(x - 3)$. Thus

$\lim_{x\to 3} \frac{2x^2 + kx - 9}{x^2 - 4x + 3}$ will exist when the numerator contains a factor of $x - 3$. This occurs when $2(3)^2 + k(3) - 9 = 0$, which requires $k = -3$. The fraction is then

$\frac{2x^2 - 3x - 9}{(x - 1)(x - 3)} = \frac{(2x + 3)(x - 3)}{(x - 1)(x - 3)} = \frac{2x + 3}{x - 1}$

provided $x \neq 3$. The limit of the fraction as $x \to 3$ is $\frac{3(3) + 3}{3 - 1} = \frac{9}{2}$.

59. Find $\lim_{x\to 3} f(x)$, where $f(x) = \frac{x^2 - 9}{x - 3}$.

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	6.001	6.01	6.1

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2 - 9}{x - 3} = 6.$$

60. Find $\lim_{x\to -2} g(x)$, where $g(x) = \frac{x^2 - 4}{x + 2}$.

x	−2.1	−2.01	−2.001	−1.999	−1.99	−1.9
$g(x)$	−4.1	−4.01	−4.001	−3.999	−3.99	−3.9

$$\lim_{x\to -2} g(x) = \lim_{x\to -2} \frac{x^2 - 4}{x + 2} = -4.$$

61. Find $\lim_{x\to 1} f(x)$, where $f(x) = \frac{5x^2 - 7x + 2}{x^2 - 1}$.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1.316	1.482	1.498	1.502	1.517	1.667

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{5x^2 - 7x + 2}{x^2 - 1} = 1.5 = \frac{3}{2}.$$

62. Find $\lim_{x\to 3} g(x)$, where $g(x) = \frac{x^2 - 9}{x^2 + x - 6}$.

x	−3.1	−3.01	−3.001
$g(x)$	1.196	1.1996	1.19996

x	−2.999	−2.99	−2.9
$g(x)$	1.20004	1.2004	1.204

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$$\lim_{x\to -3} g(x) = \lim_{x\to -3} \frac{x^2 - 9}{x^2 + x - 6} = 1.2 = \frac{6}{5}.$$

63. **(a)** $\lim_{x\to -2} \frac{3x}{(x+2)^3}$ does not exist since

$$\lim_{x\to -2^+} \frac{3x}{(x+2)^3} = -\infty$$

and $\lim_{x\to -2^-} \frac{3x}{(x+2)^3} = \infty.$

(b) Since $(x+2)^3 = 0$ when $x = -2$, $x = -2$ is the vertical asymptote of the graph of $F(x)$.

(c) The two answers are related. Since $x = -2$ is a vertical asymptote, we know that $\lim_{x\to -2} F(x)$ does not exist.

64. $G(x) = \frac{-6}{(x-4)^2}$

(a) $\lim_{x\to 4} G(x) = -\infty$ (does not exist), since by looking at the graph of $G(x)$, we see that as x gets closer to 4 from either the right or the left, $g(x)$ gets smaller.

(b) Since $(x-4)^2 = 0$ when $x = 4$, $x = 4$ is the vertical asymptote of the graph of $G(x)$.

(c) The two answers are related. Since $x = 4$ is a vertical asymptote, we know the $\lim_{x\to 4} G(x)$ does not exist.

67. **(a)** $\lim_{x\to -\infty} e^x = 0$ since, as the graph goes further to the left, e^x gets closer to 0.

(b) The graph of e^x has a horizontal asymptote at $y = 0$ since $\lim_{x\to -\infty} e^x = 0$.

68. **(a)** $y = xe^{-x}$

From the graph, it appears that

$$\lim_{x\to\infty} xe^{-x} = 0.$$

x	1	10	50
y	0.37	0.00045	9.64×10^{-21}

(b) $y = x^2e^{-x}$

From the graph, it appears that

$$\lim_{x\to\infty} x^2e^{-x} = 0.$$

x	1	10	50
y	0.37	0.0045	4.82×10^{-19}

(c) $\lim_{x\to\infty} x^ne^{-x} = 0.$

69. **(a)** $\lim_{x\to 0^+} \ln x = -\infty$ (does not exist) since, as the graph gets closer to $x = 0$, the value of $\ln x$ get smaller.

(b) The graph of $y = \ln x$ has a vertical asymptote at $x = 0$ since $\lim_{x\to 0^+} \ln x = -\infty$.

70. **(a)** $y = x\ln x$

From the graph, it appears that

$$\lim_{x\to 0^+} x\ln x = 0.$$

x	1	0.5	0.1	0.01	0.001
y	0	−0.347	−0.230	−0.0461	−0.0069

(b) $y = x(\ln x)^2$

From the graph it appears that

$$\lim_{x\to 0^+} x(\ln x)^2 = 0.$$

x	1	0.5	0.1	0.01	0.001
y	0	0.240	0.53	0.212	0.048

(c) $\lim_{x\to 0^+} x(\ln x)^n = 0.$

73. $\lim_{x\to 1} \frac{x^4 + 4x^3 - 9x^2 + 7x - 3}{x - 1}$

(a)

x	1.01	1.001	1.0001	0.99	0.999	0.9999
$f(x)$	5.0908	5.009	5.0009	4.9108	4.991	4.9991

As $x \to 1^-$ and as $x \to 1^+$, we see that $f(x) \to 5$.

(b) Graph

$$y = \frac{x^4 + 4x^3 - 9x^2 + 7x - 3}{x - 1}$$

on a graphing calculator. One suitable choice for the viewing window is $[-6, 6]$ by $[-10, 40]$ with Xscl = 1, Yscl = 10.

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Because $x - 1 = 0$ when $x = 1$, we know that the function is undefined at this x-value. The graph does not show an asymptote at $x = 1$. This indicates that the rational expression that defines this function is not written in lowest terms, and that the graph should have an open circle to show a "hole" in the graph at $x = 1$. The graphing calculator doesn't show the hole, but if we try to find the value of the function at $x = 1$, we see that it is undefined. (Using the TABLE feature on a TI-84 Plus, we see that for $x = 1$, the y-value is listed as "ERROR.")

By viewing the function near $x = 1$, and using the ZOOM feature, we see that as x gets close to 1 from the left or the right, y gets close to 5, suggesting that

$$\lim_{x\to 1} \frac{x^4 + 4x^3 - 9x^2 + 7x - 3}{x - 1} = 5.$$

74. $\lim_{x\to 2} \frac{x^4 + x - 18}{x^2 - 4}$

(a)

x	2.01	2.001	2.0001	1.99	1.999	1.9999
$f(x)$	8.29	8.25	8.25	8.21	8.25	8.25

(b) Graph

$$y = \frac{x^4 + x - 18}{x^2 - 4}.$$

One suitable choice for the viewing window is $[-5, 5]$ by $[0, 20]$ Because $x^2 - 4 = 0$ when $x = -2$ or $x = 2$, we know that the function is undefined at these two x-values. The graph shows an asymptote at $x = -2$. There should be open circle to show a "hole" in the graph at $x = 2$. The graphing calculator doesn't show the hole, but if we try to find the value of the function at $x = 2$, we see that it is undefined. (Using the TABLE feature on a TI-84 Plus, we see that for $x = 2$, the y-value is listed as "ERROR.")

By viewing the function near $x = 2$ and using the ZOOM feature, we verify that the required limit is 8.25. (We may not be able to get this value exactly.)

75. $\lim_{x\to -1} \frac{x^{1/3} + 1}{x + 1}$

(a)

x	-1.01	-1.001	-1.0001
$f(x)$	0.33223	0.33322	0.33332

x	-0.99	-0.999	-0.9999
$f(x)$	0.33445	0.33344	0.33334

We see that as $x \to -1^-$ and as $x \to -1^+$,

$f(x) \to 0.3333$ or $\frac{1}{3}$.

(b) Graph $y = \frac{x^{1/3} + 1}{x + 1}$.

One suitable choice for the viewing window is $[-5, 5]$ by $[-2, 2]$

Because $x + 1 = 0$ when $x = -1$, we know that the function is undefined at this x-value. The graph does not show an asymptote at $x = -1$. This indicates that the rational expression that defined this function is not written lowest terms, and that the graph should have an open circle to show a "hole" in the graph at $x = -1$. The graphing calculator doesn't show the hole, but if we try to find the value of the function at $x = -1$, we see that it is undefined. (Using the TABLE feature on a TI-83, we see that for $x = -1$, the y-value is listed as "ERROR.")

By viewing the function near $x = -1$ and using the ZOOM feature, we see that as x gets close to -1 from the left or right, y gets close to 0.3333, suggesting that

$$\lim_{x\to -1} \frac{x^{1/3} + 1}{x + 1} = 0.3333 \text{ or } \frac{1}{3}.$$

76. $\lim_{x\to 4} \frac{x^{3/2} - 8}{x + x^{1/2} - 6}$

(a)

x	4.1	4.01	4.001	4.0001
$f(x)$	2.4179	2.4018	2.4002	2.4

x	3.9	3.99	3.999	3.9999
$f(x)$	2.3819	2.3982	2.3998	2.4

(b) Graph

$$y = \frac{x^{3/2} - 8}{x + x^{1/2} - 6}.$$

This function is undefined at $x = 4$ because this value would make the denominator equal to 0. However, by viewing the function near $x = 4$ and using the ZOOM feature, we verify that the required limit is 2.4.

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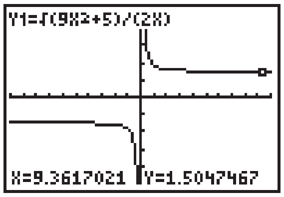
77. $\lim_{x\to\infty} \frac{\sqrt{9x^2+5}}{2x}$

Graph the functions on a graphing calculator. A good choice for the viewing window is $[-10, 10]$ by $[-5, 5]$.



(a) The graph appears to have horizontal asymptotes at $y = \pm 1.5$. We see that as $x \to \infty, y \to 1.5,$ so we determine that

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+5}}{2x} = 1.5.$$

(b) As $x \to \infty$,

$$\frac{\sqrt{9x^2+5}}{2x} \to \frac{3|x|}{2x}.$$

Since $x > 0, |x| = x$, so

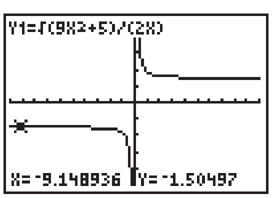
$$\frac{3|x|}{2x} = \frac{3x}{2x} = \frac{3}{2}.$$

Thus,

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+5}}{2x} = \frac{3}{2} \text{ or } 1.5.$$

78. $\lim_{x\to-\infty} \frac{\sqrt{9x^2+5}}{2x}$

Graph this function on a graphing calculator. A good choice for the viewing window is $[-10,10]$ by $[-5,5]$.



(a) The graph appears to have horizontal asymptotes at $y = \pm 1.5$. We see that as $x \to -\infty, y \to -1.5,$ so we determine that

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+5}}{2x} = -1.5.$$

(b) As $x \to -\infty$,

$$\frac{\sqrt{9x^2+5}}{2x} \to \frac{3|x|}{2x}.$$

Since $x < 0, |x| = -x$, so

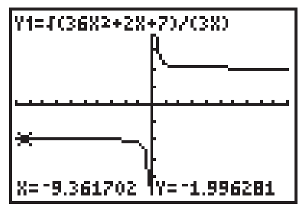
$$\frac{3|x|}{2x} = \frac{3(-x)}{2x} = -\frac{3}{2}.$$

Thus,

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+5}}{2x} = -\frac{3}{2} \text{ or } -1.5.$$

79. $\lim_{x\to-\infty} \frac{\sqrt{36x^2+2x+7}}{3x}$

Graph this function on a graphing calculator. A good choice for the viewing window is $[-10, 10]$ by $[-5,5]$.



(a) The graph appears to have horizontal asymptotes at $y = \pm 2$. We see that as $x \to -\infty, y \to -2,$ so we determine that

$$\lim_{x\to-\infty} \frac{\sqrt{36x^2+2x+7}}{3x} = -2.$$

(b) As $x \to -\infty$,

$$\frac{\sqrt{36x^2+2x+7}}{3x} \to \frac{6|x|}{3x}.$$

Since $x < 0, |x| = -x$, so

$$\frac{6|x|}{3x} = \frac{6(-x)}{3x} = -2.$$

Thus,

$$\lim_{x\to-\infty} \frac{\sqrt{36x^2+2x+7}}{3x} = -2.$$

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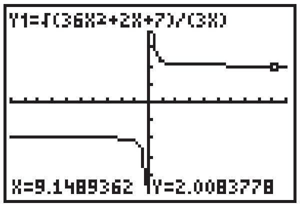
80. $\lim\limits_{x\to\infty} \dfrac{\sqrt{36x^2 + 2x + 7}}{3x}$

Graph this function on a graphing calculator. A good choice for the viewing window is $[-10, 10]$ by $[-5, 5]$.



(a) The graph appears to have horizontal asymptotes at $y = \pm 2$. We see that as $x \to \infty$, $y \to 2$, so we determine that

$$\lim_{x\to\infty} \frac{\sqrt{36x^2 + 2x + 7}}{3x} = 2.$$

(b) As $x \to \infty$,

$$\frac{\sqrt{36x^2 + 2x + 7}}{3x} \to \frac{6\,|x|}{3x}.$$

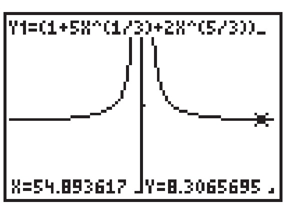
Since $x > 0, |x| = x$, so

$$\frac{6\,|x|}{3x} = \frac{6x}{3x} = 2.$$

Thus, $\lim\limits_{x\to\infty} \dfrac{\sqrt{36x^2 + 2x + 7}}{3x} = 2.$

81. $\lim\limits_{x\to\infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5}$

Graph this function on a graphing calculator. A good choice for the viewing window is $[-20, 20]$ by $[0, 20]$ with Xscl $= 5$, Yscl $= 5$.



(a) The graph appears to have a horizontal asymptote at $y = 8$. We see that as $x \to \infty$, $y \to 8$, so we determine that

$$\lim_{x\to\infty} \frac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} = 8.$$

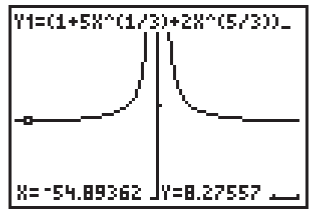
(b) As $x \to \infty$,

$$\frac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} \to \frac{8x^5}{x^5} = 8.$$

Thus, $\lim\limits_{x\to\infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} = 8.$

82. $\lim\limits_{x\to-\infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5}$

Graph this function on a graphing calculator. A good choice for the viewing window is $[-60, 60]$ by $[0, 20]$ with Xscl $= 10$, Yscl $= 10$.



(a) The graph appears to have a horizontal asymptote at $y = 8$. We see that as $x \to -\infty$, $y \to 8$, so we determine that

$$\lim_{x\to-\infty} \frac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} = 8.$$

(b) As $\dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} \to \dfrac{8x^5}{x^5} = 8.$

Thus, $\lim\limits_{x\to-\infty} \dfrac{(1 + 5x^{1/3} + 2x^{5/3})^3}{x^5} = 8.$

84. **(a)** $\lim\limits_{x\to 12} G(t)$

As t approaches 12 from either direction, the value of $G(t)$ for the corresponding point on the graph approaches 3.

Thus, $\lim\limits_{x\to 12} G(t) = 3$ which represents 3 million gallons.

(b) $\lim\limits_{x\to 16^+} G(t) = 1.5$

$\lim\limits_{x\to 16^-} G(t) = 2$

Since $\lim\limits_{x\to 16^+} G(t) \neq \lim\limits_{x\to 16^-} G(t)$,

$\lim\limits_{x\to 16} G(t)$ does not exist.

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(c) $G(16)$ is the value of function $G(t)$ when $t = 16$. This value occurs at the solid dot on the graph $G(16) = 2$ which represents 2 million gallons.

(d) The tipping point occurs at the break in the graph, when $t = 16$ months.

85. **(a)** $\lim_{x\to 94} T(x) = 7.25$ cents

(b) $\lim_{x\to 13^-} T(x) = 7.25$ cents

(c) $\lim_{x\to 13^+} T(x) = 7.5$ cents

(d) $\lim_{x\to 13} T(x) =$ does not exist

(e) $T(13) = 7.5$ cents

86. **(a)** $\lim_{t\to 2014^-} C(t) = 46$ cents

(b) $\lim_{t\to 2014^+} C(t) = 49$ cents

(c) $\lim_{t\to 2014} C(t) =$ does not exist

(d) $C(2014) = 49$ cents

87. $C(x) = 15{,}000 + 6x$

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{15{,}000 + 6x}{x} = \frac{15{,}000}{x} + 6$$

$$\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} \frac{15{,}000}{x} + 6 = 0 + 6 = 6$$

This means that the average cost approaches \$6 as the number of DVDs produced becomes very large.

88. $C(x) = 0.0147x + 167.55$

$$\begin{aligned}\overline{C}(x) &= \frac{C(x)}{x}\\ &= \frac{0.0417x + 167.55}{x}\\ &= 0.0417 + \frac{167.55}{x}\end{aligned}$$

$$\lim_{x\to\infty} C(x) = 0.0417$$

The average cost approaches \$0.0417 per mile as the number of miles becomes very large.

89. $P(s) = \dfrac{63s}{s+8}$

$$\begin{aligned}\lim_{s\to\infty} \frac{63s}{s+8} &= \lim_{s\to\infty} \frac{\frac{63s}{s}}{\frac{s}{s} + \frac{8}{s}}\\ &= \lim_{s\to\infty} \frac{63}{1 + \frac{8}{s}}\\ &= \frac{63}{1+0}\\ &= 63\end{aligned}$$

The number of items of work a new employee produces gets closer and closer to 63 as the number of days of training increases.

90.
$$\begin{aligned}&\lim_{n\to\infty}\left[R\left[\frac{1-(1+i)^{-n}}{i}\right]\right]\\ &= \frac{R}{i}\lim_{n\to\infty}\left[1-(1+i)^{-n}\right]\\ &= \frac{R}{i}\left[\lim_{n\to\infty} 1 - \lim_{n\to\infty}(1+i)^{-n}\right]\\ &= \frac{R}{i}\left[1-0\right] = \frac{R}{i}\end{aligned}$$

91.
$$\begin{aligned}&\lim_{n\to\infty}\left[\frac{R}{i-g}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]\right]\\ &= \frac{R}{i-g}\lim_{n\to\infty}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]\\ &= \frac{R}{i-g}\left[\lim_{n\to\infty} 1 - \lim_{n\to\infty}\left(\frac{1+g}{1+i}\right)^n\right]\end{aligned}$$

assuming $i > g$,

$$\begin{aligned}&= \frac{R}{i-g}\left[1-0\right]\\ &= \frac{R}{i-g}\end{aligned}$$

92. **(a)** $N(65) = 71.8e^{-8.96e^{(-0.0685(65))}}$
≈ 64.68

To the nearest whole number, this species of alligator has approximately 65 teeth after 65 days of incubation by this formula.

(b) Since $\lim_{t\to\infty}(-8.96e^{-0.0685t}) = -8.96 \cdot 0 = 0$, it follows that

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$$\lim_{t\to\infty} 71.8e^{-8.96e^{(-0.0685t)}} = 71.8e^0 = 71.8 \cdot 1 = 71.8$$

So, to the nearest whole number, $\lim_{t\to\infty} N(t) \approx 72$. Therefore, by this model a newborn alligator of this species will have about 72 teeth.

93. (a)

$$D(t) = 155(1 - e^{-0.0133t})$$

$$D(20) = 155\left(1 - e^{-0.0133(20)}\right) = 155(1 - e^{-0.266}) \approx 36.2$$

The depth of the sediment layer deposited below the bottom of the lake in 1970 was 36.2 cm.

(b)

$$\begin{aligned}\lim_{t\to\infty} D(t) &= \lim_{t\to\infty} 155(1 - e^{-0.0133t}) \\ &= 155 \lim_{t\to\infty} (1 - e^{-0.0133t}) \\ &= 155\left(\lim_{t\to\infty} 1 - \lim_{t\to\infty} e^{-0.0133t}\right) \\ &= 155(1) - 155 \lim_{t\to\infty} e^{-0.0133t} \\ &= 155 - 155(0) = 155\end{aligned}$$

Thus,

$$\lim_{t\to\infty} D(t) = 155.$$

Going back in time (t is years before 1990), the depth of the sediment approaches 155 cm.

94.

$$A(h) = \frac{0.17h}{h^2 + 2}$$

$$\begin{aligned}\lim_{x\to\infty} A(h) &= \lim_{x\to\infty} \frac{0.17h}{h^2 + 2} \\ &= \lim_{x\to\infty} \frac{\frac{0.17h}{h^2}}{\frac{h^2}{h^2} + \frac{2}{h^2}} \\ &= \lim_{x\to\infty} \frac{\frac{0.17}{h}}{1 + \frac{2}{h^2}} \\ &= \frac{0}{1 + 0} = 0\end{aligned}$$

This means that the concentration of the drug in the bloodstream approaches 0 as the number of hours after injection increases.

95. (a) $p_2 = \frac{1}{2} + \left(0.7 - \frac{1}{2}\right)[1 - 2(0.2)]^2 = 0.572$

(b) $p_4 = \frac{1}{2} + \left(0.7 - \frac{1}{2}\right)[1 - 2(0.2)]^4 = 0.526$

(c) $p_8 = \frac{1}{2} + \left(0.7 - \frac{1}{2}\right)[1 - 2(0.2)]^8 = 0.503$

(d)

$$\begin{aligned}\lim_{n\to\infty} P_n &= \lim_{n\to\infty}\left[\frac{1}{2} + \left(p_0 - \frac{1}{2}\right)(1 - 2p)^n\right] \\ &= \frac{1}{2} + \lim_{n\to\infty}\left(p_0 - \frac{1}{2}\right)(1 - 2p)^n \\ &= \frac{1}{2} + \left(p_0 - \frac{1}{2}\right)\lim_{n\to\infty}(1 - 2p)^n \\ &= \frac{1}{2} + \left(p_0 - \frac{1}{2}\right)\cdot 0 = \frac{1}{2}\end{aligned}$$

The number in parts (a), (b), and (c) represent the probability that the legislator will vote yes on the second, fourth, and eighth votes. In (d), as the number of roll calls increases, the probability gets close to 0.5, but is never less than 0.5.

3.2 Continuity

Your Turn 1

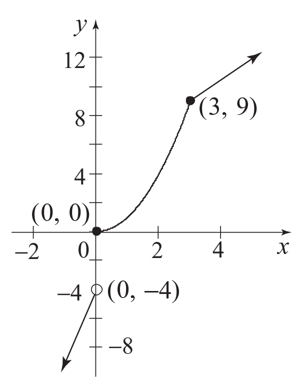
$f(x) = \sqrt{5x + 3}$

The square root function is discontinuous wherever $5x + 3 < 0$. There is a discontinuity when

$5a + 3 < 0$, or $a < -\frac{3}{5}$.

Your Turn 2

$$f(x) = \begin{cases} 5x - 4 & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 3 \\ x + 6 & \text{if } x > 3 \end{cases}$$



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$$\lim_{x\to 0^-} f(x) = -4$$
$$\lim_{x\to 0^+} f(x) = 0$$

Because $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, the limit doesn't exist, so f is discontinuous at $x = 0$.

3.2 Warmup Exericses

W1.

If $x \neq 2$, $\dfrac{2x^2 - 11x + 14}{x^2 - 5x + 6} = \dfrac{(x-2)(2x-7)}{(x-2)(x-3)}$
$= \dfrac{2x-7}{x-3}$.

Thus $\lim_{x\to 2} \dfrac{2x^2 - 11x + 14}{x^2 - 5x + 6} = \lim_{x\to 2} \dfrac{2x-7}{x-3}$
$= \dfrac{2(2)-7}{2-3} = 3$.

W2.

If $x \neq 4$, $\dfrac{3x^2 - 4x - 32}{x^2 - 6x + 8} = \dfrac{(x-4)(3x+8)}{(x-4)(x-2)}$
$= \dfrac{3x+8}{x-2}$.

Thus $\lim_{x\to 4} \dfrac{3x^2 - 4x - 32}{x^2 - 6x + 8} = \lim_{x\to 4} \dfrac{3x+8}{x-2}$
$= \dfrac{3(4)+8}{4-2} = 10$.

W3. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x+1) = 4$
$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x-2) = 4$
Thus $\lim_{x\to 3} f(x) = 4$.

W4. $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} 2x - 2 = 8$
$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} 4x + 1 = 21$
Thus $\lim_{x\to 5} f(x)$ does not exist.

W5. Since in an open interval containing 6, $f(x)$ is defined as $4x + 1$,
$\lim_{x\to 6} f(x) = \lim_{x\to 6} (4x+1) = 25$.

3.2 Exercises

1. Discontinuous at $x = -1$

(a) $f(-1)$ does not exist.

(b) $\lim_{x\to -1^-} f(x) = \dfrac{1}{2}$

(c) $\lim_{x\to -1^+} f(x) = \dfrac{1}{2}$

(d) $\lim_{x\to -1} f(x) = \dfrac{1}{2}$ (since (a) and (b) have the same answers)

(e) $f(-1)$ does not exist.

2. Discontinuous at $x = -1$

(a) $f(-1) = 2$

(b) $\lim_{x\to -1^-} f(x) = 2$

(c) $\lim_{x\to -1^+} f(x) = 4$

(d) $\lim_{x\to -1} f(x)$ does not exist (since parts (a) and (b) have different answers).

(e) $\lim_{x\to -1} f(x)$ does not exist.

3. Discontinuous at $x = 1$

(a) $f(1) = 2$

(b) $\lim_{x\to 1^-} f(x) = -2$

(c) $\lim_{x\to 1^+} f(x) = -2$

(d) $\lim_{x\to 1} f(x) = -2$ (since (a) and (b) have the same answers)

(e) $\lim_{x\to 1} f(x) \neq f(1)$

4. Discontinuous at $x = -2$ and $x = 3$.

(a) $f(-2) = 1$ $\qquad f(3) = 1$

(b) $\lim_{x\to -2^-} f(x) = -1$ $\quad \lim_{x\to 3^-} f(x) = -1$

(c) $\lim_{x\to -2^+} f(x) = -1$ $\quad \lim_{x\to 3^+} f(x) = -1$

(d) $\lim_{x\to -2} f(x) = -1$ (since parts (a) and (b) have the same answer)
$\lim_{x\to 3} f(x) = -1$ (since parts (a) and (b) have the same answer)

(e) $\lim_{x\to -2} f(x) \neq f(-2)$ $\quad \lim_{x\to 3} f(x) \neq f(3)$

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5. Discontinuous at $x = -5$ and $x = 0$

(a) $f(-5)$ does not exist. $f(0)$ does not exist.

(b) $\lim\limits_{x \to -5^-} f(x) = \infty$ (limit does not exist)

$\lim\limits_{x \to 0^-} f(x) = 0$

(c) $\lim\limits_{x \to -5^+} f(x) = -\infty$ (limit does not exist)

$\lim\limits_{x \to 0^+} f(x) = 0$

(d) $\lim\limits_{x \to -5} f(x)$ does not exist, since the answers to (a) and (b) are different. $\lim\limits_{x \to 0} f(x) = 0$, since the answers to (a) and (b) are the same.

(e) $f(-5)$ does not exist and $\lim\limits_{x \to -5} f(x)$ does not exist. $f(0)$ does not exist.

6. Discontinuous at $x = 0$ and $x = 2$

(a) $f(0)$ does not exist. $f(2)$ does not exist.

(b) $\lim\limits_{x \to 0^-} f(x) = -\infty$ (limit does not exist)

$\lim\limits_{x \to 2^-} f(x) = -2$

(c) $\lim\limits_{x \to 0^+} f(x) = -\infty$ (limit does not exist)

$\lim\limits_{x \to 2^+} f(x) = -2$

(d) $\lim\limits_{x \to 0} f(x) = -\infty$ (limit does not exist)

$\lim\limits_{x \to 2} f(x) = -2$ (since parts (a) and (b) have the same answer)

(e) $f(0)$ does not exist and $\lim\limits_{x \to 0} f(x)$ does not exist. $f(2)$ does not exist.

7. $f(x) = \dfrac{5 + x}{x(x - 2)}$

$f(x)$ is discontinuous at $x = 0$ and $x = 2$ since the denominator equals 0 at these two values.

$\lim\limits_{x \to 0} f(x)$ does not exist since $\lim\limits_{x \to 0^-} f(x) = \infty$ and $\lim\limits_{x \to 0^+} f(x) = -\infty$.

$\lim\limits_{x \to 2} f(x)$ does not exist since

$\lim\limits_{x \to 2^-} f(x) = -\infty$ and $\lim\limits_{x \to 2^+} f(x) = \infty$.

8. $f(x) = \dfrac{-2x}{(2x + 1)(3x + 6)}$

$f(x)$ is discontinuous at $x = -\frac{1}{2}$ and $x = -2$ since the denominator equals 0 at these two values.

$\lim\limits_{x \to -2} f(x)$ does not exist since

$\lim\limits_{x \to -2^-} = +\infty$ and $\lim\limits_{x \to -2^+} f(x) = -\infty$.

$\lim\limits_{x \to -\frac{1}{2}} f(x)$ does not exist since

$\lim\limits_{x \to -\frac{1}{2}^-} f(x) = -\infty$ and $\lim\limits_{x \to \frac{1}{2}^+} f(x) = +\infty$.

9. $f(x) = \dfrac{x^2 - 4}{x - 2}$

$f(x)$ is discontinuous at $x = 2$ since the denominator equals zero at that value.

Since for $x \neq 2$

$$\frac{x^2 - 4}{x - 2} = \frac{(x + 2)(x - 2)}{x - 2} = x + 2,$$

$\lim\limits_{x \to 2} f(x) = 2 + 2 = 4.$

10. $f(x) = \dfrac{x^2 - 25}{x + 5}$

$f(x)$ is discontinuous at $x = -5$ since the denominator equals zero at that value.

Since for $x \neq -5$

$$\frac{x^2 - 25}{x + 5} = \frac{(x + 5)(x - 5)}{x + 5} = x - 5,$$

$\lim\limits_{x \to -5} f(x) = -5 - 5 = -10.$

11. $p(x) = x^2 - 4x + 11$

Since $p(x)$ is a polynomial function, it is continuous everywhere and thus discontinuous nowhere.

12. $q(x) = -3x^3 + 2x^2 - 4x + 1$

Since $q(x)$ is a polynomial function, it is continuous everywhere and thus discontinuous nowhere.

13. $p(x) = \dfrac{|x + 2|}{x + 2}$

$p(x)$ is discontinuous at $x = -2$ since the denominator is zero at that value.

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since $\lim_{x \to -2^-} p(x) = -1$ and $\lim_{x \to -2^+} p(x) = 1$, $\lim_{x \to -2} p(x)$ does not exist.

14. $r(x) = \dfrac{|5 - x|}{x - 5}$

$r(x)$ is discontinuous at $x = 5$ since the denominator is zero at that value.

Since $\lim_{x \to 5^+} r(x) = 1$ and $\lim_{x \to 5-} r(x) = -1$, $\lim_{x \to 5} r(x)$ does not exist.

15. $k(x) = e^{\sqrt{x-1}}$

The function is undefined for $x < 1$, so the function is discontinuous for $a < 1$. The limit as x approaches any $a < 1$ does not exist because the function is undefined for $x < 1$.

16. $j(x) = e^{1/x}$

$j(x)$ is discontinuous at $x = 0$ since the function is undefined there.

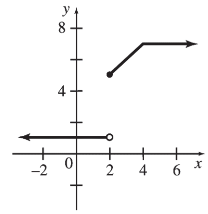
$\lim_{x \to 0} j(x)$ does not exist since $\lim_{x \to 0^-} j(x) = 0$ and $\lim_{x \to 0^+} j(x) = \infty$.

17. As x approaches 0 from the left or the right, $\left|\frac{x}{x-1}\right|$ approaches 0 and $r(x) = \ln\left|\frac{x}{x-1}\right|$ goes to $-\infty$. So $\lim_{x \to 0} r(x)$ does not exist. As x approaches 1 from the left or the right, $\left|\frac{x}{x-1}\right|$ goes to ∞ and so does $r(x) = \ln\left|\frac{x}{x-1}\right|$. So $\lim_{x \to 1} r(x)$ does not exist.

18. As x approaches -2 from the left or the right, $\left|\frac{x+2}{x-3}\right|$ approaches 0 and $r(x) = \ln\left|\frac{x+2}{x-3}\right|$ goes to $-\infty$. So $\lim_{x \to -2} r(x)$ does not exist. As x approaches 3 from the left or the right, $\left|\frac{x+2}{x-3}\right|$ goes to ∞ and so does $r(x) = \ln\left|\frac{x+2}{x-3}\right|$. So $\lim_{x \to 3} r(x)$ does not exist.

19. $f(x) = \begin{cases} 1 & \text{if } x < 2 \\ x + 3 & \text{if } 2 \le x \le 4 \\ 7 & \text{if } x > 4 \end{cases}$

(a)



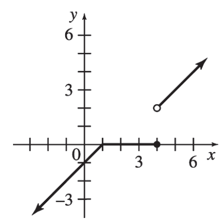
(b) $f(x)$ is discontinuous at $x = 2$.

(c) $\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 5$

20. $f(x) = \begin{cases} x - 1 & \text{if } x < 2 \\ 0 & \text{if } 1 \le x \le 4 \\ x - 2 & \text{if } x > 4 \end{cases}$

(a)



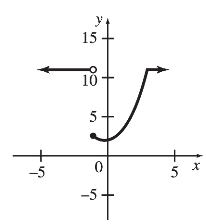
(b) $f(x)$ is discontinuous at $x = 4$.

(c) $\lim_{x \to 4^-} f(x) = 0$

$\lim_{x \to 4^+} f(x) = 2$

21. $g(x) = \begin{cases} 11 & \text{if } x < -1 \\ x^2 + 2 & \text{if } -1 \le x \le 3 \\ 11 & \text{if } x > 3 \end{cases}$

(a)



(b) $g(x)$ is discontinuous at $x = -1$.

(c) $\lim_{x \to -1^-} g(x) = 11$

$\lim_{x \to -1^+} g(x) = (-1)^2 + 2 = 3$

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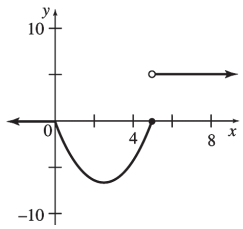
22. $g(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 - 5x & \text{if } 0 \le x \le 5 \\ 5 & \text{if } x > 5 \end{cases}$

(a)



(b) $g(x)$ is discontinuous at $x = 5$.

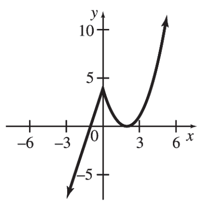
(c) $\lim_{x \to 5^-} g(x) = 5^2 - 5(5)$

$= 0$

$\lim_{x \to 5^+} g(x) = 5$

23. $h(x) = \begin{cases} 4x + 4 & \text{if } x \le 0 \\ x^2 - 4x + 4 & \text{if } x > 0 \end{cases}$

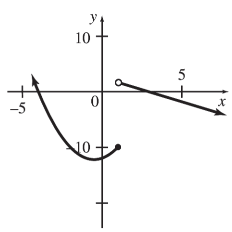
(a)



(b) There are no points of discontinuity.

24. $h(x) = \begin{cases} x^2 + x - 12 & \text{if } x \le 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

(a)



(b) $h(x)$ is discontinuous at $x = 1$.

(c) $\lim_{x \to 1^-} h(x) = 1^2 + 1 - 12 = -10$

$\lim_{x \to 1^+} h(x) = 3 - 1 = 2$

25. Find k so that $kx^2 = x + k$ for $x = 2$.

$$k(2)^2 = 2 + k$$
$$4k = 2 + k$$
$$3k = 2$$
$$k = \frac{2}{3}$$

26. Find k so that $x^3 + k = kx - 5$ for $x = 3$.

$$3^3 + k = 3k - 5$$
$$27 + k = 3k - 5$$
$$32 = 2k$$
$$16 = k$$

27. $\dfrac{2x^2 - x - 15}{x - 3} = \dfrac{(2x + 5)(x - 3)}{x - 3} = 2x + 5$

Find k so that $2x + 5 = kx - 1$ for $x = 3$.

$$2(3) + 5 = k(3) - 1$$
$$6 + 5 = 3k - 1$$
$$11 = 3k - 1$$
$$12 = 3k$$
$$4 = k$$

28. $\dfrac{3x^2 + 2x - 8}{x + 2} = \dfrac{(3x - 4)(x + 2)}{x + 2} = 3x - 4$

Find k so that $3x - 4 = 3x + k$ for $x = -2$.

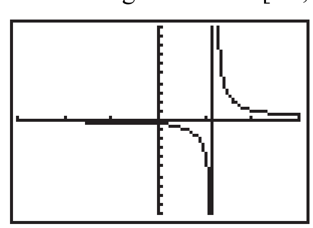
$$3(-2) - 4 = 3(-2) + k$$
$$-6 - 4 = -6 + k$$
$$-10 = -6 + k$$
$$-4 = k$$

31. $f(x) = \dfrac{x^2 + x + 2}{x^3 - 0.9x^2 + 4.14x - 5.4} = \dfrac{P(x)}{Q(x)}$

(a) Graph

$$Y_1 = \frac{P(x)}{Q(x)} = \frac{x^2 + x + 2}{x^3 - 0.9x^2 + 4.14x - 5.4}$$

on a graphing calculator. A good choice for the viewing window is $[-3,3]$ by $[-10,10]$.



The graph has a vertical asymptote at $x = 1.2$, which indicates that f is discontinuous at $x = 1.2$.

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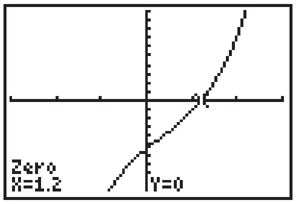
(b) Graph

$$Y_2 = Q(x) = x^3 - .09x^2 + 4.14x - 5.4$$

using the same viewing window.



We see that this graph has one x-intercept, 1.2. This indicates that 1.2 is the only real solution of the equation $Q(x) = 0$.

This result verifies our answer from part (a) because a rational function of the form

$$f(x) = \frac{P(x)}{Q(x)}$$

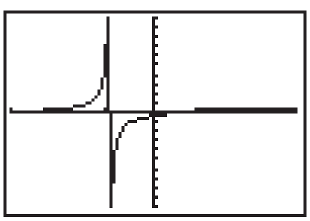
will be discontinuous wherever $Q(x) = 0$.

32. $f(x) = \dfrac{x^2 + 3x - 2}{x^3 - 0.9x^2 + 4.14x + 5.4} = \dfrac{P(x)}{Q(x)}$

(a) Graph

$$Y_1 = \frac{P(x)}{Q(x)} = \frac{x^2 + 3x - 2}{x^3 - 0.9x^2 + 4.14x + 5.4}$$

on a graphing calculator. A good choice for the viewing window is $[-3,3]$ by $[-10,10]$.

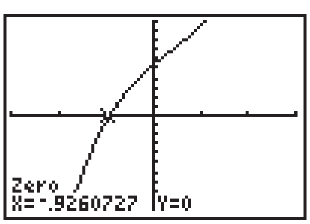


The graph has a vertical asymptote at $x \approx -0.9$. It is difficult to read this value accurately from the graph.

(b) Graph

$$Y_2 = Q(x) = x^3 - 0.9x^2 + 4.14x + 5.4$$

using the same viewing window.



We see that this graph has one x-intercept, ≈ -0.926. This indicates that -0.926 is the only real solution of the equation $Q(x) = 0$.

A rational function of the form $f(x) = \dfrac{P(x)}{Q(x)}$

will be discontinuous wherever $Q(x) = 0$, so we see that f is discontinuous at $x \approx -0.926$.

The result in part (b) is consistent with the result in part (a), but the result in part (b) is more accurate.

33. $$g(x) = \frac{x + 4}{x^2 + 2x - 8} = \frac{x + 4}{(x - 2)(x + 4)} = \frac{1}{x - 2}, x \neq -4$$

If $g(x)$ is defined so that $g(-4) = \frac{1}{-4-2} = -\frac{1}{6}$, then the function becomes continuous at -4. It cannot be made continuous at 2. The correct answer is (a).

34. f is discontinuous at $x = -6$, -4, and 3 because the limit does not exist there. Also, $f(-6)$ and $f(3)$ are undefined.

f is discontinuous at $x = 4$ because even though $\lim\limits_{x \to 4} f(x) = 1$, $f(4)$ is undefined.

f is discontinuous at $x = 0$ because $\lim\limits_{x \to 0} f(x) \neq f(0)$.

35. **(a)** $\lim\limits_{x \to 6} P(x)$

As x approaches 6 from the left or the right, the value of $P(x)$ for the corresponding point on the graph approaches 500.

Thus, $\lim\limits_{x \to 6} P(x) = \$500$.

(b) $\lim\limits_{x \to 10^-} P(x) = \$1500$

because, as x approaches 10 from the left, $P(x)$ approaches \$1500.

(c) $\lim\limits_{x \to 10^+} P(x) = \$1000$ because, as x approaches 10 from the right, $P(x)$ approaches \$1000.

(d) Since $\lim\limits_{x \to 10^+} P(x) \neq \lim\limits_{x \to 10^-} P(x)$, $\lim\limits_{x \to 10} P(x)$ does not exist.

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(e) From the graph, the function is discontinuous at $x = 10$. This may be the result of a change of shifts.

(f) From the graph, the second shift will be as profitable as the first shift when 15 units are produced.

36. In dollars,

$C(x) = 4x$ if $0 < x \leq 150$

$C(x) = 3x$ if $150 < x \leq 400$

$C(x) = 2.5x$ if $400 < x$.

(a) $C(130) = 4(130) = \$520$

(b) $C(150) = 4(150) = \$600$

(c) $C(210) = 3(210) = \$630$

(d) $C(400) = 3(400) = \$1200$

(e) $C(500) = 2.5(500) = \$1250$

(f) C is discontinuous at $x = 150$ and $x = 400$ because those represent points of price change.

37. In dollars,

$$F(x) = \begin{cases} 1.25x & \text{if } 0 < x \leq 100 \\ 1.00x & \text{if } x > 100. \end{cases}$$

(a) $F(80) = 1.25(80) = \$100$

(b) $F(150) = 1.00(150) = \$150$

(c) $F(100) = 1.25(100) = \$125$

(d) F is discontinuous at $x = 100$.

38. $C(t)$ is a step function. The average cost per day is $A(t) = \frac{C(t)}{t}$ for integer t. Thus, $A(t)$ is also a step function.

(a) $A(4) = \dfrac{36(4)}{4} = \$36$

(b) $A(5) = \dfrac{36(5)}{5} = \$36$

(c) $A(6) = \dfrac{180}{6} = \$30$

(d) $A(7) = \dfrac{180}{7} \approx \$25.71$

(e) $A(8) = \dfrac{180 + 36}{8} = \$27$

(f) $\lim_{t \to 5^-} A(t) = 36$ because as t approaches 5 from the left, $A(t)$ is equal to 36.

(g) $\lim_{t \to 5^+} A(t) = 36$ because just to the right of 5, $A(t)$ is equal to 36.

(h) $A(t)$ is discontinuous at 1, 2, 3, 4, 7, 8, 9, 10, 11.

39. $C(x)$ is a step function.

(a) $\lim_{x \to 3^-} C(x) = \$1.40$

(b) $\lim_{x \to 3^+} C(x) = \$1.61$

(c) $\lim_{x \to 3} C(x)$ does not exist.

(d) $C(3) = \$1.40$

(e) $\lim_{x \to 8.5^-} C(x) = \$2.66$

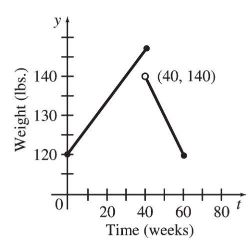
(f) $\lim_{x \to 8.5^+} C(x) = \$2.66$

(g) $\lim_{x \to 8.5} C(x) = \$2.66$

(h) $C(8.5) = \$2.66$

(i) $C(x)$ is discontinuous at 1, 2, 3, …, 11, 12

40. (a) Since $t = 0$ weeks the woman weighs 120 lbs. and at $t = 40$ weeks she weighs 147 lbs. graph the line beginning at coordinate (0, 120) and ending at (40, 147), with closed circles at these points. Since immediately after giving birth, she loses 14 lbs. and continues to lose 13 more lbs. over the following 20 weeks, graph the line between the points (40, 133) and (60, 120) with an open circle at (40, 133) and a closed circle at (60, 120).



(b) From the graph, we see that

$$\lim_{t \to 40^-} w(t) = 147 \neq 133 = \lim_{t \to 40^+} w(t),$$

where $w(t)$ is the weight in pounds t weeks after conception. Therefore, w is discontinuous at $t = 40$.

41.

$$W(t) = \begin{cases} 48 + 3.64t + 0.6363t^2 + 0.00963t^3, & 1 \leq t \leq 28 \\ -1{,}004 + 65.8t, & 28 < t \leq 56 \end{cases}$$

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(a) $W(25) = 48 + 3.64(25) + 0.6363(25)^2 + 0.00963(25)^3 \approx 687.156$

A male broiler at 25 days weighs about 687 grams.

(b) $W(t)$ is not a continuous function.

At $t = 28$

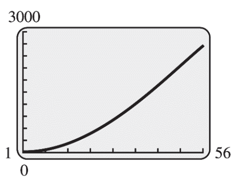
$$\lim_{t\to 28^-} W(t) = \lim_{t\to 28^-} 48 + 3.64t + 0.6363t^2 + 0.00963t^3$$
$$= 48 + 3.64(28) + 0.6363(28)^2 + 0.00963(28)^3$$
$$\approx 860.18$$

and $\lim_{t\to 28^-} W(t) \neq \lim_{t\to 28^+} (-1004 + 65.8t) = -1004 + 65.8(28) = 838.4$

so $\lim_{t\to 28^-} W(t) \neq \lim_{t\to 28^+} W(t)$

Thus $W(t)$ is discontinuous.

(c)



3.3 Rates of Change

Your Turn 1

$A(t) = 11.14(1.023)^t$

Average rate of change from $t = 0$ (2000) to $t = 10$ (2010) is

$$\frac{A(10) - A(0)}{10 - 0} = \frac{11.14(1.023)^{10} - 11.14(1.023)^0}{10} = \frac{13.98 - 11.14}{10} = \frac{2.84}{10} \approx 0.0284,$$

or 0.0284 million.

The U.S. Asian population increased, on average, by 284,000 people per year.

Your Turn 2

$$\frac{A(2013) - A(2011)}{2103 - 2011} = \frac{65.80 - 68.35}{2} = -1.325$$

The average change per year is a decrease of \$1.325 billion.

Your Turn 3

For $t = 2$, the instantaneous velocity is $\lim_{h\to 0} \frac{s(2+h) - s(2)}{h}$ feet per second.

$$\begin{aligned} s(2 + h) &= 2(2 + h)^2 - 5(2 + h) + 40 \\ &= 2(4 + 4h + h^2) - 10 - 5h + 40 \\ &= 8 + 8h + h^2 - 10 - 5h + 40 \\ &= h^2 + 3h + 38 \end{aligned}$$

$$\begin{aligned} s(2) &= 2(2)^2 - 5(2) + 40 \\ &= 2(4) - 10 + 40 \\ &= 38 \end{aligned}$$

$$\begin{aligned} \lim_{h\to 0} \frac{s(2 + h) - s(2)}{h} &= \lim_{h\to 0} \frac{h^2 + 3h + 38 - 38}{h} \\ &= \lim_{h\to 0} \frac{h^2 + 3h}{h} \\ &= \lim_{h\to 0} \frac{h(h + 3)}{h} \\ &= \lim_{h\to 0} h + 3 = 3 \end{aligned}$$

or 3 feet per second.

Your Turn 4

$C(x) = x^2 - 2x + 12$

The instantaneous rate of change of cost when $x = 4$ is

$$\lim_{h\to 0} \frac{C(h + 4) - C(4)}{h}$$
$$= \lim_{h\to 0} \frac{[(h + 4)^2 - 2(h + 4) + 12] - [4^2 - 2(4) + 12]}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 8h + 16 - 2h - 8 + 12 - 16 + 8 - 12}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 6h}{h} = \lim_{h\to 0} \frac{h(h + 6)}{h}$$
$$= \lim_{h\to 0} h + 6 = 0 + 6 = 6$$

When $x = 4$, the cost increases at a rate of \$6 per unit.

Your Turn 5

$A(t) = 11.14(1.023)^t$, $t = 10$ corresponds to 2010

$$\lim_{h\to 0} \frac{11.14(1.023)^{10+h} - 11.14(1.023)^{10}}{h}$$

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Use the TABLE feature on a TI-84 Plus calculator.

h	$\dfrac{11.14(1.023)^{10+h} - 11.14(1.023)^{10}}{h}$
1	0.32164
0.1	0.31836
0.01	0.31803
0.001	0.318
0.0001	0.318
0.00001	0.318

The limit seems to be approaching 0.318 million. The instantaneous rate of change in the U.S. Asian population is about 318,000 people per year in 2010.

3.3 Warmup Exercises

W1. $f(x) = 2x^2 + 3x + 4$

$$\begin{aligned} f(5+h) &= 2(5+h)^2 + 3(5+h) + 4 \\ &= 2(25 + 10h + h^2) + 15 + 3h + 4 \\ &= 50 + 20h + 2h^2 + 19 + 3h \\ &= 2h^2 + 23h + 69 \end{aligned}$$

W2. $f(x) = 2x^2 + 3x + 4$

$$\frac{f(2+h) - f(2)}{h}$$
$$= \frac{2(2+h)^2 + 3(2+h) + 4 - (2(2)^2 + 3(2) + 4)}{h}$$
$$= \frac{2h^2 + 8h + 8 + 6 + 3h + 4 - 18}{h}$$
$$= \frac{2h^2 + 11h}{h} = 2h + 11$$

W3. $f(x) = \dfrac{2}{x+1}$

$$f(3+h) = \frac{2}{(3+h)+1} = \frac{2}{4+h}$$

W4. $f(x) = \dfrac{2}{x+1}$

$$\frac{f(4+h) - f(4)}{h}$$
$$= \frac{1}{h}\cdot\left[\frac{2}{(4+h)+1} - \frac{2}{5}\right]$$
$$= \frac{1}{h}\cdot\left[\frac{(2)(5) - (2)(5+h)}{(5)(5+h)}\right]$$
$$= \frac{1}{h}\cdot\left(\frac{-2h}{(5)(5+h)}\right) = -\frac{2}{(5)(5+h)}$$

3.3 Exercises

1. $y = x^2 + 2x = f(x)$ between $x = 1$ and $x = 3$

Average rate of change

$$= \frac{f(3) - f(1)}{3 - 1}$$
$$= \frac{15 - 3}{2}$$
$$= 6$$

2. $y = -4x^2 - 6 = f(x)$ between $x = 2$ and $x = 6$

Average rate of change

$$= \frac{f(6) - f(2)}{6 - 2}$$
$$= \frac{(-150) - (-22)}{6 - 2}$$
$$= \frac{-150 + 22}{4}$$
$$= \frac{-128}{4} = -32$$

3. $y = -3x^3 + 2x^2 - 4x + 1 = f(x)$ between $x = -2$ and $x = 1$

Average rate of change

$$= \frac{f(1) - f(-2)}{1 - (-2)}$$
$$= \frac{(-4) - (-41)}{1 - (-2)} = \frac{-45}{3} = -15$$

4. $y = 2x^3 - 4x^2 + 6x = f(x)$ between $x = -1$ and $x = 4$

Average rate of change

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$$= \frac{f(4) - f(-1)}{4 - (-1)}$$
$$= \frac{88 - (-12)}{5}$$
$$= \frac{100}{5}$$
$$= 20$$

5. $y = \sqrt{x} = f(x)$ between $x = 1$ and $x = 4$

Average rate of change

$$= \frac{f(4) - f(1)}{4 - 1}$$
$$= \frac{2 - 1}{3}$$
$$= \frac{1}{3}$$

6. $y = \sqrt{3x - 2} = f(x)$ between $x = 1$ and $x = 2$

$$\text{Average rate of change} = \frac{f(2) - f(1)}{2 - 1}$$
$$= \frac{2 - 1}{2 - 1}$$
$$= \frac{1}{1} = 1$$

7. $y = e^x = f(x)$ between $x = -2$ and $x = 0$

Average rate of change

$$= \frac{f(0) - f(-2)}{0 - (-2)}$$
$$= \frac{1 - e^{-2}}{2}$$
$$\approx 0.4323$$

8. $y = \ln x = f(x)$ between $x = 2$ and $x = 4$

$$\text{Average rate of change} = \frac{f(4) - f(2)}{4 - 2}$$
$$= \frac{\ln 4 - \ln 2}{2}$$
$$\approx 0.3466$$

9.

$$\lim_{h \to 0} \frac{s(6 + h) - s(6)}{h}$$
$$= \lim_{h \to 0} \frac{(6 + h)^2 + 5(6 + h) + 2 - [6^2 + 5(6) + 2]}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + 17h + 68 - 68}{h} = \lim_{h \to 0} \frac{h^2 + 17h}{h}$$
$$= \lim_{h \to 0} \frac{h(h + 17)}{h} = \lim_{h \to 0} (h + 17) = 17$$

The instantaneous velocity at $t = 6$ is 17.

10. $s(t) = t^2 + 5t + 2$

$$\lim_{h \to 0} \frac{s(1 + h) - s(1)}{h}$$
$$= \lim_{h \to 0} \frac{[(1 + h)^2 + 5(1 + h) + 2] - [(1)^2 + 5(1) + 2]}{h}$$
$$= \lim_{h \to 0} \frac{[1 + 2h + h^2 + 5 + 5h + 2] - [1 + 5 + 2]}{h}$$
$$= \lim_{h \to 0} \frac{8 + 7h + h^2 - 8}{h} = \lim_{h \to 0} \frac{7h + h^2}{h}$$
$$= \lim_{h \to 0} \frac{h(7 + h)}{h} = \lim_{h \to 0} (7 + h) = 7$$

The instantaneous velocity at $t = 1$ is 7.

11. $s(t) = 5t^2 - 2t - 7$

$$\lim_{h \to 0} \frac{s(2 + h) - s(2)}{h}$$
$$= \lim_{h \to 0} \frac{[5(2 + h)^2 - 2(2 + h) - 7] - [5(2)^2 - 2(2) - 7]}{h}$$
$$= \lim_{h \to 0} \frac{[20 + 20h + 5h^2 - 4 - 2h - 7] - [20 - 4 - 7]}{h}$$
$$= \lim_{h \to 0} \frac{9 + 18h + 5h^2 - 9}{h} = \lim_{h \to 0} \frac{18h + 5h^2}{h}$$
$$= \lim_{h \to 0} \frac{h(18 + 5h)}{h} = \lim_{h \to 0} (18 + 5h) = 18$$

The instantaneous velocity at $t = 2$ is 18.

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12. $s(t) = 5t^2 - 2t - 7$

$$\lim_{h\to 0} \frac{s(3+h) - s(3)}{h}$$

$$= \lim_{h\to 0} \frac{[5(3+h)^2 - 2(3+h) - 7] - [5(3)^2 - 2(3) - 7]}{h}$$

$$= \lim_{h\to 0} \frac{[45 + 30h + 5h^2 - 6 - 2h - 7] - [45 - 6 - 7]}{h}$$

$$= \lim_{h\to 0} \frac{32 + 28h + 5h^2 - 32}{h} = \lim_{h\to 0} \frac{28h + 5h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(28 + 5h)}{h} = \lim_{h\to 0} (28 + 5h) = 28$$

The instantaneous velocity at $t = 3$ is 28.

13. $s(t) = t^3 + 2t + 9$

$$\lim_{x\to 0} \frac{s(1+h) - s(1)}{h}$$

$$= \lim_{h\to 0} \frac{[(1+h)^3 + 2(1+h) + 9] - [(1)^3 + 2(1) + 9]}{h}$$

$$= \lim_{h\to 0} \frac{[1 + 3h + 3h^2 + h^3 + 2 + 2h + 9] - [1 + 2 + 9]}{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 3h^2 + 5h + 12 - 12}{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 3h^2 + 5h}{h} = \lim_{h\to 0} \frac{h(h^2 + 3h + 5)}{h}$$

$$= \lim_{h\to 0} (h^2 + 3h + 5) = 5$$

The instantaneous velocity at $t = 1$ is 5.

14.

$s(t) = t^3 + 2t + 9$

$$\lim_{h\to 0} \frac{s(4+h) - s(4)}{h}$$

$$= \lim_{h\to 0} \frac{(4+h)^3 + 2(4+h) + 9 - [4^3 + 2(4) + 9]}{h}$$

$$= \lim_{h\to 0} \frac{4^3 + 3(4^2)h + 3(4h^2) + h^3 + 8 + 2h + 9 - (4^3 + 8 + 9)}{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 12h^2 + 48h + 2h}{h} = \lim_{h\to 0} \frac{h^3 + 12h^2 + 50h}{h}$$

$$= \lim_{h\to 0} \frac{h(h^2 + 12h + 50)}{h} = \lim_{h\to 0} (h^2 + 12h + 50) = 50$$

The instantaneous velocity at $t = 4$ is 50.

15. $f(x) = x^2 + 2x$ at $x = 0$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{(0+h)^2 + 2(0+h) - [0^2 + 2(0)]}{h}$$

$$= \lim_{h\to 0} \frac{h^2 + 2h}{h}$$

$$= \lim_{h\to 0} \frac{h(h+2)}{h}$$

$$= \lim_{h\to 0} h + 2 = 2$$

The instantaneous rate of change at $x = 0$ is 2.

16. $s(t) = -4t^2 - 6$ at $t = 2$

$$\lim_{h\to 0} \frac{s(2+h) - s(2)}{h}$$

$$= \lim_{h\to 0} \frac{-4(2+h)^2 - 6 - [-4(2)^2 - 6]}{h}$$

$$= \lim_{h\to 0} \frac{-4(4 + 4h + h^2) - 6 + 16 + 6}{h}$$

$$= \lim_{h\to 0} \frac{-16h - 4h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-16 - 4h)}{h}$$

$$= \lim_{h\to 0} (-16 - 4h) = -16$$

The instantaneous rate of change at $t = 2$ is -16.

17. $g(t) = 1 - t^2$ at $t = -1$

$$\lim_{h\to 0} \frac{g(-1+h) - g(-1)}{h}$$

$$= \lim_{h\to 0} \frac{1 - (-1+h)^2 - [1 - (-1)^2]}{h}$$

$$= \lim_{h\to 0} \frac{1 - (1 - 2h + h^2) - 1 + 1}{h}$$

$$= \lim_{h\to 0} \frac{2h - h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2-h)}{h}$$

$$= \lim_{h\to 0} (2 - h) = 2$$

The instantaneous rate of change at $t = -1$ is 2.

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18. $F(x) = x^2 + 2$ at $x = 0$

$$\lim_{h\to 0} \frac{F(0+h) - F(0)}{h}$$
$$= \lim_{h\to 0} \frac{(0+h)^2 + 2 - [0^2 + 2]}{h}$$
$$= \lim_{h\to 0} \frac{h^2}{h}$$
$$= \lim_{h\to 0} h$$
$$= 0$$

The instantaneous rate of change at $x = 0$ is 0.

19. $f(x) = x^x$ at $x = 2$

h	
0.01	$\frac{f(2+0.01) - f(2)}{0.01} = \frac{2.01^{2.01} - 2^2}{0.01} = 6.84$
0.001	$\frac{f(2+0.001) - f(2)}{0.001} = \frac{2.001^{2.001} - 2^2}{0.001} = 6.779$
0.0001	$\frac{f(2+0.0001) - f(2)}{0.00001} = \frac{2.0001^{2.0001} - 2^2}{0.0001} = 6.773$
0.00001	$\frac{f(2+0.00001) - f(2)}{0.00001} = \frac{2.00001^{2.00001} - 2^2}{0.00001} = 6.7727$
0.000001	$\frac{f(2+0.000001) - f(2)}{0.000001} = \frac{2.000001^{2.000001} - 2^2}{0.000001} = 6.7726$

The instantaneous rate of change at $x = 2$ is 6.773.

20. $f(x) = x^x$ at $x = 3$

h	
0.01	$\frac{f(3+0.01) - f(3)}{0.01} = \frac{3.01^{3.01} - 3^3}{0.01} = 57.3072$
0.001	$\frac{f(3+0.001) - f(3)}{0.001} = \frac{3.001^{3.001} - 3^3}{0.001} = 56.7265$
0.00001	$\frac{f(3+0.00001) - f(3)}{0.00001} = \frac{3.00001^{3.00001} - 3^3}{0.00001} = 56.6632$
0.000001	$\frac{f(3+0.000001) - f(3)}{0.000001} = \frac{3.000001^{3.000001} - 3^3}{0.000001} = 56.6626$
0.0000001	$\frac{f(3+0.0000001) - f(3)}{0.0000001} = \frac{3.0000001^{3.0000001} - 3^3}{0.0000001} = 56.6625$

The instantaneous rate of change at $x = 3$ is 56.66.

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21. $f(x) = x^{\ln x}$ at $x = 2$

h	
0.01	$\frac{f(2+0.01)-f(2)}{0.01} = \frac{2.01^{\ln 2.01} - 2^{\ln 2}}{0.01} = 1.1258$
0.001	$\frac{f(2+0.001)-f(2)}{0.001} = \frac{2.001^{\ln 2.001} - 2^{\ln 2}}{0.001} = 1.1212$

h	
0.0001	$\frac{f(2+0.0001)-f(2)}{0.0001} = \frac{2.0001^{\ln 2.0001} - 2^{\ln 2}}{0.0001} = 1.1207$
0.00001	$\frac{f(2+0.00001)-f(2)}{0.00001} = \frac{2.00001^{\ln 2.00001} - 2^{\ln 2}}{0.00001} = 1.1207$

The instantaneous rate of change at $x = 2$ is 1.121.

22. $f(x) = x^{\ln x}$ at $x = 3$

h	
0.01	$\frac{f(3+0.01)-f(3)}{0.01} = \frac{3.01^{\ln 3.01} - 3^{\ln 3}}{0.01} = 2.4573$
0.001	$\frac{f(3+0.001)-f(3)}{0.001} = \frac{3.001^{\ln 3.001} - 3^{\ln 3}}{0.001} = 2.4495$
0.00001	$\frac{f(3+0.00001)-f(3)}{0.00001} = \frac{3.00001^{\ln 3.00001} - 3^{\ln 3}}{0.00001} = 2.4486$

The instantaneous rate of change at $x = 3$ is 2.449.

24. If the instantaneous rate of change of $f(x)$ with respect to x is positive when $x = 1$, the function would be increasing.

25. $P(x) = 2x^2 - 5x + 6$

(a) $P(4) = 18$
$P(2) = 4$

Average rate of change of profit

$$= \frac{P(4) - P(2)}{4 - 2} = \frac{18 - 4}{2} = \frac{14}{2} = 7,$$

which is \$700 per item.

(b) $P(3) = 9$
$P(2) = 4$

Average rate of change of profit

$$= \frac{P(3) - P(2)}{3 - 2} = \frac{9 - 4}{1} = 5$$

which is \$500 per item.

(c) $\lim_{h\to 0} \frac{P(2+h) - P(2)}{h}$

$$= \lim_{h\to 0} \frac{2(2+h)^2 - 5(2+h) + 6 - 4}{h}$$
$$= \lim_{h\to 0} \frac{8 + 8h + 2h^2 - 10 - 5h + 2}{h}$$
$$= \lim_{h\to 0} \frac{2h^2 + 3h}{h} = \lim_{h\to 0} \frac{h(2h+3)}{h}$$
$$= \lim_{h\to 0} (2h + 3) = 3,$$

which is \$300 per item.

(d) $\lim_{h\to 0} \frac{P(4+h) - P(4)}{h}$

$$= \lim_{h\to 0} \frac{2(4+h)^2 - 5(4+h) + 6 - 18}{h}$$
$$= \lim_{h\to 0} \frac{32 + 16h + 2h^2 - 20 - 5h - 12}{h}$$
$$= \lim_{h\to 0} \frac{2h^2 + 11h}{h}$$
$$= \lim_{h\to 0} \frac{h(2h+11)}{h}$$
$$= \lim_{h\to 0} 2h + 11 = 11,$$

which is \$1100 per item.

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26. $R = 10x - 0.002x^2$

(a) Average rate of change $= \dfrac{R(1001) - R(1000)}{1001 - 1000} = \dfrac{8005.998 - 8000}{1} = 5.998$

The average rate of change is $5998.

(b) Marginal revenue $= \lim\limits_{h \to 0} \dfrac{R(1000 + h) - R(1000)}{h}$

$$= \lim_{h \to 0} \frac{[10(1000 + h) - 0.002(1000 + h)^2] - [10(1000) - 0.002(1000)^2]}{h}$$

$$= \lim_{h \to 0} \frac{[10{,}000 + 10h - 0.002(1{,}000{,}000 + 2000h + h^2)] - 8000}{h}$$

$$= \lim_{h \to 0} \frac{10{,}000 + 10h - 2000 - 4h - 0.002h^2 - 8000}{h}$$

$$= \lim_{h \to 0} \frac{6h - 0.002h^2}{h} = \lim_{h \to 0} \frac{h(6 - 0.002h)}{h} = \lim_{h \to 0} (6 - 0.002h) = 6$$

The marginal revenue is $6000. This is approximately the revenue generated by the 1000th unit produced.

(c) Additional revenue $= R(1001) - R(1000) = [10(1001) - 0.002(1001)^2] - [10(1000) - 0.002(1000)^2]$

$= 8005.998 - 8000$

The additional revenue is $5998.

(d) The answers to parts (a) and (c) are the same and are approximately equal to the answer to part (b).

27. $N(p) = 80 - 5p^2, 1 \le p \le 4$

(a) Average rate of change of demand is

$$\frac{N(3) - N(2)}{3 - 2} = \frac{35 - 60}{1}$$
$= -25$ boxes per dollar.

(b) Instantaneous rate of change when p is 2 is

$$\lim_{h \to 0} \frac{N(2 + h) - N(2)}{h}$$
$$= \lim_{h \to 0} \frac{80 - 5(2 + h)^2 - [80 - 5(2)^2]}{h}$$
$$= \lim_{h \to 0} \frac{80 - 20 - 20h - 5h^2 - (80 - 20)}{h}$$
$$= \lim_{h \to 0} \frac{-5h^2 - 20h}{h} = -20 \text{ boxes per dollar.}$$

Around the $2 point, a $1 price increase (say, from $1.50 to $2.50) causes a drop in demand of about 20 boxes.

(c) Instantaneous rate of change when p is 3 is

$$\lim_{h \to 0} \frac{80 - 5(3 + h)^2 - [80 - 5(3)^2]}{h}$$
$$= \lim_{h \to 0} \frac{80 - 45 - 30h - 5h^2 - 80 + 45}{h}$$
$$= \lim_{h \to 0} \frac{-30h - 5h^2}{h}$$
$= -30$ boxes per dollar.

(d) As the price increases, the demand decreases; this is an expected change.

28. $A(t) = 1000(1.03)^t$

(a) Average rate of change in the total amount from $t = 0$ to $t = 5$:

$$\frac{A(5) - A(0)}{5 - 0} = \frac{1000(1.03)^5 - 1000(1.03)^0}{5}$$
$= 31.8548,$

which is $31.85 per year.

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(b) Average rate of change in the total amount from $t = 5$ to $t = 10$:

$$\frac{A(10) - A(5)}{10 - 5} = \frac{1000(1.03)^{10} - 1000(1.03)^5}{5} = 36.9285,$$

which is \$36.93 per year.

(c) Instantaneous rate of change for $t = 5$:

$$\lim_{h \to 0} \frac{1000(1.03)^{5+h} - 1000(1.03)^5}{h}$$

Use the TABLE feature on a TI-84 Plus calculator to estimate the limit.

h	$\frac{1000(1.03)^{5+h} - 1000(1.03)^5}{h}$
1	34.7782
0.1	34.3174
0.01	34.2718
0.001	34.2673
0.0001	34.2668
0.00001	34.2668

The limit seems to be approaching 34.27. So, the instantaneous rate of change for $t = 5$ is about \$34.27 per year.

29. $A(t) = 1000e^{0.03t}$

(a) Average rate of change in the total amount from $t = 0$ to $t = 5$:

$$\frac{A(5) - A(0)}{5 - 0} = \frac{1000e^{0.03(5)} - 1000e^{0.03(0)}}{5} = 32.3668,$$

which is \$32.37 per year.

(b) Average rate of change in the total amount from $t = 5$ to $t = 10$:

$$\frac{A(10) - A(5)}{10 - 5} = \frac{1000e^{0.03(10)} - 1000e^{0.03(5)}}{5} = 37.6049,$$

which is \$37.60 per year.

(c) Instantaneous rate of change for $t = 5$:

$$\lim_{h \to 0} \frac{1000e^{0.03(5+h)} - 1000e^{0.03(5)}}{h}$$

Use the TABLE feature on a TI-84 Plus calculator to estimate the limit.

h	$\frac{1000e^{0.03(5+h)} - 1000e^{0.03(5)}}{h}$
1	35.3831
0.1	34.9074
0.01	34.8603
0.001	34.8556
0.0001	34.8551
0.00001	34.8550

The limit seems to be approaching 34.855. So, the instantaneous rate of change for $t = 5$ is about \$34.86 per year.

30. **(a)** $\frac{4.0 - 6.6}{2000 - 1994} \approx -0.43$

(b) $\frac{4.7 - 4.0}{2006 - 2000} \approx 0.12$

(c) $\frac{8.2 - 4.7}{2012 - 2006} \approx 0.58$

31. Let $P(t)$ = the price per gallon of gasoline for the month t, where $t = 1$ represents January, $t = 2$ represents February, and so on.

(a) $P(1) = 339$ (cents)
$P(3) = 378$ (cents)

Average change in price from January to March:

$$\frac{P(3) - P(1)}{3 - 1} = \frac{378 - 339}{2} = 19.5$$

On average, the price of gasoline increased about 19.5 cents per gallon per month.

(b) $P(3) = 378$ (cents), $P(12) = 336$ (cents)

Average change in price from March to December:

$$\frac{P(12) - P(3)}{12 - 3} = \frac{336 - 378}{9} \approx -4.7$$

On average, the price of gasoline decreased about 4.7 cents per gallon per month.

(c) $P(1) = 339$ (cents), $P(12) = 336$ (cents)

Average change in price from January to December:

$$\frac{P(12) - P(1)}{12 - 1} = \frac{336 - 339}{12} \approx -0.3$$

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On average, the price of gasoline decreased about 0.3 cents per gallon per month.

32. **(a)** For the period 2008 to 2012

$$\frac{287.6 - 381.6}{4} = -23.5$$

Decreases by an average of \$23.5 billion per year.

(b) For the period 2012 to 2020:

$$\frac{493.3 - 287.6}{8} \approx 25.7$$

Increases by an average of \$25.7 billion per year.

33. **(a)** $$\frac{A(15) - A(0)}{15} = \frac{11.14\left(1.023^{15} - 1.023^{0}\right)}{15} \approx 0.302$$

A gives the population in millions so this is an average rate of change of 302,000 people per year.

(b) Use the TABLE feature on a TI-84 Plus calculator to estimate the limit.

h	$\frac{11.14(1.023)^{15+h} - 11.14(1.023)^{15}}{h}$
1	0.3604
0.1	0.3567
0.01	0.3563
0.001	0.3563

The limit seems to be approaching 0.356. So, the instantaneous rate of change of the Asian population in 2015 is about 356,000 people per year.

34. Let $P(t) =$ world population estimated in billions for year t.

(a) $P(1990) = 5.3$

If replacement-level fertility is reached in 2010, $P(2050) = 8.6$.

Average rate of change

$$= \frac{P(2050) - P(1990)}{2050 - 1990}$$
$$= \frac{8.6 - 5.3}{60} = 0.055$$

On average, the population will increase 55 million per year.

If replacement-level fertility is reached in 2030, $P(2050) = 9.2$.

Average rate of change

$$= \frac{P(2050) - P(1990)}{2050 - 1990}$$
$$= \frac{9.2 - 5.3}{60} = 0.065$$

On average, the population will increase 65 million per year.

If replacement-level fertility is reached in 2050, $P(2050) = 9.8$.

Average rate of change

$$= \frac{P(2050) - P(1990)}{2050 - 1990}$$
$$= \frac{9.8 - 5.3}{60} = 0.075$$

On average, the population will increase 75 million per year. The projection for replacement-level fertility by 2010 predicts the smallest rate of change in world population.

(b) If replacement-level fertility is reached in 2010

$$P(2090) = 9.3$$
$$P(2130) = 9.6$$

Average rate of change

$$= \frac{P(2130) - P(2090)}{2130 - 2090}$$
$$= \frac{9.6 - 9.3}{40}$$
$$= 0.0075$$

On average, the population will increase 7.5 million per year.

If replacement-level fertility is reached in 2030,

$$P(2090) = 10.3$$
$$P(2130) = 10.6$$

Average rate of change

$$= \frac{P(2130) - P(2090)}{2130 - 2090}$$
$$= \frac{10.6 - 10.3}{40}$$
$$= 0.0075$$

On average, the population will increase 7.5 million per year.

If replacement-level fertility is reached in 2050,

$$P(2090) = 11.35$$
$$P(2130) = 11.75$$

Average rate of change

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$$= \frac{P(2130) - P(2090)}{2130 - 2090}$$
$$= \frac{11.75 - 11.35}{40}$$
$$= 0.01$$

On average, the population will increase 10 million per year. From 2090 – 2130 the three projections show almost the same rate of change in world population.

35. **(a)** 2006 to 2008:
$$\frac{47{,}800 - 48{,}600}{2} = -400 \text{ per year}$$
(b) 2008 to 2010:
$$\frac{47{,}500 - 47{,}800}{2} = -150 \text{ per year}$$
(c) 2007 to 1011:
$$\frac{49{,}273 - 56{,}000}{4} \approx -1682 \text{ per year}$$

36. $L(t) = -0.01t^2 + 0.788t - 7.048$

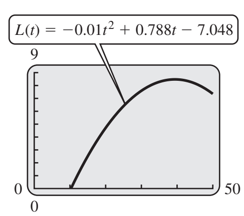
(a) $$\frac{L(28) - L(22)}{28 - 22} = \frac{7.176 - 5.448}{6} = 0.288$$

The average rate of growth during weeks 22 through 28 is 0.288 mm per week.

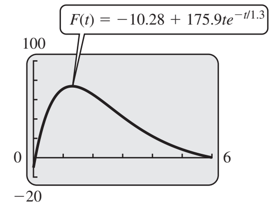
(b)
$$\lim_{h \to 0} \frac{L(t + h) - L(t)}{h}$$
$$= \lim_{h \to 0} \frac{L(22 + h) - L(22)}{h}$$
$$= \lim_{h \to 0} \frac{[-0.01(22 + h)^2 + 0.788(22 + h) - 7.048] - 5.448}{h}$$
$$= \lim_{h \to 0} \frac{-0.01(h^2 + 44h + 484) + 17.336 + 0.788h - 12.496}{h}$$
$$= \lim_{h \to 0} \frac{-0.01h^2 + 0.348h}{h}$$
$$= \lim_{h \to 0} (-0.01h + 0.348)$$
$$= 0.348$$

The instantaneous rate of growth at exactly 22 weeks is 0.348 mm per week.

(c)



37. **(a)**



(b) The average rate of change during the first hour is
$$\frac{F(1) - F(0)}{1 - 0} \approx 81.51$$
kilojoules per hour per hour.

(c) Store $F(t)$ in a function menus of a graphing calculator. Store $\frac{Y_1(1+X) - Y_1(1)}{X}$ as Y_2 in the function menu, where Y_1 represents $F(t)$. Substitute small values for X in Y_2 perhaps with use of a table feature of the graphing calculator. As X is allowed to get smaller, Y_2 approaches 18.81 kilojoules per hour per hour.

(d) Through use of a MAX/feature program of a graphing calculator, the maximum point seen in part (a) is estimated to occur at approximately $t = 1.3$ hours.

38. **(a)** The average rate of change of $M(t)$ on the interval $[105, 115]$ is
$$\frac{M(115) - M(105)}{115 - 105} = \frac{0.8}{10} = 0.08$$
kilograms per day.

(b) Calculate $\lim_{h \to 0} \frac{M(105 + h) - M(105)}{h}$

$$M(105 + h) = 27.5 + 0.3(105 + h) - 0.001(105 + h)^2$$
$$= 27.5 + 31.5 + 0.3h - (11.025 + 0.21h + 0.001h^2)$$
$$= 47.975 + 0.09h - 0.001h^2$$
$$M(105) = 47.975$$

So, the instantaneous rate of change of $M(t)$ at $t = 105$ is

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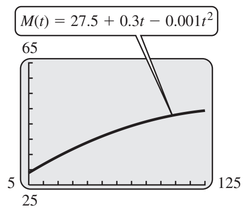
$$\lim_{h\to 0}\left(\frac{47.975 + 0.09h - 0.001h^2 - 47.975}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{0.09h - 0.001h^2}{h}\right)$$
$$= \lim_{h\to 0}(0.09 - 0.001h)$$
$$= 0.09 \text{ kilograms per day.}$$

(c)



39. Let $I(t)$ represent immigration (in thousands) in year t.

(a) $$\frac{I(1960) - I(1910)}{1960 - 1910} = \frac{265 - 1042}{50} = -15.54$$

The average rate of change is $-15,540$ immigrants per year.

(b) $$\frac{I(2010) - I(1960)}{2010 - 1960} = \frac{1043 - 265}{50} = 15.56$$

The average rate of change is 15,560 immigrants per year.

(c) $$\frac{I(2010) - I(1910)}{2010 - 1910} = \frac{1043 - 1042}{100} = 0.01$$

The average rate of change is 10 immigrants per year.

(d) $$\frac{-15{,}540 + 15{,}560}{2} = \frac{20}{2} = 10$$

They are equal. This will not be true for all time periods. (It is true only for time periods of equal length.)

(e) 2013 is 53 years after 1960.

$1{,}043{,}000 + 53(15{,}560/\text{year}) \approx 1{,}090{,}000$

The predicted number of immigrants in 2013 is about 1,090,000 immigrants. The predicted value is about 99,000 more than the actual number of 990,553.

40. Let $D(t)$ represent the percent of students (8th, 10th, or 12th graders) who have used marijuana by the year t.

(a) 8th graders:

$$\frac{D(2010) - D(2007)}{2010 - 2007} = \frac{17.3 - 14.2}{3} = 1.03 \text{ percent per year}$$
$$\frac{D(2013) - D(2010)}{2013 - 2010} = \frac{16.5 - 17.3}{3} = -0.27 \text{ percent per year}$$
$$\frac{D(2013) - D(2007)}{2013 - 2007} = \frac{16.5 - 14.2}{6} = 0.38 \text{ percent per year}$$

(b) 10th graders:

$$\frac{D(2010) - D(2007)}{2010 - 2007} = \frac{33.4 - 31}{3} = 0.80 \text{ percent per year}$$
$$\frac{D(2013) - D(2010)}{2013 - 2010} = \frac{35.8 - 33.4}{3} = 0.80 \text{ percent per year}$$
$$\frac{D(2013) - D(2007)}{2013 - 2007} = \frac{35.8 - 31}{6} = 0.80 \text{ percent per year}$$

(c) 12th graders:

$$\frac{D(2010) - D(2007)}{2010 - 2007} = \frac{43.8 - 41.8}{3} = 0.67 \text{ percent per year}$$
$$\frac{D(2013) - D(2010)}{2013 - 2010} = \frac{45.5 - 43.8}{3} = 0.57 \text{ percent per year}$$
$$\frac{D(2013) - D(2007)}{20013 - 2007} = \frac{45.5 - 41.8}{6} = 0.62 \text{ percent per year}$$

41. (a) $$\frac{T(3000) - T(1000)}{3000 - 1000} = \frac{23 - 15}{2000} = \frac{8}{2000} = \frac{4}{1000}$$

From 1000 to 3000 ft, the temperature changes about 4° per 1000 ft; the temperature rises (on the average).

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(b) $\dfrac{T(5000) - T(1000)}{5000 - 1000} = \dfrac{22 - 15}{4000}$
$= \dfrac{7}{4000}$
$= \dfrac{1.75}{1000}$

From 1000 to 5000 ft, the temperature changes about 1.75° per 1000 ft; the temperature rises (on the average).

(c) $\dfrac{T(9000) - T(3000)}{9000 - 3000} = \dfrac{15 - 23}{6000}$
$= \dfrac{-8}{6000}$
$= \dfrac{-\frac{4}{3}}{1000}$

From 3000 to 9000 ft, the temperature changes about $-\frac{4}{3}^{\circ}$ per 1000 ft; the temperature falls (on the average).

(d) $\dfrac{T(9000) - T(1000)}{9000 - 1000} = \dfrac{15 - 15}{8000} = 0$

From 1000 to 9000 ft, the temperature changes about 0° per 1000 ft; the temperature stays constant (on the average).

(e) The temperature is highest at 3000 ft and lowest at 1000 ft. If 7000 ft is changed to 10,000 ft, the lowest temperature would be at 10,000 ft.

(f) The temperature at 9000 ft is the same as 1000 ft.

42. **(a)** $\dfrac{s(2) - s(0)}{2 - 0} = \dfrac{10 - 0}{2} = 5 \text{ ft/sec}$

(b) $\dfrac{s(4) - s(2)}{4 - 2} = \dfrac{14 - 10}{2} = 2 \text{ ft/sec}$

(c) $\dfrac{s(6) - s(4)}{6 - 4} = \dfrac{20 - 14}{2} = 3 \text{ ft/sec}$

(d) $\dfrac{s(8) - s(6)}{8 - 6} = \dfrac{30 - 20}{2} = 5 \text{ ft/sec}$

(e) (i) $\dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$
$= \dfrac{f(4 + 2) - f(4 - 2)}{(2)(2)}$
$= \dfrac{f(6) - f(2)}{4}$
$= \dfrac{20 - 10}{4}$
$= \dfrac{10}{4} = 2.5 \text{ ft/sec}$

(ii) $\dfrac{2 + 3}{2} = 2.5 \text{ ft/sec}$

(f) (i) $\dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$
$= \dfrac{f(6 + 2) - f(6 - 2)}{(2)(2)}$
$= \dfrac{f(8) - f(4)}{4}$
$= \dfrac{30 - 14}{4}$
$= \dfrac{16}{4} = 4 \text{ ft/sec}$

(ii) $\dfrac{3 + 5}{2} = 4 \text{ ft/sec}$

43. **(a)** Average rate of change from 0.5 to 1:

$$\frac{f(1) - f(0.5)}{1 - 0.5} = \frac{55 - 30}{0.5} = 50 \text{ mph}$$

Average rate of change from 1 to 1.5:

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{80 - 55}{0.5} = 50 \text{ mph}$$

Estimate of instantaneous velocity is

$$\frac{50 + 50}{2} = 50 \text{ mph.}$$

(b) Average rate of change from 1.5 to 2:

$$\frac{f(2) - f(1.5)}{2 - 1.5} = \frac{104 - 80}{0.5} = 48 \text{ mph}$$

Average rate of change from 2 to 2.5

$$\frac{f(2.5) - f(2)}{2.5 - 2} = \frac{124 - 104}{0.5} = 40 \text{ mph}$$

Estimate of instantaneous velocity is

$$\frac{48 + 40}{2} = 44 \text{ mph.}$$

44. $s(t) = t^2 + 5t + 2$

(a) Average velocity

$= \dfrac{s(6) - s(4)}{6 - 4}$
$= \dfrac{68 - 38}{6 - 4}$
$= \dfrac{30}{2} = 15 \text{ ft/sec}$

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(b) Average velocity

$$= \frac{s(5) - s(4)}{5 - 4}$$
$$= \frac{52 - 38}{5 - 4}$$
$$\frac{14}{1} = 14 \text{ ft/sec}$$

(c) $\lim_{h \to 0} \frac{s(4 + h) - s(4)}{h}$

$$= \lim_{h \to 0} \frac{(4 + h)^2 + 5(4 + h) + 2 - 38}{h}$$
$$= \lim_{h \to 0} \frac{16 + 8h + h^2 + 20 + 5h + 2 - 38}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + 13h}{h}$$
$$= \lim_{h \to 0} \frac{h(h + 13)}{h}$$
$$= \lim_{h \to 0} (h + 13)$$
$$= 13 \text{ ft/sec}$$

3.4 Definition of the Derivative

Your Turn 1

(a) $f(x) = x^2 - x, \; x = -2$ and $x = 1.$

Slope of secant line

$$= \frac{f(1) - f(-2)}{1 - (-2)} = \frac{0 - 6}{3} = -2$$

Use the point-slope form and the point $(1, f(1))$, or $(1, 0)$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -2(x - 1)$$
$$y = -2x + 2$$

(b) slope of tangent at $(-2, 6)$

$$= \lim_{h \to 0} \frac{f(-2 + h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{[(-2 + h)^2 - (-2 + h)] - [(-2)^2 - (-2)]}{h}$$
$$= \lim_{h \to 0} \frac{\left[4 - 4h + h^2 + 2 - h\right] - \left[4 + 2\right]}{h}$$
$$= \lim_{h \to 0} \frac{-5h + h^2}{h} = \lim_{h \to 0} (-5 + h)$$
$$= -5$$

The equation of the tangent line is

$$y - 6 = (-5)(x - (-2))$$
$$y = 6 - 5x - 10$$
$$y = -5x - 4.$$

Your Turn 2

$$f(x) = x^2 - x$$
$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{[(x + h)^2 - (x + h)] - [x^2 - x]}{h}$$
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$$
$$= \lim_{h \to 0} \frac{2xh - h + h^2}{h} = \lim_{h \to 0} \frac{h(2x - 1 + h)}{h}$$
$$= \lim_{h \to 0} (2x - 1 + h) = 2x - 1 + 0$$
$$= 2x - 1$$
$$f'(-2) = 2(-2) - 1 = -5$$

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Your Turn 3

$$\begin{aligned}
f(x) &= x^3 - 1\\
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{[(x+h)^3-1]-[x^3-1]}{h}\\
&= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-1-x^3+1}{h}\\
&= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}\\
&= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}\\
&= \lim_{h\to 0}(3x^2+3xh+h^2)\\
&= 3x^2+0+0\\
&= 3x^2\\
f'(-1) &= 3(-1)^2 = 3
\end{aligned}$$

Your Turn 4

$$\begin{aligned}
f(x) &= -\frac{2}{x}\\
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\left(-\frac{2}{x+h}\right)-\left(-\frac{2}{x}\right)}{h}\\
&= \lim_{h\to 0}\left[-\frac{2}{x+h}+\frac{2}{x}\right]\cdot\frac{1}{h}\\
&= \lim_{h\to 0}\frac{-2x+2x+2h}{x(x+h)}\cdot\frac{1}{h}\\
&= \lim_{h\to 0}\frac{2h}{x(x+h)}\cdot\frac{1}{h} = \lim_{h\to 0}\frac{2}{x(x+h)}\\
&= \frac{2}{x(x+0)}\\
&= \frac{2}{x^2}
\end{aligned}$$

Your Turn 5

$$\begin{aligned}
f(x) &= 2\sqrt{x}\\
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{2\sqrt{x+h}-2\sqrt{x}}{h}\\
&= \lim_{h\to 0}\frac{2\sqrt{x+h}-2\sqrt{x}}{h}\cdot\frac{2\sqrt{x+h}+2\sqrt{x}}{2\sqrt{x+h}+2\sqrt{x}}\\
&= \lim_{h\to 0}\frac{4(x+h)-4x}{h\left(2\sqrt{x+h}+2\sqrt{x}\right)}\\
&= \lim_{h\to 0}\frac{4x+4h-4x}{h\left(2\sqrt{x+h}+2\sqrt{x}\right)}\\
&= \lim_{h\to 0}\frac{4h}{h\left(2\sqrt{x+h}+2\sqrt{x}\right)}\\
&= \lim_{h\to 0}\frac{4}{2\sqrt{x+h}+2\sqrt{x}}\\
&= \frac{4}{2\sqrt{x}+2\sqrt{x}} = \frac{4}{4\sqrt{x}}\\
&= \frac{1}{\sqrt{x}}
\end{aligned}$$

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Your Turn 6

$$\begin{aligned}
C(x) &= 10x - 0.002x^2 \\
C'(x) &= \lim_{h\to 0}\frac{C(x+h)-C(x)}{h} \\
&= \lim_{h\to 0}\frac{10(x+h)-0.002(x+h)^2-10x+0.002x^2}{h} \\
&= \lim_{h\to 0}\frac{10x+10h-0.002x^2-0.004xh-0.002h^2-10x+0.002x^2}{h} \\
&= \lim_{h\to 0}\frac{10h-0.004xh-0.002h^2}{h} \\
&= \lim_{h\to 0}\frac{h(10-0.004x-0.002h)}{h} \\
&= \lim_{h\to 0}(10-0.004x-0.002h) \\
&= 10-0.004x+0 \\
&= 10-0.004x \\
C'(100) &= 10-0.004(100) = 10-0.4 = 9.60
\end{aligned}$$

The rate of change of the cost when $x = 100$ is \$9.60.

Your Turn 7

$f(x) = 2\sqrt{x}$ at $x = 4$

From Your Turn 5, we have

$$f'(x) = \frac{1}{\sqrt{x}}.$$

At $x = 4$: $f'(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$ and $f(4) = 2\sqrt{4} = 2(2) = 4$

Slope of the tangent line at $(4, f(4))$, or $(4, 4)$ is $\frac{1}{2}$.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 4 &= \tfrac{1}{2}(x - 4) \\
y - 4 &= \tfrac{1}{2}x - 2 \\
y &= \tfrac{1}{2}x + 2
\end{aligned}$$

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3.4 Warmup Exercises

W1.

$f(x) = 3x^2 - 2x - 5$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{3(x+h)^2 - 2(x+h) - 5 - (3x^2 - 2x - 5)}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - (3x^2 - 2x - 5)}{h}$$
$$= \frac{6xh + 3h^2 - 2h}{h} = 6x + 3h - 2$$

W2.

$$f(x) = \frac{3}{x-2}$$
$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\cdot\left[\frac{3}{x+h-2} - \frac{3}{x-2}\right]$$
$$= \frac{1}{h}\cdot\left[\frac{(3)(x-2)-(3)(x+h-2)}{(x+h-2)(x-2)}\right]$$
$$= \frac{1}{h}\cdot\left[\frac{3x-6-3x-3h+6}{(x+h-2)(x-2)}\right]$$
$$= \frac{1}{h}\cdot\left[\frac{-3h}{(x+h-2)(x-2)}\right]$$
$$= -\frac{3}{(x+h-2)(x-2)}$$

W3. The line $5x + 6y = 7$ has slope $-5/6$ so any line parallel to this line has equation $y = -(5/6)x + b$. If the line passes through $(4, -1)$, then $-1 = -(5/6)4 + b$

$$b = 1 + 5/3 = 7/3$$

and the equation of the line is $y = -(5/6)x + 7/3$.

W4. The line through $(6, 2)$ and $(-2, 5)$ has slope equal to $\frac{2-5}{6-(-2)} = \frac{-3}{8} = -\frac{3}{8}$. This line has equation $y = -(3/8)x + b$. Since it passes through $(6, 2)$, $2 = -(3/8)(6) + b$

$$b = 2 + 9/4 = 17/4$$

and the equation of the line is $y = -(3/8)x + 17/4$.

3.4 Exercises

1. (a) $f(x) = 5$ is a horizontal line and has slope 0; the derivative is 0.

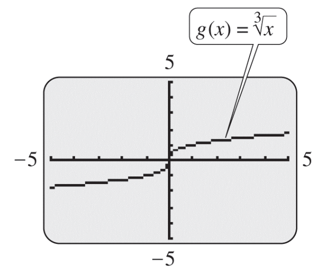
(b) $f(x) = x$ has slope 1; the derivative is 1.

(c) $f(x) = -x$ has slope of -1, the derivative is -1.

(d) $x = 3$ is vertical and has undefined slope; the derivative does not exist.

(e) $y = mx + b$ has slope m; the derivative is m.

2. (a)



The line tangent to $g(x) = \sqrt[3]{x}$ at $x = 0$ is a vertical line. Since the slope of a vertical line is undefined, $g'(0)$ does not exist.

3. $f(x) = \frac{x^2-1}{x+2}$ is not differentiable when $x + 2 = 0$ or $x = -2$ because the function is undefined and a vertical asymptote occurs there.

4. If the rate of change of $f(x)$ is zero when $x = a$, the tangent line at that point must have a slope of zero. Thus, the tangent line is horizontal at that point.

5. Using the points $(5,3)$ and $(6,5)$, we have

$$m = \frac{5-3}{6-5} = \frac{2}{1}$$
$$= 2.$$

6. Using the points $(2,2)$ and $(-2,6)$, we have

$$m = \frac{6-2}{-2-2} = \frac{4}{-4} = -1.$$

7. Using the points $(-2,2)$ and $(2,3)$, we have

$$m = \frac{3-2}{2-(-2)} = \frac{1}{4}.$$

8. Using the points $(3, -1)$ and $(-2, 3)$, we have

$$m = \frac{3-(-1)}{-2-3} = \frac{4}{-5} = -\frac{4}{5}.$$

9. Using the points $(-3, -3)$ and $(0, -3)$, we have

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$$m = \frac{-3-(-3)}{0-3} = \frac{0}{-3} = 0.$$

10. The line tangent to the curve at (4, 2) is a vertical line. A vertical line has undefined slope.

11. $f(x) = 3x - 7$

Step 1 $f(x+h)$
$= 3(x+h) - 7$
$= 3x + 3h - 7$

Step 2 $f(x+h) - f(x)$
$= 3x + 3h - 7 - (3x - 7)$
$= 3x + 3h - 7 - 3x + 7$
$= 3h$

Step 3 $\frac{f(x+h)-f(x)}{h} = \frac{3h}{h} = 3$

Step 4 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} 3 = 3$

$f'(-2) = 3,\ f'(0) = 3\ f'(3) = 3$

12. $f(x) = -2x + 5$

Step 1 $f(x+h)$
$= -2(x+h) + 5$
$= -2x - 2h + 5$

Step 2 $f(x+h) - f(x)$
$= -2x - 2h + 5 - (-2x + 5)$
$= -2x - 2h + 5 + 2x - 5$
$= -2h$

Step 3 $\frac{f(x+h)-f(x)}{h} = \frac{-2h}{h} = -2$

Step 4 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} -2 = -2$

$f'(-2) = -2,\ f'(0) = -2\ f'(3) = -2$

13. $f(x) = -4x^2 + 9x + 2$

Step 1 $f(x+h)$
$= -4(x+h)^2 + 9(x+h) + 2$
$= -4(x^2 + 2xh + h^2) + 9x + 9h + 2$
$= -4x^2 - 8xh - 4h^2 + 9x + 9h + 2$

Step 2 $f(x+h) - f(x)$
$= -4x^2 - 8xh - 4h^2 + 9x + 9h + 2$
$-(-4x^2 + 9x + 2)$
$= -8xh - 4h^2 + 9h$
$= h(-8x - 4h + 9)$

Step 3 $\frac{f(x+h)-f(x)}{h}$
$= \frac{h(-8x - 4h + 9)}{h}$
$= -8x - 4h + 9$

Step 4 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} (-8x - 4h + 9)$
$= -8x + 9$

$f'(-2) = -8(-2) + 9 = 25$
$f'(0) = -8(0) + 9 = 9$
$f'(3) = -8(3) + 9 = -15$

14. $f(x) = 6x^2 - 5x - 1$

Step 1 $f(x+h)$
$= 6(x+h)^2 - 5(x+h) - 1$
$= 6(x^2 + 2xh + h^2) - 5x - 5h - 1$
$= 6x^2 + 12xh + 6h^2 - 5x - 5h - 1$

Step 2 $f(x+h) - f(x)$
$= 6x^2 + 12xh + 6h^2 - 5x - 5h - 1$
$- 6x^2 + 5x + 1$
$= 6h^2 + 12xh - 5h$
$= h(6h + 12x - 5)$

Step 3 $\frac{f(x+h)-f(x)}{h} = \frac{h(6h + 12x - 5)}{h}$
$= 6h + 12x - 5$

Step 4 $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{h\to 0} (6h + 12x - 5)$
$= 12x - 5$

$f'(-2) = 12(-2) - 5 = -29$
$f'(0) = 12(0) - 5 = -5$
$f'(3) = 12(3) - 5 = 31$

15. $f(x) = \frac{12}{x}$

$f(x+h) = \frac{12}{x+h}$

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$$f(x+h) - f(x) = \frac{12}{x+h} - \frac{12}{x}$$
$$= \frac{12x - 12(x+h)}{x(x+h)}$$
$$= \frac{12x - 12x - 12h}{x(x+h)}$$
$$= \frac{-12h}{x(x+h)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-12h}{hx(x+h)}$$
$$= \frac{-12}{x(x+h)}$$
$$= \frac{-12}{x^2 + xh}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-12}{x^2 + xh}$$
$$= \frac{-12}{x^2}$$

$$f'(-2) = \frac{-12}{(-2)^2} = \frac{-12}{4} = -3$$

$f'(0) = \frac{-12}{0^2}$ which is undefined so $f'(0)$ does not exist.

$$f'(3) = \frac{-12}{3^2}$$
$$= \frac{-12}{9} = -\frac{4}{3}$$

16. $f(x) = \frac{3}{x}$

$$f(x+h) = \frac{3}{x+h}$$

$$f(x+h) - f(x) = \frac{3}{x+h} - \frac{3}{x}$$
$$= \frac{3x - 3(x+h)}{x(x+h)}$$
$$= \frac{-3h}{x(x+h)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-3h}{hx(x+h)}$$
$$= \frac{-3}{x(x+h)}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{-3}{x(x+h)}$$
$$= \frac{-3}{x^2}$$
$$f'(-2) = -\frac{3}{4}$$

$f'(0) = -\frac{3}{0}$ which is undefined, so $f'(0)$ does not exist.

$$f'(3) = -\frac{3}{3^2} = -\frac{1}{3}$$

17. $f(x) = \sqrt{x}$

Steps 1-3 are combined.

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$
$$= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$
$$= \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$
$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$f'(-2) = \frac{1}{2\sqrt{-2}}$ which is undefined so $f'(-2)$ does not exist.

$f'(0) = \frac{1}{2\sqrt{0}} = \frac{1}{0}$ which is undefined so $f'(0)$ does not exist.

$$f'(3) = \frac{1}{2\sqrt{3}}$$

18. $f(x) = -3\sqrt{x}$

Steps 1-3 are combined.

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{-3\sqrt{x+h} + 3\sqrt{x}}{h}$$

Rationalize the numerator.

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$$= \frac{-3\sqrt{x+h}+3\sqrt{x}}{h} \cdot \frac{-3\sqrt{x+h}-3\sqrt{x}}{-3\sqrt{x+h}-3\sqrt{x}}$$
$$= \frac{9(x+h)-9x}{h(-3\sqrt{x+h}-3\sqrt{x})}$$
$$= \frac{9x+9h-9x}{h(-3\sqrt{x+h}-3\sqrt{x})}$$
$$= \frac{9}{-3\sqrt{x+h}-3\sqrt{x}} = \frac{3}{-\sqrt{x+h}-\sqrt{x}}$$
$$f'(x) = \lim_{h\to 0} \frac{3}{-\sqrt{x+h}-\sqrt{x}}$$
$$= \frac{3}{-\sqrt{x}-\sqrt{x}} = \frac{3}{-2\sqrt{x}}$$

$f'(-2) = \dfrac{3}{-2\sqrt{-2}}$ which is undefined so $f'(-2)$ does not exist.

$f'(0) = \dfrac{3}{-2\sqrt{0}} = \dfrac{3}{0}$ which is undefined so $f'(0)$ does not exist.

$$f'(3) = \frac{3}{-2\sqrt{3}} = -\frac{3}{2\sqrt{3}}$$

19. $f(x) = 2x^3 + 5$

Steps 1-3 are combined.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{2(x+h)^3+5-(2x^3+5)}{h}$$
$$= \frac{2(x^3+3x^2h+3xh^2+h^3)+5-2x^3-5}{h}$$
$$= \frac{2x^3+6x^2h+6xh^2+2h^3+5-2x^3-5}{h}$$
$$= \frac{6x^2h+6xh^2+2h^3}{h}$$
$$= \frac{h(6x^2+6xh+2h^2)}{h}$$
$$= 6x^2+6xh+2h^2$$
$$f'(x) = \lim_{h\to 0}(6x^2+6xh+2h^2) = 6x^2$$
$$f'(-2) = 6(-2)^2 = 24$$
$$f'(0) = 6(0)^2 = 0$$
$$f'(3) = 6(3)^2 = 54$$

20. $f(x) = 4x^3 - 3$

Steps 1-3 are combined.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{4(x+h)^3-3-(4x^3-3)}{h}$$
$$= \frac{4(x^3+3x^2h+3xh^2+h^3)-3-4x^3+3}{h}$$
$$= \frac{4x^3+12x^2h+12xh^2+4h^3-3-4x^3+3}{h}$$
$$= \frac{12x^2h+12xh^2+4h^3}{h}$$
$$= \frac{h(12x^2+12xh+4h^2)}{h}$$
$$= 12x^2+12xh+4h^2$$
$$f'(x) = \lim_{h\to 0}(12x^2+12xh+4h^2) = 12x^2$$
$$f'(-2) = 12(-2)^2 = 48$$
$$f'(0) = 12(0)^2 = 0$$
$$f'(3) = 12(3)^2 = 108$$

21. (a) $f(x) = x^2 + 2x;\ x = 3, x = 5$

$$\text{Slope of secant line} = \frac{f(5)-f(3)}{5-3}$$
$$= \frac{(5)^2+2(5)-[(3)^2+2(3)]}{2}$$
$$= \frac{35-15}{2}$$
$$= 10$$

Now use $m = 10$ and $(3, f(3)) = (3,15)$ in the point-slope form.

$$y - 15 = 10(x-3)$$
$$y - 15 = 10x - 30$$
$$y = 10x - 30 + 15$$
$$y = 10x - 15$$

(b) $f(x) = x^2 + 2x;\ x = 3$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{[(x+h)^2+2(x+h)]-(x^2+2x)}{h}$$
$$= \frac{(x^2+2hx+h^2+2x+2h)-(x^2+2x)}{h}$$
$$= \frac{2hx+h^2+2h}{h} = 2x+h+2$$
$$f'(x) = \lim_{h\to 0}(2x+h+2) = 2x+2$$

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$f'(3) = 2(3) + 2 = 8$ is the slope of the tangent line at $x = 3$.

Use $m = 8$ and (3,15) in the point-slope form.

$$y - 15 = 8(x - 3)$$
$$y = 8x - 9$$

22. (a) $f(x) = 6 - x^2;\ x = -1,\ x = 3$

$$\text{Slope of secant line} = \frac{f(3) - f(-1)}{3 - (-1)}$$
$$= \frac{6 - (3)^2 - [6 - (-1)^2]}{4}$$
$$= \frac{-3 - 5}{4}$$
$$= -2$$

Now use $m = -2$ and $(-1, f(-1)) = (-1, 5)$ in the point-slope form.

$$y - 5 = -2[x - (-1)]$$
$$y - 5 = -2x - 2$$
$$y = -2x - 2 + 5$$
$$y = -2x + 3$$

(b) $f(x) = 6 - x^2;\ x = -1$

$$\frac{f(x + h) - f(x)}{h}$$
$$= \frac{[6 - (x + h)^2] - [6 - (x)^2]}{h}$$
$$= \frac{[6 - (x^2 + 2xh + h^2)] - [6 - x^2]}{h}$$
$$= \frac{6 - x^2 - 2xh - h^2 - 6 + x^2}{h}$$
$$= \frac{-2xh - h^2}{h} = \frac{h(-2x - h)}{h} = -2x - h$$

$$f'(x) = \lim_{h \to 0}(-2x - h) = -2x$$

$f'(-1) = -2(-1) = 2$ is the slope of the tangent line at $x = -1$. Use $m = 2$ and $(-1, 5)$ in the point-slope form.

$$y - 5 = 2(x + 1)$$
$$y - 5 = 2x + 2$$
$$y = 2x + 7$$

23. (a) $f(x) = \frac{5}{x};\ x = 2, x = 5$

$$\text{Slope of secant line} = \frac{f(5) - f(2)}{5 - 2}$$
$$= \frac{\frac{5}{5} - \frac{5}{2}}{3} = \frac{1 - \frac{5}{2}}{3}$$
$$= -\frac{1}{2}$$

Now use $m = -\frac{1}{2}$ and $(5, f(5)) = (5, 1)$ in the point-slope form.

$$y - 1 = -\frac{1}{2}[x - 5]$$
$$y - 1 = -\frac{1}{2}x + \frac{5}{2}$$
$$y = -\frac{1}{2}x + \frac{5}{2} + 1$$
$$y = -\frac{1}{2}x + \frac{7}{2}$$

(b) $f(x) = \frac{5}{x};\ x = 2$

$$\frac{f(x + h) - f(x)}{h} = \frac{\frac{5}{x+h} - \frac{5}{x}}{h}$$
$$= \frac{\frac{5x - 5(x+h)}{(x+h)x}}{h}$$
$$= \frac{5x - 5x - 5h}{h(x + h)(x)}$$
$$= \frac{-5h}{h(x + h)x}$$
$$= \frac{-5}{(x + h)x}$$

$$f'(x) = \lim_{h \to 0} \frac{-5}{(x + h)(x)} = -\frac{5}{x^2}$$

$f'(2) = \frac{-5}{2^2} = -\frac{5}{4}$ is the slope of the tangent line at $x = 2$.

Now use $m = -\frac{5}{4}$ and $\left(2, \frac{5}{2}\right)$ in the point-slope form.

$$y - \frac{5}{2} = -\frac{5}{4}(x - 2)$$
$$y - \frac{5}{2} = -\frac{5}{4}x + \frac{10}{4}$$
$$y = -\frac{5}{4}x + 5$$

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24. (a) $f(x) = -\frac{3}{x+1};\ x = 1,\ x = 5$

$$\text{Slope of secant line} = \frac{f(5) - f(1)}{5 - 1} = \frac{-\frac{3}{(5+1)} - \left[-\frac{3}{(1+1)}\right]}{4} = \frac{-\frac{1}{2} + \frac{3}{2}}{4} = \frac{1}{4}$$

Now use $m = \frac{1}{4}$ and $(1, f(1)) = \left(1, -\frac{3}{2}\right)$ in the point-slope form.

$$y - \left(-\frac{3}{2}\right) = \frac{1}{4}(x - 1)$$
$$y + \frac{3}{2} = \frac{1}{4}x - \frac{1}{4}$$
$$y = \frac{1}{4}x - \frac{1}{4} - \frac{3}{2}$$
$$y = \frac{1}{4}x - \frac{7}{4}$$

(b) $f(x) = \dfrac{-3}{x+1};\ x = 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{-3}{(x+h)+1} - \frac{-3}{x+1}}{h} = \frac{\frac{-3(x+1)+3(x+h+1)}{(x+1)(x+h+1)}}{h} = \frac{\frac{-3x-3+3x+3h+3}{(x+1)(x+h+1)}}{h} = \frac{3h}{h(x+1)(x+h+1)}$$

$$f'(x) = \lim_{h\to 0} \frac{3}{(x+1)(x+h+1)} = \frac{3}{(x+1)^2}$$

$f'(1) = \frac{3}{(1+1)^2} = \frac{3}{4}$ is the slope of the tangent line at $x = 1$. Use $m = \frac{3}{4}$ and $\left(1, -\frac{3}{2}\right)$ in the point-slope form.

$$y - \left(-\frac{3}{2}\right) = \frac{3}{4}(x - 1)$$
$$y + \frac{3}{2} = \frac{3}{4}x - \frac{3}{4}$$
$$y = \frac{3}{4}x - \frac{9}{4}$$

25. (a) $f(x) = 4\sqrt{x};\ x = 9, x = 16$

$$\text{Slope of secant line} = \frac{f(16) - f(9)}{16 - 9} = \frac{4\sqrt{16} - 4\sqrt{9}}{7} = \frac{16 - 12}{7} = \frac{4}{7}$$

Now use $m = \frac{4}{7}$ and $(9, f(9)) = (9, 12)$ in the point-slope form.

$$y - 12 = \frac{4}{7}(x - 9)$$
$$y - 12 = \frac{4}{7}x - \frac{36}{7}$$
$$y = \frac{4}{7}x - \frac{36}{7} + 12$$
$$y = \frac{4}{7}x + \frac{48}{7}$$

(b) $f(x) = 4\sqrt{x};\ x = 9$

$$\frac{f(x+h) - f(x)}{h} = \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} \cdot \frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}} = \frac{16(x+h) - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})}$$

$$f'(x) = \lim_{h\to 0} \frac{16(x+h) - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})} = \lim_{h\to 0} \frac{16h}{h(4\sqrt{x+h} + 4\sqrt{x})} = \lim_{h\to 0} \frac{4}{(\sqrt{x+h} + \sqrt{x})} = \frac{4}{2\sqrt{x}} = \frac{2}{\sqrt{x}}$$

$f'(9) = \frac{2}{\sqrt{9}} = \frac{2}{3}$ is the slope of the tangent line at $x = 9$.

Use $m = \frac{2}{3}$ and $(9, 12)$ in the point-slope form.

$$y - 12 = \frac{2}{3}(x - 9)$$
$$y = \frac{2}{3}x + 6$$

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26. (a) $f(x) = \sqrt{x};\ x = 25,\ x = 36$

$$\text{Slope of secant line} = \frac{f(36) - f(25)}{36 - 25} = \frac{\sqrt{36} - \sqrt{25}}{11} = \frac{6 - 5}{11} = \frac{1}{11}$$

Now use $m = \frac{1}{11}$ and $(25,\ f(25)) = (25, 5)$ in the point-slope form.

$$y - 5 = \frac{1}{11}(x - 25)$$
$$y - 5 = \frac{1}{11}x - \frac{25}{11}$$
$$y = \frac{1}{11}x - \frac{25}{11} + 5$$
$$y = \frac{1}{11}x + \frac{30}{11}$$

(b) $f(x) = \sqrt{x}; x = 25$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$f'(x) = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{2 \cdot 5} = \frac{1}{10}$$

Use $m = \frac{1}{10}$ and $(25, 5)$ in the point-slope form.

$$y - 5 = \frac{1}{10}(x - 25)$$
$$y - 5 = \frac{1}{10}x - \frac{25}{10}$$
$$y = \frac{1}{10}x + \frac{5}{2}$$

27. $f(x) = -4x^2 + 11x$

$$\frac{f(x+h) - f(x)}{h} = \frac{-4(x+h)^2 + 11(x+h) - (-4x^2 + 11x)}{h} = \frac{-8xh - 4h^2 + 11h}{h}$$

$$f'(x) = \lim_{h\to 0}(-8x - 4h + 11) = -8x + 11$$
$$f'(2) = -8(2) + 11 = -5$$
$$f'(16) = -8(16) + 11 = -117$$
$$f'(-3) = -8(-3) + 11 = 35$$

28. $f(x) = 6x^2 - 4x$

$$\frac{f(x+h) - f(x)}{h} = \frac{6(x+h)^2 - 4(x+h) - (6x^2 - 4x)}{h} = \frac{12xh + 6h^2 - 4h}{h} = 12x + 6h - 4$$

$$f'(x) = \lim_{h\to 0}(12x + 6h - 4) = 12x - 4$$
$$f'(2) = 12(2) - 4 = 24 - 4 = 20$$
$$f'(16) = 12(16) - 4 = 192 - 4 = 188$$
$$f'(-3) = 12(-3) - 4 = -36 - 4 = -40$$

29. $f(x) = e^x$

$$\frac{f(x+h) - f(x)}{h} = \frac{e^{x+h} - e^x}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$f'(2) \approx 7.3891;\ f'(16) \approx 8{,}886{,}111;\ f'(-3) \approx 0.0498$

30. $f(x) = \ln|x|$

$$\frac{f(x+h) - f(x)}{h} = \frac{\ln|x+h| - \ln|x|}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{\ln|x+h| - \ln|x|}{h}$$
$$f'(2) = 0.5$$
$$f'(16) = 0.0625$$
$$f'(-3) = -0.\overline{3}$$

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31. $f(x) = -\dfrac{2}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{-2}{x+h} - \left(\frac{-2}{x}\right)}{h} = \frac{\frac{-2x+2(x+h)}{(x+h)x}}{h} = \frac{2h}{h(x+h)x} = \frac{2}{(x+h)x}$$

$$f'(x) = \lim_{h\to 0}\frac{2}{(x+h)x} = \frac{2}{x^2}$$

$$f'(2) = \frac{2}{2^2} = \frac{1}{2}$$

$$f'(16) = \frac{2}{16^2} = \frac{2}{256} = \frac{1}{128}.$$

$$f'(-3) = \frac{2}{(-3)^2} = \frac{2}{9}$$

32. $f(x) = \dfrac{6}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{6}{x+h} - \frac{6}{x}}{h} = \frac{6x-6(x+h)}{hx(x+h)} = \frac{-6}{x(x+h)}$$

$$f'(x) = \lim_{h\to 0}\frac{-6}{x(x+h)} = \frac{-6}{x^2}$$

$$f'(2) = \frac{-6}{(2)^2} = \frac{-6}{4} = -\frac{3}{2}$$

$$f'(16) = \frac{-6}{16^2} = \frac{-6}{256} = -\frac{3}{128}$$

$$f'(-3) = \frac{-6}{(-3)^2} = \frac{-6}{9} = -\frac{2}{3}$$

33. $f(x) = \sqrt{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$f'(x) = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$f'(2) = \frac{1}{2\sqrt{2}}$$

$$f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

$f'(-3) = \dfrac{1}{2\sqrt{-3}}$ is not a real number, so

$f'(-3)$ does not exist.

34. $f(x) = -3\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{-3\sqrt{x+h}+3\sqrt{x}}{h} = \frac{-3\sqrt{x+h}+3\sqrt{x}}{h}\cdot\frac{-3\sqrt{x+h}-3\sqrt{x}}{-3\sqrt{x+h}-3\sqrt{x}} = \frac{9(x+h)-9x}{h(-3\sqrt{x+h}-3\sqrt{x})} = \frac{9}{-3\sqrt{x+h}-3\sqrt{x}} = \frac{3}{-\sqrt{x+h}-\sqrt{x}}$$

$$f'(x) = \lim_{h\to 0}\frac{3}{-\sqrt{x+h}-\sqrt{x}} = \frac{3}{-\sqrt{x}-\sqrt{x}} = -\frac{3}{2\sqrt{x}}$$

$$f'(2) = -\frac{3}{2\sqrt{2}}$$

$$f'(16) = \frac{-3}{2\sqrt{16}} = -\frac{3}{8}$$

$f'(-3) = \frac{-3}{2\sqrt{-3}}$, which is not a real number, so

$f'(-3)$ does not exist.

35. At $x = 0$, the graph of $f(x)$ has a sharp point. Therefore, there is no derivative for $x = 0$.

36. No derivative exists at $x = -6$ because the function is not defined at $x = -6$.

37. For $x = -3$ and $x = 0$, the tangent to the graph of $f(x)$ is vertical. For $x = -1$, there is a gap in the graph of $f(x)$. For $x = 2$, the function $f(x)$ does not exist. For $x = 3$ and $x = 5$, the graph of $f(x)$ has sharp points. Therefore, no derivative exists for $x = -3$, $x = -1$, $x = 0$, $x = 2$, $x = 3$, and $x = 5$.

38. For $x = -5$ and $x = 0$, the function $f(x)$ is not defined. For $x = -3$ and $x = 2$ the graph of $f(x)$ has sharp points. For $x = 4$, the tangent to the

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graph is vertical. Therefore, no derivative exists for $x = -5,\ x = -3,\ x = 0,\ x = 2,$ or $x = 4.$

39. **(a)** The rate of change of $f(x)$ is positive when $f(x)$ is increasing, that is, on $(a, 0)$ and (b, c).

(b) The rate of change of $f(x)$ is negative when $f(x)$ is decreasing, that is, on $(0, b,)$.

(c) The rate of change is zero when the tangent to the graph is horizontal, that is, at $x = 0$ and $x = b.$

40. The zeros of graph (b) correspond to the turning points of graph (a), the points where the derivative is zero. Graph (a) gives the distance, while graph (b) gives the velocity.

41. The zeros of graph (b) correspond to the turning points of graph (a), the points where the derivative is zero. Graph (a) gives the distance, while graph (b) gives the velocity.

42. $f(x) = x^x,\ a = 2$

(a)

h	
0.01	$\frac{f(2+0.01)-f(2)}{0.01} = \frac{2.01^{2.01}-2^2}{0.01} = 6.84$
0.001	$\frac{f(2+0.001)-f(2)}{0.001} = \frac{2.001^{2.001}-2^2}{0.001} = 6.779$
0.0001	$\frac{f(2+0.0001)-f(2)}{0.0001} = \frac{2.0001^{2.0001}-2^2}{0.0001} = 6.773$
0.00001	$\frac{f(2+0.00001)-f(2)}{0.00001} = \frac{2.00001^{2.00001}-2^2}{0.00001} = 6.7727$
0.000001	$\frac{f(2+0.000001)-f(2)}{0.000001} = \frac{2.000001^{2.000001}-2^2}{0.000001} = 6.7726$

It appears that $f'(2) \approx 6.773.$

(b) Graph the function on a graphing calculator and move the cursor to an x-value near $x = 2.$ A good choice for the initial viewing window is $[0, 3]$ by $[0, 10]$.

Now zoom in on the function several times. Each time you zoom in, the graph will look less like a curve and more like a straight line. When the graph appears to be a straight line, use the TRACE feature to select two points on the graph, and record their coordinates. Use these two points to compute the slope. The result will be very close to the most accurate value found in part (a), which is 6.773.

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43. $f(x) = x^x, a = 3$

(a)

h	
0.01	$\frac{f(3 + 0.01) - f(3)}{0.01} = \frac{3.01^{3.01} - 3^3}{0.01} = 57.3072$
0.001	$\frac{f(3 + 0.001) - f(3)}{0.001} = \frac{3.001^{3.001} - 3^3}{0.001} = 56.7265$
0.00001	$\frac{f(3 + 0.00001) - f(3)}{0.00001} = \frac{3.00001^{3.00001} - 3^3}{0.00001} = 56.6632$
0.000001	$\frac{f(3 + 0.000001) - f(3)}{0.000001} = \frac{3.000001^{3.000001} - 3^3}{0.000001} = 56.6626$
0.0000001	$\frac{f(3 + 0.0000001) - f(3)}{0.0000001} = \frac{3.0000001^{3.0000001} - 3^3}{0.0000001} = 56.6625$

It appears that $f'(3) \approx 56.66$.

(b) Graph the function on a graphing calculator and move the cursor to an x-value near $x = 3$. A good choice for the initial viewing window is [0, 4] by [0, 60] with $\text{Xscl} = 1$, $\text{Yscl} = 10$.

Now zoom in on the function several times. Each time you zoom in, the graph will look less like a curve and more like a straight line. Use the TRACE feature to select two points on the graph, and record their coordinates. Use these two points to compute the slope. The result will be close to the most accurate value found in part (a), which is 56.66.

Note: In this exercise, the method used in part (a) gives more accurate results than the method used in part (b).

44. $f(x) = x^{1/x}, \ a = 2$

(a)

h	
0.01	$\frac{f(2 + .01) - f(2)}{0.01} = \frac{2.01^{1/2.01} - 2^{1/2}}{0.01} = 0.1071$
0.001	$\frac{f(2 + 0.001) - f(2)}{0.001} = \frac{2.001^{1/2.001} - 2^{1/2}}{0.001} = 0.1084$
0.0001	$\frac{f(2 + 0.0001) - f(2)}{0.0001} = \frac{2.0001^{1/2.0001} - 2^{1/2}}{0.0001} = 0.1085$
0.00001	$\frac{f(2 + 0.00001) - f(2)}{0.00001} = \frac{2.00001^{1/2.00001} - 2^{1/2}}{0.00001} = 0.1085$
0.000001	$\frac{f(2 + 0.000001) - f(2)}{0.000001} = \frac{2.000001^{1/2.000001} - 2^{1/2}}{0.000001} = 0.1085$

It appears that $f'(2) = 0.1085$.

(b) Graph this function on a graphing calculator and move the cursor to an x-value near $x = 2$. A good choice for the initial viewing window is [0, 5] by [0, 3].

Follow the procedure outlined in the solution for Exercise 42, part (b). The final result will be close to the value found in part (a) of this exercise, which is 0.1085.

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45. $f(x) = x^{1/x},\ a = 3$

(a)

h	
0.01	$\frac{f(3+0.01)-f(3)}{0.01} = \frac{3.01^{1/3.01} - 3^{1/3}}{0.01} = -0.0160$
0.001	$\frac{f(3+0.001)-f(3)}{0.001} = \frac{3.001^{1/3.001} - 3^{1/3}}{0.001} = -0.0158$
0.0001	$\frac{f(3+0.0001)-f(3)}{0.0001} = \frac{3.0001^{1/3.0001} - 3^{1/3}}{0.0001} = -0.0158$

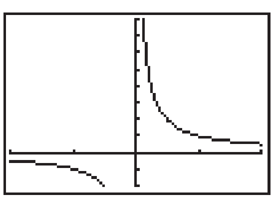
It appears that $f'(3) = -0.0158$.

(b) Graph the function on a graphing calculator and move the cursor to an x-value near $x = 3$. A good choice for the initial viewing window is $[0, 5]$ by $[0, 3]$.

Follow the procedure outlined in the solution for Exercise 43, part (b). Note that near $x = 3$, the graph is very close to a horizontal line, so we expect that it slope will be close to 0. The final result will be close to the value found in part (a) of this exercise, which is -0.0158.

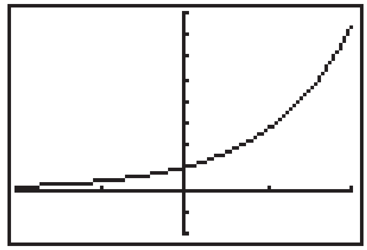
46. For Column A, let $h = 0.01$ $f(x) = \ln|x|$

Graph $y = \frac{\ln|x+0.01| - \ln|x|}{0.01}$.



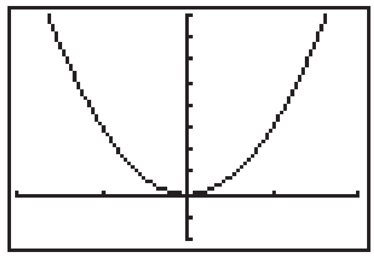
$f(x) = e^x$

Graph $y = \frac{e^{x+0.01} - e^x}{0.01}$.



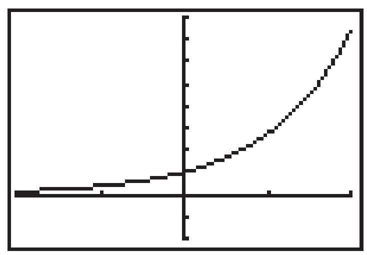
$f(x) = x^3$

Graph $y = \frac{(x+0.01)^3 - x^3}{0.01}$.

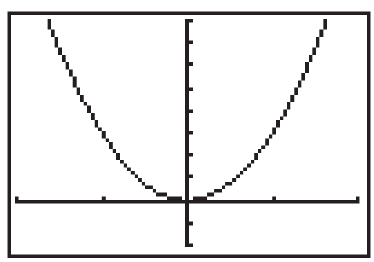


Column B

Graph $y = e^x$



Graph $y = 3x^2$



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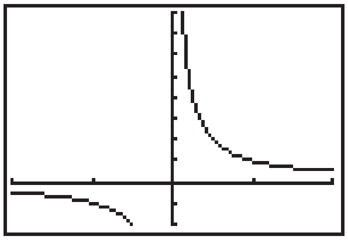
Graph $y = \frac{1}{x}$



We observe that the graph of

$$y = \frac{\ln|x + 0.01| - \ln|x|}{0.01}$$

is very similar to the graph of

$$y = \frac{1}{x},$$

the graph of

$$y = \frac{e^{x + 0.01} - e^x}{0.01}$$

is very similar to the graph of $y = e^x$, and the graph of

$$y = \frac{(x + 0.01)^3 - x^3}{0.01}$$

is very similar to the graph of

$$y = 3x^2.$$

Thus the derivative of In x is $\frac{1}{x}$, the derivative of e^x is e^x, and the derivative of x^3 is $3x^2$.

48. (a)

$$f(x) = -4x^2 + 11x$$

$$\begin{aligned} f(x + h) &= -4(x + h)^2 + 11(x + h) \\ &= -4(x^2 + 2xh + h^2) + 11(x + h) \\ &= -4x^2 - 8xh - 4h^2 + 11x + 11h \end{aligned}$$

$$\begin{aligned} &f(x + h) - f(x) \\ &= -4x^2 - 8xh - 4h^2 + 11x + 11h + 4x^2 - 11x \\ &= -8xh - 4h^2 + 11h \end{aligned}$$

$$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{-8xh - 4h^2 + 11h}{h} \\ &= -8x + 4h + 11 \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h \to 0} (-8x + 4h + 11) \\ &= -8x + 11 \end{aligned}$$

$$f'(3) = -8(3) + 11 = -13$$

$$\begin{aligned} &\frac{f(3 + 0.1) - f(3)}{0.1} \\ &= \frac{f(3.1) - f(3)}{0.1} \\ &= \frac{-4(3.1)^2 + 11(3.1) - (-4(3)^2 + 11(3))}{0.1} \\ &= \frac{-4(9.61) + 11(3.1) + 4(9) - 11(3)}{0.1} \\ &= \frac{-38.44 + 34.1 + 36 - 33}{0.1} \\ &= \frac{-1.34}{0.1} = -13.4 \end{aligned}$$

$$\begin{aligned} &\frac{f(3 + 0.1) - f(3 - 0.1)}{2(0.1)} \\ &= \frac{f(3.1) - f(2.9)}{0.2} \\ &= \frac{-4(3.1)^2 + 11(3.1) - (-4(2.9)^2 + 11(2.9))}{0.2} \\ &= \frac{-4(9.61) + 11(3.1) + 4(8.41) - 11(2.9)}{0.2} \\ &= \frac{-38.44 + 34.1 + 33.64 - 31.9}{0.2} \\ &= \frac{-2.6}{0.2} = -13 \end{aligned}$$

(b) $f'(3) = -8(3) + 11 = -13$

$$\begin{aligned} &\frac{f(3 + 0.01) - f(3)}{0.01} \\ &= \frac{f(3.01) - f(3)}{0.01} \\ &= \frac{-4(3.01)^2 + 11(3.01) - (-4(3)^2 + 11(3))}{0.01} \\ &= \frac{-4(9.0601) + 11(3.01) + 4(9) - 11(3)}{0.01} \\ &= \frac{-36.2404 + 33.11 + 36 - 33}{0.01} \\ &= \frac{-0.1304}{0.01} = -13.04 \end{aligned}$$

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$$\frac{f(3+0.01)-f(3-0.01)}{2(0.01)}$$
$$= \frac{f(3.01)-f(2.99)}{0.02}$$
$$= \frac{-4(3.01)^2+11(3.01)-(-4(2.99)^2+11(2.99))}{0.02}$$
$$= \frac{-4(9.0601)+11(3.01)+4(8.9401)-11(2.99)}{0.02}$$
$$= \frac{-36.2404+33.11+35.7604-32.89}{0.02}$$
$$= \frac{-0.26}{0.02} = -13$$

(c)
$$f(x) = \frac{-2}{x}$$
$$f(x+h) = \frac{-2}{x+h}$$
$$f(x+h)-f(x) = \frac{-2}{x+h} - \left(\frac{-2}{x}\right)$$
$$= \frac{-2}{x+h} + \frac{2}{x}$$
$$= \frac{-2x+2(x+h)}{x(x+h)}$$
$$= \frac{2h}{x(x+h)}$$
$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{2h}{x(x+h)}}{h}$$
$$= \frac{2h}{x(x+h)} \cdot \frac{1}{h}$$
$$= \frac{2}{x(x+h)}$$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{2}{x(x+h)} = \frac{2}{x^2}$$
$$f'(3) = \frac{2}{3^2} = \frac{2}{9} \approx 0.222222$$
$$\frac{f(3+0.1)-f(3)}{0.1} = \frac{f(3.1)-f(3)}{0.1}$$
$$= \frac{\frac{-2}{3.1} - \frac{-2}{3}}{0.1}$$
$$\approx .215054$$
$$\frac{f(3+0.1)-f(3-0.1)}{2(0.1)} = \frac{f(3.1)-f(2.9)}{0.2}$$
$$= \frac{\frac{-2}{3.1} - \frac{-2}{2.9}}{0.2}$$
$$= 0.222469$$

(d) $f'(3) = \frac{2}{3^2} = \frac{2}{9} \approx 0.222222$
$$\frac{f(3+.01)-f(3)}{0.01} = \frac{f(3.01)-f(3)}{0.01}$$
$$= \frac{\frac{-2}{3.01} - \frac{-2}{3}}{0.01}$$
$$\approx 0.221484$$
$$\frac{f(3+0.01)-f(3-0.01)}{2(0.01)} = \frac{f(3.01)-f(2.99)}{0.02}$$
$$= \frac{\frac{-2}{3.01} - \frac{-2}{2.99}}{0.02}$$
$$\approx 0.222225$$

(e) $f(x) = \sqrt{x}$
$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$
$$= \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$$
$$= \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$
$$= \frac{1}{h(\sqrt{x+h}+\sqrt{x})}$$
$$f'(x) = \lim_{h\to 0} \frac{1}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}$$
$$f'(3) = \frac{1}{2\sqrt{3}} \approx 0.288675$$
$$\frac{f(3+0.1)-f(3)}{0.1} = \frac{f(3.1)-f(3)}{0.1}$$
$$= \frac{\sqrt{3.1}-\sqrt{3}}{0.1}$$
$$\approx 0.286309$$
$$\frac{f(3+0.1)-f(3-0.1)}{2(0.1)} = \frac{f(3.1)-f(2.9)}{0.2}$$
$$= \frac{\sqrt{3.1}-\sqrt{2.9}}{0.2}$$
$$\approx 0.288715$$

(f) $f'(3) = \frac{1}{2\sqrt{3}} \approx 0.288675$
$$\frac{f(3+0.01)-f(3)}{0.01} = \frac{f(3.01)-f(3)}{0.01}$$
$$= \frac{\sqrt{3.01}-\sqrt{3}}{0.01}$$
$$\approx 0.288435$$

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$$\frac{f(3+0.01)-f(3-0.01)}{2(0.01)} = \frac{f(3.01)-f(2.99)}{0.02}$$
$$= \frac{\sqrt{3.01}-\sqrt{2.99}}{0.02}$$
$$\approx 0.288676$$

49. $D(p) = -2p^2 - 4p + 300$

D is demand; p is price.

(a) Given that $D'(p) = -4p - 4$, the rate of change of demand with respect to price is $-4p - 4$, the derivative of the function $D(p)$.

(b) $D'(10) = -4(10) - 4$
$= -44$

The demand is decreasing at the rate of about 44 items for each increase in price of $1.

50. $P(x) = 1000 + 32x - 2x^2$

(a) $8000 is 8 thousands, so $x = 8$.

$$P'(8) = 32 - 4(8) = 32 - 32 = 0$$

No, the firm should not increase production, since the marginal profit is 0.

(b) $6000, $x = 6$

$$P'(6) = 32 - 4(6) = 32 - 24 = 8$$

Yes, the first should increase production, since the marginal profit is positive, $8000.

(c) $12,000, $x = 12$

$$P'(12) = 32 - 4(12)$$
$$= 32 - 48 = -16$$

No, because the marginal profit is negative, −$16,000.

(d) $20,000, $x = 20$

$$P'(20) = 32 - 4(20)$$
$$= 32 - 80 = -48$$

No, because the marginal profit is negative, −$48,000.

51. $R(x) = 20x - \frac{x^2}{500}$

(a) $R'(x) = 20 - \frac{1}{250}x$

At $y = 1000$,

$$R'(1000) = 20 - \frac{1}{250}(1000)$$
$$= \$16 \text{ per table}.$$

(b) The marginal revenue for the l00lst table is approximately $R'(1000)$. From (a), this is about $16.

(c) The actual revenue is

$$R(1001) - R(1000) = 20(1001) - \frac{1001^2}{500}$$
$$- \left[20(1000) - \frac{1000^2}{500}\right]$$
$$= 18{,}015.998 - 18{,}000$$
$$= \$15.998 \text{ or } \$16.$$

(d) The marginal revenue gives a good approximation of the actual revenue from the sale of the 100lst table.

52. $C(x) = -0.00375x^2 + 1.5x + 1000, 0 \le x \le 180$

(a) The marginal cost is given by

$$C'(x) = -0.0075x + 1.5,\ 0 \le x \le 180$$

(b) $C'(100) = -0.0075(100) + 1.5 = 0.75$

This represents the fact that the cost of producing the next (l0lst) taco is approximately $0.75.

(c) The exact cost to produce the l0lst taco is given by

$$C(101) - C(100)$$
$$= [-0.00375(101)^2 + 1.5(101) + 1000]$$
$$-[-0.00375(100)^2 + 1.5(100) + 1000]$$
$$= (-38.25375 + 151.5 + 1000)$$
$$-(37.5 + 150 + 1000)$$
$$= 0.74625$$

That is, the exact cost of producing the l0lst taco is $0.74625.

(d) The exact cost of producing the l0lst taco is $0.00375 less than the approximate cost. They are very close.

(e) $C(x) = ax^2 + bx + c$
$C'(x) = 2ax + b$

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$$[C(x+1) - C(x)] - C'(x)$$
$$= [a(x+1)^2 + b(x+1) + c]$$
$$- [ax^2 + bx + c] - (2ax + b)$$
$$= ax^2 + 2ax + a + bx + b + c$$
$$-ax^2 - bx - c - 2ax - b$$
$$= a$$

(f) $C(x) = ax^2 + bx + c$

$$C'(x) = 2ax + b$$
$$C(x+1) - C(x)$$
$$= [a(x+1)^2 + b(x+1) + c]$$
$$-[ax^2 + bx + c]$$
$$= ax^2 + 2ax + a + bx$$
$$+ b + c - ax^2 - bx - c$$
$$= 2ax + a + b$$
$$C'\left(x + \frac{1}{2}\right) = 2a\left(x + \frac{1}{2}\right) + b$$
$$= 2ax + a + b$$

Thus, $C(x+1) - C(x) = C'\left(x + \frac{1}{2}\right)$.

53. (a) $f(x) = 0.0000329x^3 - 0.00450x^2 + 0.0613x + 2.34$

$f(10) = 2.54$

$f(20) = 2.03$

$f(30) = 1.02$

(b)

$$Y_1 = 0.0000329x^3 - 0.00450x^2 + 0.0613x + 2.34$$

$nDeriv(Y_1, x, 0) \approx 0.061$

$nDeriv(Y_1, x, 10) \approx -0.019$

$nDeriv(Y_1, x, 20) \approx -0.079$

$nDeriv(Y_1, x, 30) \approx -0.120$

$nDeriv(Y_1, x, 35) \approx -0.133$

54. (a) From the graph, V_{mp} is just about at the turning point of the curve. Thus, the slope of the tangent line is approximately zero. The power expenditure is not changing.

(b) From the graph, the slope of the tangent line at V_{mr} is approximately 0.54. The power expended is increasing 0.54 unit per unit increase in speed.

(c) The power level first decreases to V_{mp}, then increases at greater rates.

(d) V_{mr} is the point which produces the smallest slope of a line.

55. The derivative at (2, 4000) can be approximated by the slope of the line through (0, 2000) and (2, 4000).
The derivative is approximately

$$\frac{4000 - 2000}{2 - 0} = \frac{2000}{2} = 1000.$$

Thus the shellfish population is increasing at a rate of 1000 shellfish per unit time.

The derivative at about (10,10,300) can be approximated by the slope of the line through (10,10,300) and (13,12,000). The derivative is approximately

$$\frac{12,000 - 10,300}{13 - 10} = \frac{1700}{3} \approx 570.$$

The shellfish population is increasing at a rate of about 570 shellfish per unit time. The derivative at about (13, 11,250) can be approximated by the slope of the line through (13, 11,250) and (16, 12,000). The derivative is approximately

$$\frac{12,000 - 11,000}{16 - 11} = \frac{1000}{5} \approx 200.$$

The shellfish population is increasing at a rate of 200 shellfish per unit time.

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56. $I(t) = 27 + 72t - 1.5t^2$

(a)
$$\begin{aligned} I'(t) &= \lim_{h\to 0} \frac{I(t+h) - I(t)}{h} \\ &= \lim_{h\to 0} \frac{27 + 72t + 72h - 1.5t^2 - 3th - 1.5h^2 - 27 - 72t + 1.5t^2}{h} \\ &= \lim_{h\to 0} \frac{72h - 3th - 1.5h^2}{h} \\ &= \lim_{h\to 0} 72 - 3t - 1.5h \\ &= 72 - 3t \\ I'(5) &= 72 - 3(5) \\ &= 72 - 15 \\ &= 57 \end{aligned}$$

The rate of change of the intake of food 5 minutes into a meal is 57 grams per minute.

(b)
$$\begin{aligned} I'(24) &\overset{?}{=} 0 \\ 72 - 3(24) &\overset{?}{=} 0 \\ 0 &= 0 \end{aligned}$$

24 minutes after the meal starts the rate of food consumption is 0.

(c) After 24 minutes the rate of food consumption is negative according to the function where a rate of zero is more accurate. A logical range for this function is

$$0 \le t \le 24.$$

57. (a) Set $M(v) = 150$ and solve for v.

$$\begin{aligned} 0.0312443v^2 - 101.39v + 82,264 &= 150 \\ 0.0312443v^2 - 101.39v + 82,114 &= 0 \end{aligned}$$

Solve using the quadratic formula.

Let D equal the discriminant.

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-101.39)^2 - 4(0.0312443)(82,114) \\ &\approx 17.55 \\ v &= \frac{101.39 \pm \sqrt{D}}{2(0.0312443)} \end{aligned}$$

$v \approx 1690$ meter per second or

$v \approx 1560$ meters per second.

Since the functions is defined only for $v \ge 1620,$ the only solution is 1690 meters per second.

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(b) Calculate $\lim_{h\to 0} \dfrac{M(1700+h)-M(1700)}{h}$

$$\begin{aligned} M&(1700+h)\\ &= 0.0312443(1700+h)^2 - 101.39(1700+h) + 82{,}264\\ &= 90{,}296.027 + 106.23062h + 0.0312443h^2 - 172{,}363 - 101.39h + 82{,}264\\ &= 0.01312443h^2 + 4.84062h + 197.027 \end{aligned}$$

$M(1700) = 197.027$, so the derivative of $M(v)$ at $v = 1700$ is

$$\begin{aligned} &\lim_{h\to 0}\left(\frac{0.0312443h^2 + 4.84062h + 197.027 - 197.027}{h}\right)\\ &= \lim_{h\to 0}\left(\frac{0.0312443h^2 + 4.84062h}{h}\right)\\ &= \lim_{h\to 0}\left(0.0312443h + 4.84062\right)\\ &= 4.84062\\ &\approx 4.84 \text{ days per meter per second} \end{aligned}$$

The increase in velocity for this cheese from 1700 m/s to 1701 m/s indicates that the approximate age of the cheese has increased by 4.84 days.

58. The slope of the tangent line to the graph at the first point is found by finding two points on the tangent line.

$$\begin{aligned} (x_1, y_1) &= (1000, 13.5)\\ (x_2, y_2) &= (0, 18.5)\\ m &= \frac{18.5 - 13.5}{0 - 1000} = \frac{5}{-1000} = -0.005 \end{aligned}$$

At the second point, we have

$$\begin{aligned} (x_1, y_1) &= (1000, 13.5)\\ (x_2, y_2) &= (2000, 21.5).\\ m &= \frac{21.5 - 13.5}{2000 - 1000}\\ &= \frac{8}{1000}\\ &= 0.008 \end{aligned}$$

At the third point, we have

$$\begin{aligned} (x_1, y_1) &= (5000, 20)\\ (x_2, y_2) &= (3000, 22.5).\\ m &= \frac{22.5 - 20}{3000 - 5000}\\ &= \frac{2.5}{-2000}\\ &= -0.00125 \end{aligned}$$

At 500 ft, the temperature decreases 0.005° per foot. At about 1500 ft, the temperature increases 0.008° per foot. At 5000 ft, the temperature decreases 0.00125° per foot.

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59. **(a)** From the graph, $T(0.5) \approx 185.$ A tangent drawn at this point appears to intersect the $T = 1$ vertical line at about 320, so the tangent has a slope of about

$$\frac{T(1) - T(0.5)}{1 - 0.5} = \frac{320 - 185}{0.5} = 270.$$

$T'(0.5) \approx 270$; at 9:00 AM the temperature is increasing at about 270° per hour.

(b) A tangent drawn to the curve at $T = 3$ appears to intersect the $T = 2$ line at 480 and the $T = 4$ line at 180, so the tangent has a slope of about

$$\frac{T(4) - T(2)}{4 - 2} = \frac{180 - 480}{2} = -150.$$

$T'(3) \approx -150$; at 11:30 AM the temperature is decreasing at about 150° per hour.

(c) At $T = 4$ the graph appears to have a horizontal tangent, so $T'(4) \approx 0$; the temperature is staying constant at 12:30 PM.

(d) At about 11:15 AM.

60. **(a)** The slope of the graph at 16 looks horizontal. Thus, the derivative for a 16 ounce bat is about 0 mph per oz .

The slope of the graph at $x = 25$ can be estimated using the points (25, 63.4) and (26, 62.8).

$$\text{slope} = \frac{62.8 - 63.4}{26 - 25} = -0.6$$

Thus the derivative is for a 25-ounce bat is about -0.6 mph per oz.

(b) The optimal bat is 16 oz.

61. **(a)** At 40 oz the tangent looks horizontal; thus the derivative for a 40-ounce bat is about 0 mph per oz.

The slope of the graph at $x = 30$ can be estimated using the points (30, 79) and (32, 80).

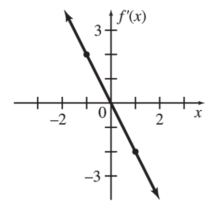
$$\text{slope} = \frac{80 - 79}{32 - 30} = 0.5$$

Thus, the derivative for a 30 ounce bat is about 0.5 mph per oz .

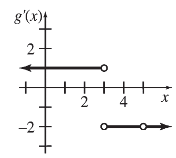
(b) The optimal bat is 40 oz.

3.5 Graphical Differentiation

Your Turn 1



Your Turn 2



3.5 Warmup Exercises

W1. The tangent at (2, 13) goes through the points (0, 9) and (4, 17), so the tangent has slope

$$\frac{17 - 9}{4 - 0} = 2.$$

W2. The tangent at (3, 4) goes through the point (1, 14), so the tangent has slope

$$\frac{4 - 14}{3 - 1} = -5.$$

3.5 Exercises

3. Since the x-intercepts of the graph of f' occur whenever the graph of f has a horizontal tangent line, Y_1 is the derivative of Y_2. Notice that Y_1 has 2 x-intercepts; each occurs at an x-value where the tangent line to Y_2 is horizontal.

Note also that Y_1 is positive whenever Y_2 is increasing, and that Y_1 is negative whenever Y_2 is decreasing.

4. Since the x-intercepts of the graph f' occur whenever the graph of f has a horizontal tangent line, Y_2 is the derivative of Y_1. Notice that Y_2 has 2 x-intercepts; each occurs at an x-value where the tangent line to Y_1 is horizontal.

Note also that Y_2 is negative whenever Y_1 is decreasing, and Y_2 is positive whenever Y_1 is increasing.

5. Since the x-intercepts of the graph of f' occur whenever the graph of f has a horizontal tangent line, Y_2 is the derivative of Y_1. Notice that Y_2 has

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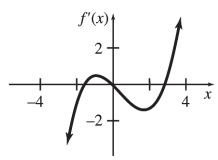
1 x-intercept which occurs at the x-value where the tangent line to Y_1 is horizontal. Also notice that the range on which Y_1 is increasing, Y_2 is positive and the range on which it is decreasing, Y_2 is negative.

6. Since the x-intercepts of the graph f' occur whenever the graph of f has a horizontal tangent line, Y_1 is the derivative of Y_2. Notice that Y_1 has 4 x-intercepts; each occurs at an x-value where the tangent line to Y_2 is horizontal.

Note also that Y_1 is negative whenever Y_2 is decreasing and Y_1 is positive whenever Y_2 is increasing.

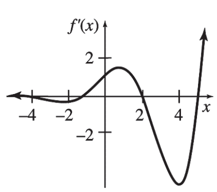
7. To graph f', observe the intervals where the slopes of tangent lines are positive and where they are negative to determine where the derivative is positive and where it is negative. Also, whenever f has a horizontal tangent, f' will be 0, so the graph of f' will have an x-intercept. The x-values of the three turning point on the graph of f become the three x-intercepts of the graph of f.

Estimate the magnitude of the slope at several points by drawing tangents to the graph of f.



8. To graph f', observe the intervals where the slopes of lines are positive and where they are negative to determine where the derivative is positive and where it is negative. Also, whenever f has a horizontal tangent, f' will be 0.

Estimate the magnitude of the slope at several points by drawing tangents to the graph of f.



9. On the interval $(-\infty, -2)$, the graph of f is a horizontal line, so its slope is 0. Thus, on this interval, the graph of f' is $y = 0$ on $(-\infty, -2)$. On the interval $(-2, 0)$, the graph of f is a straight line, so its slope is constant. To find this slope, use the points $(-2, 2)$ and $(0, 0)$.

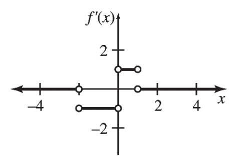
$$m = \frac{2 - 0}{-2 - 0} = \frac{2}{-2} = -1$$

On the interval $(0, 1)$, the slope is also constant. To find this slope, use the points $(0, 0)$ and $(1, 1)$.

$$m = \frac{1 - 0}{1 - 0} = 1$$

On the interval $(1, \infty)$, the graph is again a horizontal line, so $m = 0$. The graph of f' will be made up of portions of the y-axis and the lines $y = -1$ and $y = 1$.

Because the graph of f has "sharp points" or "corners" at $x = -2$, $x = 0$, and $x = 1$, we know that $f'(-2)$, $f'(0)$, and $f'(1)$ do not exist. We show this on the graph of f' by using open circles at the endpoints of the portions of the graph.



10. On the interval $(-\infty, -3)$, the graph of f is a straight line, so its slope is constant. To find this slope, use the points $(-6, -3)$ and $(-3, 0)$.

$$m = \frac{0 - (-3)}{-3 - (-6)} = \frac{3}{3} = 1$$

On the interval $(3, \infty)$, the slope of f is also constant. To find this slope, use the points$(3, 0)$ and $(6, -3)$.

$$m = \frac{-3 - 0}{6 - 3} = \frac{-3}{3} = -1$$

Thus, we have $f'(x) = 1$ on $(-\infty, -3)$ and $f'(x) = -1$ on $(3, \infty)$. Because the graph of f has sharp points at $x = -3$ and $x = 3$, we know that $f'(-3)$ and $f'(3)$ do not exist. We show this on the graph of $f'(x)$ by using open circles.

We also observe that the slopes of tangent lines are negative on $(-3, 0)$, that the graph has a horizontal tangent at $x = 0$, and that the slopes of tangent lines are positive on $(0, 3)$. Thus, f' is negative on $(-3, 0)$, 0 at $x = 0$, and positive on $(0, 3)$. Furthermore, by drawing tangents, we see that on $(-3, 3)$, f' increases from -1 to 1.

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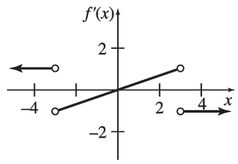


11. On the interval $(-\infty, -2)$, the graph of f is a straight line, so its slope is constant. To find this slope, use the points $(-4, 2)$ and $(-2, 0)$.

$$m = \frac{0 - 2}{-2 - (-4)} = \frac{-2}{2} = -1$$

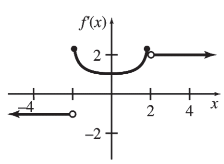
On the interval $(2, \infty)$, the slope of f is also constant. To find this slope, use the points $(2, 0)$ and $(3, 2)$.

$$m = \frac{2 - 0}{3 - 2} = \frac{2}{1} = 2$$

Thus, we have $f'(x) = -1$ on $(-\infty, -2)$ and $f'(x) = 2$ on $(2, \infty)$.

Because f is discontinuous at $x = -2$ and $x = 2$, we know that $f'(-2)$ and $f'(2)$ do not exist, which we indicate with open circles at $(-2, -1)$ and $(2, 2)$ on the graph of f'.

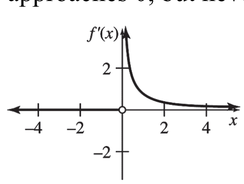
On the interval $(-2, 2)$ all tangent lines have positive slopes, so the graph of f' will be above the y-axis. Notice that the slope of f (and thus the y-value of f') decreases on $(-2, 0)$ and increases on $(0, 2)$ with a minimum value on this interval of about 1 at $x = 0$.



12. On the interval $(-\infty, 0)$, the graph of f is a horizontal line, so its slope is 0. Thus, the graph of f' is $y = 0$ (the x-axis) on $(-\infty, 0)$.

Since f is discontinuous at $x = 0$, $f'(0)$ does not exist. Thus, the graph of f' has an open circle at $x = 0$.

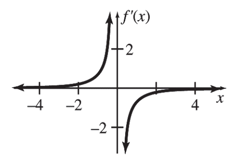
On the interval $(0, \infty)$, the slopes are positive but decreasing at a slower rate as x gets larger. Therefore, the value of f' will be positive but decreasing on this interval. This value approaches 0, but never becomes 0.



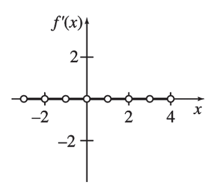
13. We observe that the slopes of tangent lines are positive on the interval $(-\infty, 0)$ and negative on the interval $(0, \infty)$, so the value of f' will be positive on $(-\infty, 0)$ and negative on $(0, \infty)$. Since f is undefined at $x = 0$, $f'(0)$ does not exist.

Notice that the graph of f becomes very flat when $|x| \to \infty$. The *value* of f approaches 0 and also the *slope* approaches 0. Thus, $y = 0$ (the x-axis) is a horizontal asymptote for both the graph of f and the graph of f'.

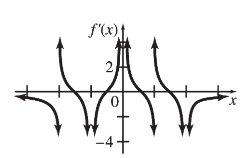
As $x \to 0^-$ and $x \to 0^+$, the graph of f gets very steep, so $|f'(x)| \to \infty$. Thus, $x = 0$ (the y-axis) is a vertical asymptote for both the graph of f and the graph of f'.



14. The graph of f is a step function. (This is the greatest integer function, $f(x) = [[x]]$.) The graph is made up of an infinite series of horizontal line segments. Thus, the derivative will be 0 everywhere it is defined. However, since f is discontinuous wherever x is an integer, $f'(x)$ does not exist at any integer.



15. The slope of $f(x)$ is undefined at $x = -2, -1, 0, 1,$ and 2, and the graph approaches vertical (unbounded slope) as x approaches those values. Accordingly, the graph of $f'(x)$ has vertical asymptotes at $x = -2, -1, 0, 1,$ and 2. $f(x)$ has turning points (zero slope) at $x = -1.5, -0.5, 0.5,$ and 1.5, so the graph of $f'(x)$ crosses the x-axis at those values. Elsewhere, the graph of $f'(x)$ is negative where $f(x)$ is decreasing and positive where $f(x)$ is increasing.



16. The slope of $f(x)$ is undefined at $x = -1$ and 1. The graph approaches vertical (unbounded slope) as x approaches -1 from the left and 1 from the

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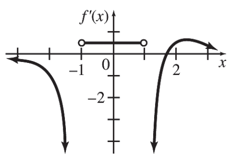
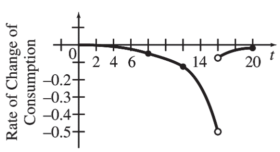
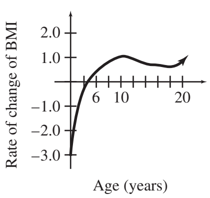
right. Accordingly, the graph of $f'(x)$ has vertical asymptotes for $x = -1$ approached from the left and for $x = 1$ approached from the right. $f(x)$ has a turning point (zero slope) near $x = 1.7$, so the graph of $f'(x)$ crosses the x-axis near $x = 1.7$. Elsewhere, the graph of $f'(x)$ is negative where $f(x)$ is decreasing and positive where $f(x)$ is increasing. Between $x = -1$ and $x = 1$, the slope of $f(x)$ is 0.5; therefore, $f'(x) = 0.5$ between -1 and 1.



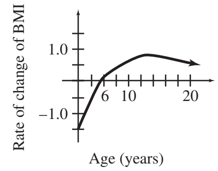
17. The graph of G decreases steadily with varying degrees of steepness. The steepness increases (that is, the slopes of the tangent lines becomes more negative) between $t = 12$ and $t = 16$. Since G is discontinuous at $t = 16, G'(16)$ doesn't exist. The graph continues to decrease after $t = 16$, but the slopes of the tangent lines become less negative as the curve gets flatter. So, the derivative values are increasing toward 0.



18. **(a)** The curve slants downward up to about age 4.25, where it turns and begins to rise. There is a slight decline in steepness between ages 10 and 18. Correspondingly, the graph of the rate of change lies below the horizontal axis to the left of 4.25 years and above the horizontal axis to the right of that point.



(b) The curve slants downward up to about age 5.75, where it turns and begins to rise. There is a slight decline in steepness between ages 13 and 20. Correspondingly, the graph of the rate of change lies below the horizontal axis to the left of 5.75 years and above the horizontal axis to the right of that point.

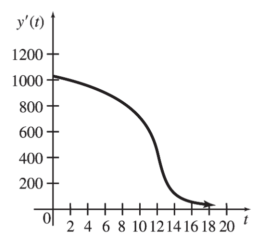


19. The growth rate of the function $y = f(t)$ is given by the derivative of this function $y' = f(t)$. We use the graph of f to sketch the graph of f'. First, notice as x increases, y increases throughout the domain of f, but at a slower and slower rate.
The slope of f is positive but always decreasing, and approaches 0 as t gets large. Thus, y' will always be positive and decreasing. It will approach but never reach 0.

To plot point on the graph of f', we need to estimate the slope of f at several points. From the graph of f, we obtain the values given in the following table.

t	y'
2	1000
10	700
13	250

Use these points to sketch the graph.



20. Let $P(V)$ represent the power corresponding to a given value of the tern's speed, V.
The rate of change of power as a function of time is given by the derivative of this function, $P'(V)$. We use the graph of P to sketch the graph of P'.

First, we observe that the graph of P has one turning point, at $V = V_{mp} \approx 8$. At this value of V, the graph has a horizontal tangent, so the graph of V' has an x-intercept at this value of V. Since the slopes of tangent lines are negative on the interval $(0, V_{mp})$ and positive when $V > V_{mp}$, the value of V' is negative on $(0, V_{mp})$ and positive when $V > V_{mp}$.

Use the tangents drawn on the graph and additional tangent as needed to estimate the slope at several points on the graph of V to improve the accuracy of the graph of V'.

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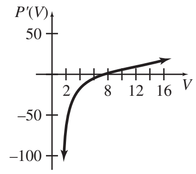
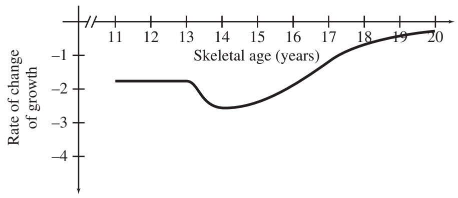
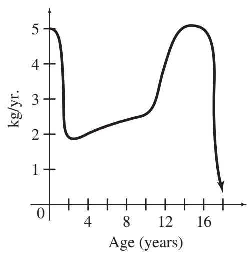


21.

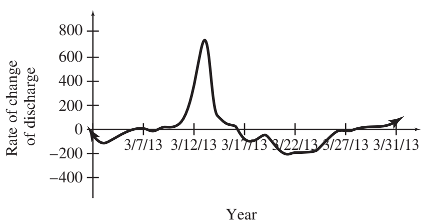


About 9 cm; about 2.6 cm less per year

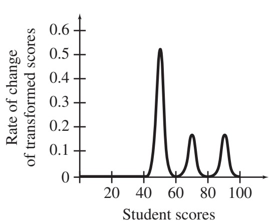
22.



23.



24.



Chapter 3 Review Exercises

1. True

2. True

3. True

4. False; for example, if $f(x) = \frac{x^2-4}{x+2}$,

$\lim_{x\to-2} f(x) = -4$, but the graph of $f(x) = \frac{x^2-4}{x+2}$ has a hole at the point $(-2, -4)$.

5. True

6. False; for example, the rational function $f(x) = \frac{5}{x+1}$ is discontinuous at $x = -1$.

7. False; the derivative gives the instantaneous rate of change of a function.

8. True

9. True

10. True

11. False; the slope of the tangent line gives the instantaneous rate of change.

12. False; for example, the function $f(x) = |x|$ is continuous at $x = 0$, but $f'(0)$ does not exist. The graph of $f(x) = |x|$ has a "corner" at $x = 0$.

16. The derivative can be used to find the instantaneous rate of change at a point on a function, and the slope of a tangent line at a point on a function.

17. **(a)** $\lim_{x\to-3^-} = 4$

(b) $\lim_{x\to-3^+} = 4$

(c) $\lim_{x\to-3} = 4$ (since parts (a) and (b) have the same answer)

(d) $f(-3) = 4$, since $(-3, 4)$ is a point of the graph.

18. **(a)** $\lim_{x\to-1^-} g(x) = -2$

(b) $\lim_{x\to-1^+} g(x) = 2$

(c) $\lim_{x\to-1} g(x)$ does not exist since parts (a) and (b) have different answers.

(d) $g(-1) = -2$, since $(-1, -2)$ is a point on the graph.

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19. **(a)** $\lim_{x \to 4^-} f(x) = \infty$

(b) $\lim_{x \to 4^+} f(x) = -\infty$

(c) $\lim_{x \to 4} f(x)$ does not exist since limits in (a) and (b) do not exist.

(d) $f(4)$ does not exist since the graph has no point with an x-value of 4.

20. **(a)** $\lim_{x \to 2^-} h(x) = 1$

(b) $\lim_{x \to 2^+} h(x) = 1$

(c) $\lim_{x \to 2} h(x) = 1$

(d) $h(2)$ does not exists since the graph has no point with an x-value of 2.

21. $\lim_{x \to -\infty} g(x) = \infty$ since the y-value gets very large as the x-value gets very small.

22. $\lim_{x \to \infty} f(x) = -3$ since the line $y = -3$ is a horizontal asymptote for the graph.

23. $\lim_{x \to 6} \frac{2x+7}{x+3} = \frac{2(6)+7}{6+3} = \frac{19}{9}$

24. Let $f(x) = \frac{2x+5}{x+3}$.

x	-3.1	-3.01	-3.001
$f(x)$	12	102	1002

x	-2.9	-2.99	-2.999
$f(x)$	-8	-98	-998

As x approaches -3 from the left, $f(x)$ gets infinitely larger. As x approaches -3 from the right, $f(x)$ gets infinitely smaller. Therefore, $\lim_{x \to -3} \frac{2x+5}{x+3}$ does not exist.

25.
$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x-4)(x+4)}{x-4} = \lim_{x \to 4} (x+4) = 4 + 4 = 8$$

26.
$$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2} = \lim_{x \to 2} \frac{(x+5)(x-2)}{x-2} = \lim_{x \to 2} (x+5) = 2 + 5 = 7$$

27.
$$\lim_{x \to -4} \frac{2x^2 + 3x - 20}{x + 4} = \lim_{x \to -4} \frac{(2x-5)(x+4)}{x+4} = \lim_{x \to -4} (2x-5) = 2(-4) - 5 = -13$$

28.
$$\lim_{x \to 3} \frac{3x^2 - 2x - 21}{x - 3} = \lim_{x \to 3} \frac{(3x+7)(x-3)}{x-3} = \lim_{x \to 3} (3x+7) = 9 + 7 = 16$$

29.
$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3} = \lim_{x \to 9} \frac{x - 9}{(x-9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

30.
$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4} = \lim_{x \to 16} \frac{x - 16}{(x-16)(\sqrt{x} + 4)} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{\sqrt{16} + 4} = \frac{1}{4 + 4} = \frac{1}{8}$$

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31. $\lim_{x\to\infty} \frac{2x^2+5}{5x^2-1} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{5}{x^2}}{\frac{5x^2}{x^2}-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{2+\frac{5}{x^2}}{5-\frac{1}{x^2}}$

$= \frac{2+0}{5-0}$

$= \frac{2}{5}$

32. $\lim_{x\to\infty} \frac{x^2+6x+8}{x^3+2x+1} = \lim_{x\to\infty} \frac{\frac{x^2}{x^3}+\frac{6x}{x^3}+\frac{8}{x^3}}{\frac{x^3}{x^3}+\frac{2x}{x^3}+\frac{1}{x^3}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{6}{x^2}+\frac{8}{x^3}}{1+\frac{2}{x^2}+\frac{1}{x^3}}$

$= \frac{0+0+0}{1+0+0} = 0$

33. $\lim_{x\to-\infty}\left(\frac{3}{8}+\frac{3}{x}-\frac{6}{x^2}\right)$

$= \lim_{x\to-\infty}\frac{3}{8} + \lim_{x\to-\infty}\frac{3}{x} - \lim_{x\to-\infty}\frac{6}{x^2}$

$= \frac{3}{8}+0-0$

$= \frac{3}{8}$

34. $\lim_{x\to-\infty}\left(\frac{9}{x^4}+\frac{10}{x^2}-6\right)$

$= \lim_{x\to-\infty}\frac{9}{x^4} + \lim_{x\to-\infty}\frac{10}{x^2} - \lim_{x\to-\infty}6$

$= 0+0-6 = -6$

35. As shown on the graph, $f(x)$ is discontinuous at x_2 and x_4.

36. As shown on the graph, $f(x)$ is discontinuous at x_1 and x_4.

37. $f(x)$ is discontinuous at $x = 0$ and $x = -\frac{1}{3}$ since that is where the denominator of $f(x)$ equals 0. $f(0)$ and $f\left(-\frac{1}{3}\right)$ do not exist.

$\lim_{x\to 0} f(x)$ does not exist since $\lim_{x\to 0^+} f(x) = -\infty$, but $\lim_{x\to 0^-} f(x) = \infty$. $\lim_{x\to -\frac{1}{3}} f(x)$ does not exist since $\lim_{x\to -\frac{1}{3}^-} = -\infty$, but $\lim_{x\to -\frac{1}{3}^+} f(x) = \infty$.

38. $f(x) = \frac{7-3x}{(1-x)(3+x)}$

The function is discontinuous at $x = -3$ and $x = 1$ because those values make the denominator of the fraction equal to zero.

$\lim_{x\to -3} f(x)$ does not exist since $\lim_{x\to -3^-} f(x) = -\infty$ and $\lim_{x\to -3^+} f(x) = \infty$.

$\lim_{x\to 1} f(x)$ does not exist since $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1^+} f(x) = -\infty$.

$f(-3)$ and $f(1)$ do not exist since there is no point of the graph that has an x-value of -3 or 1.

39. $f(x)$ is discontinuous at $x = -5$ since that is where the denominator of $f(x)$ equals 0. $f(-5)$ does not exist.

$\lim_{x\to -5} f(x)$ does not exist since $\lim_{x\to -5^-} f(x) = \infty$, but $\lim_{x\to -5^+} f(x) = -\infty$.

40. $f(x) = \frac{x^2-9}{x+3}$

The function is discontinuous at $x = -3$ since this value makes the denominator of the fraction equal to zero.

$$\lim_{x\to -3} \frac{x^2-9}{x+3} = \lim_{x\to -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x\to -3}(x-3) = -3-3 = -6$$

$f(-3)$ does not exist since there is no point on the graph with an x-value of -3.

41. $f(x) = x^2 + 3x - 4$ is continuous everywhere since f is a polynomial function.

42. $f(x) = 2x^2 - 5x - 3$ has no points of discontinuity since it is a polynomial function, which is continuous everywhere.

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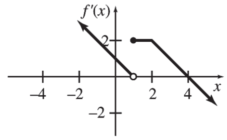
43. **(a)**

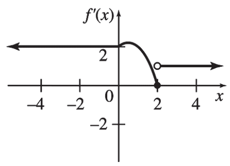


(b) The graph is discontinuous at $x = 1$.

(c) $\lim_{x\to1^-} f(x) = 0;\ \lim_{x\to1^+} f(x) = 2$

44. $f(x) = \begin{cases} 2 & \text{if} \quad x < 0 \\ -x^2 + x + 2 & \text{if} \quad 0 \le x \le 2 \\ 1 & \text{if} \quad x > 2 \end{cases}$

(a)



(b) The graph is discontinuous at $x = 2$.

(c) $\lim_{x\to2^-} f(x) = -4 + 2 + 2 = 0$

$\lim_{x\to2^+} f(x) = 1$

45. $f(x) = \dfrac{x^4 + 2x^3 + 2x^2 - 10x + 5}{x^2 - 1}$

(a) Find the values of $f(x)$ when x is close to 1.

x	y
1.1	2.6005
1.01	2.06
1.001	2.006
1.0001	2.0006
0.99	1.94
0.999	1.994
0.9999	1.9994

It appears that $\lim_{x\to1} f(x) = 2$.

(b) Graph

$$y = \frac{x^4 + 2x^3 + 2x^2 - 10x + 5}{x^2 - 1}$$

on a graphing calculator. One suitable choice for the viewing window is $[-2, 6]$ by $[-10, 10]$. Because $x^2 - 1 = 0$ when $x = -1$ or $x = 1$, this function is discontinuous at these two x-values. The graph shows a vertical asymptote at $x = -1$ but not at $x = 1$. The graph should have an open circle to show a "hole" in the graph at $x = 1$. The graphing calculator doesn't show the hole, but trying to find the value of the function of $x = 1$ will show that this value is undefined.

By viewing the function near $x = 1$ and using the ZOOM feature, we see that as x gets close to 1 from the left or the right, y gets close to 2, suggesting that

$$\lim_{x\to1} \frac{x^4 + 2x^3 + 2x^2 - 10x + 5}{x^2 - 1} = 2.$$

46. $f(x) = \dfrac{x^4 + 3x^3 + 7x^2 + 11x + 2}{x^3 + 2x^2 - 3x - 6}$

(a) Find values of $f(x)$ when x is close to -2.

x	$f(x)$
−2.01	−12.62
−2.001	−12.96
−2.0001	−13
−1.99	−13.41
−1.999	−13.04
−1.9999	−13

It appears that $\lim_{x\to-2} f(x) = -13$.

(b) Graph

$$y = \frac{x^4 + 3x^3 + 7x^2 + 11x + 2}{x^3 + 2x^2 - 3x - 6}$$

on a graphing calculator. One suitable choice for the viewing window is $[-5, 5]$ by $[-10, 10]$. By viewing the function near $x = -2$, we see that as x gets close to -2 from the left on the right, y gets close to -13, suggesting that

$$\lim_{x\to-2} \frac{x^4 + 3x^3 + 7x^2 + 11x + 2}{x^3 + 2x^2 - 3x - 6} = -13.$$

47. $y = 6x^3 + 2 = f(x)$; from $x = 1$ to $x = 4$

$$f(4) = 6(4)^3 + 2 = 386$$
$$f(1) = 6(1)^3 + 2 = 8$$

Average rate of change:

$$= \frac{386 - 8}{4 - 1} = \frac{378}{3} = 126$$

$y' = 18x$

Instantaneous rate of change at $x = 1$:

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$$f'(1) = 18(1) = 18$$

48. $y = -2x^3 - 3x^2 + 8 = f(x)$

$$f(6) = -2(6)^3 - 3(6)^2 + 8 = -532$$
$$f(-2) = -2(-2)^3 - 3(-2)^2 + 8 = 12$$

Average rate of change:

$$= \frac{f(6) - f(-2)}{6 - (-2)}$$
$$= \frac{-532 - 12}{6 + 2} = \frac{-544}{8} = -68$$
$$y' = -6x^2 - 6x$$

Instantaneous rate of change at $x = -2$:

$$f'(-2) = -6(-2)^2 - 6(-2) = -6(4) + 12 = -12$$

49. $y = \frac{-6}{3x - 5} = f(x)$; from $x = 4$ to $x = 9$

$$f(9) = \frac{-6}{3(9) - 5} = \frac{-6}{22} = -\frac{3}{11}$$
$$f(4) = \frac{-6}{3(4) - 5} = -\frac{6}{7}$$

Average rate of change:

$$= \frac{\frac{-3}{11} - \left(-\frac{6}{7}\right)}{9 - 4} = \frac{\frac{-21+66}{77}}{5} = \frac{45}{5(77)} = \frac{9}{77}$$
$$y' = \frac{(3x - 5)(0) - (-6)(3)}{(3x - 5)^2} = \frac{18}{(3x - 5)^2}$$

Instantaneous rate of change at $x = 4$:

$$f'(4) = \frac{18}{(3 \cdot 4 - 5)^2} = \frac{18}{7^2} = \frac{18}{49}$$

50. $y = \frac{x + 4}{x - 1} = f(x)$

$$f(5) = \frac{5 + 4}{5 - 1} = \frac{9}{4}$$
$$f(2) = \frac{2 + 4}{2 - 1} = 6$$

Average rate of change:

$$= \frac{\frac{9}{4} - 6}{5 - 2} = \frac{\frac{-15}{4}}{3} = -\frac{5}{4}$$
$$y' = \frac{(x - 1)(1) - (x + 4)(1)}{(x - 1)^2}$$
$$= \frac{x - 1 - x - 4}{(x - 1)^2} = \frac{-5}{(x - 1)^2}$$

Instantaneous rate of change at $x = 2$:

$$f'(2) = \frac{-5}{(2 - 1)^2} = \frac{-5}{1} = -5$$

51. **(a)** $f(x) = 3x^2 - 5x + 7$; $x = 2$, $x = 4$

Slope of secant line

$$= \frac{f(4) - f(2)}{4 - 2}$$
$$= \frac{[3(4)^2 - 5(4) + 7] - [3(2)^2 - 5(2) + 7]}{2}$$
$$= \frac{35 - 9}{2}$$
$$= 13$$

Now use $m = 13$ and $2, f(2) = (2, 9)$ in the point-slope form.

$$y - 9 = 13(x - 2)$$
$$y - 9 = 13x - 26$$
$$y = 13x - 26 + 9$$
$$y = 13x - 17$$

(b) $f(x) = 3x^2 - 5x + 7$; $x = 2$

$$\frac{f(x + h) - f(x)}{h}$$
$$= \frac{[3(x + h)^2 - 5(x + h) + 7] - [3x^2 - 5x + 7]}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 5x - 5h + 7 - 3x^2 + 5x - 7}{h}$$
$$= \frac{6xh + 3h^2 - 5h}{h}$$
$$= 6x + 3h - 5$$

$$f'(x) = \lim_{h \to 0} 6x + 3h - 5$$
$$= 6x - 5$$
$$f'(2) = 6(2) - 5$$
$$= 7$$

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Now use $m = 7$ and $(2, f(2)) = (2,9)$ in the point-slope form.

$$y - 9 = 7(x - 2)$$
$$y - 9 = 7x - 14$$
$$y = 7x - 14 + 9$$
$$y = 7x - 5$$

52. (a) $f(x) = \frac{1}{x}; x = \frac{1}{2}, x = 3$

$$\text{Slope of secant line} = \frac{f(3) - f\left(\frac{1}{2}\right)}{3 - \frac{1}{2}}$$
$$= \frac{\frac{1}{3} - 2}{3 - \frac{1}{2}} = \frac{2 - 12}{18 - 3}$$
$$= \frac{-10}{15} = -\frac{2}{3}$$

Now use $m = -\frac{2}{3}$ and $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, 2\right)$ in the point-slope form.

$$y - 2 = -\frac{2}{3}\left(x - \frac{1}{2}\right)$$
$$y - 2 = -\frac{2}{3}x + \frac{1}{3}$$
$$y = -\frac{2}{3}x + \frac{1}{3} + 2$$
$$y = -\frac{2}{3}x + \frac{7}{3}$$

(b) $f(x) = \frac{1}{x}; x = \frac{1}{2}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$
$$= \frac{x - (x+h)}{xh(x+h)} = -\frac{h}{xh(x+h)}$$
$$= -\frac{1}{x(x+h)}$$

$$f'(x) = \lim_{h \to 0} -\frac{1}{x(x+h)}$$
$$= -\frac{1}{x^2}$$
$$f'\left(\frac{1}{2}\right) = -\frac{1}{\left(\frac{1}{2}\right)^2}$$
$$= -4$$

Now use $m = -4$ and $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, 2\right)$ in the point-slope form.

$$y - 2 = -4\left(x - \frac{1}{2}\right)$$
$$y - 2 = -4x + 2$$
$$y = -4x + 2 + 2$$
$$y = -4x + 4$$

53. (a) $f(x) = \frac{12}{x-1}; x = 3, x = 7$

$$\text{Slope of secant line} = \frac{f(7) - f(3)}{7 - 3}$$
$$= \frac{\frac{12}{7-1} - \frac{12}{3-1}}{4}$$
$$= \frac{2 - 6}{4}$$
$$= -1$$

Now use $m = -1$ and $(3, f(x)) = (3,6)$ in the point-slope form.

$$y - 6 = -1(x - 3)$$
$$y - 6 = -x + 3$$
$$y = -x + 3 + 6$$
$$y = -x + 9$$

(b) $f(x) = \frac{12}{x-1}; x = 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{12}{x+h-1} - \frac{12}{x-1}}{h}$$
$$= \frac{12(x-1) - 12(x+h-1)}{h(x-1)(x+h-1)}$$
$$= \frac{-12h}{h(x-1)(x+h-1)}$$
$$= -\frac{12}{(x-1)(x+h-1)}$$

$$f'(x) = \lim_{h \to 0} -\frac{12}{(x-1)(x+h-1)}$$
$$= -\frac{12}{(x-1)^2}$$
$$f'(3) = -\frac{12}{(3-1)^2}$$
$$= -3$$

Now use $m = -3$ and $(3, f(x)) = (3,6)$ in the point-slope form.

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$$y - 6 = -3(x - 3)$$
$$y - 6 = -3x + 9$$
$$y = -3x + 9 + 6$$
$$y = -3x + 15$$

54. (a) $f(x) = 2\sqrt{x-1};\ x = 5, x = 10$

$$\text{Slope of secant line} = \frac{f(10) - f(5)}{10 - 5} = \frac{2\sqrt{10-1} - 2\sqrt{5-1}}{5} = \frac{2(3) - 2(2)}{5} = \frac{2}{5}$$

Now use $m = \frac{2}{5}$ and $(5, f(x)) = (5,4)$ in the point-slope form.

$$y - 4 = \frac{2}{5}(x - 5)$$
$$y - 4 = \frac{2}{5}x - 2$$
$$y = \frac{2}{5}x - 2 + 4$$
$$y = \frac{2}{5}x + 2$$

(b) $f(x) = 2\sqrt{x-1};\ x = 5$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{2\sqrt{x+h-1} - 2\sqrt{x-1}}{h}$$
$$= \frac{2(\sqrt{x+h-1} - \sqrt{x-1})(\sqrt{x+h-1} + \sqrt{x-1})}{h(\sqrt{x+h-1} + \sqrt{x-1})}$$
$$= \frac{2(x+h-1-x+1)}{h(\sqrt{x+h-1} + \sqrt{x-1})} = \frac{2h}{h(\sqrt{x+h-1} + \sqrt{x-1})}$$
$$= \frac{2}{\sqrt{x+h-1} + \sqrt{x-1}}$$

$$f'(x) = \lim_{h \to 0} \frac{2}{\sqrt{x+h-1} + \sqrt{x-1}} = \frac{2}{2\sqrt{x-1}} = \frac{1}{\sqrt{x-1}}$$
$$f'(5) = \frac{1}{\sqrt{5-1}} = \frac{1}{2}$$

Now use $m = \frac{1}{2}$ and $(5, f(x)) = (5,4)$ in the point-slope form.

$$y - 4 = \frac{1}{2}(x - 5)$$
$$y - 4 = \frac{1}{2}x - \frac{5}{2}$$
$$y = \frac{1}{2}x - \frac{5}{2} + 4$$
$$y = \frac{1}{2}x + \frac{3}{2}$$

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55. $y = 4x^2 + 3x - 2 = f(x)$

$$\begin{aligned}
y' &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[4(x+h)^2 + 3(x+h) - 2] - [4x^2 + 3x - 2]}{h} \\
&= \lim_{h\to 0} \frac{4(x^2 + 2xh + h^2) + 3x + 3h - 2 - 4x^2 - 3x + 2}{h} \\
&= \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 + 3x + 3h - 2 - 4x^2 - 3x + 2}{h} \\
&= \lim_{h\to 0} \frac{8xh + 4h^2 + 3h}{h} \\
&= \lim_{h\to 0} \frac{h(8x + 4h + 3)}{h} \\
&= \lim_{h\to 0} (8x + 4h + 3) \\
&= 8x + 3
\end{aligned}$$

56. $y = 5x^2 - 6x + 7 = f(x)$

$$\begin{aligned}
y' &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[5(x+h)^2 - 6(x+h) + 7] - [5x^2 - 6x + 7]}{h} \\
&= \lim_{h\to 0} \frac{5(x^2 + 2xh + h^2) - 6x - 6h + 7 - 5x^2 + 6x}{h} \\
&= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 - 6x - 6h + 7 - 5x^2 + 6x + 7}{h} \\
&= \lim_{h\to 0} \frac{10xh + 5h^2 - 6h}{h} = \lim_{h\to 0} \frac{h(10x + 5h - 6)}{h} \\
&= \lim_{h\to 0} (10x + 5h - 6) = 10x - 6
\end{aligned}$$

57. $f(x) = (\ln x)^x$, $x_0 = 3$

(a)

h	
0.01	$\frac{f(3 + 0.01) - f(3)}{0.01} = \frac{(\ln 3.01)^{3.01} - (\ln 3)^3}{0.01} = 1.3385$
0.001	$\frac{f(3 + 0.001) - f(3)}{0.001} = \frac{(\ln 3.001)^{3.001} - (\ln 3)^3}{0.001} = 1.3323$

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0.0001	$\frac{f(3+0.0001)-f(3)}{0.0001} = \frac{(\ln 3.0001)^{3.0001}-(\ln 3)^3}{0.0001} = 1.3317$
0.00001	$\frac{f(3+0.00001)-f(3)}{0.00001} = \frac{(\ln 3.00001)^{3.00001}-(\ln 3)^3}{0.00001} = 1.3317$

It appears that $f'(3) \approx 1.332$.

(b) Using a graphing calculator will confirm this result.

58. $f(x) = x^{\ln x}, x_0 = 2$

(a)

h	
0.01	$\frac{f(2+0.01)-f(2)}{0.01} = \frac{2.01^{\ln 2.01}-2^{\ln 2}}{0.01} = 1.1258$
0.001	$\frac{f(2+0.001)-f(2)}{0.001} = \frac{2.001^{\ln 2.001}-2^{\ln 2}}{0.001} = 1.1212$
0.0001	$\frac{f(2+0.0001)-f(2)}{0.0001} = \frac{2.0001^{\ln 2.0001}-2^{\ln 2}}{0.0001} = 1.1207$
0.00001	$\frac{f(2+0.00001)-f(2)}{0.00001} = \frac{2.00001^{\ln 2.00001}-2^{\ln 2}}{0.00001} = 1.1207$

It appears that $f'(2) = 1.121$.

(b) Graph the function on a graphing calculator and move the cursor to an x-value near $x = 2$. A good choice for the viewing window is [0, 10] by [0, 10].

Zoom in on the function until the graph looks like a straight line. Use the TRACE feature to select two points on the graph, and use these points to compute the slope. The result will be close to the most accurate value found in part (a), which is 1.121.

59. On the interval $(-\infty, 0)$, the graph of f is a straight line, so its slope is constant. To find this slope, use the points $(-2, 2)$ and $(0, 0)$.

$$m = \frac{0-2}{0-(-2)} = \frac{-2}{2} = -1$$

Thus, the value of f' will be -1 on this interval.

The graph of f has a sharp point at 0, so $f'(0)$ does not exist. To show this, we use an open circle on the graph of f' at $(0, -1)$.

We also observe that the slope of f is positive but decreasing from $x = 0$ to about $x = 1$, and then

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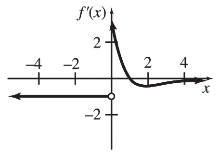
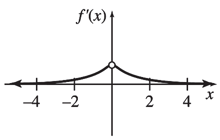
negative from there on. As $x \to \infty, f(x) \to 0$ and also $f'(x) = 0$.

Use this information to complete the graph of f'.



60. On the intervals $(-\infty, 0)$ and $(0, \infty)$, the slope of any tangent line will be positive, so the derivative will be positive. Thus, the graph of f' will lie above the y-axis. The slope of f and thus the value of f' approaches 0 when $x \to -\infty$ and $x \to \infty$ and approaches some particular but unknown positive value > 1 when $x \to 0^-$ and $x \to 0^+$.

Because f is discontinuous at $x = 0$, we know that $f'(0)$ does not exist, which we indicate with an open circle at $x = 0$ on the graph of f'.



61. $\lim\limits_{x\to\infty} \dfrac{cf(x) - dg(x)}{f(x) - g(x)}$

$$= \frac{\lim\limits_{x\to\infty} [cf(x) - dg(x)]}{\lim\limits_{x\to\infty} [f(x) - g(x)]}$$

$$= \frac{\lim\limits_{x\to\infty} [cf(x)] - \lim\limits_{x\to\infty} [dg(x)]}{\lim\limits_{x\to\infty} [f(x)] - \lim\limits_{x\to\infty} [g(x)]}$$

$$= \frac{c \lim\limits_{x\to\infty} [f(x)] - d \lim\limits_{x\to\infty} [g(x)]}{\lim\limits_{x\to\infty} [f(x)] - \lim\limits_{x\to\infty} [g(x)]}$$

$$= \frac{c \cdot c - d \cdot d}{c - d} = \frac{(c + d)(c - d)}{c - d}$$

$$= c + d$$

The answer is (e).

62. $R(x) = 5000 + 16x - 3x^2$

(a) $R'(x) = 16 - 6x$

(b) Since x is in hundreds of dollars, \$1000 corresponds to $x = 10$.

$$R'(10) = 16 - 6(10) = 16 - 60 = -44$$

An increase of \$100 spent on advertising when advertising expenditures are \$1000 will result in the revenue decreasing by \$44.

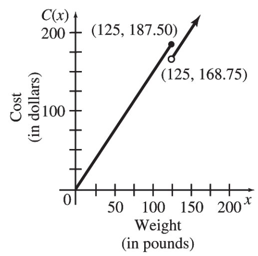
63. $C(x) = \begin{cases} 1.50x & \text{for } 0 < x \le 125 \\ 1.35x & \text{for } x > 125 \end{cases}$

(a) $C(100) = 1.50(100) = \$150$

(b) $C(125) = 1.50(125) = \$187.50$

(c) $C(140) = 1.35(140) = \$189$

(d)



(e) By reading the graph, $C(x)$ is discontinuous at $x = \$125$.

The average cost per pound is given by $\overline{C}(x) = \frac{C(x)}{x}$.

$$\overline{C}(x) = \begin{cases} 1.50 & \text{for } 0 < x \le 125 \\ 1.35 & \text{for } x > 125 \end{cases}$$

(f) $\overline{C}(100) = \$1.50$

(g) $\overline{C}(125) = \$1.50$

(h) $\overline{C}(140) = \$1.35$

The marginal cost is given by

$$C(x) = \begin{cases} 1.50 & \text{for } 0 < x \le 125 \\ 1.35 & \text{for } x > 125. \end{cases}$$

(i) $C'(100) = 1.50$; the l01st pound will cost \$1.50.

(j) $C'(140) = 1.35$; the 141st pound will cost \$1.35.

64. $P(x) = 15x + 25x^2$

(a) $P(6) = 15(6) + 25(6)^2 = 90 + 900 = 990$

$P(7) = 15(7) + 25(7)^2 = 105 + 1225 = 1330$

Average rate of change:

$$= \frac{P(7) - P(6)}{7 - 6} = \frac{1330 - 990}{1}$$

$$= 340 \text{ cents or } \$3.40$$

(b) $P(6) = 990$

$$\begin{aligned} P(6.5) &= 15(6.5) + 25(6.5)^2 \\ &= 97.5 + 1056.25 \\ &= 1153.75 \end{aligned}$$

Average rate of change:

$$\begin{aligned} &= \frac{P(6.5) - P(6)}{6.5 - 6} \\ &= \frac{1153.75 - 990}{0.5} \\ &= 327.5 \text{ cents or } \$3.28 \end{aligned}$$

(c) $P(6) = 990$

$$\begin{aligned} P(6.1) &= 15(6.1) + 25(6.1)^2 \\ &= 91.5 + 930.25 \\ &= 1021.75 \end{aligned}$$

Average rate of change:

$$\begin{aligned} &= \frac{P(6.1) - P(6)}{6.1 - 6} \\ &= \frac{1021.75 - 990}{0.1} \\ &= 317.5 \text{ cents or } \$3.18 \end{aligned}$$

(d)

$$\begin{aligned} P'(x) &= 15 + 50x \\ P'(6) &= 15 + 50(6) \\ &= 15 + 300 \\ &= 315 \text{ cents or } \$3.15 \end{aligned}$$

(e) $P'(20) = 15 + 50(20) = 1015¢$ or $\$10.15$

(f) $P'(30) = 15 + 50(30) = 1515¢$ or $\$15.15$

(g) The domain of x is $[0, \infty)$ since pounds cannot be measured with negative numbers.

(h) Since $P'(x) = 15 + 50x$ gives the marginal profit, and $x \geq 0$, $P'(x)$ can never be negative.

(i)

$$\begin{aligned} \overline{P}(x) &= \frac{P(x)}{x} \\ &= \frac{15x + 25x^2}{x} \\ &= 15 + 25x \end{aligned}$$

(j) $\overline{P}'(x) = 25$

(k) The marginal average profit cannot change since $\overline{P}'(x)$ is constant. The profit per pound never changes, no matter now many pounds are sold.

65. (b) The value of x for which the average cost is smallest is $x = 7.5$. This can be found by drawing a line from the origin to any point of $C(x)$. At $x = 7.5$, you will get a line with the smallest slope.

(c) The marginal cost equals the average cost at the point where the average cost is smallest.

66. (a)

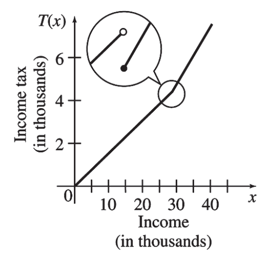
$$\begin{aligned} \lim_{x \to 29{,}300^-} T(x) &= (29{,}300)(0.15) \\ &= \$4395 \end{aligned}$$

(b)

$$\begin{aligned} \lim_{x \to 29{,}300^+} T(x) &= 4350 + (0.27)(29{,}300 - 29{,}300) \\ &= \$4350 \end{aligned}$$

(c) $\lim_{x \to 29{,}300} T(x)$ does not exist since parts (a) and (b) have different answers.

(d)



(e) The graph is discontinuous at $x = 29{,}300$.

(f) For $0 \leq x \leq 29{,}300$,

$$A(x) = \frac{T(x)}{x} = \frac{0.15x}{x} = 0.15.$$

For $x > 29{,}300$,

$$\begin{aligned} A(x) &= \frac{T(x)}{x} \\ &= \frac{4350 + (0.27)(x - 29{,}300)}{x} \\ &= \frac{0.27x - 3561}{x} \\ &= 0.27 - \frac{3561}{x}. \end{aligned}$$

(g) $\lim_{x \to 29{,}300^-} A(x) = 0.15$

(h)

$$\begin{aligned} \lim_{x \to 29{,}300^+} A(x) &= 0.27 - \frac{3561}{29{,}300} \\ &= 0.14846 \end{aligned}$$

(i) $\lim_{x \to 29{,}300} A(x)$ does not exist since parts (g) and (h) have different answers.

(j) $\lim_{x \to \infty} A(x) = 0.27 - 0 = 0.27$

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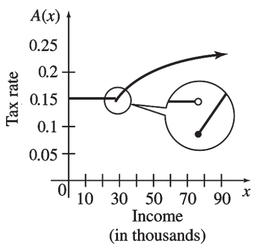
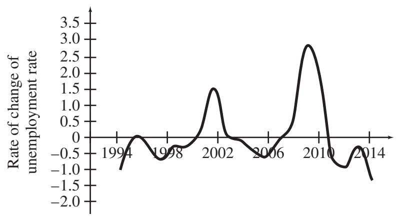
(k)



67.



The annual unemployment rate in 2012 was about 8.2%. Our sketch of the rate of change of the unemployment rate
indicates that the rate of change in 2012 was approximately -0.5% per year.

68. (a) The slope of the tangent line at $x = 2000$ is about 0.13; the number of people aged 65 and over with Alzheimer's Disease is going up at a rate of about 0.13 million per year.

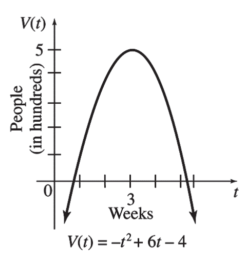
(b) The slope of the tangent line at x = 2040 is about 0.34; the number of people aged 65 and over with Alzheimer's Disease is going up at a rate of about 0.34 million per year.

(c)
$$\frac{A(2040) - A(2000)}{2040 - 2000} = \frac{11.1 - 4.6}{40}$$
$$\approx 0.16$$

The average rate of change in the number of people 65 and over with Alzheimer's Disease over this interval is about 0.16 million people per year.

69. $V(t) = -t^2 + 6t - 4$

(a)



(b) The x-intercepts of the parabola are 0.8 and 5.2, so a reasonable domain would be [0.8, 5.2], which represents the time period from 0.8 to 5.2 weeks.

(c) The number of cases reaches a maximum at the vertex;

$$x = \frac{-b}{2a} = \frac{-6}{-2} = 3$$

$$V(3) = -3^2 + 6(3) - 4 = 5$$

The vertex of the parabola is $(3, 5)$. This represents a maximum at 3 weeks of 500 cases.

(d) The rate of change function is

$$V'(t) = -2t + 6.$$

(e) The rate of change in the number of cases at the maximum is

$$V'(3) = -2(3) + 6 = 0.$$

(f) The sign of the rate of change up to the maximum is $+$ because the function is increasing. The sign of the rate of change after the maximum is $-$ because the function is decreasing.

70. (a) The rate can be estimated by estimating the slope of the tangent line to the curve at the given time. Answers will vary depending on the points used to determine the slope of the tangent line.

(i) The tangent line to the curve at 17:37 can be found by using the endpoints at (17:35.5, 0) and (17:40, 400). The rate the whale is descending at 17:37 is about

$$\frac{400 - 0}{17{:}40 - 17{:}35.5} = \frac{400}{4.5}$$
$$\approx 90 \text{ meters per minute.}$$

(ii) The tangent line to the curve at 17:39 can be found by using the endpoints at (17:36.2, 0) and (17:40.8, 400). The rate the whale is descending at 17:39 is about

$$\frac{400 - 0}{17{:}40.8 - 17{:}36.2} = \frac{400}{4.6}$$
$$\approx 85 \text{ meters per minute.}$$

(b) The whale appears to have 5 distinct rates at which it is descending.

Interval	Rate (meters per minute)
17:35–17:35.3	$\frac{f(17{:}35.3) - f(17{:}35)}{17{:}35.3 - 17{:}35} = \frac{10 - 0}{0.3} \approx 13$

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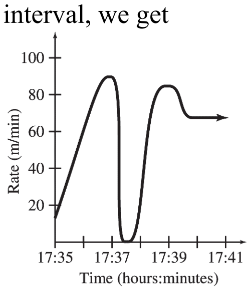
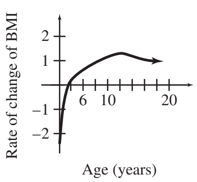
17:36–17:37	$\frac{f(17{:}37) - f(17{:}36)}{17{:}37 - 17{:}36} = \frac{130 - 40}{1} \approx 90$
17:37.3–17:37.7	$\frac{f(17{:}37.7) - f(17{:}37.3)}{17{:}37.7 - 17{:}37.3} = \frac{150 - 150}{0.4} = 0$
17:38.3–17:39.5	$\frac{f(17{:}39.5) - f(17{:}38.3)}{17{:}39.5 - 17{:}38.3} = \frac{300 - 200}{1.2} \approx 83$
17:40–17:41	$\frac{f(14{:}41) - f(14{:}40)}{17{:}41 - 17{:}40} = \frac{400 - 333}{1} = 67$

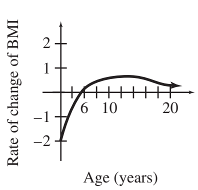
Making smooth transitions between each interval, we get



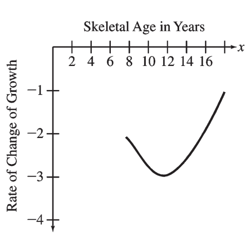
71. (a)



(b)



72.

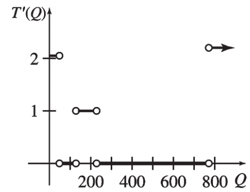


For a 10-year old girl, the remaining growth is about 14 cm and the rate of change is about -2.75 cm per year.

73. (a) The graph is discontinuous nowhere.

(b) The graph is not differentiable where the graph makes a sudden change, namely at $x = 50$, $x = 130$, $x = 230$, and $x = 770$.

(c)



Extended Application: A Model for Drugs Administered Intravenously

1. (a) Since the half-life of the drug is 9 hours, the exponential decay constant is

$$k = -\frac{\ln 2}{9} \approx -0.077.$$

Since the initial amount injected is 500 mg, the model for the amount in the bloodstream (in mg) after t hours is

$$A(t) = 500e^{-0.077t}$$

(b) The average rate of change over the t-interval from 0 to 2 is

$$\frac{A(2) - A(0)}{2} = \frac{500\left(e^{-0.077(2)} - e^{-0.077(0)}\right)}{2} = -35.68 \text{ mg/hr}$$

For the interval from $t = 9$ to $t = 11$, we get

$$\frac{A(11) - A(9)}{2} = \frac{500\left(e^{-0.077(11)} - e^{-0.077(9)}\right)}{2} = -17.84 \text{ mg/hr}$$

2. (a) Since the half-life of the drug is 3 hours, the exponential decay constant is

$$k = -\frac{\ln 2}{3} \approx -0.23.$$

With an infusion rate of 350 mg/hr, the model is

$$A(t) = \frac{350}{0.23}(1 - e^{-0.23t}) \approx 1522(1 - e^{-0.23t})$$

(b) The average rate of change over the t-interval from 0 to 3 is

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$$\frac{A(3)-A(0)}{3} = \frac{1522\left[\left(1-e^{-0.23(3)}\right)-\left(1-e^{-0.23(0)}\right)\right]}{3} = \frac{1522\left(e^{-0.23(0)}-e^{-0.23(3)}\right)}{3} = 252.9 \text{ mg/hr}$$

For the interval from $t = 3$ to $t = 6$, we get

$$\frac{A(6)-A(3)}{3} = \frac{1522\left(e^{-0.23(3)}-e^{-0.23(6)}\right)}{3} = 126.8 \text{ mg/hr}$$

3. The exponential decay constant for a half-life of 9 hours is as found in Exercise 1: $k = -0.077$. Following Example 4, for a steady level of 240 mg, we want the infusion rate to satisfy

$$250 = \frac{r}{-k},$$

so, $r = 250(-k) = 240(0.077) = 19.25$ mg/hr. With this value of r, $\frac{r}{-k} = 250$, so the model is

$$A(t) = 250e^{-0.077t} + 250(1 - e^{-0.077t})$$

4.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
$A(t)$	500	481.116	462.945	445.46	428.636	412.447	396.87	381.881	367.458	353.579
t	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5
$A(t)$	340.225	327.376	315.011	303.114	291.666	280.65	270.05	259.851	250.037	240.593
t	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5
$A(t)$	231.507	222.763	214.35	206.254	198.464	190.968	183.756	176.816	170.138	163.712
t	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5
$A(t)$	157.529	151.579	145.854	140.346	135.045	129.945	125.037	120.314	115.77	111.398
t	20	20.5	21	21.5	22	22.5	23	23.5	24	
$A(t)$	107.191	103.142	99.247	95.498	91.891	88.421	85.081	81.868	78.776	

5.

t	0	0.5	1	1.5	2	2.5	3	3.5
$A(t)$	0	165.341	312.72	444.089	561.186	665.563	758.601	841.532
t	4	4.5	5	5.5	6	6.5	7	7.5
$A(t)$	915.454	981.345	1040.079	1092.432	1139.097	1180.694	1217.771	1250.821
t	8	8.5	9	9.5	10			
$A(t)$	1280.28	1306.539	1329.945	1350.809	1369.406			

6.

t	0	0.5	1	1.5	2	2.5	3	3.5
$A(t)$	250	250	250	250	250	250	250	250
t	4	4.5	5	5.5	6	6.5	7	7.5
$A(t)$	250	250	250	250	250	250	250	250
t	8	8.5	9	9.5	10			
$A(t)$	250	250	250	250	250			

The results show that the chosen infusion rate has the desired effect of keeping the drug at a constract level of 250 mg in the bloodstream.

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