

Chapter 1: Precalculus Review

1. (Sec 1.1)

Find the smallest and the largest values of x that satisfy each of the following inequalities:

A) $|3x - 7| \leq 8$

B) $|2x - 5| \leq 13$

Ans:

A) $x_{\min} = -\frac{1}{3}, x_{\max} = 5$

B) $x_{\min} = -4, x_{\max} = 9$

2. (Sec 1.1)

Suppose that $f(x)$ is even and $g(x)$ is odd. Determine whether the following functions are even, odd, or neither:

A) $f(x) + \cos(g(x))$

B) $f(x) - g(x)$

C) $f(x)^3 + g(x)^2$

D) $\frac{f(x)g(x)}{1 + f(x)g(x)^2}$

Ans: A) Even

B) Neither

C) Even

D) Odd

3. (Sec 1.1)

What are the smallest and largest values of x satisfying $|x - 5| \leq 4$ and $|x - 4| > 2$?

Ans: Smallest: 1, largest: 9

4. (Sec 1.1)

Find the greatest and smallest values of x such that x satisfies the inequalities

$$|2x - 5| \leq 7 \text{ and } |x - 1| \geq \frac{1}{2}.$$

Ans: $x_{\min} = -1$ $x_{\max} = 6$

5. (Sec 1.1)

State whether the following functions are even, odd, or neither:

A) $\sin(x^2 + 1)$

B) $\sin(x^3 + x)$

C) $\frac{1}{1 - (x - 2)^2}$

D) $\sin(\cos x^3)$

Ans: A) Even

B) Odd

C) Neither

D) Even

6. (Sec 1.1)

Find the radius of the circle with center $(-2, 1)$ passing through $(0, 2)$.

Ans: $\sqrt{5}$

7. (Sec 1.1)

Suppose that $|2x - 3| \leq 5$. What are the minimum and maximum possible values of $|3x - 7|$?

Ans: Minimum 0, maximum 10

8. (Sec 1.1)

Complete the statement:

if $|2x - 1| < 2$ and $|3 - x| < 5$ then,

A) $\left|x - \frac{1}{2}\right| < 1$

B) $|3x - 4| < 7$

C) $\left|x - \frac{1}{2}\right| > 1$

D) $|x - 4| > 1$

E) $|x - 4| < 1$

Ans: A

9. (Sec 1.1)

Find the greatest and smallest values of $|x|$ such that x satisfies the inequalities $|3x - 1| \leq 14$ and $|x + 1| \geq 2$.

Ans: Minimum 1, maximum 5

10. (Sec 1.1)

Find the minimum and maximum values of $|6x-1|$ if $|4x+1| \leq 3$.

Ans: Minimum 0, maximum 7

11. (Sec 1.1)

State whether the following functions are even, odd or neither:

A) $\frac{\cos x}{1+x^2}$

B) $\frac{x^3}{\sqrt{1+\cos x}}$

C) $x^3 + x + 1$

D) $\cos(x-1)$

Ans: A) Even

B) Odd

C) Neither

D) Neither

12. (Sec 1.1)

Find the interval over which the function $f(x) = \frac{1}{|x+2|}$ is increasing.Ans: $(-\infty, -2]$

13. (Sec 1.1)

Find the greatest and the smallest values of $|x|$ such that x satisfies the inequalities

$|x-2| \leq 4$ and $|x-2| \geq 3$.

Ans: 6, 1

14. (Sec 1.1)

Complete the statement: if $|x-2| < \frac{3}{4}$ and $|2x+1| < 5$ then which of the following must be true:

A) $|8x-13| > 3$.

B) $|8x-13| < 3$.

C) $|8x+13| < 3$.

D) $|8x-3| > 13$.

E) $|3x+8| < 13$

Ans: B

15. (Sec 1.1)

Find the minimum and maximum values of $|3x+2|$ for x satisfying $|2x-1| \leq 5$ and $|x-1| \geq 1$.

Ans: Min = 0, max = 11

16. (Sec 1.1)

Determine whether the following functions are even, odd or neither:

A) $y = 1 + \sin^2 x$

B) $y = x + \frac{1}{x}$

C) $y = 2^{x^2-1}$

D) $y = \frac{1}{x^3 + x^2 + 1}$

Ans: A) Even

B) Odd

C) Even

D) Neither

17. (Sec 1.1)

Find the interval over which the function $f(x) = \frac{1}{x^2+1}$ is increasing.

Ans: $(-\infty, 0]$

18. (Sec 1.1)

Find an equation of the circle passing through $(2, 7)$ and whose center is located at the midpoint of the line segment joining $(3, -5)$ and $(-11, 3)$.

Ans: $(x+4)^2 + (y+1)^2 = 100$

19. (Sec 1.1)

Find domain and range of the function $f(x) = \frac{1}{\sqrt{x^2-2}}$.

Ans: Domain: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Range: $(0, \infty)$

20. (Sec 1.2)

Find the maximum value of $|x^2 + 6x - 7|$ for $|x+3| \leq 4$.

Ans: 16

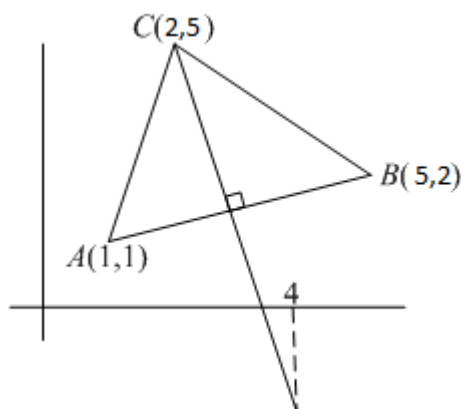
21. (Sec 1.2)

Find the point on the line $y = 2x + 1$ closest to the point $(1,1)$. What is the distance between the two points?

Ans: $\left(\frac{1}{5}, \frac{7}{5}\right), \frac{2\sqrt{5}}{5}$

22. (Sec 1.2)

Referring to the figure, find the equation of the line containing the altitude from Point C to side AB of the triangle. What is the y -coordinate of the point on this line with $x = 4$?



Ans: -3

23. (Sec 1.2)

What is the distance between the maximum point on the parabola $y = -x^2 + 2x + 1$ and the minimum point on the parabola $y = x^2 + 6x + 7$.

Ans: $\sqrt{32} = 4\sqrt{2}$

24. (Sec 1.2)

Find the maximum value of $|x^2 - 4x - 5|$ for $|x - 2| \leq 3$.

Ans: 9

25. (Sec 1.2)

Find the point of intersection of the following lines:

The line parallel to $x - 2y = 5$ and passing through the midpoint of the segment connecting the points $(1,5)$ and $(3,3)$.

The line perpendicular to $2x - y = 5$ and passing through $(0,1)$.

Ans: $(-2, 2)$

26. (Sec 1.2)

Find the point on the line $y = x + 1$ that is closest to the point $(1,1)$. What is the distance between the two points?

Ans: $\left(\frac{1}{2}, \frac{3}{2}\right), \frac{1}{\sqrt{2}}$

27. (Sec 1.2)

What is the maximum value of $|x^2 + x - 6|$ if $|x + 1| < 3$

Ans: $\frac{25}{4}$

28. (Sec 1.2)

Find an equation of the line perpendicular to $x + 2y + 1 = 0$ and passing through $(-2,1)$.

What is the y -coordinate of the point on this line with $x = -6$?

Ans: -7

29. (Sec 1.2)

A line is perpendicular to $2x + y - 7 = 0$ and passes through $(4,6)$. Find the y -coordinate of the point on the line where $x = -8$.

Ans: 0

30. (Sec 1.2)

Find the constants a and b of the line $ax + by = 1$ with x -intercept at $x = \frac{1}{6}$, and which is perpendicular to the line $x + 3y = 7$.

Ans: $a = 6, b = -2$

31. (Sec 1.2)

Find the point on the line $y = x + 2$ closest to the point $(1,5)$.

Ans: $(2,4)$

32. (Sec 1.2)

What is the maximum possible value of $|5 + 4x - x^2|$ if $|x - 2| < 3$?

Ans: 9

33. (Sec 1.2)

For which values of c does the parabola $x^2 - 6cx + (3c - 1)^2$ have no real roots?

Ans: $c < \frac{1}{6}$

34. (Sec 1.2)

What is the maximum possible value of $|x^2 + 2x - 8|$ for $|x + 1| \leq 3$?

Ans: 9

35. (Sec 1.2)

Find the point of intersection of the following lines:

The line of slope 2 passing through the midpoint of the segment joining the points (1,1) and (3,7).

The line perpendicular to $x + 3y = 7$ and passing through (0,3).

Ans: (-3,-6)

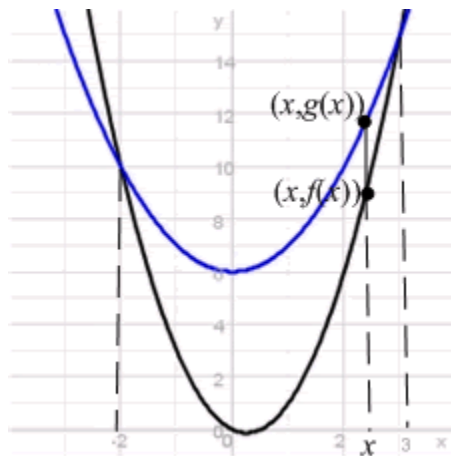
36. (Sec 1.2)

Complete the square and find the maximum value of $f(x) = \frac{1}{x^2 + 2x + 2}$.

Ans: 1

37. (Sec 1.2)

Let $f(x) = 2x^2 - x$, $g(x) = x^2 + 6$. Find the value of x such that the points $(x, f(x))$ and $(x, g(x))$ are farthest for $-2 < x < 3$.



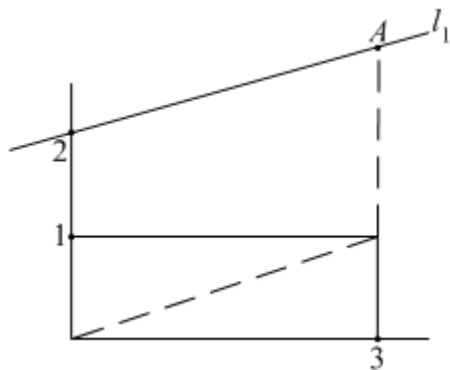
Ans: $x = \frac{1}{2}$

38. (Sec 1.2)

Find the point on the line $y = 3x - 10$ closest to the origin. What is the distance between the two points?

Ans: (3,-1), $\sqrt{10}$

39. (Sec 1.2)

The line l_1 in the figure is parallel to the diagonal of the rectangle.Find the y -coordinate of A .

Ans: 3

40. (Sec 1.2)

Find x such that $(x, 4)$ lies on the line of slope $m = -3$ through the point $\left(-1, \frac{23}{2}\right)$.Ans: $x = \frac{3}{2}$

41. (Sec 1.3)

Let $f(x) = \frac{1}{2-x}$ and $g(x) = \sin x$. Calculate the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

A) $(f \circ g)(x) = \frac{1}{2 - \sin x}, \{x : 0 \leq x \leq \pi\}, (g \circ f)(x) = \sin\left(\frac{1}{2-x}\right), \{x : x < 2\}$

B) $(f \circ g)(x) = \frac{1}{2 - \sin x}, R, (g \circ f)(x) = \sin\left(\frac{1}{2-x}\right), \{x : x \neq 2\}$

C) $(f \circ g)(x) = \sin\left(\frac{1}{2-x}\right), \{x : x \neq 2\}, (g \circ f)(x) = \frac{1}{2 - \sin x}, R$

D) $(f \circ g)(x) = \frac{1}{2 - \sin x}, R, (g \circ f)(x) = \frac{1}{\sin(2-x)}, \{x : x \neq 2\}$

E) $(f \circ g)(x) = \frac{1}{2 - \sin x}, R, (g \circ f)(x) = \sin\left(\frac{1}{2-x}\right), R$

Ans: B

42. (Sec 1.3)

Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{x^2 + 2x + c^2}{x^2 + 1}$.

Find all values of c so that the domain of $f \circ g$ is all R .

Ans: $c > 1, c < -1$

43. (Sec 1.3)

The domain and the range of the function $y = \frac{1}{1 + \sqrt{1 - x^2}}$ are:

- A) Domain $\{x : x \leq 1\}$ Range: $\left\{x : \frac{1}{2} \leq x \leq 1\right\}$
- B) Domain: $\{x : x > 0\}$ Range: $\{x : -1 \leq x \leq 1\}$
- C) Domain $\{x : -1 \leq x \leq 1\}$ Range: R
- D) Domain $\{x : -1 \leq x \leq 1\}$ Range: $\left\{x : \frac{1}{2} \leq x \leq 1\right\}$
- E) Domain: R Range: $\{x : -1 \leq x \leq 1\}$

Ans: D

44. (Sec 1.3)

The domain and range of the function $f(x) = \sin \sqrt{\pi^2 - x^2}$ are:

- A) Domain: $\{x : -\pi \leq x \leq \pi\}$ Range: $\{x : 0 \leq x \leq 1\}$
- B) Domain: $\{x : -\pi \leq x \leq \pi\}$ Range: $\{x : -1 \leq x \leq 1\}$
- C) Domain: $\{x : x \leq \pi \text{ or } x \geq \pi\}$ Range: $\{x : -1 \leq x \leq 1\}$
- D) Domain: R Range: $\{x : 0 \leq x \leq 1\}$
- E) Domain: $\{x : -1 \leq x \leq 1\}$ Range: $\{x : -\pi \leq x \leq \pi\}$

Ans: A

45. (Sec 1.3)

Given $f(x) = \sqrt{\frac{x}{1-x}}$ and $g(x) = x^2$. Calculate the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

- A) $(f \circ g)(x) = \frac{x}{\sqrt{1-x^2}}, \{x: -1 < x < 1\}, (g \circ f)(x) = \frac{x}{1-x} \{x: -1 < x < 1\}$
- B) $(f \circ g)(x) = \frac{x}{1-x}, \{x: x \neq 1\}, (g \circ f)(x) = \frac{|x|}{\sqrt{1-x^2}} \{x: -1 < x < 1\}$
- C) $(f \circ g)(x) = \frac{\sqrt{x}}{1-\sqrt{x}}, \{x: 0 \leq x < 1\}, (g \circ f)(x) = \left| \frac{x}{1-x} \right| \{x: x \neq 1\}$
- D) $(f \circ g)(x) = \frac{|x|}{\sqrt{1-x^2}}, \{x: -1 < x < 1\}, (g \circ f)(x) = \frac{x}{1-x} \{x: 0 \leq x < 1\}$
- E) $(f \circ g)(x) = \frac{|x|}{\sqrt{1-x^2}}, \{x: x \neq 1\}, (g \circ f)(x) = \frac{x}{1-x} \{x: -1 < x < 1\}$

Ans: D

46. (Sec 1.3)

Let $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x^2 - cx + (2c - 3)$. Find all values of c such that the domain of $f \circ g$ is all R .

Ans: $2 < c < 6$

47. (Sec 1.3)

Let $f(x) = \sqrt{1-x^2}$ and $g(x) = \sin x$.

Calculate the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

- A) $(f \circ g)(x) = \cos x, R; (g \circ f)(x) = \sin \sqrt{1-x^2}, \{x: -1 \leq x \leq 1\}$
- B) $(f \circ g)(x) = |\cos x|, R; (g \circ f)(x) = \sin^2 \sqrt{1-x}, \{x: x < 1\}$
- C) $(f \circ g)(x) = |\cos x|, R; (g \circ f)(x) = \sin \sqrt{1-x^2}, \{x: -1 \leq x \leq 1\}$
- D) $(f \circ g)(x) = \sqrt{1-\sin x}, R; (g \circ f)(x) = \sin^2 \sqrt{1-x}, \{x: x < 1\}$
- E) $(f \circ g)(x) = |\cos x|, \{x: -1 \leq x \leq 1\}; (g \circ f)(x) = \sin \sqrt{1-x^2}, R$

Ans: C

48. (Sec 1.3)

Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 2cx$. Find all values of c such that the domain of $f \circ g$ is all R .

Ans: $-1 \leq c \leq 1$

49. (Sec 1.3)

The domain and range of $f(x) = \sqrt{1 + \sin\left(\frac{1}{\sqrt{x}}\right)}$ are:

- A) Domain: $\left\{x : \frac{1}{\pi} < x\right\}$ Range: $\{x : 0 \leq x \leq \sqrt{2}\}$
 B) Domain: $\{x : x > 0\}$ Range: $\{x : 0 \leq x \leq \sqrt{2}\}$
 C) Domain: $\left\{x : \frac{2}{\pi} < x\right\}$ Range: $\{x : 1 \leq x \leq \sqrt{2}\}$
 D) Domain: $\{x : x \neq 0\}$ Range: $\{x : 0 \leq x\}$
 E) Domain: $\{x : 0 \leq x \leq \sqrt{2}\}$ Range: $\{x : x \neq 0\}$

Ans: B

50. (Sec 1.3)

Let $f(x) = \frac{1}{1+|x|}$ and $g(x) = x^{-3}$. Compute the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

- A) $(f \circ g)(x) = \frac{1}{1+|x|^{-3}}, \{x : x \neq 0, -1\}; (g \circ f)(x) = (1+|x|)^3, \mathbb{R}$
 B) $(f \circ g)(x) = \frac{|x|^3}{|x|^3 + 1}, \{x : x \neq -1\}; (g \circ f)(x) = (1+|x|)^3, \mathbb{R}$
 C) $(f \circ g)(x) = \frac{1}{1+|x|^{-3}}, \{x : x \neq 0, -1\}; (g \circ f)(x) = 1+|x|^3, \mathbb{R}$
 D) $(f \circ g)(x) = \frac{|x|^3}{|x|^3 + 1}, \{x : x \neq -1\}; (g \circ f)(x) = (1+|x|)^3, \{x : x \neq 1\}$
 E) $(f \circ g)(x) = \frac{1}{1+|x|^{-3}}, \mathbb{R}; (g \circ f)(x) = (1+|x|)^3, \{x : x \neq 0, -1\}$

Ans: A

51. (Sec 1.3)

Let $f(x) = \frac{1}{\sqrt{x+1}}$ and $g(x) = x^2 + 2cx + 8$.

Find all values of c such that the domain of $f \circ g$ is all of $\mathbb{R}$.

Ans: $-3 < c < 3$

52. (Sec 1.3)

The domain and range of $y = \frac{1}{2 - \sin x}$ are:

A) Domain: R Range: $\left\{x : \frac{1}{3} \leq x \leq 1\right\}$

B) Domain: $\left\{x : \frac{1}{3} \leq x \leq 1\right\}$ Range: R

C) Domain: $\left\{x : 0 \leq x \leq \frac{\pi}{2}\right\}$ Range: R

D) Domain: R Range: R

E) Domain: $\left\{x : 0 \leq x \leq \frac{\pi}{2}\right\}$ Range: $\left\{x : \frac{1}{3} \leq x \leq 1\right\}$

Ans: A

53. (Sec 1.3)

Let $f(x) = \frac{5}{x^2 + 2}$ and $g(x) = x^{-3}$. Calculate the composite function $f \circ g$ and determine its domain and range.

Ans: $(f \circ g)(x) = \frac{5}{x^{-6} + 2}$

Domain: $\{x : x \neq 0\}$

Range: $\left\{y : 0 < y < \frac{5}{2}\right\}$

54. (Sec 1.3)

Let $f(x) = 2 \sec x$ and $g(x) = x^2 + \frac{\pi}{2}$. Calculate the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

Ans: $(f \circ g)(x) = 2 \sec\left(x^2 + \frac{\pi}{2}\right)$

Domain: $\left\{x : x \neq \pm\sqrt{n\pi} \text{ where } n \text{ is any nonnegative integer}\right\}$

$(g \circ f)(x) = 4 \sec^2 x + \frac{\pi}{2}$

Domain: $\left\{x : x \neq \pm \frac{(2n+1)\pi}{2} \text{ where } n \text{ is any integer}\right\}$

55. (Sec 1.4)

Find $\cos \theta$, $\tan \theta$, and $\sin \theta$ if $\sec \theta = 3$ and θ is acute.

Ans: $\cos \theta = \frac{1}{3}, \tan \theta = \sqrt{8}, \sin \theta = \frac{\sqrt{8}}{3}$

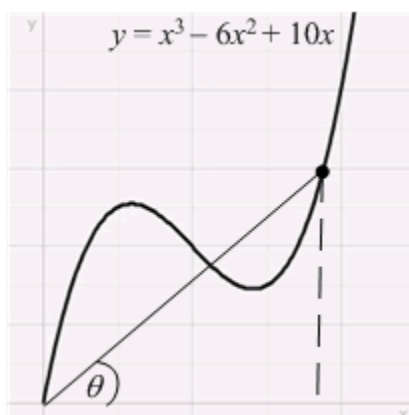
56. (Sec 1.4)

Solve the following equation for x if $0 \leq x < 2\pi$:

$$\sin 2x + \cos\left(x - \frac{\pi}{2}\right) = 0.$$

Ans: $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

57. (Sec 1.4)

Find the minimum value of the angle θ between the segment joining a point on the graph of $y = x^3 - 6x^2 + 10x$ with the origin, and the x -axis for $x \geq 0$.

Ans: $\theta = \frac{\pi}{4}$

58. (Sec 1.4)

Solve the following equation for x if $0 \leq x < 2\pi$:

$$2\sin\left(x + \frac{\pi}{3}\right) - \sin x = \frac{\sqrt{6}}{2}.$$

Ans: $x = \frac{\pi}{4}, \frac{7\pi}{4}$

59. (Sec 1.4)

Find $\sin \theta$, $\sec \theta$, and $\tan \theta$ if $\csc \theta = 2$ and $0 < \theta < \frac{\pi}{2}$.

Ans: $\sin \theta = \frac{1}{2}, \sec \theta = \frac{2}{\sqrt{3}}, \tan \theta = \frac{1}{\sqrt{3}}$

60. (Sec 1.4)

Solve the following equation for x if $0 \leq x < 2\pi$: $\cos 2x + 3\sin\left(\frac{\pi}{2} - x\right) = 1$.

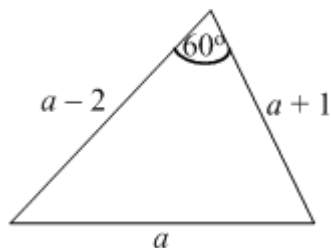
Ans: $\frac{\pi}{3}, \frac{5\pi}{3}$

61. (Sec 1.4)

Find the points of intersection of the graphs of $y = \cos 2x$ and $y = 2\cos^2 x - \sin 4x$ for $0 \leq x \leq 2\pi$.

Ans: $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

62. (Sec 1.4)

Referring to the figure, find the value of a .

Ans: $a = 7$

63. (Sec 1.4)

Find $\cos \theta$ and $\tan \theta$ if $\sin \theta = \frac{4}{5}$ and $\frac{\pi}{2} \leq \theta < \pi$.

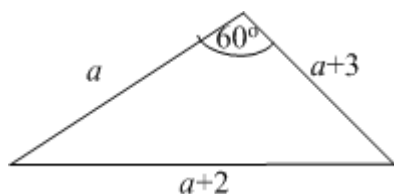
Ans: $\cos \theta = -\frac{3}{5}, \tan \theta = -\frac{4}{3}$

64. (Sec 1.4)

Find $\cos \theta$, $\sec \theta$, and $\tan \theta$ if $\sin \theta = \frac{1}{3}$ and θ is acute.

Ans: $\cos \theta = \frac{2\sqrt{2}}{3}, \sec \theta = \frac{3}{2\sqrt{2}}, \tan \theta = \frac{1}{2\sqrt{2}}$

65. (Sec 1.4)

Referring to the figure, compute the value of a .Ans: $a = 5$

66. (Sec 1.4)

Solve the following equation for x if $0 \leq x < 2\pi$:

$$2\cos 2x + 4\sin\left(x - \frac{\pi}{2}\right) + 3 = 0.$$

Ans: $\frac{\pi}{3}, \frac{5\pi}{3}$

67. (Sec 1.4)

Find the points of intersection of the graphs of $y = \cos x$ and $y = 1 - \sin \frac{x}{2}$ for

$$0 \leq x < 2\pi.$$

Ans: $(0, 1), \left(\frac{\pi}{3}, \frac{1}{2}\right), \left(\frac{5\pi}{3}, \frac{1}{2}\right)$

68. (Sec 1.4)

Find the points where the graphs of $y = \cos 2x$ and $y = 2 - 3\sin x$ intersect for

$$0 \leq x \leq \pi.$$

Ans: $\left(\frac{\pi}{6}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, \frac{1}{2}\right), \left(\frac{\pi}{2}, -1\right)$ $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

69. (Sec 1.4)

Find $\tan \theta$, $\cos \theta$, and $\sin \theta$ if $\sec \theta = 2$ and θ is acute.Ans: $\tan \theta = \sqrt{3}, \cos \theta = \frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2}$

70. (Sec 1.4)

Solve the following equation for x if $0 \leq \theta \leq \pi$: $\tan^2 \theta - 4\sin^2 \theta = 0$.Ans: $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

71. (Sec 1.4)

Find the points of intersection of the two graphs $y = \sin 2x$ and $y = 2 \cos^2 x$ for $0 \leq x < 2\pi$.

Ans: $\left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{4}, 1\right), \left(\frac{5\pi}{4}, 1\right), \left(\frac{3\pi}{2}, 0\right)$

72. (Sec 1.4)

Use the addition formula to compute $\cos\left(\frac{5\pi}{12}\right)$ exactly.

Ans: $\cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2}}{4}(\sqrt{3}-1)$

73. (Sec 1.4)

Find $\csc \theta$ if $\cot \theta = \frac{1}{3}$ and $\cos \theta < 0$.

Ans: $\csc \theta = -\frac{\sqrt{10}}{3}$

74. (Sec 1.5)

Find all vertical and horizontal asymptotes for the function $f(x) = \frac{3x^2 + 2x - 8}{x^2 + 4x - 12}$.

Ans: Vertical asymptotes: $x = 2, x = -6$

Horizontal asymptote: $y = 3$

75. (Sec 1.5)

Find the set of solutions to the inequality $(6x^2 - 12)(2x^2 - 18) > 0$.

Ans: $(-\infty, -3) \cup (-\sqrt{2}, \sqrt{2}) \cup (3, \infty)$