questions and problems

Narrow It Down: Multiple-Choice Questions

1. How does the altitude of the Sun at noon on the same day in the Northern Hemisphere's summer compare for two observers at latitudes 12° north and 54° north, respectively?
   a. It is the same for both observers because they are in the same hemisphere, experiencing summer.
   b. It cannot be determined without knowing their longitudes.
   c. It is 42° higher for the observer at 12° north because of the difference in latitude.
   d. It is 42° higher for the observer at 54° north because of the difference in latitude.
   e. The relative altitude of the Sun cannot be determined for the two locations without knowing the exact date.
2. Which observers on Earth can see Polaris on a clear night?
   a. all observers on Earth
   b. only observers above the Arctic Circle
   c. only observers in the Western Hemisphere
   d. only observers in the Southern Hemisphere
   e. only observers in the Northern Hemisphere
3. You are observing distant object A of 30 arcseconds in diameter. From your understanding of the small-angle formula, and compared with your observation of A, which of these statements is always true?
   a. A larger object at the same distance will appear the same size.
   b. A smaller object at the same distance will appear the same size.
   c. A smaller object at a greater distance will appear smaller.
   d. A smaller, less distant object will appear larger.
   e. A larger, less distant object will appear smaller.
4. The Sun is highest on the sky at noon on
   a. the winter solstice.
   b. the spring equinox.
   c. the summer solstice.
   d. the autumn equinox.
   e. any day, because the Sun reaches the same altitude daily regardless of season.
5. Which statement about constellations is true?
   a. Any group of stars can be called a constellation.
   b. A constellation includes a group of stars within specific boundaries in the same region of the sky.
   c. The stars that form a constellation must be in a configuration resembling an animal or human.
   d. The stars within a constellation are all located at the same distance from Earth.
   e. The stars within a constellation are all about the same brightness.
6. From which location are the same constellations above the horizon at any time of year?
   a. the North Pole
   b. the equator
   c. Rio de Janeiro, Brazil
   d. New York City
   e. No such location exists.
7. Warmer summertime temperatures in the Northern Hemisphere are due partly to
   a. longer days.
   b. a lower angle of the Sun's rays.
   c. Earth's being closer to the Sun in summer.
   d. the Sun radiating more energy in summer.
   e. the tilt of the Northern Hemisphere away from the Sun.
8. Which statement about Moon phases is true?
   a. In waxing phases, the lit portion of the Moon faces the eastern horizon.
   b. The new Moon has its whole face illuminated as seen from Earth.
   c. The Moon rises at sunset every day.

   d. The waning gibbous phase follows the full Moon.
   e. In waning phases, the lit portion of the Moon faces the western horizon.
9. You observe the full Moon just rising in the east. What time of day is it?
   a. sunrise (about 6:00 A.M.)
   b. noon (about 12:00 P.M.)
   c. sunset (about 6:00 P.M.)
   d. midnight (about 12:00 A.M.)
   e. midafternoon (about 3:00 P.M.)
10. Synodic and sidereal months differ because of
   a. the Moon's orbit.
   b. Earth's orbit.
   c. the Sun's orbit.
   d. the fact that Earth's year is not exactly 365 days.
   e. the different number of days in each calendar month.
11. The direct (and most typical) seasonal motion of the planets as observed from Earth is
   a. west to east with respect to the background stars.
   b. east to west with respect to the background stars.
   c. east to west at the same rate as the background stars.
   d. north to south with respect to the background stars.
   e. south to north with respect to the background stars.
12. Using only Stonehenge to calibrate astronomical motions, early people would *not* have been able to tell which of the following? Choose all that apply.
   a. when to plant
   b. when the longest day had come
   c. when Mars would appear
   d. when the Moon would be full
   e. when winter would begin
13. Which Greek philosopher is most closely associated with first rejecting supernatural explanations and arguing that reason alone could explain phenomena?
   a. Thales
   b. Aristotle
   c. Socrates
   d. Hipparchus
   e. Pythagoras
14. For what significant contribution is Eratosthenes most famous?
   a. inventing trigonometry
   b. measuring the circumference of Earth
   c. constructing the geocentric model of the Universe
   d. creating the first catalog of bright stars
   e. defining the four basic elements
15. Which of the following statements about parallax is true?
   a. Our two eyes enable us to use parallax to determine distances to objects.
   b. Earth's orbit provides astronomers an opportunity to use parallax.
   c. Most stars do not appear to shift position, because they are too far away for parallax to be observed.
   d. If all stars were on the surface of a celestial sphere and located at the same distance, we would observe no stellar parallax.
   e. all of the above
16. Which of the following was/were elements of Ptolemy's geocentric model? Choose all that apply.
   a. It provided a true explanation for why we don't feel a constant strong wind on Earth.
   b. It included epicycles.
   c. It accounted for retrograde motion.
   d. It supposed that Mercury and Venus orbit the Sun.
   e. It assumed that all planetary orbits were ellipses.
17. A lunar eclipse can occur at which Moon phase(s)? Choose all that apply.
   a. new Moon

b. first quarter
c. full Moon
d. third quarter
e. all of the above

18. Which of the following statements about solar eclipses is/are correct? Choose all that apply.
    a. A total eclipse is possible because the Sun and Moon sometimes appear to be identical in size.
    b. Solar eclipses can only occur at full Moon.
    c. Not all solar eclipses achieve totality.
    d. Prediction of solar eclipses became possible only with the advent of computers.
    e. A solar eclipse is visible to everyone on Earth equally.
19. True/False: Total eclipses can occur only when both the Moon and the Sun simultaneously pass through the line of nodes.
20. Eclipses are possible only when both Sun and Moon are at specific positions relative to Earth. How many times each month does this alignment occur?
    a. one
    b. two
    c. three
    d. four
    e. It varies widely.

To the Point: Qualitative and Discussion Questions

21. What are some factors that led to the advancement of human civilization and culture?
22. What likely uses did the ancients have for megaliths?
23. Name at least one important contribution associated with each of the following Greek thinkers: Thales of Miletus, Pythagoras, Plato, Aristotle, Eudoxus, Aristarchus, Eratosthenes, Hipparchus.
24. If the Moon crosses the meridian at midnight, what phase must the Moon be in?
25. Suppose that a month ago you saw the star Betelgeuse in the constellation Orion just rising at the eastern horizon at 8:00 P.M. Describe its position at the same time today.
26. Define retrograde motion and explain how Ptolemy's model represented it.
27. Define the celestial sphere. How is it a useful (if imaginary) tool?
28. From what location on Earth can you see every part of the celestial sphere over the course of the year?
29. How does the Sun's path across the sky differ in summer versus winter?
30. What is an analemma, and what gives it its characteristic shape?
31. If Earth's axis had no tilt relative to the plane of its orbit, how would the seasons differ from those we experience today?
32. If Earth's axis had a more significant tilt relative to the plane of its orbit, how would the seasons differ from those we experience today?
33. Explain the difference between sidereal and synodic months.
34. Describe and compare the models of the Universe defined by Aristotle and Ptolemy.
35. How did Aristotle use the lack of measurable parallax to disprove the heliocentric model championed by Aristarchus? Comment on the flaw in Aristotle's logic.

Going Further: Quantitative Questions

36. How many arcseconds are there in 4°?
37. How many degrees are there between the horizon and the zenith?
38. From your location, the Sun is at an altitude of 80° as it crosses the meridian on the summer solstice. Describe its altitude as it crosses the meridian one month later.
39. A star is at the zenith for an observer at latitude 44° north. What is its declination on the celestial sphere? (Note that astronomers use a "+" before the number for north declination and a "–" before the number for south declination.)
40. You observe the Moon's position on the sky at the same time on two consecutive days. Across how many degrees of sky has its position moved?
41. How many days are there between new Moon and full Moon?
42. You observe Mars with an angular diameter of 18″. What is its distance from Earth in kilometers? (Hint: The diameter of Mars is 6,792 km.)
43. A globular star cluster has an angular diameter of 20′. It is 25,000 light-years away. What is its diameter in light-years?
44. An object at a distance of 200 meters is 0.5 meter wide. What is its corresponding angular width in arcseconds? in arcminutes?

45. Comet Hale-Bopp has a core diameter of 40 km. At its closest approach to Earth, it was about 137 million km away. How large in arcseconds did its core appear to observers at that distance?

If your instructor assigns homework in smartwork5, access it at the Digital Landing Page for *At Play in the Cosmos*: **digital.wwnorton.com/cosmos**

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ADAM FRANK

GERCEIDA JONES

New York University

ELISHA POLOMSKI

St. Cloud State University

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CHAPTER 1

Getting Started: Science, Astronomy, and Being Human

INSTRUCTOR'S NOTES

Chapter 1 is an introduction to the history and human context of astronomical science. Major topics include:

- Tracing the history of human beings' knowledge of astronomical phenomena and how it is applied across cultures and eras.
- Size scales and scientific notation in astronomy.
- The major categories of structure in the Universe.
- Using the scientific method to increase our understanding about the Universe and how it differs from pseudoscience.
- Understanding the importance of science in making informed decisions in our high-tech society.

This course is generally taken by nonscience majors to fulfill a general science requirement. Before the students come to class, I have them fill out what I call a "First Day Questionnaire," which is used as an icebreaker on the first day of class. This not only gives me an idea of their science and math background but also serves as a tool for the student to get a quick glance at their classmates. Although the questionnaire reveals the skill level overall is very good, some students are still afraid of math and science. The discomfort that students anticipate about astronomy comes from their previous experience in former classes. They assume the class will be boring and a lot of math and physics will be required. Therefore, I ease their fears by addressing the issues that concern them. The icebreaker and introducing astronomy by discussing current events seem to open them up to wanting to learn more and to participate.

Astronomy operates on many sizes and time scales, from subatomic to large-scale structures and from nanoseconds to billions of years. Although I am comfortable in working with this wide range of units, most students are not. Section 1.2 discusses numbers and human time in a way that students can relate to. The review of scientific notation is easy to follow by looking at the examples in Going Further 1.1. Most of the quantitative problem solving in this textbook can be accomplished through proportional reasoning, such as how many times larger is Jupiter than Earth. In addition to asking questions about scientific notation and unit conversion, I introduce some basic concepts of how area or volume changes with size.

Depending on the school's curriculum, this may be the last formal science course your students will take. Science can be fun, and I try to impart that before they leave. It's not all chalk and talk. Science should be an experience of active engagement. This textbook has been designed to achieve that very goal. Some of the learning outcomes will be to vet the scientific literacy of a source, learn how science is done, and know the difference between science and pseudoscience. The seeds are sown in this first chapter as the student starts to discover that humans have been investigating the heavens since prehistoric times by way of the monoliths left behind as evidence for archaeologists. Many relics span the globe, giving us a snapshot of our past achievements as humans on this Earth for millions of years. Astronomy is no different; patterns in the night sky were beneficial to ancient people. They monitored the regular cycles of the Sun, Moon, and stars to take control of their destinies. Therefore, science is all about getting at the truth that leads to verifiable conclusions.

DISCUSSION POINTS

- Have students look at the 5,000-year-old monument in Figure 1.1. Ask them if they have seen similar shapes or structures in other parts of the country or world.
- Have students think about the time of year given in Figure 1.1. Discuss how the alignment of the structure relates to the seasons of the yearly calendar. Ask students when the Newgrange astronomical event happens and why.
- The night sky is our window into time and space. Modern astronomy is the result of many thousands of years of observations and the genius of many individuals and cultures. As astronomers continue to collect data from this vast universe and find both expected and unexpected trends, it is important for students to recognize the trends that exist in their own lives. A simple exercise students can do to gauge their actual versus desired outcome is to track the amount of time spent studying and asking questions during a professor's office hours in order to gain a more realistic view of their course grade.
- Discuss with your students how both science and technology dominate every facet of our lives. Ask them if an interest in different fields seem contradictory, such as a mix of science, art, and religion. Discuss the benefits and challenges of science and technology.
- Ask students if they are familiar with the scientific method. If so, have them explain it and what happens if the experiment fails. Is astrology a science or a pseudoscience? Why or why not?
- Discuss climate change and the depletion of natural resources. Ask students whether the threat of nuclear weapons and biological warfare pose any major challenges. If so, what are they?
- Every generation thinks it's special. We live in a vast universe of which we have very little understanding. However, it is a critical historical moment in the sense that science and technology have allowed society to advance beyond our wildest dreams. Hollywood used to be more fiction than fact, but the scientific revolution has turned that all around, much to our surprise.

LEARNING ASTRONOMY BY DOING ASTRONOMY

The following activities are available in the *Learning Astronomy by Doing Astronomy: Collaborative Lecture Activities* workbook. Each LADA activity is designed and classroom tested to illuminate a particular astronomical concept or principle. LADA is available for purchase as a stand-alone, or it is packaged with a Norton astronomy textbook, at no extra cost.

Activity 1

MATHEMATICAL AND SCIENTIFIC METHODS

This activity is a review of the math that you will encounter in this course. You will be using scientific notation as a way to express very big or very, very small numbers.

- Text reference: Section 1.2, Section 1.4
- Pay close attention to Going Further 1.1

Activity 27

A COSMIC CALENDAR

In this activity students will learn how to organize the natural history of the universe and the human journey.

- Text reference: Section 1.1

INTERACTIVE SIMULATIONS

Almost 50 new, easy-to-use interactive simulations allow students to play with physical phenomena. This Instructor's Manual provides learning goals outlining ways in which students can use each simulation to further his or her understanding of each topic.

SCALE OF THE UNIVERSE

- Introduces the students to astronomical size scales and relative size of the most common astronomical objects.
- Introduces students to the types of objects in the Universe (i.e., nebulae, planetary systems, planets, stars, star clusters, forming stars, exploding stars, galaxies, galaxy clusters, etc.)
- Allows students to familiarize themselves with the "meter sticks" astronomers use to measure distances (e.g., astronomical unit and light year)

CHAPTER 1

END-OF-CHAPTER SOLUTIONS

NARROW IT DOWN: MULTIPLE-CHOICE QUESTIONS

1. (**b**) It is inefficient to express a two digit number in scientific notation, for example, 50 as 5.0×10^1.
2. (**c**) The coefficient 14 holds the one place to the left of the decimal point and in this case rounded to the nearest tenth in standard scientific notation.
3. (**b**) This is a large positive number; you must count from the last 0 to your left leaving one number in front of the decimal point.
4. (**d**) An order of magnitude is a number rounded to the nearest power of ten.
5. (**d**) If *x* is four orders of magnitude ($10 \times 10 \times 10 \times 10$) greater than *y* (10), then *x* is $10^4/10^0$ or $10^{4-(0)}$, which is 10^4.
6. (**c**) The order of magnitude is $D_{\text{Sun}}/D_{\text{Earth}} = 10^9\text{m} / 10^7\text{ m} = 10^{9-7} = 10^2$
7. (**d**) This answer should be 8, not 7. In other words, the gas giant Jupiter is 8 orders of magnitude larger than the size of a human. $10^8/10^0 = 10^{8-(0)} = 10^8$
8. (**a**) There are seven orders of magnitude between the diameter of the Sun and the typical distance between stars. $D^*/D_{\text{Sun}} = 10^{16}\text{ m}/10^9\text{ m} = 10^{16-9} = 10^7$
9. (**c**) In other words, the human being is six orders of magnitude (6 powers of 10) taller than a grain of dust: $10^0/10^{-6} = 10^{0-(-6)} = 10^6$
10. (**b**) This can be easily demonstrated on the celestial sphere model.
11. (**b**, **c**, **& e**) Note: There must be consistency with the text ($L \sim 10^{24}$ m, pp. 1–19).
12. (**a–d**) The point of this question is that science is important in everything!
13. (**b**) There are approximately 25 years in a generation.
    $2{,}000 - 1{,}900 = 100$ years
14. (**e**) The human is 10 powers of ten taller.
    $10^{0-(-10)} = 10^{10}$ m
15. (**d**) One of the earliest practical uses of astronomy was the planting of crops by the appearance of specific constellations.
16. (**a–e**) These are all characteristics of a pseudoscience.
17. (**b & d**) The point of this question is that science is exact, and religion is based on faith.
18. (**c**) The usual starting point for the scientific method is an observed phenomenon followed by an educated guess (hypothesis), repeated experimentation, recorded results, and then a carefully constructed proposition that explains every piece of data (theory).

19. (**True**) Religious beliefs are based on faith whereas science is based on evidence.
20. (**True**) The laws of physics are the same everywhere in the Universe.

TO THE POINT: QUALITATIVE AND DISCUSSION QUESTIONS

21. As science continues to evolve, astronomers and other scientists are willing to modify or update their findings based on new data.
22. An aspect of modern science that is not accounted for is one's intuition or flashes of insight, although this phenomenon can lead to a hypothesis and ultimately a new discovery by accident.
23. Pseudoscience makes claims that cannot be substantiated through the scientific method. On the other hand, scientific ideas provide evidence or facts by repeatable experimental results.
24. Science has a specific language just like any other discipline. Because most people are afraid of science, experts would need to communicate how to acquire scientific knowledge in layman's terms. Suggestions of books or articles would break the process down for the layperson. In this way, both sides share a common goal.
25. The criteria for a successful scientific theory is a carefully constructed proposition that explains the data, and has been repeatedly tested and confirmed through observation and experimentation.
26. Science has benefitted mankind in several ways: (1) modern medicine has eradicated diseases allowing a person to live a long, healthy life, (2) transportation advances and rapid communication via the internet have connected people and resources more efficiently, and (3) astronomical observations have helped us gain a better understanding of the Universe.
27. As a result of technological advancement, the world faces many challenges, such as climate change, depletion of resources, exhaustion of many of the world's fisheries, the threat of the use of nuclear weapons, and biological terrorism.
28. What is death? Does God exist? These questions go beyond ordinary physics into the metaphysical realm.
29. The basis for astrology is making predictions derived from the apparent positions of the Sun, Moon, and planets among the constellations and their influence on our lives. In contrast, astronomy is a modern science born out of astrology that provides explanations for observed phenomena and teaches us about the Universe.
30. At Newgrange, people were able to mark the winter solstice as the Sun lit the 24-meter passageway and the central chamber once a year. The Moon also marks the calendar in its monthly cycle of 29½ days. The constellations signaled both the passage of a month and the Babylonian calendar year of 12 months. Astronomical observations benefited ancient societies by helping them keep track of time and seasons for practical purposes, including agriculture, religion, and ceremonies and as an aid to navigation.
31. A solstice is a point on the celestial sphere where the ecliptic is farthest north (6/21) or south (12/21) of the celestial equator and the moment at which the Sun returns to that point each year.
32. The writer is trying to alert the reader to the difficulties of science and technology, in particular, climate change. It is a known fact that burning fossil fuels increases the levels of CO_2 in the atmosphere. Carbon

dioxide concentration levels have risen since the 1800s from 275 ppm to 380 ppm today. This in turn has impacted our planet. Scientific data shows us an increase in global temperatures (1.5°F or 0.8°C) over the past 150 years, changes in rainfall patterns and biological habitats, and a rise in sea level that will create climate refugees. What can't be predicted is the long-term effects that climate change will have on our planet over the next century.

33. The night sky is our own window into space and time. When I look up at the starry sky, I feel a deep sense of belonging, a connection to something greater than myself. How did the vast universe come to be, and where are we going? We now know that the expansion of the Universe is accelerating.
34. Religious stories about the creation of Earth and the heavens have been around for thousands of years. At one point, modern empirical science was considered in direct conflict with the beliefs of the Catholic church. This incompatibility still exists today in public discussion regarding how we came to be: Big Bang and evolution versus creationism. In classroom surveys, most students believe in evolution, a few believe the creation story, and even fewer believe in intelligent design or divinely guided evolution. Poll your students on their beliefs and have a classroom discussion on this topic. I did; the results were amazing!
35. I disagree with Martin Rees's statement that humanity has a 50-50 chance of making it into the next century. Here's why. Understanding how science works allows one to make informed decisions. Ingenuity and necessity have always been the mother of invention.

GOING FURTHER: QUANTITATIVE QUESTIONS

36. **Set up:** Add a decimal point behind the last zero and count the number of zeroes leading up to the coefficient.

    Solve: 50,000.

    5.0×10^4 years

    Review: Scientific notation is a shorthand way to write very large or small numbers.
37. **Set up:** Cost of one candy bar is \$1 or 1.0×10^0.

    Cost of luxury car rental is \$10,000 or \$$1.0 \times 10^4$.

    Solve: $10^4/10^0 = 10^{4-(0)} = 10^4$

    Review: The rental of a luxury car is four orders of magnitude more costly.
38. **Set up:** The diameter of Jupiter is about 10^8 m.

    The diameter of Earth is 10^7 m.

    Solve: $D_{Jup}/D_{Ear} = 10^8 \text{ m}/10^7 \text{ m} = 10^{8-7} = 10^1$

    Review: Jupiter is 10 times larger than Earth.
39. **Set up:** The size of a dust particle is 10^{-6} m.

    The size of a proton is 10^{-15} m.

    Solve: Dust particle/proton $= 10^{-6} \text{ m}/10^{-15} \text{ m}$

    $= 10^{-6-(-15)}$

$= 10^9$

Review: A proton is nine orders of magnitude smaller than a dust particle.

40. **Set up:** 1,500 generations would represent how many years?

Each generation on average is 25 years.

Solve:

$$1{,}500 \text{ generations} \times \frac{25 \text{ years}}{1 \text{ generation}} = 37{,}500 \text{ years}$$
$$= 3.75 \times 10^4 \text{ years}$$

Review: The text states that the average number of years in a generation is 25. The word *each* in a word problem means multiply. Question 43 is representative of that statistic.

41. **Set up:** The size of a planetary system is 10^{13} m.

The size of our Milky Way Galaxy is 10^{21} m.

The size of an atom is 10^{-9} m.

The size of a proton is 10^{-15} m.

Solve: galaxy/planetary system $= 10^{21}$ m/10^{13} m

$= 10^{21-13}$

$= 10^8$

atom/proton $= 10^{-9}$ m/10^{-15} m

$= 10^{-9-(-15)}$

$= 10^6$

G/PS vs. atom/proton $= 10^8/10^6$

$= 10^{8-6}$

$= 10^2$

Review: The number of powers of 10 between the sizes of the galaxy and planetary system is larger by two orders of magnitude.

42. **Set up:** *Homo sapiens* first appeared on Earth 1,000,000 (1.0×10^6) years ago. Modern man, defined as culturally and anatomically correct, appeared 50,000 (5.0×10^4) years ago.

Solve: 50,000/1,000,000 = 0.05 or 1/20

Review: The ratio of time between modern man and *Homo sapiens* is 1 : 20.

43. **Set up:** The span of time between my daughter and myself is 37 years. The span between my mother and myself is 17 years. The span of time between my mother and her mother (my grandmother) is 19 years.

Solve: 37 + 17 + 19 = 73 years

By taking the average of the three numbers, 73/3 = 24.3 years.

Review: Answers will vary depending on the span of time between one generation to the next.

44. **Set up:** A million billion billion written in scientific notation

Solve: $10^6 \times 10^9 \times 10^9 = 1.0 \times 10^{24}$

Review: In scientific notation, when numbers are multiplied, the exponents are added.

45. **Set up:** Copernicus published his Sun-centered model in 1543. The number of seconds that have passed to the end of 2015 would be:

Solve: $2015 - 1543 = 472$ years

$$472\ \text{years} \times \frac{365\ \text{days}}{1\ \text{year}} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{60\ \text{min}}{1\ \text{h}} \times \frac{60\ \text{s}}{1\ \text{min}}$$

$$= 1.49 \times 10^{10}\ \text{s}$$

or, use the hint: $3.16 \times 10^{7}\ \text{s/year}$

$$472\ \text{years} \times \frac{3.16 \times 10^{7}\ \text{s}}{1\ \text{year}} = 1.49 \times 10^{10}\ \text{s}$$

Review: There are several ways in which to approach this problem. If a hint was not given, the student would have to rely upon his knowledge of units of time. This is time consuming but puts units of time into perspective; both ways check the other.

CHAPTER 2

A Universe Made, A Universe Discovered

INSTRUCTOR'S NOTES

Chapter 2 discusses the motion of objects visible in our sky and man's attempt to understand it. Major topics include:

- Humanity's fascination with the cosmos over several millennia and how it has led to the evolution of modern astronomy.
- How observing the daily motion of the Sun and stars helps to explain why we see what we see.
- The phases of the Moon in its orbital position around Earth.
- An explanation of planetary motion.
- An explanation as to how myths have shaped humanity's pursuit of knowledge in the sciences and have laid the groundwork for future discoveries.
- How the Greeks established mathematical models to explain physical phenomena instead of relying on myths.

The great thing about astronomy is that you are never too young or too old to observe the night sky. Most people won't become astrophysicists, but the point of this section is that anyone can become an amateur astronomer. This chapter explores the basic motions of the Sun, Moon, stars, and planets in our night sky. It is a fact that most people have never seen really dark skies. If there is an area on your campus that does not have a lot of light pollution, take some binoculars or, even better, a telescope and introduce your students to the objects in the sky. You can also download the iPhone App Sky Guide for about $2. It is an excellent tool to use in locating celestial objects in the night sky. You can begin your classroom discussion about what was seen, where, and the object's motion across the sky.

DISCUSSION POINTS

- The ideal way to explain the motion of the heavens to students is by introducing the concept of the celestial sphere model. This model will help students visualize the position of objects in the sky (some seasonal, others circumpolar) relative to the horizon in an east-to-west daily motion determined by your location on Earth. Make sure students understand that the Sun's path depends on their latitude, longitude, and hemisphere (Figure 2.4). Point out that the celestial sphere is a projection of Earth outward with an equator, poles, and so on, illustrated in Figure 2.6. Note that there are two ways to describe a day: solar or sidereal. Our sky appears to be as an inverted bowl above our heads. Because Earth spins from west to east, objects appear to rise in the east and set in the west. Because the sky appears to be this inverted bowl, the distance between objects on the sky are measured in terms of angular size (see Figure 2.7). Use the small-angle formula in Going Further 2.1 to aid students in gaining an understanding between the difference in an object's apparent size and its (real) physical size.

- Most students already know that the Sun changes its position in the sky during the year (Figure 2.9). They are familiar with the seasons. You may want to review with them the summer and winter solstice and the equinoxes. *Analemma* (Figure 2.8) will definitely be a new word for them.
- Explain how this shape is created with the Sun's motion over the course of a year. Make sure that students understand that seasons are determined by Earth's tilt, as shown in Figure 2.10, and not by how close or far Earth is from the Sun. Ask students how the changing length of days is related to seasons (Figure 2.11). Explain the shape of Earth's orbit around the Sun and give students definitions for *obliquity* and *precession*. Discuss the constellations of the zodiac that lie on either side of the ecliptic (see Figures 2.12 and 2.13).
- At this point, you can talk about the difference between astronomy and astrology. I give my students an astrology test from the newspaper to show the inaccuracy of this pseudoscience. I also tell them that astronomy was born out of astrology and that Kepler cast die (charted signs) for prominent members of society to earn extra money. There are some constellations that are visible all year long as are circumpolar stars. Use the celestial sphere model to show where the constellations and planets lie along the ecliptic and the Sun's annual path against the background of stars.
- We are now ready to discuss the challenges of the Earth-Moon-Sun system as seen from our perspective on Earth (Figure 2.14). This is an excellent opportunity to introduce the scientific method or model. I introduce the phases of the Moon to my students in several ways: animations, a physical demo involving students as the Sun and me as the Moon, and direct observations of the sky. They learn that the Moon's cycle is a natural clock that will allow them to predict when the Moon is highest in the sky. Rising and setting times are testable (Figure 2.15). I explain to students that the 29.5 days for the Moon to cycle through its phases is called the synodic month (see Figure 2.16) versus the sidereal month. The biggest misconception students have about Moon phases is that somehow the cause is Earth casting a shadow on the Moon. You can then introduce the concept of lunar (Figure 2.17) and solar eclipses (Figure 2.18). Figures 2.19–2.22 show the particulars of the Earth-Moon-Sun system. I explain that the reason we don't see eclipses all the time is because the Moon is either 5° above or below the ecliptic.
- Discuss basic planetary motion with students (Figures 2.23 and 2.24). I give them a historical perspective of how astronomy has evolved as a way for the student to see science at work. We are constantly improving our models to match the new data. The Greeks could not explain planetary motion without their retrograde loops (see Figure 2.25). I explain the concept of retrograde motion as described in the textbook. Emphasize to students that it is strictly an appearance.
- Students are very engaged in conversation when discussing astronomical myths versus real science (Figure 2.26). I discuss with my students that cultures around the world have observed astronomical phenomena for

more than 5,000 years. Evidence of their curiosity is found in the structures that dot the landscape of various regions, such as Newgrange and Stonehenge as shown in Figure 2.27. Early human evolution and astronomy can be seen in the cave paintings and tablets that describe astronomical events (Figures 2.28 and 2.29).

- The Greeks were the first to rely on making models of nature instead of using myths as an explanation. Discuss with students the various models by influential Greek scholars. Aristotle proposed the geocentric model of the universe involving crystalline spheres (Figure 2.30). Point out the many contributions made by Greek philosophers, such as the invention of trigonometry, determining Earth's circumference, and explaining stellar parallax (Figure 2.31). Ptolemy's geocentric model (Figure 2.32) is different from Aristotle's. Ask the students in what ways they differ. I find that when I incorporate the history of the philosopher's or scientist's life, students become very interested in learning these confusing concepts.

LEARNING ASTRONOMY BY DOING ASTRONOMY

The following activities are available in the *Learning Astronomy by Doing Astronomy: Collaborative Lecture Activities* workbook. Each LADA activity is designed and classroom tested to illuminate a particular astronomical concept or principle. LADA is available for purchase as a stand-alone, or it is packaged with a Norton astronomy textbook at no extra cost.

Activity 3

WHERE ON EARTH ARE YOU?

In this activity you will learn about Earth's coordinate system, its projection onto the celestial sphere, and how Earth's orbit around the Sun determines the seasons.

- Text reference: Section 2.2
- LADA reference: Activity 1

Activity 18

FINDING DISTANCES TO STARS USING PARALLAX MEASUREMENTS

In this activity you will determine the relationship between distance and the apparent motion of a nearby object when viewed from two vantage points.

- Text reference: Section 2.6

Activity 19

STUDYING THE PHASES OF THE MOON FROM A PRIVILEGED VIEW

In this activity the student will develop spatial reasoning skills. They will understand the connection between the Earth-Moon-Sun system and how Moon phases and eclipses are caused.

- Text reference: Section 2.3

INTERACTIVE SIMULATIONS

Almost 50 new, easy-to-use interactive simulations allow students to play with physical phenomena. This Instructor's Manual provides learning goals outlining ways in which students can use each simulation to further his or her understanding of each topic.

MOTIONS OF THE SKY

- Students will learn that the latitude of an observer determines the location of the rising and setting sun on the horizon.
- Students will see how the position of the Sun and its path across the sky changes with the season.
- Students will see the direct connection between the movement of the Sun across the sky and the rotation of the Earth.

SMALL ANGLE FORMULA

- Students will learn that all objects have two sizes associated with them: a physical size (intrinsic) and an angular size (apparent).
- Students will learn that there is a simple mathematical relationship between the apparent angular size of an object, its physical size, and the distance to the object (i.e., an object will appear larger when closer to the observer).
- Students will learn to use knowledge of the intrinsic size of an object and its apparent angular size to determine the distance to the object.
- Students will learn to use knowledge of the distance to an object and a measure of its apparent angular size to determine the physical size of the object.

LUNAR PHASES

- Students will see that one-half of the Moon is always illuminated by the Sun.
- Students will connect phases of the Moon to the Moon-Earth-Sun angle, which determines what portion of the illuminated hemisphere Moon is visible from Earth.
- (optional) Students will learn to connect the position of the Moon in the sky with a given phase and time of day.

SEASONS

- Students will see the connection between the tilt of Earth on its axis and the cycle associated with the seasons.
- Students will see that when the Sun is high in the sky and the days are longest for northern latitudes, the North Pole of the Earth is tilted toward the Sun. This happens during the summer months.

Retrograde Motion

- Students will learn two possible explanations for the planets changing course from their mostly west-to-east seasonal progression to a brief east-to-west progression known as retrograde motion.
- Students will see that the change in direction can be explained by a Sun-centered model in which the Earth orbits closer (and faster) to the Sun, and therefore laps the outer planet during every orbit.
- Students will also see how placing the orbit of the planet on an epicycle and having it conduct two orbits, one about the Earth and the other about the center of the epicycle, can produce a very similar retrograde motion.

CHAPTER 2

END-OF-CHAPTER SOLUTIONS

NARROW IT DOWN: MULTIPLE-CHOICE QUESTIONS

1. (**c**) For the observer at latitude 12° north, the Sun will appear higher in the sky at noon than the person at the higher latitude.
2. (**e**) Polaris is considered a circumpolar star. This means it never rises or sets; its daily motion is a counter-clockwise circle around the north celestial pole.
3. (**c**) The apparent size is inversely proportional to the distance. A large object is close by and a small object is far away. In other words, moving an object farther away, *D*, reduces its angular size, *a*.
4. (**c**) This is because Earth's axis in the Northern Hemisphere is tipped most directly towards the Sun.
5. (**b**) There are 88 official constellations that cover the celestial sphere. The stars are at different distances but are in the same region of sky. The textbook definition for a *constellation* is a recognizable group of stars and the area encompassing them.
6. (**a**) An observer at the North Pole would observe the same constellations all year round. These are called circumpolar stars.
7. (**a**) The warm summertime temperature is due to Earth's axis tilted towards the Sun. The axis of tilt causes sunlight to strike the ground at a steeper angle. Earth's tilt also affects the changing length of the days; the Sun rises earlier and sets later.
8. (**d**) In the waning phase, the lit portion of the Moon faces the eastern horizon.
9. (**c**) To tell the time of day by the phase of the Moon, imagine a clock face with noon (new Moon) being at the three o'clock position, 6 P.M. (first quarter) position, midnight (full Moon) nine o'clock position, which is the highest point in the sky for that phase. If you want to determine Moonrise and Moonset times, the four phases mentioned above are six hours apart. Therefore, if the full Moon is highest in the sky at midnight, it had to rise at 6 P.M. and will set twelve hours later at 6 A.M.
10. (**a, b**) Synodic or lunar month is the time required for a complete cycle of lunar phase—29½ days. The sidereal month is the time required for the Moon to orbit Earth once with respect to the background of fixed stars—27¼ days.
11. (**a**) Earth's rotation from west to east (counter-clockwise) makes the planets appear to rise in the east and set in the west against the background of fixed stars. Normal or prograde motion is west to east motion.
12. (**c–d**) Stonehenge had practical applications for ancient peoples. They were able to tell when to plant and harvest crops and when summer and winter would begin. In addition, the burial mounds around Stonehenge were used for religious purposes.

13. (**a**) Thales of Miletus (624–547 BCE) rejected supernatural explanations for phenomena and claimed that the world could be understood through reason alone.
14. (**b**) Eratosthenes was able to measure the circumference of Earth by comparing the maximum altitude of the Sun in two cities—Alexandria and Syene (modern day Aswan), which lies at latitude 23.5° north.
15. (**b–d**) We can measure parallax if we know the precise value of the star's annual shift. (Choice **a** cannot be true because all of the stars are so far away that their parallax angles are very small. You'd need a telescope. This is why the ancient Greeks could never measure parallax with the naked eye.)
16. (**b & c**) The Ptolemaic model was used to explain the apparent motion of the planets by positioning smaller circles (epicycles) that turned upon larger circles.
17. (**c**) The Moon is directly behind the Earth (full Moon) and must be on or near the node for a lunar eclipse to occur.
18. (**a & c**) When the Sun and Moon (farthest in its orbit from Earth) appear to be almost identical in in size, you have an annular eclipse. You don't always see a total eclipse because the disk of the Moon does not block all of the Sun's disk.
19. (**False**) The Sun and Earth are always aligned, but the Moon is slightly inclined to the ecliptic plane either 5° above or below. This is the plane of Earth's orbit as it revolves around the Sun. Eclipses occur on the full Moon (lunar) and new Moon (solar). The Moon has to be near or on the node for an eclipse to occur.
20. (**b**) Due to the inclination of the Moon's orbit, it spends most of its time either above or below the ecliptic. The Moon crosses through the the nodes that lie along a nearly straight line with the Sun and Earth twice a year.

TO THE POINT: QUALITATIVE AND DISCUSSION QUESTIONS

21. Between 70,000 years ago and 50,000 years ago, the advancement of human civilization and culture started with tool making and an explosion in art production. During the Neolithic period, humans had been hunter-gatherers, but a change in climate allowed them to establish permanent settlements and an agrarian society was born. City life allowed inhabitants to develop specialized skills as leatherworkers or metalsmiths. A priestly class became our first true astronomers by keeping records of lunar phases, lunar and solar eclipses, and planetary motions. And living closer together in cities meant people could pass these records on from generation to generation.
22. Ancients built megaliths as a result of watching the motions of the skies. These huge structures had a practical role to play in society, such as knowing when to plant and harvest crops and when the rains and cold weather would come. Besides using the skies as a clock, some of the sites, such as Stonehenge, were used for religious purposes.
23. Each of the following Greek thinkers made significant contributions to society:
    a) Thales of Miletus (624–546 BCE) proposed the first known model of the universe without relying on supernatural means.
    b) Pythagoras (570–500 BCE) was an influential mathematician-philosopher known for the development of

geometry.

c) Plato (428–347 BCE) was known for the quote to his students, “Save the appearance,” meaning to develop a mathematical model for the motions of the planets.

d) Aristotle (384–322 BCE) attempted to deduce how the world worked. In Aristotle’s model, the terrestrial, or sublunar, domain was the realm of change and decay. He also argued that the cosmos was divided into two parts: Earth at the center, and surrounding Earth was a celestial domain of planets and stars.

e) Eudoxus (409–356 BCE) argued the nested sphere model composed the heavens. Each sphere was centered on Earth and rotated with a different speed.

f) Aristarchus (310–230 BCE) was the first to propose the heliocentric model in which Earth was just another planet orbiting the Sun.

g) Eratosthenes (276–195 BCE) was a philosopher and astronomer who determined the circumference of Earth.

h) Hipparchus (190–120 BCE) is credited with the development of trigonometry.

24. If the Moon crosses the meridian at midnight (setting time), the phase would be Full Moon. That means that the Moon rose at 6 P.M. and set at 6 A.M.
25. If Betelgeuse rose on the eastern horizon at 8:00 P.M. one month ago, now it would rise a bit later and a little more south of east.
26. Retrograde motion is backward motion compared to normal motion (west to east) against the background of star. Ptolemy’s model placed small circles called epicycles upon larger circles to explain retrograde motion. The back of the loop on the smaller circle is where east to west (westward) motion occurred.
27. The celestial sphere is an imaginary model on which objects in the sky appear to reside relative to Earth.

    It is useful as a navigation tool. On the sphere is marked the north and south celestial poles, the celestial equator, declination and right ascension, the ecliptic, and the vernal equinox.
28. You can see every part of the celestial sphere all year round from the equator.
29. During the summer, the path of the Sun is higher closer to the zenith than in winter. The Sun rises earlier and sets later. There is more daylight in the summer because sunlight strikes the ground at a steeper angle. There is less daylight in the winter because Earth’s rotational axis in the Northern Hemisphere points away from the Sun. This means that the Sun rises later and sets earlier, making the days shorter. A shorter day is a result of sunlight striking the ground at a shallower angle.
30. An *analemma* is the motion of the Sun mapped out in the sky over the course of a year. If you were to take a picture of the Sun at the same position every day, it would trace out a figure-eight shape in the sky. On June 21, the Sun would be close to the highest point on the figure eight (noontime Sun closest to the zenith). On December 21 the Sun would be very close to the lowest point of the figure eight, meaning that the Sun was closer to the southern horizon at noon on that day than on any other day of the year. On both March 21 and September 21, the Sun would be close to the middle of the figure eight.
31. If Earth’s orbit had no tilt relative to the plane of its orbit, there would be no seasons. Seasons are caused by the tilt of Earth’s axis, not by Earth’s distance from the Sun.
32. If Earth’s axis had a more significant tilt relative to the plane of its orbit, the seasons would be more ex-

treme.

33. The sidereal month is the time it takes for the Moon to orbit Earth with respect to the background of fixed stars, which is approximately 27¼ days. The synodic month is the time it takes the Moon to complete all eight phases, which is approximately 29½ days. It is about 2 days longer than the sidereal month.
34. Aristotle's model of the universe was based on crystalline spheres, unchanging and perfect. His model put Earth at the center (geocentric model) with each sphere guiding the planets through perfectly circular orbits at constant or uniform velocity. Ptolemy's model was geocentric too. The difference between the two is that Ptolemy was able to give an explanation for retrograde motion with his epicycles, and it worked! He placed the Sun, Moon, and planets on circles orbiting Earth. Each planet moved with uniform speed on a smaller circle, called an epicycle. It was the center of the epicycle that then moved with uniform speed around Earth. In addition to his epicycles to match observations, Ptolemy in some cases used smaller epicycles, called equants, with the main epicycle. Every planet's uniform motion was adjusted to Earth being slightly off center from the planetary orbits in order to account for better accuracy of predictions.
35. Aristotle was dead long before Aristarchus was born. Aristarchus did propose the heliocentric model. Aristotle argued that the absence of parallax for the stars in the sky implied that the Earth must be the center of the universe and not the sun. Since parallax is hard to observe due to the great distance of stars, Aristarchus was not able to explain the lack of observable parallax during his time (Aristotle's argument). He believed that the sun was at the center of the universe because it was bigger in size than the Earth.

GOING FURTHER: QUANTITATIVE QUESTIONS

36. **Set up:** Each degree is divided into 60 arcminutes (abbreviated 60′) and divide each arcminute into 60 arcseconds (abbreviated 60″).

    Solve:
    $$1^\circ = 3{,}600'' \, (60 \times 60)$$
    $$4^\circ \times \frac{3{,}600}{1^\circ} = 14{,}400'' \text{ or } 1.44 \times 10^{4}{}''$$

    Review: You can determine how many arcseconds are in one degree by remembering that 1° = 3,600″ (60 × 60). The next step is to cancel out like units in order to get the units asked for in the word problem.
37. **Set up:** The horizon = 0° altitude and the zenith = 90° altitude. The difference would be:

    Solve: 90° – 0° = 90°

    Review: The horizon is an imaginary line at 0° altitude. The zenith is directly overhead at an altitude of 90°.
38. **Set up:** The Sun is at an altitude of 80°. From summer solstice to the autumn equinox the Sun will drift more to the south.

    Solve: 180° – 90° – 80° = 10°

    Review: On the summer solstice, the Sun follows a longer and higher path through the sky. As the months pass, the Sun will start to drift southward, reaching the autumn equinox (9/21) on the celestial equator before continuing its journey southward until the winter solstice (12/21). Rising continually south of due east, the Sun sets considerably south of due west now. In the Northern Hemisphere, mid- and high-latitude observers will see the Sun quite low in the southern sky. After the winter solstice, the Sun will start the cycle all over again. The Sun's path will drift more northward.

39. **Set up:** The observer is at latitude 44° north.

    Solve: latitude $+44°$ north

    Review: The declination is analogous to latitude on Earth. It runs north and south on the celestial sphere. A plus sign is used for the Northern Hemisphere and a negative sign for Southern Hemisphere.

40. **Set up:** The number of degrees that the Moon roughly covers in the sky in one day is:

    Solve: 360° divided by 27.3 = 13.2° (sidereal month)

    360° divided by 29.5 = 12.2° (synodic month)

    Review: This means that the Moon moves an angular distance equal to its own diameter or, in other words, about 30 arc minutes in an hour.

41. **Set up:** First take the complete Moon cycle of 27⅓ or 29.5 days and divide by the number of phases, which is eight. This gives the amount of time the Moon is in each phase, 3.4125 or 3.6875 days. Between new Moon and full Moon, you can calculate the number of days:

    Solve: $3.4125 \times 4 = 13.6375$ (sidereal month)

    $3.6875 \times 4 = 14.75$ (synodic month)

    Review: Starting with the new Moon, the phases are: (1) waxing crescent, (2) first quarter, (3) waxing gibbous, (4) full Moon, (5) waning gibbous, (6) third quarter, (7) waning crescent, and back to (8) new Moon.

42. **Set up:** Mars has an angular size of 18″, and the diameter of Mars (*d*) is 6,792 km. What is its distance from Earth? Using the small-angle formula: a = 206,265″ × d/D, solve for *D*.

    Solve:
    $$D = \frac{(206{,}265'')(d)}{a''} = \frac{(206{,}265'')(6{,}792\ \text{km})}{18''}$$
    $$= 77{,}830{,}660\ \text{km} = 7.78 \times 10^{7}\ \text{km}$$

    Review: When using the small-angle formula, you can calculate any of the variables as long as you know the value of any two variables. The distance *D* and the physical size *d* must be expressed in the units of length (meters or kilometers). The variable *a* refers to our field of view and is expressed in units of degrees. The next step is to cancel out like units in order to get the desired units asked for in the word problem.

43. **Set up:** The angular diameter of a globular cluster is *a* = 20″ and *D* = 25,000 light-years. Find the diameter in light-years. Using the small-angle formula: a = 206,265″ × d/D, solve for *d*.

$a = 20'$, $1' = 60''$ and $D = 25{,}000$ light-years

$$20' \times \frac{60''}{1'} = 1{,}200''$$

Solve:

$$\begin{aligned} d &= \frac{a}{206{,}265} \times D \\ &= \frac{1{,}200''}{206{,}265''} \times 25{,}000 \text{ ly} \\ &= \left(5.82 \times 10^{-3}\right)\left(25{,}000 \text{ ly}\right) \\ &= 145.4 \text{ light-years (ly)} \end{aligned}$$

Review: When using the small-angle formula, you can calculate any of the variables as long as you know the value of any two variables. The distance *D* and the physical size *d* must be expressed in the units of length (meters or kilometers), but in this case *D* is expressed in light-years (ly). The variable *a* refers to our field of view and is expressed in units of degrees.

The next step is to cancel out like units in order to get the units asked for in the word problem.

44. **Set up:** Given *D* = 200 meters and *d* = 0.5 meters, Find the angular width, *a*, in arcseconds and in arcminutes. Using the small-angle formula: a = 206,265″ × d/D, solve for *a*.

Solve:

$$\begin{aligned} a &= 206{,}265'' \times \frac{0.5 \text{ m}}{200 \text{ m}} \\ &= 206{,}265'' \times 2.5 \times 10^{-3} \\ &= 515.66'' \text{ or } 5.2 \times 10^{2}'' \text{ (arcseconds)} \end{aligned}$$

$$\text{arcminutes} = 515.66'' \times \frac{1'}{60''} = 8.59'$$

Review: Each degree is divided into 60 arcminutes (abbreviated 60′) and each arcminute is divided into 60 arcseconds (abbreviated 60″). 1° = 3,600″ (60 × 60)

45. **Set up:** Given the core diameter of a comet, *d* = 40 km, *D* = 137 million km away, determine in arcseconds how big the core appeared on Earth. Using the small-angle formula: a = 206,265″ × d/D, solve for *a*.

Solve:

$$\begin{aligned} a &= 206{,}265'' \times \frac{40 \text{ km}}{1.37 \times 10^{8} \text{ km}} \\ &= 206{,}265'' \times 2.92 \times 10^{-7} \\ &= 0.06'' \end{aligned}$$

Review: When using the small-angle formula, you can calculate any of the variables as long as you know the value of any two variables. The distance *D* and the physical size *d* must be expressed in the units of length (meters or kilometers). The variable *a* refers to our field of view and is expressed in units of degrees. The next step is to cancel out like units in order to obtain the desired units in the word problem.