Complete Solutions Manual

for

SINGLE VARIABLE CALCULUS
EARLY TRANSCENDENTALS

SEVENTH EDITION

DANIEL ANDERSON
University of Iowa

JEFFERY A. COLE
Anoka-Ramsey Community College

DANIEL DRUCKER
Wayne State University

BROOKS/COLE
CENGAGE Learning™

Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

BROOKS/COLE
CENGAGE Learning™

For product information and technology assistance, contact us at **Cengage Learning Customer & Sales Support, 1-800-354-9706**

For permission to use material from this text or product, submit all requests online at **www.cengage.com/permissions.** Further permissions questions can be e-mailed to **permissionrequest@cengage.com.**

ISBN-13: 978-0-8400-4936-0
ISBN-10: 0-8400-4936-6

Brooks/Cole
20 Davis Drive
Belmont, CA 94002-3098
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: **www.cengage.com/global**

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Brooks/Cole, visit **www.cengage.com/brookscole**

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**

Printed in the United States of America
1 2 3 4 5 6 7 14 13 12 11 10

PREFACE

This *Complete Solutions Manual* contains solutions to all exercises in the text *Single Variable Calculus, Early Transcendentals,* Seventh Edition, by James Stewart. A student version of this manual is also available; it contains solutions to the odd-numbered exercises in each section, the review sections, the True-False Quizzes, and the Problem Solving sections, as well as solutions to all the exercises in the Concept Checks. No solutions to the projects appear in the student version. It is our hope that by browsing through the solutions, professors will save time in determining appropriate assignments for their particular class.

We use some nonstandard notation in order to save space. If you see a symbol that you don't recognize, refer to the Table of Abbreviations and Symbols on page iv.

We appreciate feedback concerning errors, solution correctness or style, and manual style. Any comments may be sent directly to jeff.cole@anokaramsey.edu, or in care of the publisher: Brooks/Cole, Cengage Learning, 20 Davis Drive, Belmont CA 94002-3098.

We would like to thank Stephanie Kuhns and Kathi Townes, of TECHarts, for their production services; and Elizabeth Neustaetter, of Brooks/Cole, Cengage Learning, for her patience and support. All of these people have provided invaluable help in creating this manual.

Jeffery A. Cole
Anoka-Ramsey Community College

James Stewart
McMaster University
and University of Toronto

Daniel Drucker
Wayne State University

Daniel Anderson
University of Iowa

ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of f
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\stackrel{\text{CAS}}{=}$	indicates the use of a computer algebra system.
$\stackrel{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\stackrel{j}{=}$	indicates the use of Formula j in the Table of Integrals in the back endpapers.
$\stackrel{\text{s}}{=}$	indicates the use of the substitution $\{u = \sin x, du = \cos x\,dx\}$.
$\stackrel{\text{c}}{=}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x\,dx\}$.

CONTENTS

11 □ INFINITE SEQUENCES AND SERIES 939

□ APPENDIXES 1077

DIAGNOSTIC TESTS

Test A Algebra

1. (a) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

(b) $-3^4 = -(3)(3)(3)(3) = -81$

(c) $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

(d) $\dfrac{5^{23}}{5^{21}} = 5^{23-21} = 5^2 = 25$

(e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(f) $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

2. (a) Note that $\sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$ and $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$. Thus $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$.

(b) $(3a^3b^3)(4ab^2)^2 = 3a^3b^3 16a^2b^4 = 48a^5b^7$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(\dfrac{x^2y^{-1/2}}{3x^{3/2}y^3}\right)^2 = \dfrac{(x^2y^{-1/2})^2}{(3x^{3/2}y^3)^2} = \dfrac{x^4y^{-1}}{9x^3y^6} = \dfrac{x^4}{9x^3y^6y} = \dfrac{x}{9y^7}$

3. (a) $3(x+6) + 4(2x-5) = 3x + 18 + 8x - 20 = 11x - 2$

(b) $(x+3)(4x-5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

(c) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} - \left(\sqrt{b}\right)^2 = a - b$

Or: Use the formula for the difference of two squares to see that $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$.

(d) $(2x+3)^2 = (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$.

Note: A quicker way to expand this binomial is to use the formula $(a+b)^2 = a^2 + 2ab + b^2$ with $a = 2x$ and $b = 3$:
$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$

(e) See Reference Page 1 for the binomial formula $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Using it, we get
$(x+2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3 = x^3 + 6x^2 + 12x + 8$.

4. (a) Using the difference of two squares formula, $a^2 - b^2 = (a+b)(a-b)$, we have
$4x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$.

(b) Factoring by trial and error, we get $2x^2 + 5x - 12 = (2x-3)(x+4)$.

(c) Using factoring by grouping and the difference of two squares formula, we have
$x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x^2-4)(x-3) = (x-2)(x+2)(x-3)$.

(d) $x^4 + 27x = x(x^3 + 27) = x(x+3)(x^2 - 3x + 9)$

This last expression was obtained using the sum of two cubes formula, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ with $a = x$ and $b = 3$. [See Reference Page 1 in the textbook.]

(e) The smallest exponent on x is $-\frac{1}{2}$, so we will factor out $x^{-1/2}$.
$3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x^2 - 3x + 2) = 3x^{-1/2}(x-1)(x-2)$

(f) $x^3y - 4xy = xy(x^2 - 4) = xy(x-2)(x+2)$

5. (a) $\dfrac{x^2+3x+2}{x^2-x-2} = \dfrac{(x+1)(x+2)}{(x+1)(x-2)} = \dfrac{x+2}{x-2}$

(b) $\dfrac{2x^2-x-1}{x^2-9} \cdot \dfrac{x+3}{2x+1} = \dfrac{(2x+1)(x-1)}{(x-3)(x+3)} \cdot \dfrac{x+3}{2x+1} = \dfrac{x-1}{x-3}$

(c) $\dfrac{x^2}{x^2-4} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} - \dfrac{x+1}{x+2} \cdot \dfrac{x-2}{x-2} = \dfrac{x^2-(x+1)(x-2)}{(x-2)(x+2)}$

$$= \frac{x^2-(x^2-x-2)}{(x+2)(x-2)} = \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$$

(d) $\dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} = \dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} \cdot \dfrac{xy}{xy} = \dfrac{y^2-x^2}{x-y} = \dfrac{(y-x)(y+x)}{-(y-x)} = \dfrac{y+x}{-1} = -(x+y)$

6. (a) $\dfrac{\sqrt{10}}{\sqrt{5}-2} = \dfrac{\sqrt{10}}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2} = \dfrac{\sqrt{50}+2\sqrt{10}}{\left(\sqrt{5}\right)^2-2^2} = \dfrac{5\sqrt{2}+2\sqrt{10}}{5-4} = 5\sqrt{2}+2\sqrt{10}$

(b) $\dfrac{\sqrt{4+h}-2}{h} = \dfrac{\sqrt{4+h}-2}{h} \cdot \dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \dfrac{4+h-4}{h\left(\sqrt{4+h}+2\right)} = \dfrac{h}{h\left(\sqrt{4+h}+2\right)} = \dfrac{1}{\sqrt{4+h}+2}$

7. (a) $x^2+x+1 = \left(x^2+x+\frac{1}{4}\right)+1-\frac{1}{4} = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

(b) $2x^2-12x+11 = 2(x^2-6x)+11 = 2(x^2-6x+9-9)+11 = 2(x^2-6x+9)-18+11 = 2(x-3)^2-7$

8. (a) $x+5 = 14-\frac{1}{2}x \quad \Leftrightarrow \quad x+\frac{1}{2}x = 14-5 \quad \Leftrightarrow \quad \frac{3}{2}x = 9 \quad \Leftrightarrow \quad x = \frac{2}{3}\cdot 9 \quad \Leftrightarrow \quad x = 6$

(b) $\dfrac{2x}{x+1} = \dfrac{2x-1}{x} \quad \Rightarrow \quad 2x^2 = (2x-1)(x+1) \quad \Leftrightarrow \quad 2x^2 = 2x^2+x-1 \quad \Leftrightarrow \quad x = 1$

(c) $x^2-x-12 = 0 \quad \Leftrightarrow \quad (x+3)(x-4) = 0 \quad \Leftrightarrow \quad x+3 = 0$ or $x-4 = 0 \quad \Leftrightarrow \quad x = -3$ or $x = 4$

(d) By the quadratic formula, $2x^2+4x+1 = 0 \quad \Leftrightarrow$

$$x = \frac{-4 \pm \sqrt{4^2-4(2)(1)}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = \frac{2\left(-2 \pm \sqrt{2}\right)}{4} = \frac{-2 \pm \sqrt{2}}{2} = -1 \pm \tfrac{1}{2}\sqrt{2}.$$

(e) $x^4-3x^2+2 = 0 \quad \Leftrightarrow \quad (x^2-1)(x^2-2) = 0 \quad \Leftrightarrow \quad x^2-1 = 0$ or $x^2-2 = 0 \quad \Leftrightarrow \quad x^2 = 1$ or $x^2 = 2 \quad \Leftrightarrow$

$x = \pm 1$ or $x = \pm\sqrt{2}$

(f) $3\,|x-4| = 10 \quad \Leftrightarrow \quad |x-4| = \frac{10}{3} \quad \Leftrightarrow \quad x-4 = -\frac{10}{3}$ or $x-4 = \frac{10}{3} \quad \Leftrightarrow \quad x = \frac{2}{3}$ or $x = \frac{22}{3}$

(g) Multiplying through $2x(4-x)^{-1/2}-3\sqrt{4-x} = 0$ by $(4-x)^{1/2}$ gives $2x-3(4-x) = 0 \quad \Leftrightarrow$

$2x-12+3x = 0 \quad \Leftrightarrow \quad 5x-12 = 0 \quad \Leftrightarrow \quad 5x = 12 \quad \Leftrightarrow \quad x = \frac{12}{5}$.

9. (a) $-4 < 5-3x \le 17 \quad \Leftrightarrow \quad -9 < -3x \le 12 \quad \Leftrightarrow \quad 3 > x \ge -4$ or $-4 \le x < 3$.

In interval notation, the answer is $[-4, 3)$.

(b) $x^2 < 2x+8 \quad \Leftrightarrow \quad x^2-2x-8 < 0 \quad \Leftrightarrow \quad (x+2)(x-4) < 0$. Now, $(x+2)(x-4)$ will change sign at the critical values $x = -2$ and $x = 4$. Thus the possible intervals of solution are $(-\infty, -2)$, $(-2, 4)$, and $(4, \infty)$. By choosing a single test value from each interval, we see that $(-2, 4)$ is the only interval that satisfies the inequality.

(c) The inequality $x(x-1)(x+2)>0$ has critical values of $-2, 0$, and 1. The corresponding possible intervals of solution are $(-\infty,-2)$, $(-2,0)$, $(0,1)$ and $(1,\infty)$. By choosing a single test value from each interval, we see that both intervals $(-2,0)$ and $(1,\infty)$ satisfy the inequality. Thus, the solution is the union of these two intervals: $(-2,0)\cup(1,\infty)$.

(d) $|x-4|<3 \quad\Leftrightarrow\quad -3<x-4<3 \quad\Leftrightarrow\quad 1<x<7$. In interval notation, the answer is $(1,7)$.

(e) $\dfrac{2x-3}{x+1}\le 1 \quad\Leftrightarrow\quad \dfrac{2x-3}{x+1}-1\le 0 \quad\Leftrightarrow\quad \dfrac{2x-3}{x+1}-\dfrac{x+1}{x+1}\le 0 \quad\Leftrightarrow\quad \dfrac{2x-3-x-1}{x+1}\le 0 \quad\Leftrightarrow\quad \dfrac{x-4}{x+1}\le 0$.

Now, the expression $\dfrac{x-4}{x+1}$ may change signs at the critical values $x=-1$ and $x=4$, so the possible intervals of solution are $(-\infty,-1)$, $(-1,4]$, and $[4,\infty)$. By choosing a single test value from each interval, we see that $(-1,4]$ is the only interval that satisfies the inequality.

10. (a) False. In order for the statement to be true, it must hold for all real numbers, so, to show that the statement is false, pick $p=1$ and $q=2$ and observe that $(1+2)^2\ne 1^2+2^2$. In general, $(p+q)^2=p^2+2pq+q^2$.

(b) True as long as a and b are nonnegative real numbers. To see this, think in terms of the laws of exponents: $\sqrt{ab}=(ab)^{1/2}=a^{1/2}b^{1/2}=\sqrt{a}\,\sqrt{b}$.

(c) False. To see this, let $p=1$ and $q=2$, then $\sqrt{1^2+2^2}\ne 1+2$.

(d) False. To see this, let $T=1$ and $C=2$, then $\dfrac{1+1(2)}{2}\ne 1+1$.

(e) False. To see this, let $x=2$ and $y=3$, then $\dfrac{1}{2-3}\ne\dfrac{1}{2}-\dfrac{1}{3}$.

(f) True since $\dfrac{1/x}{a/x-b/x}\cdot\dfrac{x}{x}=\dfrac{1}{a-b}$, as long as $x\ne 0$ and $a-b\ne 0$.

Test B Analytic Geometry

1. (a) Using the point $(2,-5)$ and $m=-3$ in the point-slope equation of a line, $y-y_1=m(x-x_1)$, we get $y-(-5)=-3(x-2) \quad\Rightarrow\quad y+5=-3x+6 \quad\Rightarrow\quad y=-3x+1$.

(b) A line parallel to the x-axis must be horizontal and thus have a slope of 0. Since the line passes through the point $(2,-5)$, the y-coordinate of every point on the line is -5, so the equation is $y=-5$.

(c) A line parallel to the y-axis is vertical with undefined slope. So the x-coordinate of every point on the line is 2 and so the equation is $x=2$.

(d) Note that $2x-4y=3 \quad\Rightarrow\quad -4y=-2x+3 \quad\Rightarrow\quad y=\frac{1}{2}x-\frac{3}{4}$. Thus the slope of the given line is $m=\frac{1}{2}$. Hence, the slope of the line we're looking for is also $\frac{1}{2}$ (since the line we're looking for is required to be parallel to the given line). So the equation of the line is $y-(-5)=\frac{1}{2}(x-2) \quad\Rightarrow\quad y+5=\frac{1}{2}x-1 \quad\Rightarrow\quad y=\frac{1}{2}x-6$.

2. First we'll find the distance between the two given points in order to obtain the radius, r, of the circle: $r=\sqrt{[3-(-1)]^2+(-2-4)^2}=\sqrt{4^2+(-6)^2}=\sqrt{52}$. Next use the standard equation of a circle, $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center, to get $(x+1)^2+(y-4)^2=52$.

3. We must rewrite the equation in standard form in order to identify the center and radius. Note that $x^2 + y^2 - 6x + 10y + 9 = 0 \quad \Rightarrow \quad x^2 - 6x + 9 + y^2 + 10y = 0$. For the left-hand side of the latter equation, we factor the first three terms and complete the square on the last two terms as follows: $x^2 - 6x + 9 + y^2 + 10y = 0 \quad \Rightarrow$ $(x-3)^2 + y^2 + 10y + 25 = 25 \quad \Rightarrow \quad (x-3)^2 + (y+5)^2 = 25$. Thus, the center of the circle is $(3, -5)$ and the radius is 5.

4. (a) $A(-7, 4)$ and $B(5, -12) \quad \Rightarrow \quad m_{AB} = \dfrac{-12 - 4}{5 - (-7)} = \dfrac{-16}{12} = -\dfrac{4}{3}$

(b) $y - 4 = -\frac{4}{3}[x - (-7)] \quad \Rightarrow \quad y - 4 = -\frac{4}{3}x - \frac{28}{3} \quad \Rightarrow \quad 3y - 12 = -4x - 28 \quad \Rightarrow \quad 4x + 3y + 16 = 0$. Putting $y = 0$, we get $4x + 16 = 0$, so the x-intercept is -4, and substituting 0 for x results in a y-intercept of $-\frac{16}{3}$.

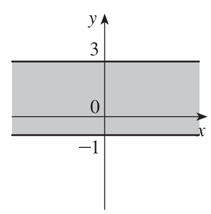
(c) The midpoint is obtained by averaging the corresponding coordinates of both points: $\left(\frac{-7+5}{2}, \frac{4+(-12)}{2}\right) = (-1, -4)$.

(d) $d = \sqrt{[5-(-7)]^2 + (-12-4)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

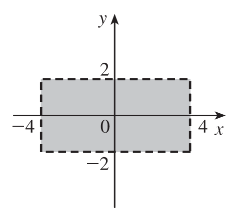
(e) The perpendicular bisector is the line that intersects the line segment $\overline{AB}$ at a right angle through its midpoint. Thus the perpendicular bisector passes through $(-1, -4)$ and has slope $\frac{3}{4}$ [the slope is obtained by taking the negative reciprocal of the answer from part (a)]. So the perpendicular bisector is given by $y + 4 = \frac{3}{4}[x - (-1)]$ or $3x - 4y = 13$.

(f) The center of the required circle is the midpoint of $\overline{AB}$, and the radius is half the length of $\overline{AB}$, which is 10. Thus, the equation is $(x+1)^2 + (y+4)^2 = 100$.

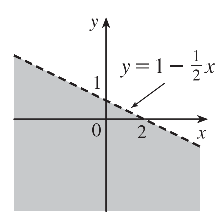
5. (a) Graph the corresponding horizontal lines (given by the equations $y = -1$ and $y = 3$) as solid lines. The inequality $y \geq -1$ describes the points (x, y) that lie on or *above* the line $y = -1$. The inequality $y \leq 3$ describes the points (x, y) that lie on or *below* the line $y = 3$. So the pair of inequalities $-1 \leq y \leq 3$ describes the points that lie on or *between* the lines $y = -1$ and $y = 3$.



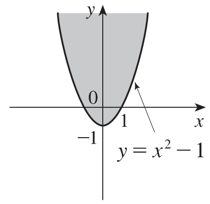
(b) Note that the given inequalities can be written as $-4 < x < 4$ and $-2 < y < 2$, respectively. So the region lies between the vertical lines $x = -4$ and $x = 4$ and between the horizontal lines $y = -2$ and $y = 2$. As shown in the graph, the region common to both graphs is a rectangle (minus its edges) centered at the origin.



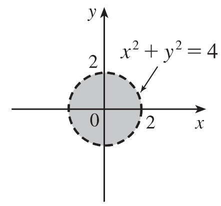
(c) We first graph $y = 1 - \frac{1}{2}x$ as a dotted line. Since $y < 1 - \frac{1}{2}x$, the points in the region lie *below* this line.



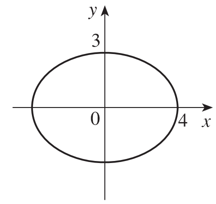
(d) We first graph the parabola $y = x^2 - 1$ using a solid curve. Since $y \geq x^2 - 1$, the points in the region lie on or *above* the parabola.



(e) We graph the circle $x^2 + y^2 = 4$ using a dotted curve. Since $\sqrt{x^2 + y^2} < 2$, the region consists of points whose distance from the origin is less than 2, that is, the points that lie *inside* the circle.



(f) The equation $9x^2 + 16y^2 = 144$ is an ellipse centered at $(0, 0)$. We put it in standard form by dividing by 144 and get $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$. The x-intercepts are located at a distance of $\sqrt{16} = 4$ from the center while the y-intercepts are a distance of $\sqrt{9} = 3$ from the center (see the graph).



Test C Functions

1. (a) Locate -1 on the x-axis and then go down to the point on the graph with an x-coordinate of -1. The corresponding y-coordinate is the value of the function at $x = -1$, which is -2. So, $f(-1) = -2$.

   (b) Using the same technique as in part (a), we get $f(2) \approx 2.8$.

   (c) Locate 2 on the y-axis and then go left and right to find all points on the graph with a y-coordinate of 2. The corresponding x-coordinates are the x-values we are searching for. So $x = -3$ and $x = 1$.

   (d) Using the same technique as in part (c), we get $x \approx -2.5$ and $x \approx 0.3$.

   (e) The domain is all the x-values for which the graph exists, and the range is all the y-values for which the graph exists. Thus, the domain is $[-3, 3]$, and the range is $[-2, 3]$.

2. Note that $f(2 + h) = (2 + h)^3$ and $f(2) = 2^3 = 8$. So the difference quotient becomes

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h} = \frac{h(12 + 6h + h^2)}{h} = 12 + 6h + h^2.$$

3. (a) Set the denominator equal to 0 and solve to find restrictions on the domain: $x^2 + x - 2 = 0 \quad \Rightarrow \quad (x - 1)(x + 2) = 0 \quad \Rightarrow \quad x = 1$ or $x = -2$. Thus, the domain is all real numbers except 1 or -2 or, in interval notation, $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

   (b) Note that the denominator is always greater than or equal to 1, and the numerator is defined for all real numbers. Thus, the domain is $(-\infty, \infty)$.

   (c) Note that the function h is the sum of two root functions. So h is defined on the intersection of the domains of these two root functions. The domain of a square root function is found by setting its radicand greater than or equal to 0. Now,

$4 - x \geq 0 \quad \Rightarrow \quad x \leq 4$ and $x^2 - 1 \geq 0 \quad \Rightarrow \quad (x-1)(x+1) \geq 0 \quad \Rightarrow \quad x \leq -1$ or $x \geq 1$. Thus, the domain of h is $(-\infty, -1] \cup [1, 4]$.

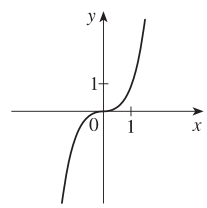
4. (a) Reflect the graph of f about the x-axis.

   (b) Stretch the graph of f vertically by a factor of 2, then shift 1 unit downward.

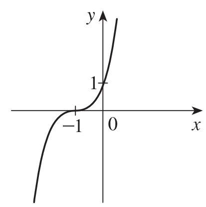
   (c) Shift the graph of f right 3 units, then up 2 units.

5. (a) Make a table and then connect the points with a smooth curve:

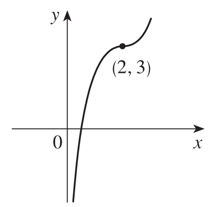
x	-2	-1	0	1	2
y	-8	-1	0	1	8



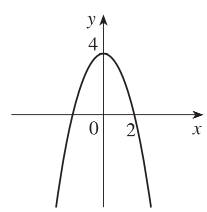
   (b) Shift the graph from part (a) left 1 unit.



   (c) Shift the graph from part (a) right 2 units and up 3 units.

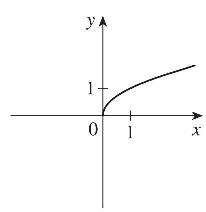


   (d) First plot $y = x^2$. Next, to get the graph of $f(x) = 4 - x^2$, reflect f about the x-axis and then shift it upward 4 units.

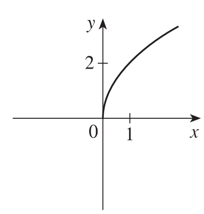


   (e) Make a table and then connect the points with a smooth curve:

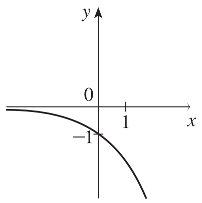
x	0	1	4	9
y	0	1	2	3



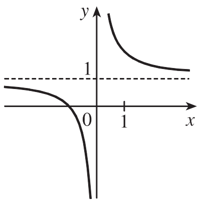
   (f) Stretch the graph from part (e) vertically by a factor of two.



(g) First plot $y = 2^x$. Next, get the graph of $y = -2^x$ by reflecting the graph of $y = 2^x$ about the x-axis.

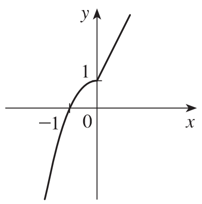


(h) Note that $y = 1 + x^{-1} = 1 + 1/x$. So first plot $y = 1/x$ and then shift it upward 1 unit.



6. (a) $f(-2) = 1 - (-2)^2 = -3$ and $f(1) = 2(1) + 1 = 3$

(b) For $x \leq 0$ plot $f(x) = 1 - x^2$ and, on the same plane, for $x > 0$ plot the graph of $f(x) = 2x + 1$.



7. (a) $(f \circ g)(x) = f(g(x)) = f(2x-3) = (2x-3)^2 + 2(2x-3) - 1 = 4x^2 - 12x + 9 + 4x - 6 - 1 = 4x^2 - 8x + 2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 = 2x^2 + 4x - 2 - 3 = 2x^2 + 4x - 5$

(c) $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(2x-3)) = g(2(2x-3)-3) = g(4x-9) = 2(4x-9) - 3$
$= 8x - 18 - 3 = 8x - 21$

Test D Trigonometry

1. (a) $300^\circ = 300^\circ \left(\dfrac{\pi}{180^\circ}\right) = \dfrac{300\pi}{180} = \dfrac{5\pi}{3}$

(b) $-18^\circ = -18^\circ \left(\dfrac{\pi}{180^\circ}\right) = -\dfrac{18\pi}{180} = -\dfrac{\pi}{10}$

2. (a) $\dfrac{5\pi}{6} = \dfrac{5\pi}{6}\left(\dfrac{180^\circ}{\pi}\right) = 150^\circ$

(b) $2 = 2\left(\dfrac{180^\circ}{\pi}\right) = \dfrac{360^\circ}{\pi} \approx 114.6^\circ$

3. We will use the arc length formula, $s = r\theta$, where s is arc length, r is the radius of the circle, and θ is the measure of the central angle in radians. First, note that $30^\circ = 30^\circ \left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{6}$. So $s = (12)\left(\dfrac{\pi}{6}\right) = 2\pi$ cm.

4. (a) $\tan(\pi/3) = \sqrt{3}$ [You can read the value from a right triangle with sides 1, 2, and $\sqrt{3}$.]

(b) Note that $7\pi/6$ can be thought of as an angle in the third quadrant with reference angle $\pi/6$. Thus, $\sin(7\pi/6) = -\frac{1}{2}$, since the sine function is negative in the third quadrant.

(c) Note that $5\pi/3$ can be thought of as an angle in the fourth quadrant with reference angle $\pi/3$. Thus, $\sec(5\pi/3) = \dfrac{1}{\cos(5\pi/3)} = \dfrac{1}{1/2} = 2$, since the cosine function is positive in the fourth quadrant.

5. $\sin\theta = a/24 \quad\Rightarrow\quad a = 24\sin\theta$ and $\cos\theta = b/24 \quad\Rightarrow\quad b = 24\cos\theta$

6. $\sin x = \frac{1}{3}$ and $\sin^2 x + \cos^2 x = 1 \quad\Rightarrow\quad \cos x = \sqrt{1 - \frac{1}{9}} = \dfrac{2\sqrt{2}}{3}$. Also, $\cos y = \frac{4}{5} \quad\Rightarrow\quad \sin y = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$.

So, using the sum identity for the sine, we have

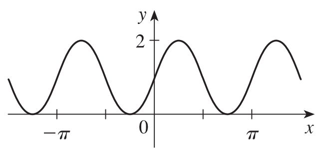
$$\sin(x+y) = \sin x\,\cos y + \cos x\,\sin y = \frac{1}{3}\cdot\frac{4}{5} + \frac{2\sqrt{2}}{3}\cdot\frac{3}{5} = \frac{4+6\sqrt{2}}{15} = \frac{1}{15}\left(4+6\sqrt{2}\right)$$

7. (a) $\tan\theta\,\sin\theta + \cos\theta = \dfrac{\sin\theta}{\cos\theta}\sin\theta + \cos\theta = \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

(b) $\dfrac{2\tan x}{1+\tan^2 x} = \dfrac{2\sin x/(\cos x)}{\sec^2 x} = 2\,\dfrac{\sin x}{\cos x}\,\cos^2 x = 2\sin x\,\cos x = \sin 2x$

8. $\sin 2x = \sin x \quad\Leftrightarrow\quad 2\sin x\,\cos x = \sin x \quad\Leftrightarrow\quad 2\sin x\,\cos x - \sin x = 0 \quad\Leftrightarrow\quad \sin x\,(2\cos x - 1) = 0 \quad\Leftrightarrow$
$\sin x = 0$ or $\cos x = \frac{1}{2} \quad\Rightarrow\quad x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$.

9. We first graph $y = \sin 2x$ (by compressing the graph of $\sin x$ by a factor of 2) and then shift it upward 1 unit.



1 □ FUNCTIONS AND MODELS

1.1 Four Ways to Represent a Function

1. The functions $f(x) = x + \sqrt{2-x}$ and $g(u) = u + \sqrt{2-u}$ give exactly the same output values for every input value, so f and g are equal.

2. $f(x) = \dfrac{x^2 - x}{x-1} = \dfrac{x(x-1)}{x-1} = x$ for $x - 1 \neq 0$, so f and g [where $g(x) = x$] are not equal because $f(1)$ is undefined and $g(1) = 1$.

3. (a) The point $(1, 3)$ is on the graph of f, so $f(1) = 3$.

   (b) When $x = -1$, y is about -0.2, so $f(-1) \approx -0.2$.

   (c) $f(x) = 1$ is equivalent to $y = 1$. When $y = 1$, we have $x = 0$ and $x = 3$.

   (d) A reasonable estimate for x when $y = 0$ is $x = -0.8$.

   (e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-2 \leq x \leq 4$, or $[-2, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-1 \leq y \leq 3$, or $[-1, 3]$.

   (f) As x increases from -2 to 1, y increases from -1 to 3. Thus, f is increasing on the interval $[-2, 1]$.

4. (a) The point $(-4, -2)$ is on the graph of f, so $f(-4) = -2$. The point $(3, 4)$ is on the graph of g, so $g(3) = 4$.

   (b) We are looking for the values of x for which the y-values are equal. The y-values for f and g are equal at the points $(-2, 1)$ and $(2, 2)$, so the desired values of x are -2 and 2.

   (c) $f(x) = -1$ is equivalent to $y = -1$. When $y = -1$, we have $x = -3$ and $x = 4$.

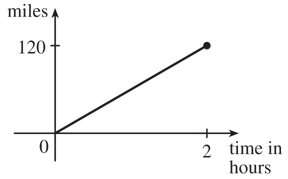
   (d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval $[0, 4]$.

   (e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-4 \leq x \leq 4$, or $[-4, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-2 \leq y \leq 3$, or $[-2, 3]$.

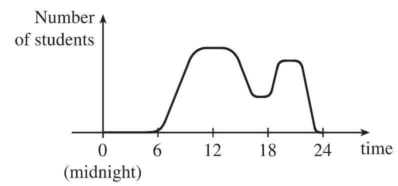
   (f) The domain of g is $[-4, 3]$ and the range is $[0.5, 4]$.

5. From Figure 1 in the text, the lowest point occurs at about $(t, a) = (12, -85)$. The highest point occurs at about $(17, 115)$. Thus, the range of the vertical ground acceleration is $-85 \leq a \leq 115$. Written in interval notation, we get $[-85, 115]$.

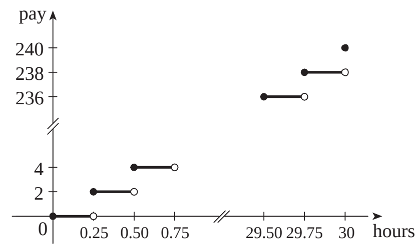
6. *Example 1:* A car is driven at 60 mi/h for 2 hours. The distance d traveled by the car is a function of the time t. The domain of the function is $\{t \mid 0 \leq t \leq 2\}$, where t is measured in hours. The range of the function is $\{d \mid 0 \leq d \leq 120\}$, where d is measured in miles.



Example 2: At a certain university, the number of students N on campus at any time on a particular day is a function of the time t after midnight. The domain of the function is $\{t \mid 0 \le t \le 24\}$, where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at once.



Example 3: A certain employee is paid $8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is $[0, 30]$ and the range of the function is $\{0, 2.00, 4.00, \ldots, 238.00, 240.00\}$.



7. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.

8. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-2, 2]$ and the range is $[-1, 2]$.

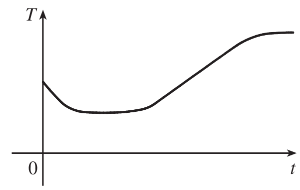
9. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$.

10. No, the curve is not the graph of a function since for $x = 0, \pm1$, and ±2, there are infinitely many points on the curve.

11. The person's weight increased to about 160 pounds at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 120 pounds for the next 5 years, then increased rapidly to about 170 pounds. The next 30 years saw a gradual increase to 190 pounds. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.

12. First, the tub was filled with water to a height of 15 in. Then a person got into the tub, raising the water level to 20 in. At around 12 minutes, the person stood up in the tub but then immediately sat down. Finally, at around 17 minutes, the person got out of the tub, and then drained the water.

13. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



14. Runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. B initially led the race, followed by C, and then A. C then passed B to lead for a while. Then A passed first B, and then passed C to take the lead and finish first. Finally, B passed C to finish in second place. All three runners completed the race.

Complete Solutions Manual
for
MULTIVARIABLE CALCULUS
SEVENTH EDITION

DAN CLEGG
Palomar College

BARBARA FRANK
Cape Fear Community College

BROOKS/COLE
CENGAGE Learning™

Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

BROOKS/COLE
CENGAGE Learning™

For product information and technology assistance, contact us at
Cengage Learning Customer & Sales Support,
1-800-354-9706

For permission to use material from this text or product,
submit all requests online at **www.cengage.com/permissions**
Further permissions questions can be e-mailed to
permissionrequest@cengage.com

ISBN-13: 978-0-8400-4947-6
ISBN-10: 0-8400-4947-1

Brooks/Cole
20 Davis Drive
Belmont, CA 94002-3098
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: **www.cengage.com/global**

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Brooks/Cole, visit **www.cengage.com/brookscole**

Purchase any of our products at our preferred online store **www.cengagebrain.com**

Printed in the United States of America
1 2 3 4 5 6 7 15 14 13 12 11

PREFACE

This *Complete Solutions Manual* contains detailed solutions to all exercises in the text *Multivariable Calculus,* Seventh Edition (Chapters 10–17 of *Calculus,* Seventh Edition, and *Calculus: Early Transcendentals,* Seventh Edition) by James Stewart. A *Student Solutions Manual* is also available, which contains solutions to the odd-numbered exercises in each chapter section, review section, True-False Quiz, and Problems Plus section as well as all solutions to the Concept Check questions. (It does not, however, include solutions to any of the projects.)

Because of differences between the regular version and the *Early Transcendentals* version of the text, some references are given in a dual format. In these cases, users of the *Early Transcendentals* text should use the references denoted by "ET."

While we have extended every effort to ensure the accuracy of the solutions presented, we would appreciate correspondence regarding any errors that may exist. Other suggestions or comments are also welcome, and can be sent to dan clegg at `dclegg@palomar.edu` or in care of the publisher: Brooks/Cole, Cengage Learning, 20 Davis Drive, Belmont CA 94002-3098.

We would like to thank James Stewart for entrusting us with the writing of this manual and offering suggestions and Kathi Townes of TECH-arts for typesetting and producing this manual as well as creating the illustrations. We also thank Richard Stratton, Liz Covello, and Elizabeth Neustaetter of Brooks/Cole, Cengage Learning, for their trust, assistance, and patience.

DAN CLEGG
Palomar College

BARBARA FRANK
Cape Fear Community College

ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of f
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
I/D	Increasing/Decreasing Test
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\stackrel{\text{CAS}}{=}$	indicates the use of a computer algebra system.
$\stackrel{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\stackrel{j}{=}$	indicates the use of Formula j in the Table of Integrals in the back endpapers.
$\stackrel{\text{s}}{=}$	indicates the use of the substitution $\{u = \sin x, du = \cos x\, dx\}$.
$\stackrel{\text{c}}{=}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x\, dx\}$.

CONTENTS

12 VECTORS AND THE GEOMETRY OF SPACE 235

13 VECTOR FUNCTIONS 313

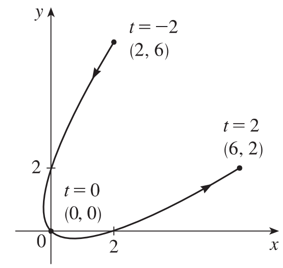
14 PARTIAL DERIVATIVES 373

10 □ PARAMETRIC EQUATIONS AND POLAR COORDINATES

10.1 Curves Defined by Parametric Equations

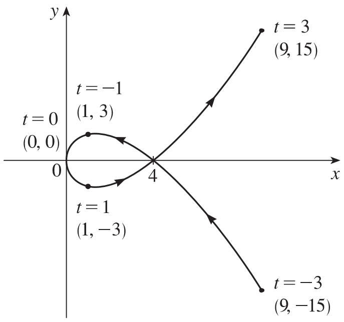
1. $x = t^2 + t, \quad y = t^2 - t, \quad -2 \le t \le 2$

t	-2	-1	0	1	2
x	2	0	0	2	6
y	6	2	0	0	2



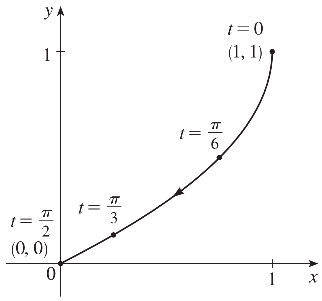
2. $x = t^2, \quad y = t^3 - 4t, \quad -3 \le t \le 3$

t	± 3	± 2	± 1	0
x	9	4	1	0
y	± 15	0	∓ 3	0



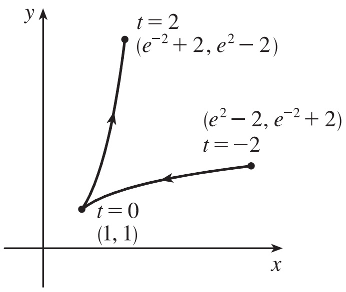
3. $x = \cos^2 t, \quad y = 1 - \sin t, \quad 0 \le t \le \pi/2$

t	0	$\pi/6$	$\pi/3$	$\pi/2$
x	1	$3/4$	$1/4$	0
y	1	$1/2$	$1 - \frac{\sqrt{3}}{2} \approx 0.13$	0

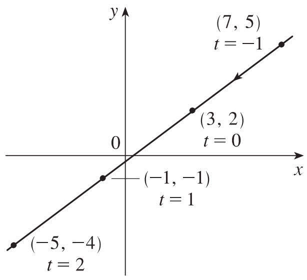


4. $x = e^{-t} + t, \quad y = e^t - t, \quad -2 \le t \le 2$

t	-2	-1	0	1	2
x	$e^2 - 2$ 5.39	$e - 1$ 1.72	1	$e^{-1} + 1$ 1.37	$e^{-2} + 2$ 2.14
y	$e^{-2} + 2$ 2.14	$e^{-1} + 1$ 1.37	1	$e - 1$ 1.72	$e^2 - 2$ 5.39



5. $x = 3 - 4t,\ y = 2 - 3t$



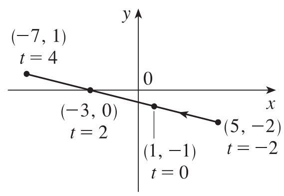
(a)

t	-1	0	1	2
x	7	3	-1	-5
y	5	2	-1	-4

(b) $x = 3 - 4t \quad \Rightarrow \quad 4t = -x + 3 \quad \Rightarrow \quad t = -\frac{1}{4}x + \frac{3}{4}$, so

$y = 2 - 3t = 2 - 3\left(-\frac{1}{4}x + \frac{3}{4}\right) = 2 + \frac{3}{4}x - \frac{9}{4} \quad \Rightarrow \quad y = \frac{3}{4}x - \frac{1}{4}$

6. $x = 1 - 2t,\ y = \frac{1}{2}t - 1,\ -2 \le t \le 4$



(a)

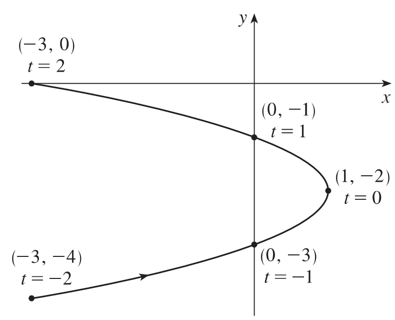
t	-2	0	2	4
x	5	1	-3	-7
y	-2	-1	0	1

(b) $x = 1 - 2t \quad \Rightarrow \quad 2t = -x + 1 \quad \Rightarrow \quad t = -\frac{1}{2}x + \frac{1}{2}$, so

$y = \frac{1}{2}t - 1 = \frac{1}{2}\left(-\frac{1}{2}x + \frac{1}{2}\right) - 1 = -\frac{1}{4}x + \frac{1}{4} - 1 \quad \Rightarrow \quad y = -\frac{1}{4}x - \frac{3}{4}$,

with $-7 \le x \le 5$

7. $x = 1 - t^2,\ y = t - 2,\ -2 \le t \le 2$



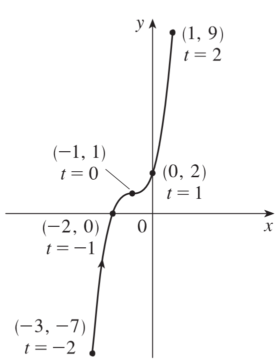
(a)

t	-2	-1	0	1	2
x	-3	0	1	0	-3
y	-4	-3	-2	-1	0

(b) $y = t - 2 \quad \Rightarrow \quad t = y + 2$, so $x = 1 - t^2 = 1 - (y + 2)^2 \quad \Rightarrow$

$x = -(y + 2)^2 + 1$, or $x = -y^2 - 4y - 3$, with $-4 \le y \le 0$

8. $x = t - 1,\ y = t^3 + 1,\ -2 \le t \le 2$



(a)

t	-2	-1	0	1	2
x	-3	-2	-1	0	1
y	-7	0	1	2	9

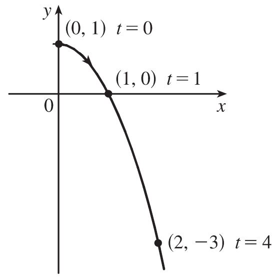
(b) $x = t - 1 \quad \Rightarrow \quad t = x + 1$, so $y = t^3 + 1 \quad \Rightarrow \quad y = (x + 1)^3 + 1$,

or $y = x^3 + 3x^2 + 3x + 2$, with $-3 \le x \le 1$

9. $x = \sqrt{t},\ y = 1 - t$

(a)

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	1	0	-1	-2	-3



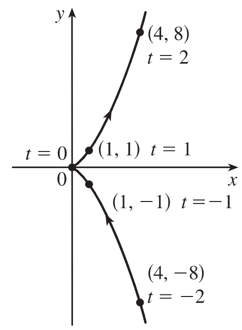
(b) $x = \sqrt{t} \quad \Rightarrow \quad t = x^2 \quad \Rightarrow \quad y = 1 - t = 1 - x^2$. Since $t \geq 0$, $x \geq 0$.

So the curve is the right half of the parabola $y = 1 - x^2$.

10. $x = t^2, y = t^3$

(a)

t	-2	-1	0	1	2
x	4	1	0	1	4
y	-8	-1	0	1	8

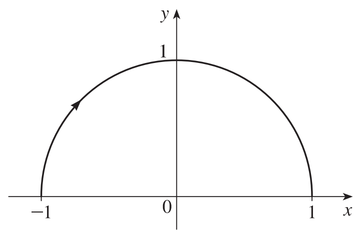


(b) $y = t^3 \quad \Rightarrow \quad t = \sqrt[3]{y} \quad \Rightarrow \quad x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{2/3}. \quad t \in \mathbb{R}, y \in \mathbb{R}, x \geq 0.$

11. (a) $x = \sin \frac{1}{2}\theta, y = \cos \frac{1}{2}\theta, -\pi \leq \theta \leq \pi$.

$x^2 + y^2 = \sin^2 \frac{1}{2}\theta + \cos^2 \frac{1}{2}\theta = 1$. For $-\pi \leq \theta \leq 0$, we have $-1 \leq x \leq 0$ and $0 \leq y \leq 1$. For $0 < \theta \leq \pi$, we have $0 < x \leq 1$ and $1 > y \geq 0$. The graph is a semicircle.

(b)

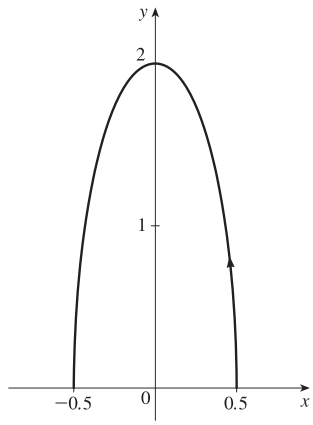


12. (a) $x = \frac{1}{2}\cos\theta, y = 2\sin\theta, 0 \leq \theta \leq \pi$.

$(2x)^2 + \left(\frac{1}{2}y\right)^2 = \cos^2\theta + \sin^2\theta = 1 \quad \Rightarrow \quad 4x^2 + \frac{1}{4}y^2 = 1 \quad \Rightarrow$

$\dfrac{x^2}{(1/2)^2} + \dfrac{y^2}{2^2} = 1$, which is an equation of an ellipse with x-intercepts $\pm\frac{1}{2}$ and y-intercepts ± 2. For $0 \leq \theta \leq \pi/2$, we have $\frac{1}{2} \geq x \geq 0$ and $0 \leq y \leq 2$. For $\pi/2 < \theta \leq \pi$, we have $0 > x \geq -\frac{1}{2}$ and $2 > y \geq 0$. So the graph is the top half of the ellipse.

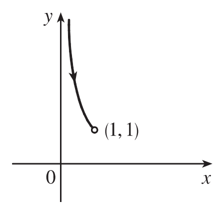
(b)



13. (a) $x = \sin t, y = \csc t, 0 < t < \frac{\pi}{2}$. $y = \csc t = \dfrac{1}{\sin t} = \dfrac{1}{x}$.

For $0 < t < \frac{\pi}{2}$, we have $0 < x < 1$ and $y > 1$. Thus, the curve is the portion of the hyperbola $y = 1/x$ with $y > 1$.

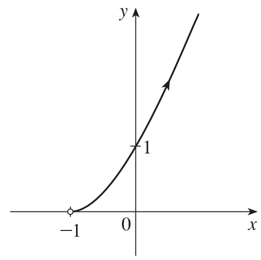
(b)



14. (a) $x = e^t - 1$, $y = e^{2t}$.

$y = (e^t)^2 = (x+1)^2$ and since $x > -1$, we have the right side of the parabola $y = (x+1)^2$.

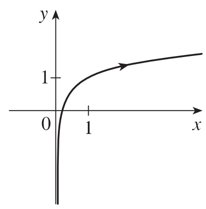
(b)



15. (a) $x = e^{2t} \quad \Rightarrow \quad 2t = \ln x \quad \Rightarrow \quad t = \frac{1}{2}\ln x$.

$y = t + 1 = \frac{1}{2}\ln x + 1$.

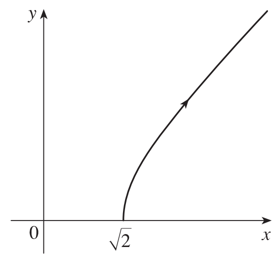
(b)



16. (a) $x = \sqrt{t+1} \quad \Rightarrow \quad x^2 = t + 1 \quad \Rightarrow \quad t = x^2 - 1$.

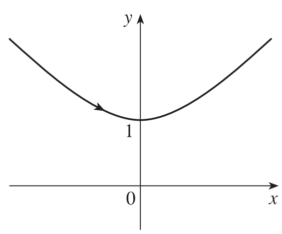
$y = \sqrt{t-1} = \sqrt{(x^2-1)-1} = \sqrt{x^2-2}$. The curve is the part of the hyperbola $x^2 - y^2 = 2$ with $x \geq \sqrt{2}$ and $y \geq 0$.

(b)



17. (a) $x = \sinh t$, $y = \cosh t \quad \Rightarrow \quad y^2 - x^2 = \cosh^2 t - \sinh^2 t = 1$. Since $y = \cosh t \geq 1$, we have the upper branch of the hyperbola $y^2 - x^2 = 1$.

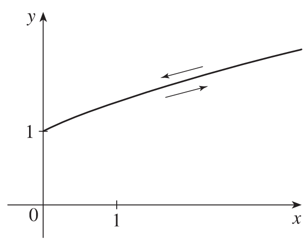
(b)



18. (a) $x = \tan^2\theta$, $y = \sec\theta$, $-\pi/2 < \theta < \pi/2$.

$1 + \tan^2\theta = \sec^2\theta \quad \Rightarrow \quad 1 + x = y^2 \quad \Rightarrow \quad x = y^2 - 1$. For $-\pi/2 < \theta \leq 0$, we have $x \geq 0$ and $y \geq 1$. For $0 < \theta < \pi/2$, we have $0 < x$ and $1 < y$. Thus, the curve is the portion of the parabola $x = y^2 - 1$ in the first quadrant. As θ increases from $-\pi/2$ to 0, the point (x, y) approaches $(0, 1)$ along the parabola. As θ increases from 0 to $\pi/2$, the point (x, y) retreats from $(0, 1)$ along the parabola.

(b)



19. $x = 3 + 2\cos t$, $y = 1 + 2\sin t$, $\pi/2 \leq t \leq 3\pi/2$. By Example 4 with $r = 2$, $h = 3$, and $k = 1$, the motion of the particle takes place on a circle centered at $(3, 1)$ with a radius of 2. As t goes from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$, the particle starts at the point $(3, 3)$ and moves counterclockwise along the circle $(x-3)^2 + (y-1)^2 = 4$ to $(3, -1)$ [one-half of a circle].

20. $x = 2\sin t$, $y = 4 + \cos t \quad \Rightarrow \quad \sin t = \dfrac{x}{2}$, $\cos t = y - 4$. $\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \left(\dfrac{x}{2}\right)^2 + (y-4)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(0, 4)$. As t goes from 0 to $\frac{3\pi}{2}$, the particle starts at the point $(0, 5)$ and moves clockwise to $(-2, 4)$ [three-quarters of an ellipse].

21. $x = 5\sin t, y = 2\cos t \quad \Rightarrow \quad \sin t = \dfrac{x}{5}, \cos t = \dfrac{y}{2}. \quad \sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \left(\dfrac{x}{5}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(0, 0)$. As t goes from $-\pi$ to 5π, the particle starts at the point $(0, -2)$ and moves clockwise around the ellipse 3 times.

22. $y = \cos^2 t = 1 - \sin^2 t = 1 - x^2$. The motion of the particle takes place on the parabola $y = 1 - x^2$. As t goes from -2π to $-\pi$, the particle starts at the point $(0, 1)$, moves to $(1, 0)$, and goes back to $(0, 1)$. As t goes from $-\pi$ to 0, the particle moves to $(-1, 0)$ and goes back to $(0, 1)$. The particle repeats this motion as t goes from 0 to 2π.

23. We must have $1 \le x \le 4$ and $2 \le y \le 3$. So the graph of the curve must be contained in the rectangle $[1, 4]$ by $[2, 3]$.

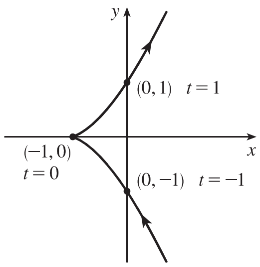
24. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

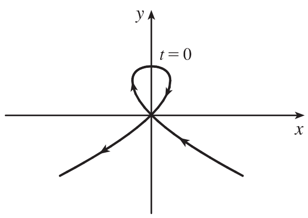
(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

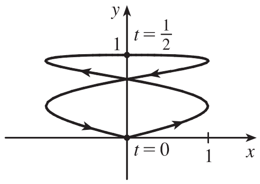
25. When $t = -1$, $(x, y) = (0, -1)$. As t increases to 0, x decreases to -1 and y increases to 0. As t increases from 0 to 1, x increases to 0 and y increases to 1. As t increases beyond 1, both x and y increase. For $t < -1$, x is positive and decreasing and y is negative and increasing. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



26. For $t < -1$, x is positive and decreasing, while y is negative and increasing (these points are in Quadrant IV). When $t = -1$, $(x, y) = (0, 0)$ and, as t increases from -1 to 0, x becomes negative and y increases from 0 to 1. At $t = 0$, $(x, y) = (0, 1)$ and, as t increases from 0 to 1, y decreases from 1 to 0 and x is positive. At $t = 1, (x, y) = (0, 0)$ again, so the loop is completed. For $t > 1$, x and y both become large negative. This enables us to draw a rough sketch. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



27. When $t = 0$ we see that $x = 0$ and $y = 0$, so the curve starts at the origin. As t increases from 0 to $\frac{1}{2}$, the graphs show that y increases from 0 to 1 while x increases from 0 to 1, decreases to 0 and to -1, then increases back to 0, so we arrive at the point $(0, 1)$. Similarly, as t increases from $\frac{1}{2}$ to 1, y decreases from 1 to 0 while x repeats its pattern, and we arrive back at the origin. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



28. (a) $x = t^4 - t + 1 = (t^4 + 1) - t > 0$ [think of the graphs of $y = t^4 + 1$ and $y = t$] and $y = t^2 \geq 0$, so these equations are matched with graph V.

(b) $y = \sqrt{t} \geq 0$. $x = t^2 - 2t = t(t-2)$ is negative for $0 < t < 2$, so these equations are matched with graph I.

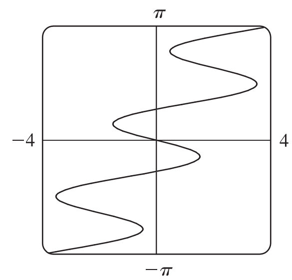
(c) $x = \sin 2t$ has period $2\pi/2 = \pi$. Note that $y(t + 2\pi) = \sin[t + 2\pi + \sin 2(t + 2\pi)] = \sin(t + 2\pi + \sin 2t) = \sin(t + \sin 2t) = y(t)$, so y has period 2π. These equations match graph II since x cycles through the values -1 to 1 twice as y cycles through those values once.

(d) $x = \cos 5t$ has period $2\pi/5$ and $y = \sin 2t$ has period π, so x will take on the values -1 to 1, and then 1 to -1, before y takes on the values -1 to 1. Note that when $t = 0$, $(x, y) = (1, 0)$. These equations are matched with graph VI.

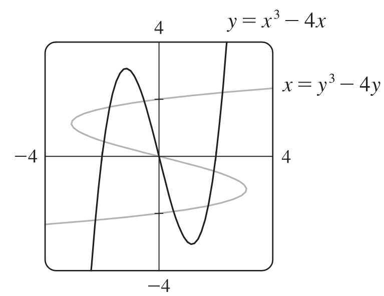
(e) $x = t + \sin 4t$, $y = t^2 + \cos 3t$. As t becomes large, t and t^2 become the dominant terms in the expressions for x and y, so the graph will look like the graph of $y = x^2$, but with oscillations. These equations are matched with graph IV.

(f) $x = \dfrac{\sin 2t}{4 + t^2}$, $y = \dfrac{\cos 2t}{4 + t^2}$. As $t \to \infty$, x and y both approach 0. These equations are matched with graph III.

29. Use $y = t$ and $x = t - 2\sin \pi t$ with a t-interval of $[-\pi, \pi]$.



30. Use $x_1 = t$, $y_1 = t^3 - 4t$ and $x_2 = t^3 - 4t$, $y_2 = t$ with a t-interval of $[-3, 3]$. There are 9 points of intersection; $(0, 0)$ is fairly obvious. The point in quadrant I is approximately $(2.2, 2.2)$, and by symmetry, the point in quadrant III is approximately $(-2.2, -2.2)$. The other six points are approximately $(\mp 1.9, \pm 0.5)$, $(\mp 1.7, \pm 1.7)$, and $(\mp 0.5, \pm 1.9)$.



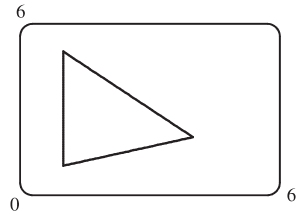
31. (a) $x = x_1 + (x_2 - x_1)t$, $y = y_1 + (y_2 - y_1)t$, $0 \leq t \leq 1$. Clearly the curve passes through $P_1(x_1, y_1)$ when $t = 0$ and through $P_2(x_2, y_2)$ when $t = 1$. For $0 < t < 1$, x is strictly between x_1 and x_2 and y is strictly between y_1 and y_2. For every value of t, x and y satisfy the relation $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, which is the equation of the line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

Finally, any point (x, y) on that line satisfies $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$; if we call that common value t, then the given parametric equations yield the point (x, y); and any (x, y) on the line between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ yields a value of t in $[0, 1]$. So the given parametric equations exactly specify the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$.

(b) $x = -2 + [3 - (-2)]t = -2 + 5t$ and $y = 7 + (-1 - 7)t = 7 - 8t$ for $0 \leq t \leq 1$.

32. For the side of the triangle from A to B, use $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (4, 2)$. Hence, the equations are

$$x = x_1 + (x_2 - x_1)\,t = 1 + (4 - 1)\,t = 1 + 3t,$$
$$y = y_1 + (y_2 - y_1)\,t = 1 + (2 - 1)\,t = 1 + t.$$



Graphing $x = 1 + 3t$ and $y = 1 + t$ with $0 \le t \le 1$ gives us the side of the triangle from A to B. Similarly, for the side BC we use $x = 4 - 3t$ and $y = 2 + 3t$, and for the side AC we use $x = 1$ and $y = 1 + 4t$.

33. The circle $x^2 + (y - 1)^2 = 4$ has center $(0, 1)$ and radius 2, so by Example 4 it can be represented by $x = 2\cos t$, $y = 1 + 2\sin t$, $0 \le t \le 2\pi$. This representation gives us the circle with a counterclockwise orientation starting at $(2, 1)$.

(a) To get a clockwise orientation, we could change the equations to $x = 2\cos t$, $y = 1 - 2\sin t$, $0 \le t \le 2\pi$.

(b) To get three times around in the counterclockwise direction, we use the original equations $x = 2\cos t$, $y = 1 + 2\sin t$ with the domain expanded to $0 \le t \le 6\pi$.

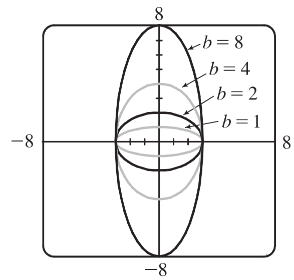
(c) To start at $(0, 3)$ using the original equations, we must have $x_1 = 0$; that is, $2\cos t = 0$. Hence, $t = \frac{\pi}{2}$. So we use $x = 2\cos t$, $y = 1 + 2\sin t$, $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$.

Alternatively, if we want t to start at 0, we could change the equations of the curve. For example, we could use $x = -2\sin t$, $y = 1 + 2\cos t$, $0 \le t \le \pi$.

34. (a) Let $x^2/a^2 = \sin^2 t$ and $y^2/b^2 = \cos^2 t$ to obtain $x = a\sin t$ and $y = b\cos t$ with $0 \le t \le 2\pi$ as possible parametric equations for the ellipse $x^2/a^2 + y^2/b^2 = 1$.

(b) The equations are $x = 3\sin t$ and $y = b\cos t$ for $b \in \{1, 2, 4, 8\}$.

(c) As b increases, the ellipse stretches vertically.



35. *Big circle:* It's centered at $(2, 2)$ with a radius of 2, so by Example 4, parametric equations are

$$x = 2 + 2\cos t, \qquad y = 2 + 2\sin t, \qquad 0 \le t \le 2\pi$$

Small circles: They are centered at $(1, 3)$ and $(3, 3)$ with a radius of 0.1. By Example 4, parametric equations are

(left) $\quad x = 1 + 0.1\cos t, \qquad y = 3 + 0.1\sin t, \qquad 0 \le t \le 2\pi$

and *(right)* $\quad x = 3 + 0.1\cos t, \qquad y = 3 + 0.1\sin t, \qquad 0 \le t \le 2\pi$

Semicircle: It's the lower half of a circle centered at $(2, 2)$ with radius 1. By Example 4, parametric equations are

$$x = 2 + 1\cos t, \qquad y = 2 + 1\sin t, \qquad \pi \le t \le 2\pi$$

To get all four graphs on the same screen with a typical graphing calculator, we need to change the last t-interval to$[0, 2\pi]$ in order to match the others. We can do this by changing t to $0.5t$. This change gives us the upper half. There are several ways to get the lower half—one is to change the "+" to a "−" in the y-assignment, giving us

$$x = 2 + 1\cos(0.5t), \qquad y = 2 - 1\sin(0.5t), \qquad 0 \le t \le 2\pi$$

36. If you are using a calculator or computer that can overlay graphs (using multiple t-intervals), the following is appropriate.

Left side: $x = 1$ and y goes from 1.5 to 4, so use

$$x = 1, \qquad y = t, \qquad 1.5 \leq t \leq 4$$

Right side: $x = 10$ and y goes from 1.5 to 4, so use

$$x = 10, \qquad y = t, \qquad 1.5 \leq t \leq 4$$

Bottom: x goes from 1 to 10 and $y = 1.5$, so use

$$x = t, \qquad y = 1.5, \qquad 1 \leq t \leq 10$$

Handle: It starts at $(10, 4)$ and ends at $(13, 7)$, so use

$$x = 10 + t, \qquad y = 4 + t, \qquad 0 \leq t \leq 3$$

Left wheel: It's centered at $(3, 1)$, has a radius of 1, and appears to go about 30° above the horizontal, so use

$$x = 3 + 1\cos t, \qquad y = 1 + 1\sin t, \qquad \tfrac{5\pi}{6} \leq t \leq \tfrac{13\pi}{6}$$

Right wheel: Similar to the left wheel with center $(8, 1)$, so use

$$x = 8 + 1\cos t, \qquad y = 1 + 1\sin t, \qquad \tfrac{5\pi}{6} \leq t \leq \tfrac{13\pi}{6}$$

If you are using a calculator or computer that cannot overlay graphs (using one t-interval), the following is appropriate. We'll start by picking the t-interval $[0, 2.5]$ since it easily matches the t-values for the two sides. We now need to find parametric equations for all graphs with $0 \leq t \leq 2.5$.

Left side: $x = 1$ and y goes from 1.5 to 4, so use

$$x = 1, \qquad y = 1.5 + t, \qquad 0 \leq t \leq 2.5$$

Right side: $x = 10$ and y goes from 1.5 to 4, so use

$$x = 10, \qquad y = 1.5 + t, \qquad 0 \leq t \leq 2.5$$

Bottom: x goes from 1 to 10 and $y = 1.5$, so use

$$x = 1 + 3.6t, \qquad y = 1.5, \qquad 0 \leq t \leq 2.5$$

To get the *x*-assignment, think of creating a linear function such that when $t = 0$, $x = 1$ and when $t = 2.5$, $x = 10$. We can use the point-slope form of a line with $(t_1, x_1) = (0, 1)$ and $(t_2, x_2) = (2.5, 10)$.

$$x - 1 = \frac{10 - 1}{2.5 - 0}(t - 0) \quad \Rightarrow \quad x = 1 + 3.6t.$$

Handle: It starts at $(10, 4)$ and ends at $(13, 7)$, so use

$$x = 10 + 1.2t, \qquad y = 4 + 1.2t, \qquad 0 \leq t \leq 2.5$$

$(t_1, x_1) = (0, 10)$ and $(t_2, x_2) = (2.5, 13)$ gives us $x - 10 = \dfrac{13 - 10}{2.5 - 0}(t - 0) \quad \Rightarrow \quad x = 10 + 1.2t$.

$(t_1, y_1) = (0, 4)$ and $(t_2, y_2) = (2.5, 7)$ gives us $y - 4 = \dfrac{7 - 4}{2.5 - 0}(t - 0) \quad \Rightarrow \quad y = 4 + 1.2t$.

Left wheel: It's centered at $(3, 1)$, has a radius of 1, and appears to go about 30° above the horizontal, so use

$$x = 3 + 1\cos\left(\tfrac{8\pi}{15}t + \tfrac{5\pi}{6}\right), \qquad y = 1 + 1\sin\left(\tfrac{8\pi}{15}t + \tfrac{5\pi}{6}\right), \qquad 0 \le t \le 2.5$$

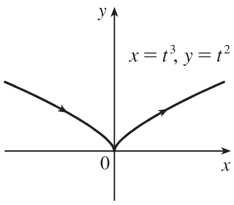
$$(t_1, \theta_1) = \left(0, \tfrac{5\pi}{6}\right) \text{ and } (t_2, \theta_2) = \left(\tfrac{5}{2}, \tfrac{13\pi}{6}\right) \text{ gives us } \theta - \tfrac{5\pi}{6} = \frac{\frac{13\pi}{6} - \frac{5\pi}{6}}{\frac{5}{2} - 0}(t - 0) \quad\Rightarrow\quad \theta = \tfrac{5\pi}{6} + \tfrac{8\pi}{15}t.$$

Right wheel: Similar to the left wheel with center $(8, 1)$, so use

$$x = 8 + 1\cos\left(\tfrac{8\pi}{15}t + \tfrac{5\pi}{6}\right), \qquad y = 1 + 1\sin\left(\tfrac{8\pi}{15}t + \tfrac{5\pi}{6}\right), \qquad 0 \le t \le 2.5$$

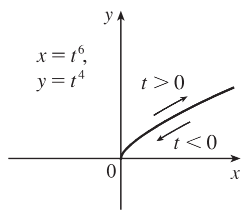
37. (a) $x = t^3 \;\Rightarrow\; t = x^{1/3}$, so $y = t^2 = x^{2/3}$.

We get the entire curve $y = x^{2/3}$ traversed in a left to right direction.



(b) $x = t^6 \;\Rightarrow\; t = x^{1/6}$, so $y = t^4 = x^{4/6} = x^{2/3}$.

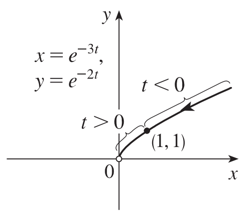
Since $x = t^6 \ge 0$, we only get the right half of the curve $y = x^{2/3}$.



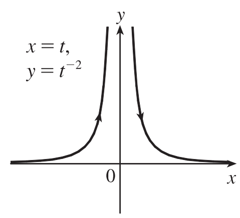
(c) $x = e^{-3t} = (e^{-t})^3 \quad [\text{so } e^{-t} = x^{1/3}]$,

$y = e^{-2t} = (e^{-t})^2 = (x^{1/3})^2 = x^{2/3}$.

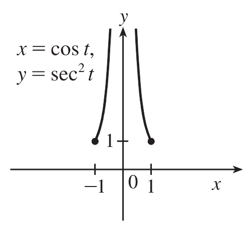
If $t < 0$, then x and y are both larger than 1. If $t > 0$, then x and y are between 0 and 1. Since $x > 0$ and $y > 0$, the curve never quite reaches the origin.



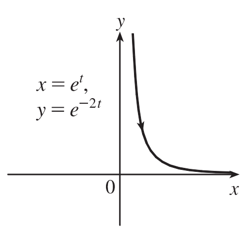
38. (a) $x = t$, so $y = t^{-2} = x^{-2}$. We get the entire curve $y = 1/x^2$ traversed in a left-to-right direction.



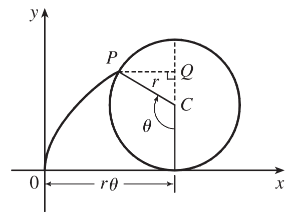
(b) $x = \cos t$, $y = \sec^2 t = \dfrac{1}{\cos^2 t} = \dfrac{1}{x^2}$. Since $\sec t \ge 1$, we only get the parts of the curve $y = 1/x^2$ with $y \ge 1$. We get the first quadrant portion of the curve when $x > 0$, that is, $\cos t > 0$, and we get the second quadrant portion of the curve when $x < 0$, that is, $\cos t < 0$.



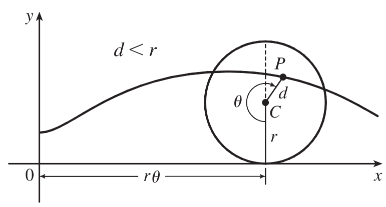
(c) $x = e^t$, $y = e^{-2t} = (e^t)^{-2} = x^{-2}$. Since e^t and e^{-2t} are both positive, we only get the first quadrant portion of the curve $y = 1/x^2$.

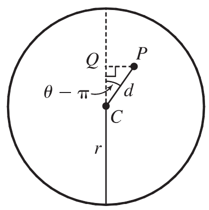


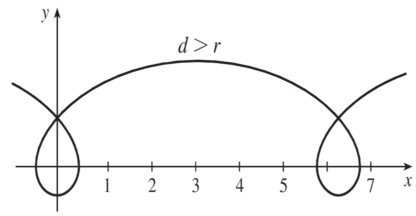
39. The case $\frac{\pi}{2} < \theta < \pi$ is illustrated. C has coordinates $(r\theta, r)$ as in Example 7, and Q has coordinates $(r\theta, r + r\cos(\pi - \theta)) = (r\theta, r(1 - \cos\theta))$ [since $\cos(\pi - \alpha) = \cos\pi\cos\alpha + \sin\pi\sin\alpha = -\cos\alpha$], so P has coordinates $(r\theta - r\sin(\pi - \theta), r(1 - \cos\theta)) = (r(\theta - \sin\theta), r(1 - \cos\theta))$ [since $\sin(\pi - \alpha) = \sin\pi\cos\alpha - \cos\pi\sin\alpha = \sin\alpha$]. Again we have the parametric equations $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$.



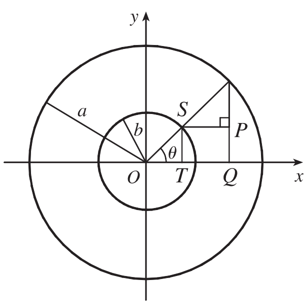
40. The first two diagrams depict the case $\pi < \theta < \frac{3\pi}{2}$, $d < r$. As in Example 7, C has coordinates $(r\theta, r)$. Now Q (in the second diagram) has coordinates $(r\theta, r + d\cos(\theta - \pi)) = (r\theta, r - d\cos\theta)$, so a typical point P of the trochoid has coordinates $(r\theta + d\sin(\theta - \pi), r - d\cos\theta)$. That is, P has coordinates (x, y), where $x = r\theta - d\sin\theta$ and $y = r - d\cos\theta$. When $d = r$, these equations agree with those of the cycloid.





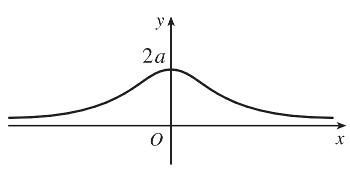


41. It is apparent that $x = |OQ|$ and $y = |QP| = |ST|$. From the diagram, $x = |OQ| = a\cos\theta$ and $y = |ST| = b\sin\theta$. Thus, the parametric equations are $x = a\cos\theta$ and $y = b\sin\theta$. To eliminate θ we rearrange: $\sin\theta = y/b \Rightarrow \sin^2\theta = (y/b)^2$ and $\cos\theta = x/a \Rightarrow \cos^2\theta = (x/a)^2$. Adding the two equations: $\sin^2\theta + \cos^2\theta = 1 = x^2/a^2 + y^2/b^2$. Thus, we have an ellipse.



42. A has coordinates $(a\cos\theta, a\sin\theta)$. Since OA is perpendicular to AB, ΔOAB is a right triangle and B has coordinates $(a\sec\theta, 0)$. It follows that P has coordinates $(a\sec\theta, b\sin\theta)$. Thus, the parametric equations are $x = a\sec\theta$, $y = b\sin\theta$.

43. $C = (2a\cot\theta, 2a)$, so the x-coordinate of P is $x = 2a\cot\theta$. Let $B = (0, 2a)$. Then $\angle OAB$ is a right angle and $\angle OBA = \theta$, so $|OA| = 2a\sin\theta$ and $A = ((2a\sin\theta)\cos\theta, (2a\sin\theta)\sin\theta)$. Thus, the y-coordinate of P is $y = 2a\sin^2\theta$.



44. (a) Let θ be the angle of inclination of segment OP. Then $|OB| = \dfrac{2a}{\cos\theta}$.

Let $C = (2a, 0)$. Then by use of right triangle OAC we see that $|OA| = 2a\cos\theta$.

Now

$$\begin{aligned}|OP| &= |AB| = |OB| - |OA| \\ &= 2a\left(\frac{1}{\cos\theta} - \cos\theta\right) = 2a\,\frac{1 - \cos^2\theta}{\cos\theta} = 2a\,\frac{\sin^2\theta}{\cos\theta} = 2a\sin\theta\,\tan\theta\end{aligned}$$

So P has coordinates $x = 2a\sin\theta\,\tan\theta\cdot\cos\theta = 2a\sin^2\theta$ and $y = 2a\sin\theta\,\tan\theta\cdot\sin\theta = 2a\sin^2\theta\,\tan\theta$.

(b)