Chapter 1

1. Estimate the slope of $f(x) = 2x^2 + 7$ at $x = 4$.
   A) 6 B) 28 C) 16 D) 56
   Ans: C Difficulty: Moderate Section: 1.1

2. Estimate the slope of $f(x) = 4x^3 + 9$ at $x = 3$.
   A) 12 B) 36 C) 108 D) 27
   Ans: C Difficulty: Moderate Section: 1.1

3. Estimate the slope of $f(x) = 2\sin x$ at $x = \dfrac{3\pi}{2}$.
   A) –2.00 B) 0.00 C) 1.00 D) 1.41
   Ans: B Difficulty: Moderate Section: 1.1

4. Estimate the slope of $f(x) = \sqrt{8x + 9}$ at $x = 2$.
   A) 0.8000 B) 4 C) 100 D) –1.6000
   Ans: A Difficulty: Moderate Section: 1.1

5. Estimate the length of the curve $y = \sqrt{x^2 + 2}$ on the interval $[0, 3]$ using three line segments. Round the answer to 3 decimal places.
   A) 3.552 B) 3.604 C) 1.902 D) 0.634
   Ans: B Difficulty: Moderate Section: 1.1

6. Estimate the length of the curve $y = 2x^2 + 4$ on the interval $[-2, 2]$ using four line segments. Round the answer to 3 decimal places.
   A) 16.492 B) 8.000 C) 16.638 D) 16.000
   Ans: C Difficulty: Moderate Section: 1.1

7. Complete the tables appropriately and use the numerical evidence to conjecture the value of $\lim_{x\to 3}\dfrac{(x-3)^2}{x^4+6x^3-54x-81}$.

x	$\dfrac{(x-3)^2}{x^4+6x^3-54x-81}$
2.9	
2.99	
2.999	
2.9999	

x	$\dfrac{(x-3)^2}{x^4+6x^3-54x-81}$
3.1	
3.01	
3.001	
3.0001	

A) 0 B) 3 C) –3 D) –81
Ans: A Difficulty: Moderate Section: 1.2

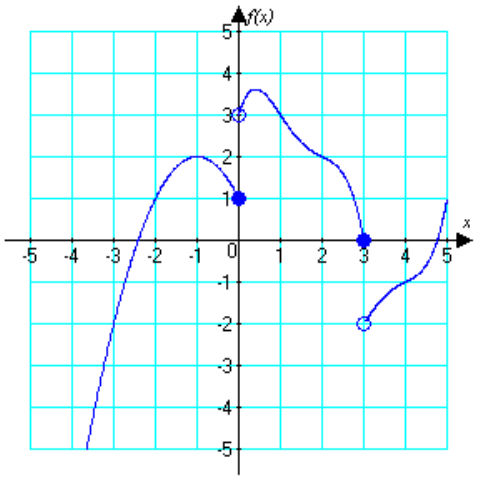
8. Complete the tables appropriately and use the numerical evidence to conjecture the value of $\lim_{x\to 2}\dfrac{5x-10}{x^2-3x+2}$.

x	$\lim_{x\to 2}\dfrac{5x-10}{x^2-3x+2}$
1.9	
1.99	
1.999	
1.9999	

x	$\lim_{x\to 2}\dfrac{5x-10}{x^2-3x+2}$
2.1	
2.01	
2.001	
2.0001	

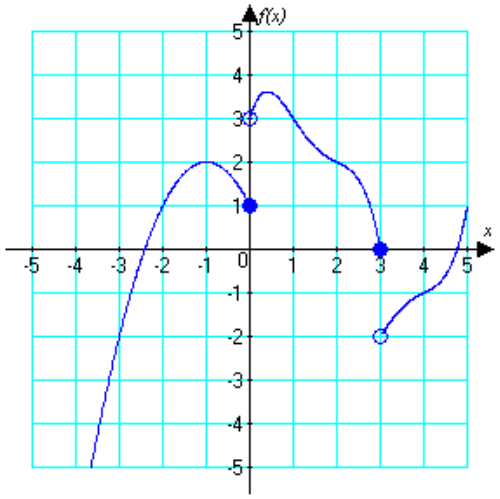
A) 0 B) 5 C) –10 D) 2
Ans: B Difficulty: Moderate Section: 1.2

9. For the function graphed below, identify
$$\lim_{x \to 0^-} f(x)$$
or state that the limit does not exist.



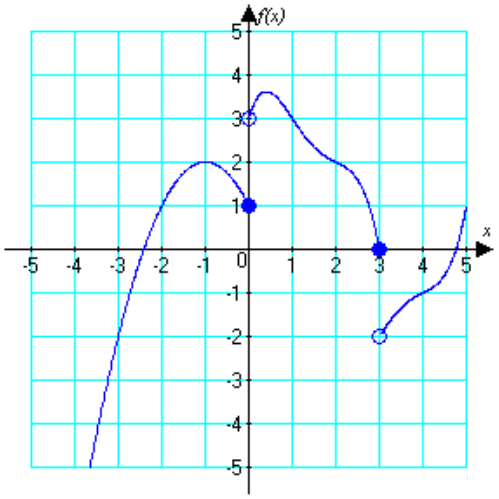
A) 1 B) 2 C) 3 D) does not exist
Ans: A Difficulty: Moderate Section: 1.2

10. For the function graphed below, identify
$$\lim_{x \to 0^+} f(x)$$
or state that the limit does not exist.



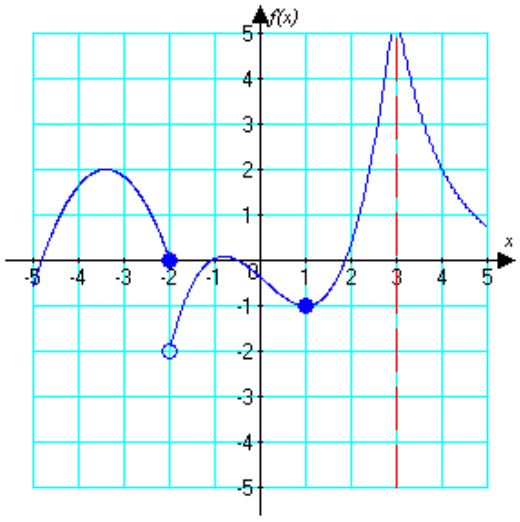
A) 0 B) 1 C) 3 D) does not exist
Ans: C Difficulty: Moderate Section: 1.2

11. For the function graphed below, identify
$\lim_{x \to 3^-} f(x)$
or state that the limit does not exist.



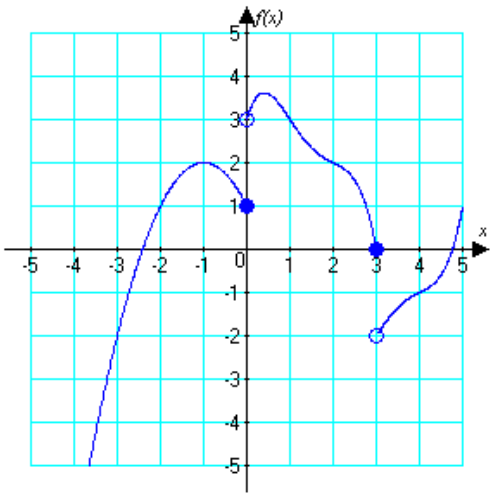
A) 0 B) 2 C) −2 D) does not exist
Ans: A Difficulty: Moderate Section: 1.2

12. For the function graphed below, identify
$\lim_{x \to -2^-} f(x)$
or state that the limit does not exist.



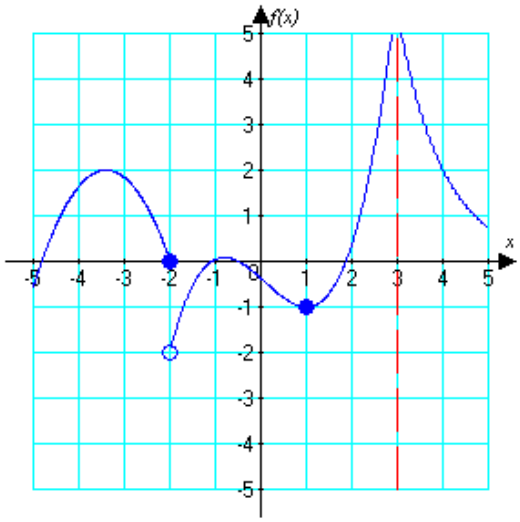
A) 0 B) −1 C) −2 D) does not exist
Ans: A Difficulty: Moderate Section: 1.2

13. For the function graphed below, identify
$\lim_{x \to 3^+} f(x)$
or state that the limit does not exist.



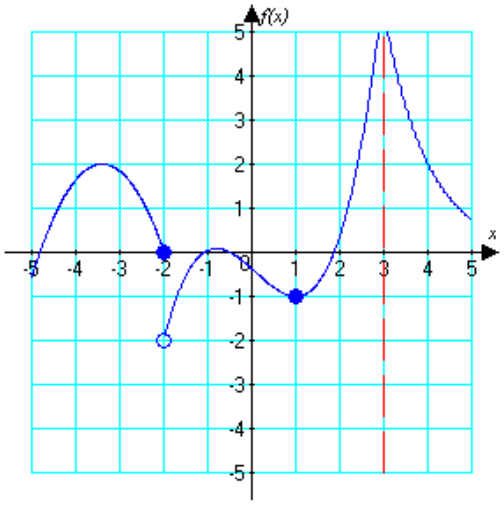
A) 0 B) 2 C) −2 D) does not exist
Ans: C Difficulty: Moderate Section: 1.2

14. For the function graphed below, identify
$\lim_{x \to -2^+} f(x)$
or state that the limit does not exist.



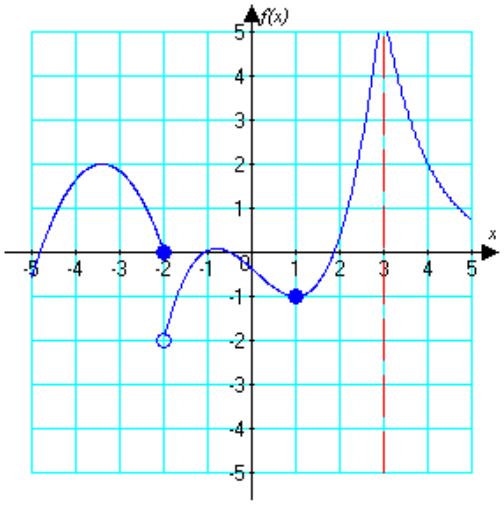
A) 0 B) −1 C) −2 D) does not exist
Ans: C Difficulty: Moderate Section: 1.2

15. For the function graphed below, identify
$\lim_{x \to 1^-} f(x)$
or state that the limit does not exist.



A) 0 B) −1 C) −2 D) does not exist
Ans: B Difficulty: Moderate Section: 1.2

16. For the function graphed below, identify
$\lim_{x \to 3^-} f(x)$
or state that the limit does not exist.

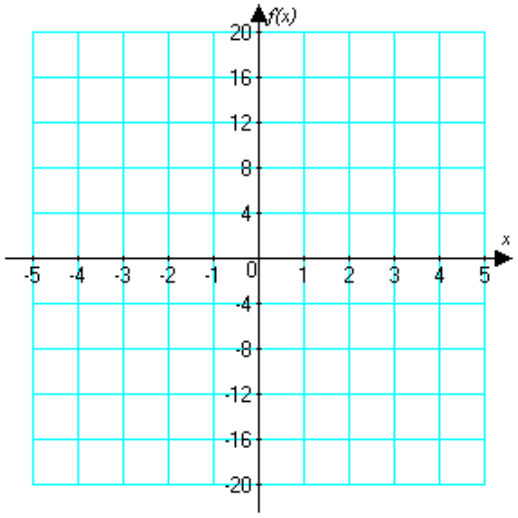


A) 0 B) −1 C) −2 D) does not exist
Ans: D Difficulty: Moderate Section: 1.2

17. Sketch the graph of

$$f(x)=\begin{cases}-4x & \text{if } x>0\\ 4x^2 & \text{if } x\le 0\end{cases}.$$

What is $\lim\limits_{x\to 4^-} f(x)$?

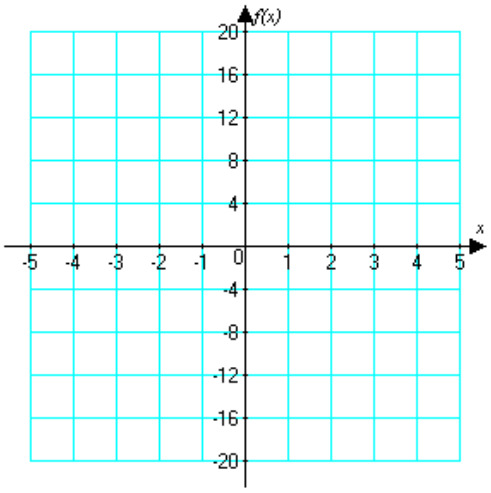


A) 0 B) –16 C) 16 D) does not exist
Ans: B Difficulty: Moderate Section: 1.2

18. Sketch the graph of

$$f(x)=\begin{cases}2x^3+2 & \text{if } x<-2\\ x^2+1 & \text{if } x\geq -2\end{cases}.$$

What is $\lim_{x\to 2^-} f(x)$?



A) –2 B) 0 C) 5 D) does not exist
Ans: C Difficulty: Moderate Section: 1.2

19. Use graphical and numerical evidence to estimate the limit.

$$\lim_{x\to 0}\frac{\sin x}{\cos x}$$

A) 0 B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) π
Ans: A Difficulty: Moderate Section: 1.2

20. Use graphical and numerical evidence to estimate the limit.

$$\lim_{x\to 8}\frac{x^2-64}{x-8}$$

A) 8 B) –8 C) –16 D) 16
Ans: D Difficulty: Moderate Section: 1.2

21. Use graphical and numerical evidence to determine if

$$\lim_{x\to 8} \frac{x^2-64}{x^2-16x+64}$$

exists. If so, state the limit.

A) The limit exists and is -8 at $x=8$.
B) The limit exists and is 8 at $x=8$.
C) The limit does not exist; the function is increasing without bound from the left and decreasing without bound from the right at $x=8$.
D) The limit does not exist; the function is decreasing without bound from the left and decreasing without bound from the right at $x=8$.
Ans: D Difficulty: Moderate Section: 1.2

22. Use graphical and numerical evidence to estimate the limit.

$$\lim_{x\to \pi/2} \frac{\cos x}{\left(x-\frac{\pi}{2}\right)}$$

A) -1 B) 0 C) $\frac{\pi}{2}$ D) π
Ans: A Difficulty: Moderate Section: 1.2

23. Use graphical and numerical evidence to determine if

$$\lim_{x\to 0} \frac{4\sin x}{x}$$

exists. If so, state the limit.

A) The limit exists and is -1 at $x=0$.
B) The limit exists and is 4 at $x=0$.
C) The limit does not exist; the function is increasing without bound at $x=0$.
D) The limit does not exist; the function is decreasing without bound at $x=0$.
Ans: B Difficulty: Moderate Section: 1.2

24. Use graphical and numerical evidence to determine if

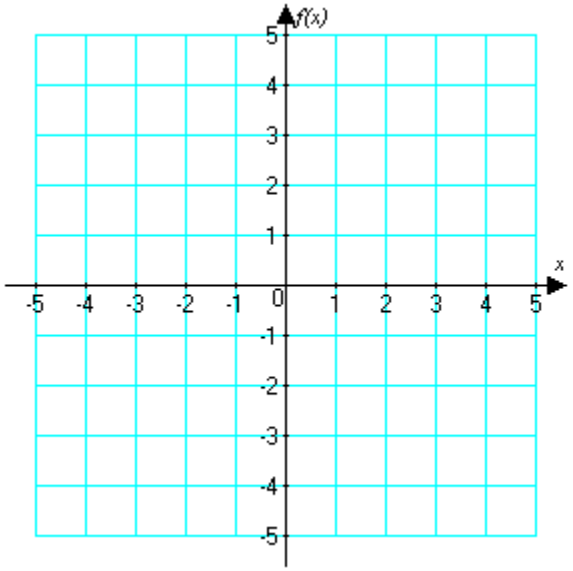
$$\lim_{x\to -8} \frac{x+8}{|x+8|}$$

exists. If so, state the limit.

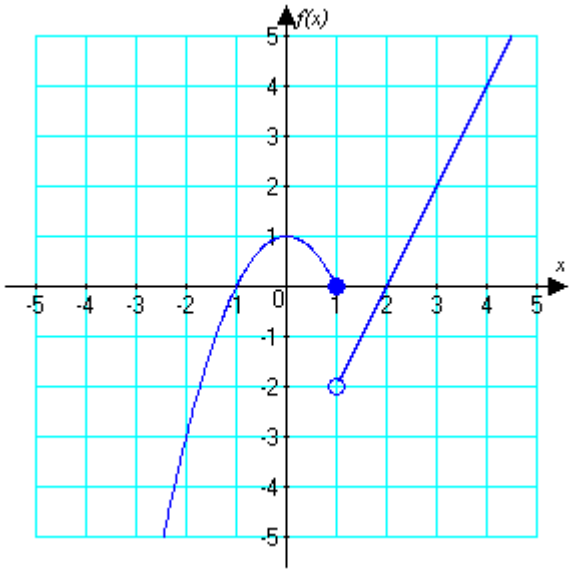
A) the limit exists and is -1 at $x=-8$
B) the limit exists and is 0 at $x=-8$
C) does not exist; the right and left limits at $x=-8$ are different
D) does not exist; the function is increasing without bound at $x=-8$
Ans: C Difficulty: Moderate Section: 1.2

25. Sketch the graph of a function with the given properties.

$f(-1) = 0,\ f(0) = 1,\ f(1) = 0,\ \lim_{x \to 1} f(x)$ does not exist



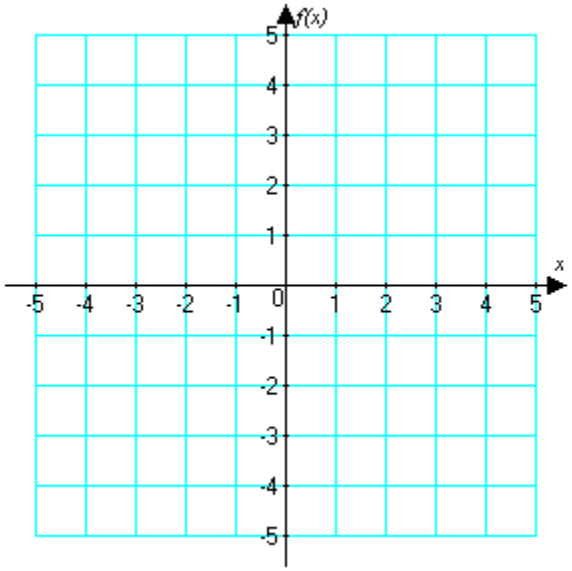
Ans: One possible function that fits the listed criteria is shown here:



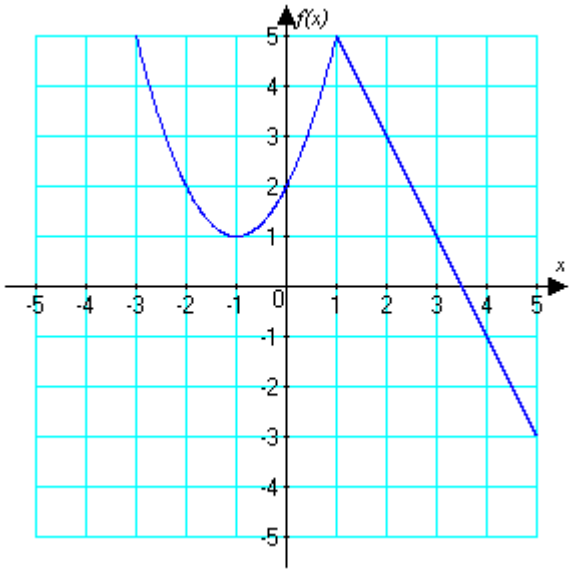
Difficulty: Moderate Section: 1.2

26. Sketch the graph of a function with the given properties.

$$f(0)=2,\ f(1)=5,\ \lim_{x\to -1} f(x)=1,\ \lim_{x\to 2} f(x)=3$$



Ans: One possible function that meets the criteria is:



Difficulty: Moderate Section: 1.2

27. A ski rental shop charges $7.00 for each hour, or portion of an hour, its ski equipment is rented for up to a maximum of $56.00 for all day. If $f(t)$ equals the total charge for the ski equipment for *t* hours, determine the limit $\lim_{t \to 6.5} f(t)$, if it exists.

A) $45.50 B) $42.00 C) $49.00 D) The limit does not exist.
Ans: C Difficulty: Moderate Section: 1.2

28. Evaluate the limit, if it exists.

$$\lim_{x \to 4} \sqrt{2x + 5}$$

A) 3 B) 13 C) $\sqrt{13}$ D) $3\sqrt{13}$
Ans: C Difficulty: Moderate Section: 1.3

29. Find the limit or explain why it does not exist.

$$\lim_{x \to 4^-} \sqrt{16 - x^2}$$

A) 4
B) 0
C) 16
D) The limit does not exist; the function is not defined for $x < 4$.
Ans: B Difficulty: Moderate Section: 1.3

30. Find the limit or explain why it does not exist.

$$\lim_{x \to 5^+} \sqrt{25 - x^2}$$

A) 5
B) 0
C) The limit does not exist; the function increases without bound as *x* approaches 5 from the right.
D) The limit does not exist; the function is not defined for $x > 5$.
Ans: D Difficulty: Moderate Section: 1.3

31. Find the limit or explain why it does not exist.

$$\lim_{x \to -2^+} \sqrt{x^2 + 3x + 2}$$

A) –2
B) 0
C) 2
D) The limit does not exist; the function is not defined for $x < -2$.
Ans: B Difficulty: Moderate Section: 1.3

32. Evaluate the limit, if it exists.

$$\lim_{x \to 3\pi/2} x^2 \cos x$$

A) 0 B) 36 C) 1 D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

33. Evaluate the limit, if it exists.

$$\lim_{x \to 3} \frac{x-7}{x^2+9}$$

A) $-\frac{2}{9}$ B) –4 C) 18 D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

34. Evaluate the limit, if it exists.

$$\lim_{x \to 3} \frac{x^2+4x-21}{x^2-10x+21}$$

A) 10 B) $-\frac{5}{2}$ C) 3 D) does not exist
Ans: B Difficulty: Moderate Section: 1.3

35. Evaluate the limit, if it exists. Assume that $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

$$\lim_{x \to 0} \frac{10 \tan x}{\sin x}$$

A) 0 B) 10 C) $\frac{1}{10}$ D) The limit does not exist.
Ans: B Difficulty: Moderate Section: 1.3

36. Evaluate the limit, if it exists.

$$\lim_{x \to 0^+} x^3 \sec^3 x$$

A) 0 B) 1 C) $\frac{\pi}{2}$ D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

37. Evaluate the limit, if it exists.

$$\lim_{x \to 0} \frac{\sqrt{x^2+3x+36}-6}{x^2+3x}$$

A) $\frac{1}{12}$ B) 6 C) 36 D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

38. Evaluate the limit, if it exists.

$$\lim_{x\to 0} \frac{6x}{2-\sqrt{x+4}}$$

A) 12 B) 24 C) –24 D) does not exist
Ans: C Difficulty: Moderate Section: 1.3

39. Evaluate the indicated limit, if it exists.

$$\lim_{x\to -3} \left(\frac{1}{x+3} + \frac{6}{x^2-9} \right)$$

A) $-\frac{1}{6}$ B) $\frac{1}{6}$ C) 0 D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

40. Evaluate $\lim_{x\to -1} f(x)$ where

$$f(x) = \begin{cases} 3x^2+3 & \text{if } x < -1 \\ 2x+3 & \text{if } x \geq -1 \end{cases}.$$

A) 0 B) 6 C) 1 D) does not exist
Ans: D Difficulty: Moderate Section: 1.3

41. Evaluate $\lim_{x\to 3} f(x)$ where

$$f(x) = \begin{cases} 4x-2 & \text{if } x < -3 \\ 14 & \text{if } -3 < x < 3 \\ 4x+2 & \text{if } x > 3 \end{cases}.$$

A) 14 B) 12 C) 10 D) does not exist
Ans: A Difficulty: Moderate Section: 1.3

42. Evaluate the limit, if it exists.

$$\lim_{h\to 0} \frac{(3+h)^3-27}{h}$$

A) 9 B) 27 C) 18 D) does not exist
Ans: B Difficulty: Moderate Section: 1.3

43. Evaluate the limit, if it exists. Assume that $\lim_{x\to 0}\frac{\sin x}{x}=1$.

$$\lim_{x\to 0}\frac{\tan 9x}{4x}$$

A) 0 B) $\frac{9}{4}$ C) $\frac{4}{9}$ D) The limit does not exist.

Ans: B Difficulty: Moderate Section: 1.3

44. For the position function $f(t)$

$$f(t)=4t^2+6 \text{ (feet)}$$

find the instantaneous velocity at time $t=3$ seconds.

A) 12 feet per second C) 24 feet per second
B) 30 feet per second D) 42 feet per second

Ans: C Difficulty: Moderate Section: 1.3

45. For the position function $f(t)$

$$f(t)=2t^3 \text{ (feet)}$$

find the instantaneous velocity at time $t=5$ seconds.

A) 150 feet per second C) 75 feet per second
B) 125 feet per second D) 50 feet per second

Ans: A Difficulty: Moderate Section: 1.3

46. Given that

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

find the limit or explain why it does not exist.

$$\lim_{x\to 0}\frac{2-2\cos^2 x}{8x^2}$$

A) 2
B) 1
C) $\frac{1}{4}$
D) The limit does not exist; the function is not defined at $x=0$.

Ans: C Difficulty: Moderate Section: 1.3

47. Given

$$\lim_{x\to a} f(x) = 1 \text{ and } \lim_{x\to a} g(x) = -5,$$

find

$$\lim_{x\to a} \left[6f(x) - 4g(x)\right].$$

A) 6 B) 11 C) 2 D) 26
Ans: D Difficulty: Moderate Section: 1.3

48. Given

$$\lim_{x\to a} f(x) = 5 \text{ and } \lim_{x\to a} g(x) = -5,$$

find

$$\lim_{x\to a} \left[2f(x)\cdot 3g(x)\right].$$

A) –25 B) –150 C) 6 D) The limit does not exist.
Ans: B Difficulty: Moderate Section: 1.3

49. Given

$$\lim_{x\to a} f(x) = 1,\ \lim_{x\to a} g(x) = -5 \text{ and } \lim_{x\to a} h(x) = 0,$$

find

$$\lim_{x\to a} \frac{\left[6f(x) + 4g(x)\right]}{h(x)}.$$

A) –4 B) –14 C) 10 D) The limit does not exist.
Ans: D Difficulty: Moderate Section: 1.3

50. Suppose that a state's tax code states that tax liability is 12% on the first 18,000 of taxable earnings and 19% on the remainder. Find the constants *a* and *b* in the tax function *T*(*x*)

$$T(x) = \begin{cases} a + 0.12x & \text{if } x \le 18{,}000 \\ b + 0.19\left(x - 18{,}000\right) & \text{if } x > 18{,}000 \end{cases}$$

such that $\lim_{x\to 0^+} T(x) = 0$ and $\lim_{x\to 18{,}000} T(x)$ exists.

A) $a \neq 0$ and $b = 0$
B) $a = 0$ and $b = 2{,}160$
C) $a = 18{,}000$ and $b = 2{,}160$
D) $a \neq 0$ and $b = 18{,}000$

Ans: B Difficulty: Moderate Section: 1.3

51. Find all discontinuities.

$$f(x) = \frac{4x - 24}{x^2 - 36}$$

A) discontinuous at $x = 0$
B) discontinuous at $x = \pm 6$
C) discontinuous at $x = 36$
D) continuous for all x

Ans: B Difficulty: Moderate Section: 1.4

52. Find all discontinuities.

$$f(x) = \frac{6x - 12}{x^2 - 4}$$

For each discontinuity that is removable, define a new function that removes the discontinuity.

Ans: discontinuous at $x = \pm 2$

The discontinuity at $x = 2$ is removable:

$$f(x) = \frac{6}{x + 2}$$

Difficulty: Moderate Section: 1.4

53. Find all discontinuities.

$$f(x) = \frac{3x}{x^2 + 2x - 15}$$

A) discontinuous at $x = -3, 5$
B) discontinuous at $x = -15$
C) discontinuous at $x = 3, -5$
D) continuous for all x

Ans: C Difficulty: Moderate Section: 1.4

54. Find all discontinuities.

$$f(x) = \frac{2x}{x^2 + 4}$$

A) discontinuous at $x = 4$
B) discontinuous at $x = -2$
C) discontinuous at $x = -2, 2$
D) continuous for all x

Ans: D Difficulty: Moderate Section: 1.4

55. Determine where f is continuous.

$$\frac{2x^2}{\sqrt{5x^3 - x^2}}$$

A) $x \neq 0$ B) $x > 0$ C) $x > \frac{1}{5}$ D) continuous on all reals

Ans: C Difficulty: Moderate Section: 1.4

56. Find all discontinuities.

$$f(x)=\begin{cases}3x & \text{if } x<1\\ 7x^2 & \text{if } x\geq 1\end{cases}$$

A) discontinuous at $x=1$
B) discontinuous at $x=3,7$
C) discontinuous at $x=-3,-7$
D) continuous for all x

Ans: A Difficulty: Moderate Section: 1.4

57. Explain why the function fails to be continuous at $x=1$ by indicating which of the conditions in the definition of continuity are not met.

$$f(x)=\begin{cases}x^2-2 & \text{if } x<1\\ 8 & \text{if } x=1\\ 3x-4 & \text{if } x>1\end{cases}$$

A) $f(1)$ exists but $\lim\limits_{x\to 1} f(x)$ does not exist
B) $\lim\limits_{x\to 1} f(x)$ exists but $f(1)$ does not exist
C) neither $f(1)$ nor $\lim\limits_{x\to 1} f(x)$ exist
D) $f(1)$ exists and $\lim\limits_{x\to 1} f(x)$ exists but $\lim\limits_{x\to 1} f(x)\neq f(1)$

Ans: D Difficulty: Moderate Section: 1.4

58. Determine the intervals where f is continuous.

$$f(x)=\sqrt{2x+12}$$

A) $(-6,\infty)$ B) $[-6,\infty)$ C) $(-\infty,-6]$ D) $(-\infty,-6)$

Ans: B Difficulty: Moderate Section: 1.4

59. Determine the intervals where f is continuous.

$$f(x)=(x-5)^{3/2}$$

A) $(-\infty,5]$ B) $[5,\infty)$ C) $[5,\infty)$ D) $(-\infty,5)$

Ans: C Difficulty: Moderate Section: 1.4

60. Determine the intervals where f is continuous.

$$f(x)=\sin(3x+3)$$

A) $(-\infty,\infty)$ B) $[3,3\pi]$ C) $[-3,3\pi]$ D) $[0,2\pi]$

Ans: A Difficulty: Moderate Section: 1.4

61. Suppose that a state's tax code states that tax liability is 11% on the first $19,000$ of taxable earnings and 19% on the remainder. Find the constants *a* and *b* in the tax function *T*(*x*) that make the function *T*(*x*) continuous.

$$T(x)=\begin{cases}0 & \text{if } x=0\\ a+0.11x & \text{if } 0<x\le 19,000\\ b+0.19\left(x-19,000\right) & \text{if } x>19,000\end{cases}$$

A) $a=0.11$ and $b=3,610$
C) $a=0$ and $b=3,610$
B) $a=0.11$ and $b=2,090$
D) $a=0$ and $b=2,090$

Ans: D Difficulty: Moderate Section: 1.4

62. Use the Intermediate Value Theorem to determine if *f* has a zero in the interval [1,7].

$$f(x)=x^2-34$$

Ans: Since *f*(*x*) is continuous on the interval [1, 7], *f*(*x*) must take on all values between *f*(1) and *f*(7). *f*(1) = –33 and *f*(7) = 15, which have opposite signs. Therefore, *f*(*x*) must equal 0 somewhere on the interval [1, 7].

Difficulty: Moderate Section: 1.4

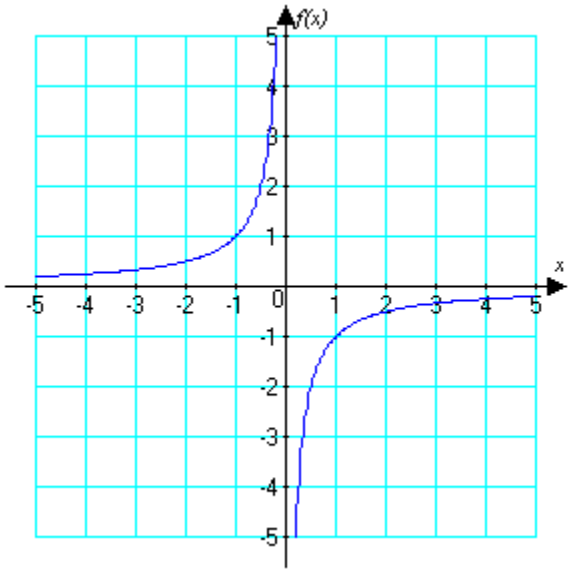
63. Use the Intermediate Value Theorem to determine if *f* has a zero in the interval [2, 7].

$$f(x)=x^3-20x-54$$

Ans: Since *f*(*x*) is continuous on the interval [2, 7], *f*(*x*) must take on all values between *f*(2) and *f*(7). *f*(2) = –86 and *f*(7) = 149, which have opposite signs. Therefore, *f*(*x*) must equal 0 somewhere on the interval [2, 7].

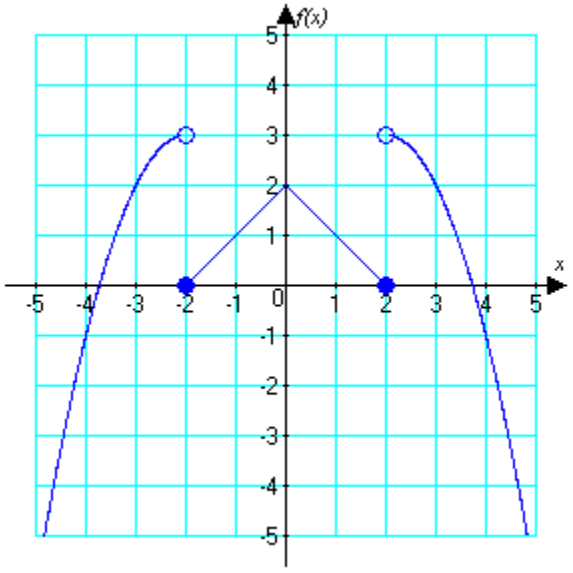
Difficulty: Moderate Section: 1.4

64. Use the graph to identify all discontinuities of *f*.



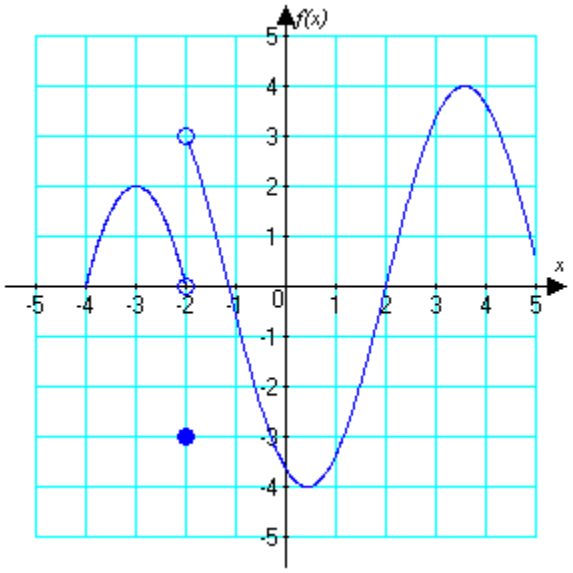
Ans: The function is discontinuous at $x = 0$.
Difficulty: Moderate Section: 1.4

65. Use the graph to identify all discontinuities of *f*.



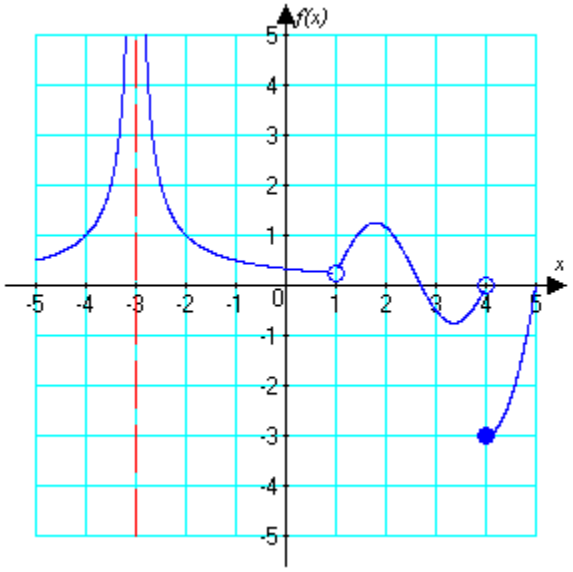
Ans: The function is discontinuous at $x = \pm 2$.
Difficulty: Moderate Section: 1.4

66. Use the graph to identify all discontinuities *f*.



Ans: The function is discontinuous at $x = -2$.
Difficulty: Moderate Section: 1.4

67. Use the graph to identify all discontinuities of *f*.



Ans: The function is discontinuous at $x = -3, 1, 4$.
Difficulty: Moderate Section: 1.4

68. Determine the values of a and b that make $f(x)$ continuous.

$$f(x)=\begin{cases}3\dfrac{\sin x}{x} & \text{if } x<0\\ a & \text{if } x=0\\ b\cos 6x & \text{if } x>0\end{cases}$$

Use $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$.

A) $a=3, b=6$

B) $a=3, b=3$

C) $a=-3, b=3$

D) No values of a and b will make $f(x)$ continuous.

Ans: B Difficulty: Moderate Section: 1.4

69. Determine if f is continuous at $x=14$ from the right.

$$f(x)=\begin{cases}4x^2 & \text{if } x<14\\ 2x-28 & \text{if } x\geq 14\end{cases}$$

A) $\lim\limits_{x\to 14} f(x)\neq f(14)$, but $f(x)$ is continuous from the right

B) $\lim\limits_{x\to 14} f(x)=f(14)$, so $f(x)$ is continuous from the right

C) $\lim\limits_{x\to 14} f(x)\neq f(14)$, so $f(x)$ is not continuous from the right

D) $\lim\limits_{x\to 14} f(x)=f(14)$, but $f(x)$ is not continuous from the right

Ans: B Difficulty: Moderate Section: 1.4

70. Determine if f is continuous at $x=4$ from the right.

$$f(x)=\begin{cases}5x^2 & \text{if } x\leq 4\\ 6x-24 & \text{if } x>4\end{cases}$$

A) $\lim\limits_{x\to 4} f(x)\neq f(d)$ but $f(x)$ is continuous from the right

B) $\lim\limits_{x\to 4} f(x)=f(d)$ so $f(x)$ is continuous from the right

C) $\lim\limits_{x\to 4} f(x)\neq f(d)$ so $f(x)$ is not continuous from the right

D) $\lim\limits_{x\to 4} f(x)=f(d)$ but $f(x)$ is not continuous from the right

Ans: C Difficulty: Moderate Section: 1.4

71. Determine the limit.

$$\lim_{x\to 3^+} \frac{2-9x}{x^2-9}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.
A) ∞ B) $-\infty$ C) 0 D) 9 E) The limit does not exist.
Ans: B Difficulty: Moderate Section: 1.5

72. Determine the limit.

$$\lim_{x\to -5} \frac{5-8x}{x^2-25}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.
A) ∞ B) $-\infty$ C) 25 D) The limit does not exist.
Ans: D Difficulty: Moderate Section: 1.5

73. Determine the limit.

$$\lim_{x\to -4} \frac{x-7}{x^2-8x+16}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.
A) $-\infty$ B) 0 C) $-\frac{11}{64}$ D) ∞ E) The limit does not exist.
Ans: C Difficulty: Moderate Section: 1.5

74. Determine the limit.

$$\lim_{x\to 10^+} \frac{3-x}{(x-10)^2}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.
A) 0 B) $-\frac{13}{400}$ C) ∞ D) $-\infty$ E) The limit does not exist.
Ans: D Difficulty: Moderate Section: 1.5

75. Determine the limit.

$$\lim_{x\to -3^-} \frac{9-x}{x+3}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.
A) 0 B) $-\frac{1}{3}$ C) ∞ D) $-\infty$ E) The limit does not exist.
Ans: D Difficulty: Moderate Section: 1.5

76. Determine the limit (answer as appropriate, with a number, ∞, $-\infty$, or does not exist).

$$\lim_{x\to \pi/2} x^4 \sec^6 x$$

A) 0 B) ∞ C) $-\infty$ D) does not exist
Ans: B Difficulty: Moderate Section: 1.5

77. Determine the limit (answer as appropriate, with a number, ∞, $-\infty$, or does not exist).

$$\lim_{x\to\infty} \ln\left(\frac{x^2+10}{x+3}\right)$$

A) $\ln\left(\frac{10}{3}\right)$ B) ∞ C) $-\infty$ D) does not exist

Ans: B Difficulty: Moderate Section: 1.5

78. Determine the limit.

$$\lim_{x\to\infty} \frac{4x^2+3x+8}{3x^2+4x+3}$$

Answer with a number, ∞, $-\infty$ or that the limit does not exist.

A) $\frac{8}{3}$ B) $\frac{4}{3}$ C) ∞ D) $-\infty$ E) The limit does not exist.

Ans: B Difficulty: Moderate Section: 1.5

79. Determine the limit (answer as appropriate, with a number, ∞, $-\infty$, or does not exist).

$$\lim_{x\to\infty} \frac{3-4/x}{9-5/x}$$

A) $\frac{4}{5}$ B) $\frac{1}{3}$ C) ∞ D) $-\infty$ E) does not exist

Ans: B Difficulty: Moderate Section: 1.5

80. Determine the limit (answer as appropriate, with a number, ∞, $-\infty$, or does not exist).

$$\lim_{x\to\infty} \frac{3x^2\cos x}{x^2+3}$$

A) 3 B) $\frac{1}{3}$ C) ∞ D) $-\infty$ E) does not exist

Ans: E Difficulty: Moderate Section: 1.5

81. Find all horizontal and vertical asymptotes of *f*(*x*).

$$f(x) = \frac{3x}{\sqrt{5+x^2}}$$

For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the vertical asymptote.

A) horizontal asymptotes at $y = \pm 3$; there are no vertical asymptotes.

B) horizontal asymptote at $y = 3$, vertical asymptote at $x = -3$; $f(x) \to \infty$ on both sides of $x = -3$

C) horizontal asymptote at $y = -3$, vertical asymptote at $x = 3$; $f(x) \to -\infty$ on both sides of $x = 3$

D) horizontal asymptotes at $y = \pm 3$, vertical asymptote at $x = 0$; $\lim\limits_{x \to 0^-} f(x) = \infty$ and $\lim\limits_{x \to 0^+} f(x) = -\infty$

Ans: A Difficulty: Moderate Section: 1.5

82. Find all horizontal and vertical asymptotes of *f*(*x*).

$$f(x) = \frac{4x}{36 - x^2}$$

For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the vertical asymptote.

A) horizontal asymptote $y = 0$; there are no vertical asymptotes.

B) horizontal asymptote at $y = 0$, vertical asymptotes at $x = \pm 6$;
$\lim\limits_{x \to -6^-} f(x) = \infty \quad \lim\limits_{x \to -6^+} f(x) = \infty$
$\lim\limits_{x \to 6^-} f(x) = -\infty \quad \lim\limits_{x \to 6^+} f(x) = -\infty$

C) horizontal asymptote at $y = 0$, vertical asymptotes at $x = \pm 6$;
$\lim\limits_{x \to -6^-} f(x) = \infty \quad \lim\limits_{x \to -6^+} f(x) = -\infty$
$\lim\limits_{x \to 6^-} f(x) = \infty \quad \lim\limits_{x \to 6^+} f(x) = -\infty$

D) horizontal asymptote at $y = 0$, vertical asymptotes at $x = \pm 6$;
$\lim\limits_{x \to -6^-} f(x) = -\infty \quad \lim\limits_{x \to -6^+} f(x) = -\infty$
$\lim\limits_{x \to 6^-} f(x) = \infty \quad \lim\limits_{x \to 6^+} f(x) = \infty$

Ans: C Difficulty: Moderate Section: 1.5

83. Find all horizontal and vertical asymptotes of $f(x)$.

$$f(x) = \sin\left(\frac{x^2+4}{x^2-4}\right)$$

For each vertical asymptote, determine whether $f(x) \to \infty$ or $f(x) \to -\infty$ on either side of the vertical asymptote.

A) horizontal asymptote at $y = 1$, vertical asymptotes at $x = \sin(\pm 2)$,
$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = \infty$
$\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$

B) horizontal asymptote at $y = 1$, vertical asymptotes at $x = \pm 2$,
$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = \infty$
$\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$

C) horizontal asymptote at $y = \sin(1)$, vertical asymptotes at $x = \pm 2$,
Limits from both sides of each vertical asymptote are undefined.

D) horizontal asymptote at $y = \sin(1)$, vertical asymptotes at $x = \pm 2$,
$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = \infty$
$\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$

Ans: C Difficulty: Moderate Section: 1.5

84. Determine all vertical and slant asymptotes.

$$y = \frac{x^3}{64 - x^2}$$

A) vertical asymptotes: $x = -8$, $x = 8$; slant asymptote: $y = -x$

B) vertical asymptote: $x = 8$; slant asymptote: $y = -8x$

C) vertical asymptote: $x = 8$; slant asymptote: $y = -x$

D) vertical asymptotes: $x = -8$, $x = 8$; slant asymptote: $y = -8x$

Ans: A Difficulty: Moderate Section: 1.5

85. Determine all vertical and slant asymptotes.

$$y = \frac{x^4}{x^3 + 6}$$

A) vertical asymptotes: $x = -\sqrt{6}$, $x = \sqrt{6}$; slant asymptote: $y = 6x$

B) vertical asymptotes: $x = -\sqrt[3]{6}$, $x = \sqrt[3]{6}$; slant asymptote: $y = x$

C) vertical asymptote: $x = -\sqrt[3]{6}$; slant asymptote: $y = x$

D) vertical asymptote: none; slant asymptote: $y = 6x$

Ans: C Difficulty: Moderate Section: 1.5

86. Suppose that the size of the pupil of a certain animal is given by $f(x)$ (mm), where x is the intensity of the light on the pupil. If $f(x) = \dfrac{80x^{-0.5} + 30}{4x^{-0.5} + 15}$, find the size of the pupil with no light and the size of the pupil with an infinite amount of light.

A) no light: 20 mm; infinite light: 2 mm
B) no light: 2 mm; infinite light: 20 mm
C) no light: 80 mm; infinite light: 0 mm
D) no light: 80 mm; infinite light: 4 mm

Ans: A Difficulty: Moderate Section: 1.5

87. Complete the table appropriately and use the numerical evidence to conjecture the value of $\lim\limits_{x \to -\infty} \dfrac{8x^4 + 8x^2 + 7}{x^4 + 2x\cos x}$.

x	$\dfrac{8x^4 + 8x^2 + 7}{x^4 + 2x\cos x}$
–10	
–100	
–1000	
–10,000	

A) 8 B) $\dfrac{7}{2}$ C) ∞ D) $-\infty$

Ans: A Difficulty: Moderate Section: 1.5

88. Consider

$$f(x) = x\left(\sqrt{49x^2 + 6} - 7x\right).$$

a. Use a graph and numerical values of the function to conjecture a value of $\lim_{x\to\infty} f(x)$.

x	$f(x)$
10^4	
10^5	
10^6	
10^7	
10^8	

b. Rewrite the function to avoid loss-of-significance error.

Ans: a. Graphs should show significant oscillation as x gets large; table should exhibit loss-of-significance error around 10^6 and larger.

b. After multiplying and dividing by the conjugate expression and reducing,

$$f(x) = \frac{6x}{\sqrt{49x^2 + 6} + 7x}$$

Difficulty: Moderate Section: 1.5

89. Find the limit exactly (Hint: multiply and divide by the conjugate expression and simplify).

$$\lim_{x\to\infty}\left(\sqrt{x^2 + 4} - x\right)$$

A) 4 B) –4 C) 0 D) the limit does not exist
Ans: C Difficulty: Moderate Section: 1.5

90. Find the limit exactly (Hint: multiply and divide by the conjugate expression and simplify).

$$\lim_{x\to\infty}\left(\sqrt{16x^2 - 2x + 1} - 4x\right)$$

A) –4 B) 0 C) 6 D) $-\frac{1}{4}$
Ans: D Difficulty: Moderate Section: 1.5

91. Find the limit exactly (Hint: multiply and divide by the conjugate expression and simplify).

$$\lim_{x\to\infty}\left(\sqrt{5x^2+7x+5}-\sqrt{5x^2+3x+1}\right)$$

A) $\sqrt{5}$ B) $\frac{2\sqrt{5}}{5}$ C) 5 D) The limit does not exist.

Ans: B Difficulty: Moderate Section: 1.5

92. Suppose the length of an animal t days after birth is given by $h(t)$.

$$h(t)=\frac{95}{3+8(0.4)^t}\text{ mm}$$

What is the length of the animal at birth?

A) 0 mm B) 95 mm C) $\frac{95}{11}$ mm D) $\frac{95}{3}$ mm

Ans: C Difficulty: Moderate Section: 1.5

93. Suppose the length of an animal t days after birth is given by $h(t)$.

$$h(t)=\frac{87}{2+8(0.4)^t}\text{ mm}$$

What is the eventual length of the animal (i.e., $h(t)$ as $t\to\infty$)?

A) 0 mm B) ∞ mm C) $\frac{87}{10}$ mm D) $\frac{87}{2}$ mm

Ans: D Difficulty: Moderate Section: 1.5

94. Find δ in terms of ε for $\lim_{x\to 0} 4x=0$.

A) $\frac{\varepsilon}{4}$ B) 4 C) 4ε D) 0

Ans: A Difficulty: Moderate Section: 1.6

95. Find δ in terms of ε for $\lim_{x\to 2}(4x+8)=16$.

A) $\frac{\varepsilon}{8}$ B) 4ε C) $\frac{\varepsilon}{4}$ D) 2ε

Ans: C Difficulty: Moderate Section: 1.6

96. Find δ in terms of ε for $\lim_{x\to -2}\frac{x^2-4}{x+2}=-4$.

A) 2 B) ε C) 4 D) $\frac{\varepsilon}{2}$

Ans: B Difficulty: Moderate Section: 1.6

97. Find δ in terms of ε for $\lim\limits_{x\to 0}\left(x^3+5\right)=5$.

A) ε^3 B) 5ε C) $\sqrt[3]{\varepsilon}$ D) $\dfrac{\varepsilon}{5}$

Ans: C Difficulty: Moderate Section: 1.6

98. Find a δ corresponding to $M = 100$ for $\lim\limits_{x\to 6^+}\dfrac{8}{x-6}=\infty$.

A) $\dfrac{2}{25}$ B) 800 C) 600 D) $\dfrac{50}{3}$

Ans: A Difficulty: Moderate Section: 1.6

99. Find a δ corresponding to $M = 100$ for $\lim\limits_{x\to 6^-}\dfrac{9}{\sqrt{36-x^2}}=\infty$.

A) $\dfrac{81}{10,000}$ B) $10,000$ C) 0.0008 D) $\dfrac{9}{100}$

Ans: C Difficulty: Moderate Section: 1.6

100. Find N corresponding to $\varepsilon = 0.1$ for $\lim\limits_{x\to -\infty}\dfrac{5x^2-5}{x^2+1}=5$.

A) $\sqrt{99}$ B) 50 C) $-\sqrt{99}$ D) –50

Ans: C Difficulty: Moderate Section: 1.6

101. Prove that the limit is correct using the appropriate definition. Show all work.

$$\lim_{x\to\infty}\left(\frac{1}{x^2+6}-4\right)=-4$$

Ans: $\left|\dfrac{1}{x^2+6}-4+4\right|<\varepsilon$ if $N=\sqrt{\dfrac{1}{\varepsilon}-6}$

Difficulty: Moderate Section: 1.6

102. Prove that the limit is correct using the appropriate definition. Show all work.

$$\lim_{x\to\infty}\left(\frac{1}{(x-9)^2}\right)=0$$

Ans: $\left|\dfrac{1}{(x-9)^2}\right|<\varepsilon$ if $M=\sqrt{\dfrac{1}{\varepsilon}}+9$

Difficulty: Moderate Section: 1.6

103. Prove that the limit is correct using the appropriate definition. Assume *k* is an integer and is greater than 0. Show all work.

$$\lim_{x\to\infty} \frac{4}{x^k} = 0$$

Ans: $\left|\frac{4}{x^k}\right| < \varepsilon$ if $M = \sqrt[k]{\frac{4}{\varepsilon}}$
Difficulty: Moderate Section: 1.6

104. Given $f(x)$, identify a specific $\varepsilon > 0$ for which no $\delta > 0$ exists to satisfy the definition of a limit.

$$f(x)\begin{cases} 7x & \text{if } x < 1 \\ x^2 + 4 & \text{if } x > 1 \end{cases} \text{ and } \lim_{x\to 1} f(x) \neq 7.$$

Ans: $\varepsilon < 1$
Difficulty: Difficult Section: 1.6

105. A metal washer of (outer) radius *r* inches weighs $4r^2$ ounces. A company manufactures 5-inch washers for different customers who have different error tolerances. If the customer demands a washer of weight $100 \pm \varepsilon$ ounces, what is the error tolerance for the radius? That is, find δ such that a radius of *r* within the interval $(5-\delta, 5+\delta)$ guarantees a weight within $(100-\varepsilon, 100+\varepsilon)$.

A) $\delta = \min\{4\varepsilon, \varepsilon\}$

B) $\delta = \max\{4\varepsilon, \varepsilon\}$

C) $\delta = \min\left\{1, \frac{\varepsilon}{44}\right\}$

D) $\delta = \max\left\{1, \frac{\varepsilon}{44}\right\}$

Ans: C Difficulty: Moderate Section: 1.6