CHAPTER 2

Section 2-1

2-1. Let *a* and *b* denote a part above and below the specification, respectively.
$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

2-2. Let *e* and *o* denote a bit in error and not in error (*o* denotes okay), respectively.

$$S = \begin{Bmatrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{Bmatrix}$$

2-3. Let *a* denote an acceptable power supply.
Let *f*, *m*, and *c* denote a power supply that has a functional, minor, or cosmetic error, respectively.
$S = \{a, f, m, c\}$

2-4. $S = \{0,1,2,...\}$ = set of nonnegative integers

2-5. Let *y* and *n* denote a web site that contains and does not contain banner ads.
The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is $S = \{yynnynyyynnynynnnnyynnyy\}$

2-6. A vector with three components can describe the three digits of the ammeter. Each digit can be 0,1,2,...,9. The sample space *S* is 1000 possible three digit integers, $S = \{000, 001, ..., 999\}$

2-7. *S* is the sample space of 100 possible two digit integers.

2-8. Let an ordered pair of numbers, such as 43 denote the response on the first and second question. Then, S consists of the 25 ordered pairs $\{11, 12, ..., 55\}$

2-9. $S = \{0,1,2,...,1E09\}$ in ppb.

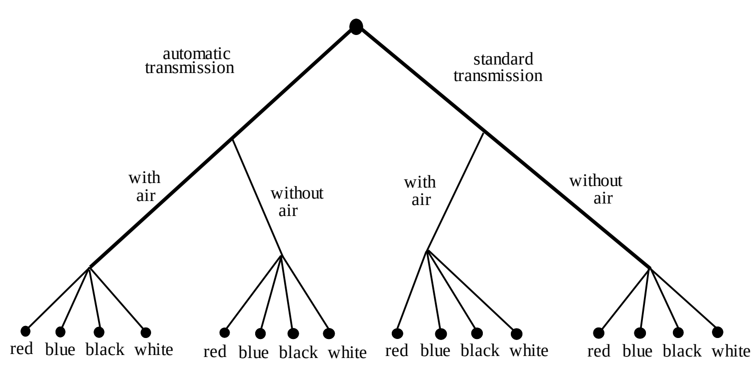
2-10. $S = \{0,1,2,...,\}$ in milliseconds

2-11. $S = \{1.0, 1.1, 1.2, \ldots 14.0\}$

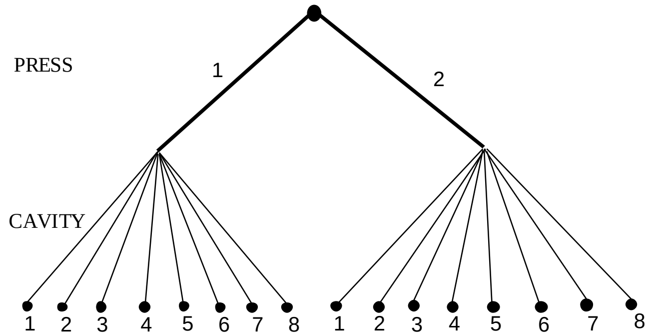
2-12. Let *s*, *m*, and *l* denote small, medium, and large, respectively. Then *S* = {*s*, *m*, *l*, *ss*, *sm*, *sl*, ….}

2-13 $S = \{0,1,2,...,\}$ in milliseconds.

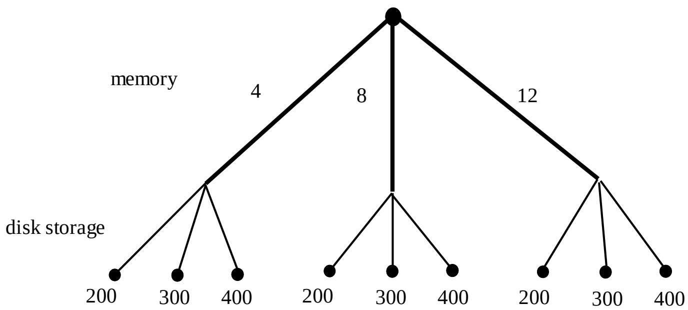
2-14.



2-15.



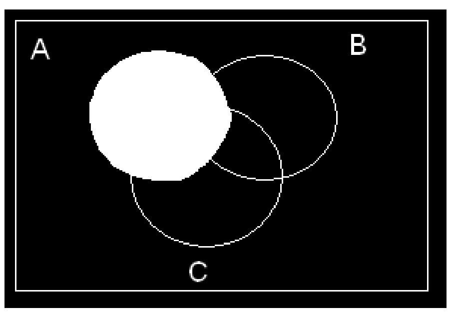
2-16.



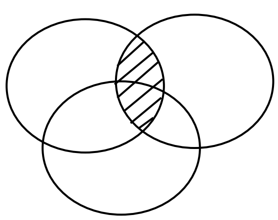
2-17. Let *c* and *b* denote connect and busy, respectively. Then $S = \{c, bc, bbc, bbbc, bbbbc, \ldots\}$

2-18. $S = \{s, fs, ffs, fffS, fffFS, fffFFS, fffFFFA\}$

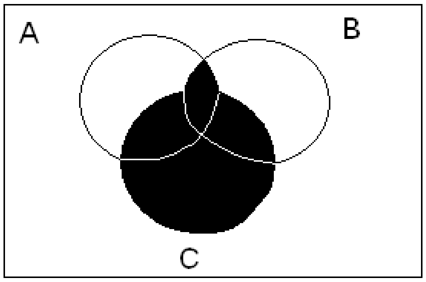
2-19. a)



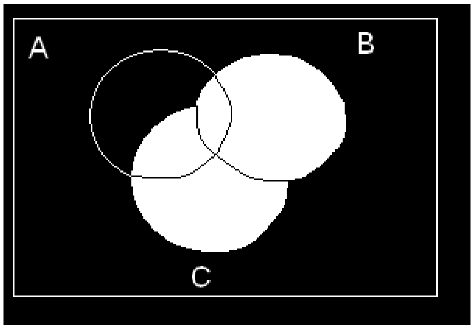
b)



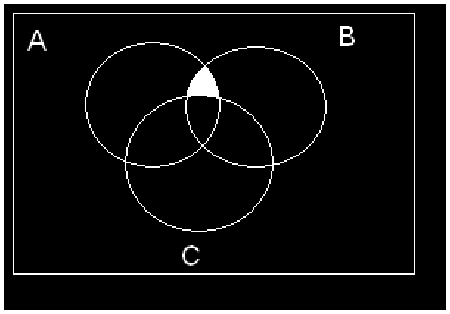
c)



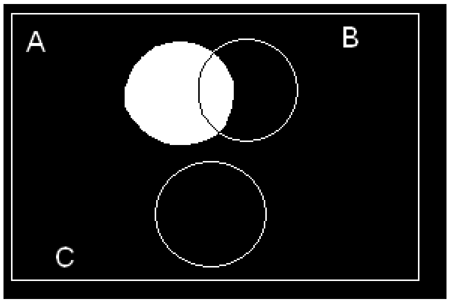
d)



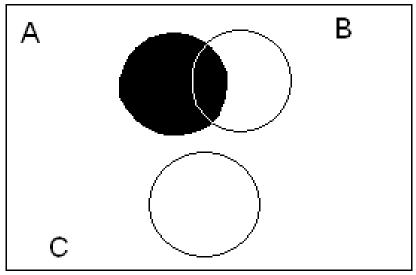
e)



2-20. a)



b)



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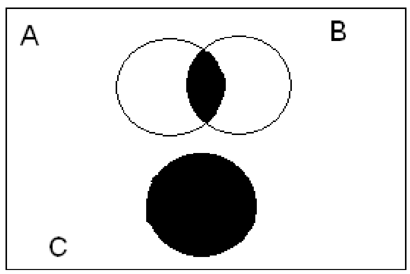

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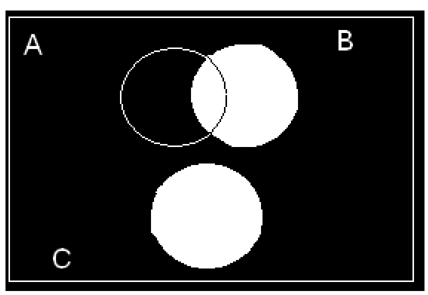

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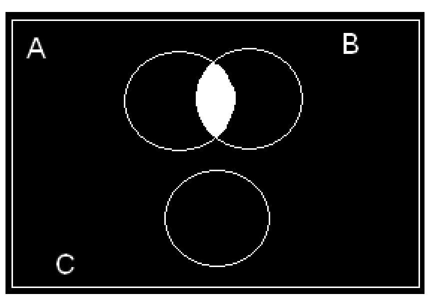
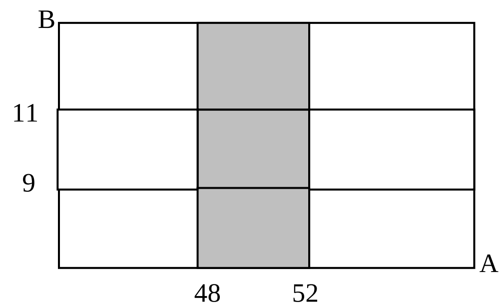


2-21. a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.
Let X denote the weight.
A is the event that $X > 11$ B is the event that $X \le 15$ C is the event that $8 \le X < 12$
$S = \{0, 1, 2, 3, \ldots\}$
b) S
c) $11 < X \le 15$ or $\{12, 13, 14, 15\}$
d) $X \le 11$ or $\{0, 1, 2, \ldots, 11\}$
e) S
f) $A \cup C$ contains the values of X such that: $X \ge 8$
Thus $(A \cup C)'$ contains the values of X such that: $X < 8$ or $\{0, 1, 2, \ldots, 7\}$
g) $\varnothing$
h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or $\varnothing$.
i) $B \cap C$ is the event $8 \le X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \ge 8$ or $\{8, 9, 10, \ldots\}$

2-22. a)



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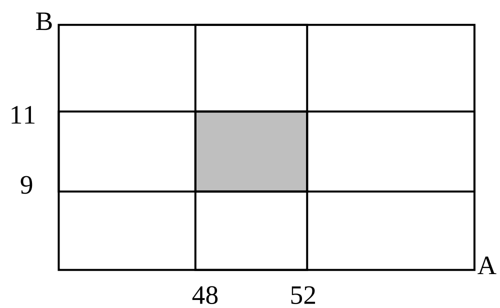
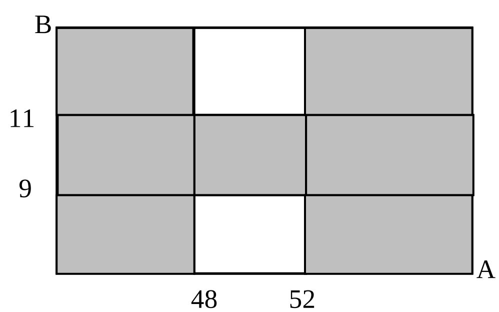
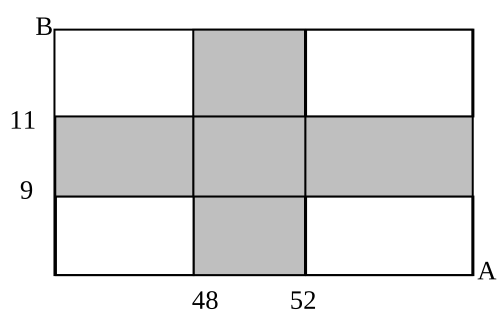


c)



d)



e) If the events are mutually exclusive, then A∩B is the null set. Therefore, the process does not produce product parts with X = 50 cm and Y = 10 cm. The process would not be successful.

2-23. Let d and o denote a distorted bit and one that is not distorted (o denotes okay), respectively.

a) $$S = \left\{ \begin{matrix} ddddd, ddddo, dddod, ddodd, \\ doddd, odddd, dddoo, ddodo, \\ doddo, odddo, ddood, dodod, \\ oddod, doodd, ododd, ooddd, \\ ddooo, dodoo, ooddo, doood, \\ ooodd, ododo, oddoo, odood, \\ doodo, doooo, odooo, oodoo, \\ ooodo, ooood, oodod, ooooo \end{matrix} \right\}$$

b) Ai's are not mutually exclusive.

$$A_1 \cap A_2 = \left\{ \begin{matrix} ddddd, ddddo, dddod, ddodd, \\ dddoo, ddodo, ddood, ddooo \end{matrix} \right\}$$

c) $$A_1 = \left\{ \begin{matrix} ddddd, ddddo, dddod, ddodd, \\ doddd, dddoo, ddodo, doddo, \\ ddood, dodod, doodd, ddooo, \\ dodoo, doood, doodo, doooo \end{matrix} \right\}$$

d) $$A_1' = \left\{ \begin{matrix} odddd \ \ odddo \ \ oddod \ \ ododd \\ ooddd \ \ ooddo \ \ ooodd \ \ ododo \\ oddoo \ \ odood \ \ odooo \ \ oodoo \\ ooodo \ \ ooood \ \ oodod \ \ ooooo \end{matrix} \right\}$$

e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{ddddd, ddddo\}$

f) $$(A_1 \cap A_2) \cup (A_3 \cap A_4) = \left\{ \begin{matrix} ddddd, ddddo, dddod, ddodd, dddoo, \\ ddodo, ddood, ddooo, doddd, odddo, \\ odddd, doddo, ooddd, ooddo \end{matrix} \right\}$$

2-24

Let w denote the wavelength. The sample space is {w | w = 0, 1, 2, …}
(a) A={w | w = 675, 676, …, 700 nm}
(b) B={ w | w = 450, 451, …, 500 nm}
(c) $A \cap B = \Phi$
(d) $A \cup B$ = {w | w = 450, 451, …, 500, 675, 676, …, 700 nm}

2-25

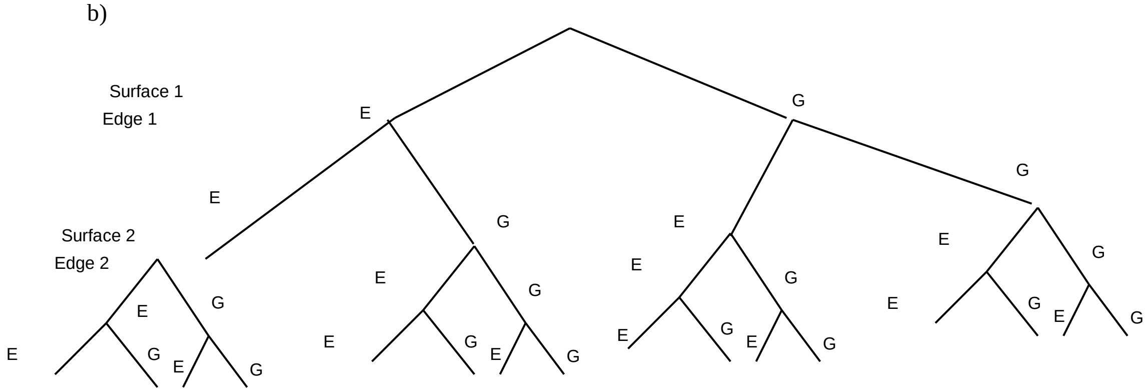
Let P and N denote positive and negative, respectively.
The sample space is {PPP, PPN, PNP, NPP, PNN, NPN, NNP, NNN}.

(a) A={ PPP }

(b) B={ NNN }
(c) $A \cap B = \Phi$
(d) $A \cup B =$ { PPP , NNN }

2-26. A $\cap$ B = 70, A′ = 15, A $\cup$ B = 95

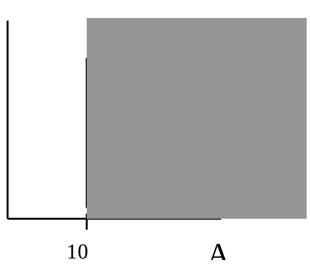
2-27. a) $A' \cap B$ = 10, B' =10, $A \cup B$ = 94
b)



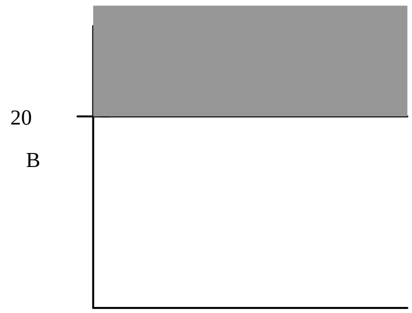
2-28. $A' \cap B$ = 55, B' =21, $A \cup B$ = 85
2-29. a) $A' = \{x \mid x \geq 72.5\}$
b) $B' = \{x \mid x \leq 52.5\}$
c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$
d) $A \cup B = \{x \mid x > 0\}$
2-30. a) {*ab*, *ac*, *ad*, *bc*, *bd*, *cd*, *ba*, *ca*, *da*, *cb*, *db*, *dc*}
b) {*ab*, *ac*, *ad*, *ae*, *af*, *ag*, *bc*, *bd*, *be*, *bf*, *bg*, *cd*, *ce*, *cf*, *cg*, *de*, *df*, *dg*, *ef*, *eg*, *fg*, *ba*, *ca*, *da*, *ea*, *fa*, *ga*, *cb*, *db*, *eb*, *fb*, *gb*, *dc*, *ec*, *fc*, *gc*, *ed*, *fd*, *gd*, *fe*, *ge*, *gf*}
c) Let *d* and *g* denote defective and good, respectively. Then *S* = {*gg*, *gd*, *dg*, *dd*}
d) S = {*gd*, *dg*, *gg*}
2-31. Let *g* denote a good board, *m* a board with minor defects, and *j* a board with major defects.
a) *S* = {*gg*, *gm*, *gj*, *mg*, *mm*, *mj*, *jg*, *jm*, *jj*}
b) *S* ={*gg*,*gm*,*gj*,*mg*,*mm*,*mj*,*jg*,*jm*}

2-32. a) The sample space contains all points in the nonnegative *X*-*Y* plane.

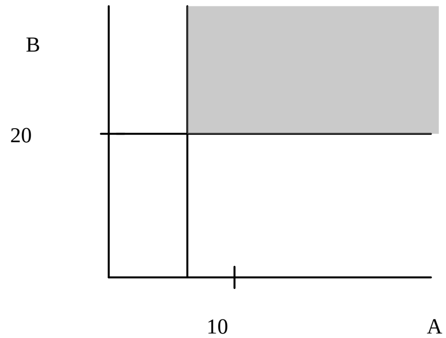
b)



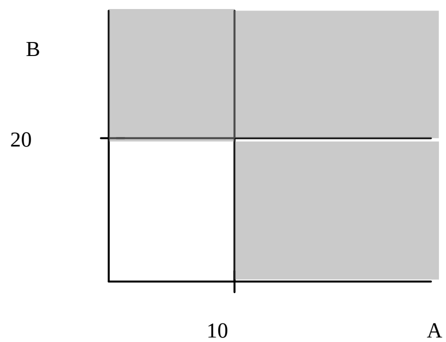
c)



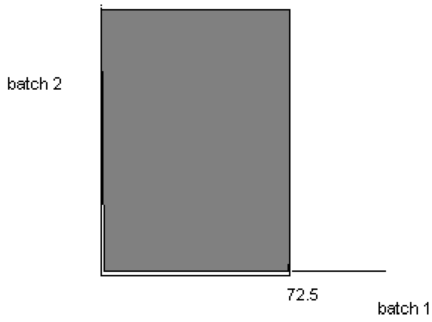
d)



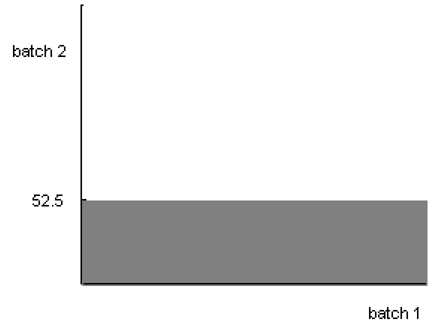
e)



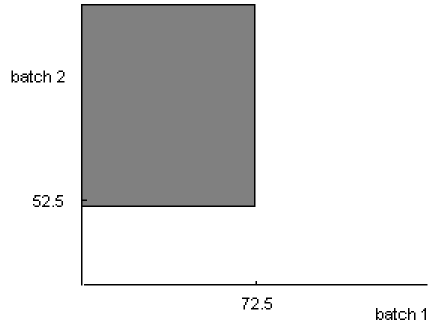
2-33. a)



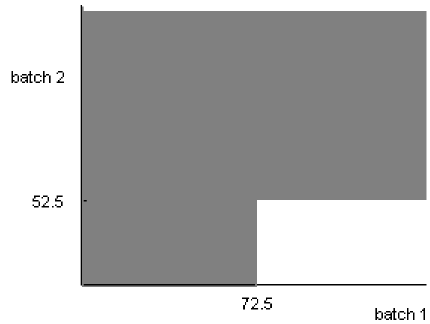
b)



c)



d)



2-34. $2^{10} = 1024$

2-35. From the multiplication rule, the answer is $5 \times 3 \times 5 \times 2 = 150$

2-36. From the multiplication rule, $3 \times 4 \times 3 = 36$

2-37. From the multiplication rule, $4 \times 4 \times 3 \times 4 = 192$

2-38. From equation 2-1, the answer is 15! = 1,307,674,368,000

2-39. From the multiplication rule and equation 2-1, the answer is 5!5! = 14,400

2-40. From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2-41. a) From equation 2-4, the number of samples of size five is $\binom{140}{5} = \frac{140!}{5!135!} = 416,965,528$

b) There are 10 ways of selecting one nonconforming chip and there are $\binom{130}{4} = \frac{130!}{4!126!} = 11,358,880$ ways of selecting four conforming chips. Therefore, the number of samples that contain exactly one nonconforming chip is $10 \times \binom{130}{4} = 113,588,800$

c) The number of samples that contain at least one nonconforming chip is the total number of samples $\binom{140}{5}$ minus the number of samples that contain no nonconforming chips $\binom{130}{5}$. That is

$$\binom{140}{5} - \binom{130}{5} = \frac{140!}{5!135!} - \frac{130!}{5!125!} = 130,721,752$$

2-42. a) If the chips are of different types, then every arrangement of 5 locations selected from the 15 results in a different layout. Therefore, $P_5^{15} = \frac{15!}{10!} = 360,360$ layouts are possible.

b) If the chips are of the same type, then every subset of 5 locations chosen from the 15 results in a different layout. Therefore, $\binom{15}{5} = \frac{15!}{5!10!} = 3003$ layouts are possible.

2-43. a) $\frac{8!}{2!6!} = 28$ sequences are possible.

b) $\frac{8!}{1!1!1!1!1!1!2!} = 20160$ sequences are possible.

c) 6! = 720 sequences are possible.

2-44. a) Every arrangement selected from the 10 different components comprises a different design. Therefore, $10! = 3,628,800$ designs are possible.

b) 7 components are the same, others are different, $\frac{10!}{7!1!1!1!} = 720$ designs are possible.

c) $\frac{10!}{3!4!} = 25200$ designs are possible.

2-45. a) From the multiplication rule, $10^3 = 1000$ prefixes are possible

b) From the multiplication rule, $8 \times 2 \times 10 = 160$ are possible

c) Every arrangement of three digits selected from the 10 digits results in a possible prefix.

$P_3^{10} = \frac{10!}{7!} = 720$ prefixes are possible.

2-46. a) From the multiplication rule, $2^8 = 256$ bytes are possible

b) From the multiplication rule, $2^7 = 128$ bytes are possible

2-47. a) The total number of samples possible is $\binom{24}{4} = \frac{24!}{4!20!} = 10,626.$ The number of samples in which exactly one tank has high viscosity is $\binom{6}{1}\binom{18}{3} = \frac{6!}{1!5!} \times \frac{18!}{3!15!} = 4896$. Therefore, the probability is

$$\frac{4896}{10626} = 0.461$$

b) The number of samples that contain no tank with high viscosity is $\binom{18}{4} = \frac{18!}{4!14!} = 3060.$ Therefore, the requested probability is $1-\frac{3060}{10626}=0.712$.

c) The number of samples that meet the requirements is $\binom{6}{1}\binom{4}{1}\binom{14}{2}=\frac{6!}{1!5!}\times\frac{4!}{1!3!}\times\frac{14!}{2!12!}=2184$.

Therefore, the probability is $\frac{2184}{10626}=0.206$

2-48. a) The total number of samples is $\binom{12}{3}=\frac{12!}{3!9!}=220$. The number of samples that result in one nonconforming part is $\binom{2}{1}\binom{10}{2}=\frac{2!}{1!1!}\times\frac{10!}{2!8!}=90.$ Therefore, the requested probability is 90/220 = 0.409.

b) The number of samples with no nonconforming part is $\binom{10}{3}=\frac{10!}{3!7!}=120.$ The probability of at least one nonconforming part is $1-\frac{120}{220}=0.455$.

2-49. The number of ways to select two parts from 40 is $\binom{40}{2}$ and the number of ways to select two defective parts from the 5 defectives ones is $\binom{5}{2}$. Therefore the probability is $\frac{\binom{5}{2}}{\binom{40}{2}}=\frac{10}{78}=0.01282$

2-50. a) $A \cap B$ = 56
b) A' = 36 + 56 = 92
c) $A \cup B$ = 40 + 12 + 16 + 44 + 56 = 168
d) $A \cup B'$ = 40+12+16+44+36=148
e) $A' \cap B'$ = 36

2-51. Total number of possible designs = 5 × 3 × 5 × 3 × 5 = 1125

2-52. a) $A \cap B$ = 1277
b) A' = 22252 – 5292 = 16960
c) $A \cup B$ = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915
d) $A \cup B'$ = 195 + 270 + 246 + 242+ 3820 + 5163 + 4728 + 3103 + 1277 = 19044
e) $A' \cap B'$ = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752

2-53. a) $A \cap B$ = 170 + 443 + 60 = 673
b) A' = 28 + 363 + 309 + 933 + 39 = 1672
c) $A \cup B$ = 1685 + 3733 + 1403 + 2 + 14 + 29 + 46 + 3 = 6915
d) $A \cup B'$ = 1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3) = 8399
e) $A' \cap B'$ = 28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3 = 1578

Section 2-2

2-54. All outcomes are equally likely
a) P(A) = 2/5
b) P(B) = 3/5
c) P(A') = 3/5
d) $P(A\cup B) = 1$
e) $P(A\cap B) = P(\varnothing)= 0$

2-55. a) P(A) = 0.5
b) P(B) = 0.7
c) P(A') = 0.5
d) $P(A\cup B) = 1$
e) $P(A\cap B) = 0.2$

2-56. a) 0.6 + 0.2 = 0.8
b) 0.2 + 0.6 = 0.8

2-57. a) 1/10
b) 6/10

2-58. a) S = {1, 2, 3, 4, 5, 6}
b) 1/6
c) 2/6
d) 5/6

2-59. a) S = {1,2,3,4,5,6,7,8}
b) 2/8
c) 6/8

2-60. The sample space is {95, 96, 97,…, 103, and 104}.
(a) Because the replicates are equally likely to indicate from 95 to 104 mL, the probability that equivalence is indicated at 100 mL is 0.1.
(b) The event that equivalence is indicated at less than 100 mL is {95, 96, 97, 98, 99}. The probability that the event occurs is 0.5.
(c) The event that equivalence is indicated between 98 and 102 mL is {98, 99, 100, 101, 102}. The probability that the event occurs is 0.5.

2-61. The sample space is {0, +2, +3, and +4}.
(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.36 + 0.34 + 0.15 = 0.85.
(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.15 + 0.36 + 0.34 = 0.85.

2-62. Total possible: 10^{16}, but only 10^8 are valid. Therefore, P(valid) = $10^8/10^{16} = 1/10^8$

2-63. 3 digits between 0 and 9, so the probability of any three numbers is 1/(10*10*10).
3 letters A to Z, so the probability of any three numbers is 1/(26*26*26). The probability your license plate is chosen is then $(1/10^3)*(1/26^3) = 5.7 \times 10^{-8}$

2-64. a) 5*5*4 = 100
b) (5*5)/100 = 25/100=1/4

2-65. (a) The number of possible experiments is $4 + 4 \times 3 + 4 \times 3 \times 3 = 52$
(b) There are 36 experiments that use all three steps. The probability the best result uses all three steps is 36/52 = 0.6923.
(c) No, it will not change. With k amounts in the first step the number of experiments is $k + 3k + 9k = 13k$. The number of experiments that complete all three steps is $9k$ out of $13k$. The probability is 9/13 = 0.6923.

2-66. a) P(A) = 85/100 = 0.85
b) P(B) = 80/100 = 0.8
c) P(A') = 15/100 = 0.15
d) $P(A\cap B)$ = 70/100 = 0.7
e) $P(A\cup B)$ = (70 + 10 + 15)/100 = 0.95
f) $P(A'\cup B)$ = (70 + 10 + 5)/100 = 0.85

2-67. a) P(A) = 30/100 = 0.30
b) P(B) = 75/100 = 0.75
c) P(A') = 1 – 0.30 = 0.70
d) $P(A\cap B)$ = 20/100 = 0.2
e) $P(A\cup B)$ = 85/100 = 0.85
f) $P(A'\cup B)$ =90/100 = 0.9

2-68. (a) The total number of transactions is 43+44+4+5+4=100

$$P(A) = \frac{44+4+4}{100} = 0.52$$

(b) $P(B) = \frac{100-5}{100} = 0.95$

(c) $P(A\cap B) = \frac{44+4+4}{100} = 0.52$

(d) $P(A\cap B') = 0$

(e) $P(A\cup B) = \frac{100-5}{100} = 0.95$

2-69. a) Because E and E' are mutually exclusive events and $E\cup E'$ = S
1 = P(S) = P($E\cup E'$) = P(E) + P(E'). Therefore, P(E') = 1 - P(E)
b) Because S and ∅ are mutually exclusive events with S = $S\cup\varnothing$
P(S) = P(S) + P(∅). Therefore, P(∅) = 0
c) Now, B = $A\cup(A'\cap B)$ and the events A and $A'\cap B$ are mutually exclusive. Therefore,
P(B) = P(A) + P($A'\cap B$). Because P($A'\cap B$) ≥ 0 , P(B) $\geq$ P(A).

2.70. a) $P(A \cap B)$ = (40 + 16)/204 = 0.2745
b) $P(A')$ = (36 + 56)/204 = 0.4510
c) $P(A \cup B)$ = (40 + 12 + 16 + 44 + 36)/204 = 0.7255
d) $P(A \cup B')$ = (40 + 12 + 16 + 44 + 56)/204 = 0.8235
e) $P(A' \cap B')$ = 56/204 = 0.2745

2-71. Total number of possible designs is 1125. The sample space of all possible designs that may be seen on five visits. This space contains $(1125)^5$ outcomes.

The number of outcomes in which all five visits are different can be obtained as follows. On the first visit any one of 1125 designs may be seen. On the second visit there are 1124 remaining designs. On the third visit there are 1123 remaining designs. On the fourth and fifth visits there are 1122 and 1121 remaining designs, respectively. From the multiplication rule, the number of outcomes where all designs are different is 1125 × 1124 × 1123 × 1122 × 1121. Therefore, the probability that a design is not seen again is

$$\frac{1125\times1124\times1123\times1122\times1121}{(1125)^5} = 0.9911$$

2-72. a) $P(A \cap B)$ = 242/22252 = 0.0109
b) $P(A')$ = (5292+6991+5640)/22252 = 0.8055

c) $P(A \cup B)$ = (195 + 270 + 246 + 242 + 984 + 3103)/22252 = 0.2265
d) $P(A \cup B')$ = (4329 + (5292 – 195) + (6991 – 270) + 5640 – 246))/22252 = 0.9680
e) $P(A' \cap B')$ = (1277 + 1558 + 666 + 3820 + 5163 + 4728)/22252 = 0.7735

2-73.

a) P(A ∩ B) = (170 + 443 + 60)/8493 = 0.0792
b) P(A′) = (28 + 363 + 309 + 933 + 39)/8493 = 1672/8493 = 0.1969
c) P(A ∪ B) = (1685+3733+1403+2+14+29+46+3)/8493 = 6915/8493 = 0.8142
d) P(A ∪ B′) = (1685 + (28 – 2) + 3733 + (363 – 14) + (309 – 29) + 1403 + (933 – 46) + (39 – 3))/8493 = 8399/8493 = 0.9889
e) P(A′ ∩ B′) = (28 – 2 + 363 – 14 + 306 – 29 + 933 – 46 + 39 – 3)/8493 = 1578/8493 = 0.1858

Section 2-3

2-74. a) P(A') = 1- P(A) = 0.6
b) P ($A \cup B$) = P(A) + P(B) - P($A \cap B$) = 0.4 + 0.2 - 0.1 = 0.5
c) P($A' \cap B$) + P($A \cap B$) = P(B). Therefore, P($A' \cap B$) = 0.2 - 0.1 = 0.1
d) P(A) = P($A \cap B$) + P($A \cap B'$). Therefore, P($A \cap B'$) = 0.4 - 0.1 = 0.3
e) P(($A \cup B$)') = 1 - P($A \cup B$) = 1 - 0.5 = 0.5
f) P($A' \cup B$) = P(A') + P(B) - P($A' \cap B$) = 0.6 + 0.2 - 0.1 = 0.7

2-75. a) P($A \cup B \cup C$) = P(A) + P(B) + P(C), because the events are mutually exclusive. Therefore,
P($A \cup B \cup C$) = 0.2+0.3+0.4 = 0.9
b) P ($A \cap B \cap C$) = 0, because $A \cap B \cap C = \emptyset$
c) P($A \cap B$) = 0 , because $A \cap B = \emptyset$
d) P($(A \cup B) \cap C$) = 0, because $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \emptyset$
e) P($A' \cap B' \cap C'$) =1-[P(A) + P(B) + P(C)] = 1-(0.2+0.3+0.4) = 0.1

2-76. (a) P(Caused by sports) = P(Caused by contact sports or by noncontact sports)
= P(Caused by contact sports) + P(Caused by noncontact sports)
= 0.46 + 0.44 = 0.9
(b) 1- P(Caused by sports) = 0.1

2.77. a) 70/100 = 0.70
b) (79+86-70)/100 = 0.95
c) No, P($A \cap B$) ≠ 0

2-78. (a) P(High strength and high conductivity)= 74/100 =0.74
(b) P(Low strength or low conductivity)
= P(Low strength) + P(Low conductivity) – P(Low strength and low conductivity)
= (10 + 1)/100 + (15 + 1)/100 – 1/100
= 0.26
(c) No, they are not mutually exclusive. Because P(Low strength) + P(Low conductivity)
= (10 + 1)/100 + (15 + 1)/100
= 0.27, which is not equal to P(Low strength or low conductivity).

2-79. a) 350/370
b) $\frac{345+5+12}{370} = \frac{362}{370}$
c) $\frac{345+5+8}{370} = \frac{358}{370}$
d) 345/370

2-80. a) 170/190 = 17/19
b) 7/190

2-81. a) P(unsatisfactory) = (5 + 10 – 2)/130 = 13/130

b) P(both criteria satisfactory) = 117/130 = 0.90, No

2-82. (a) 5/36
(b) 5/36
(c) $P(A \cap B) = P(A)P(B) = 25/1296$
(d) $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 10/36 - 25/1296 = 0.2585$

2-83. $P(A)$ = 112/204 = 0.5490, P(B) = 92/204 = 0.4510, $P(A \cap B)$ = (40+16)/204 = 0.2745
a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.5490 + 0.4510 – 0.2745 = 0.7255
b) $P(A \cap B')$ = (12 + 44)/204 = 0.2745 and $P(A \cup B') = P(A) + P(B') - P(A \cap B')$ = 0.5490 + (1 – 0.4510) – 0.2745 = 0.8235
c) $P(A' \cup B') = 1 - P(A \cap B)$ = 1 – 0.2745 = 0.7255

2-84. P(A) = 1/4 = 0.25, P(B) = 4/5 = 0.80, $P(A \cap B) = P(A)P(B)$ = (1/4)(4/5) = 1/5 = 0.20
a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.25 + 0.80 – 0.20 = 0.85
b) First $P(A \cap B') = P(A)P(B')$ = (1/4)(1/5) = 1/20 = 0.05. Then $P(A \cup B') = P(A) + P(B') - P(A \cap B')$ = 0.25 + 0.20 – 0.05= 0.40
c) $P(A' \cup B') = 1 - P(A \cap B)$ = 1 – 0.20 = 0.80

2-85. P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, $P(A \cap B)$ = 242/22252 = 0.0109,
$P(A \cap B')$ = (984+3103)/22252 = 0.1837
a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.1945 + 0.0428 – 0.0109 = 0.2264
b) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$ = 0.1945 + (1 – 0.0428) – 0.1837 = 0.9680
c) $P(A' \cup B') = 1 - P(A \cap B)$ = 1 – 0.0109 = 0.9891

2-86. P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903,
$P(A \cap B)$ = (170 + 443 + 60)/8493 = 0.0792, $P(A \cap B')$ = (1515+3290+1343)/8493 = 0.7239
a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.8031 + 0.0903 – 0.0792 = 0.8142
b) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$ = 0.8031 + (1 – 0.0903) – 0.7239 = 0.9889
c) $P(A' \cup B') = 1 - P(A \cap B)$ = 1 – 0.0792 = 0.9208

Section 2-4

2-87. a) P(A) = 85/100 b) P(B) = 80/100
c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{70/100}{80/100} = \dfrac{7}{8}$
d) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{70/100}{85/100} = \dfrac{7}{8.5}$

2-88. (a) $P(A) = \dfrac{9+30}{100} = 0.39$
(b) $P(B) = \dfrac{13+9}{100} = 0.22$
(c) $P(A | B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{9/100}{22/100} = 0.409$
(d) $P(B | A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{9/100}{39/100} = 0.23$

2-89. Let *A* denote the event that a leaf completes the color transformation and let *B* denote the event that a leaf completes the textural transformation. The total number of experiments is 300.
(a) $P(B | A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{243/300}{(243+26)/300} = 0.903$
(b) $P(A | B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{26/300}{(18+26)/300} = 0.591$

2-90. a) 0.83
b) 0.90
c) 8/9 = 0.889
d) 80/83 = 0.964
e) 80/83 = 0.964
f) 3/10 = 0.3

2-91. a) 12/100 b) 12/28 c) 34/122

2-92.
a) P(A) = 0.05 + 0.10 = 0.15
b) P(A|B) = $\frac{P(A\cap B)}{P(B)} = \frac{0.04+0.07}{0.72} = 0.153$
c) P(B) = 0.72
d) P(B|A) = $\frac{P(A\cap B)}{P(A)} = \frac{0.04+0.07}{0.15} = 0.733$
e) P(A ∩ B) = 0.04 +0.07 = 0.11
f) P(A ∪ B) = 0.15 + 0.72 – 0.11 = 0.76

2-93. Let *A* denote the event that autolysis is high and let *B* denote the event that putrefaction is high. The total number of experiments is 100.

(a) $P(B'|A) = \frac{P(A\cap B')}{P(A)} = \frac{18/100}{(14+18)/100} = 0.5625$

(b) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{14/100}{(14+59)/100} = 0.1918$

(c) $P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{9/100}{(18+9)/100} = 0.333$

2-94. a) P(gas leak) = (55 + 32)/107 = 0.813
b) P(electric failure | gas leak) = (55/107)/(87/102) = 0.632
c) P(gas leak | electric failure) = (55/107)/(72/107) = 0.764

2-95. a) 25/100
b) 24/99
c) (25/100)(24/99) = 0.0606
d) If the chips were replaced, the probability would be (25/100) = 0.25

2-96. a) 9/499 = 0.018
b) (10/500)(9/499) = 3.6×10^{-4}
c) (490/500)(489/499) = 0.96
d) 8/498 = 0.016
e) 9/498 = 0.018
f) $\left(\frac{10}{500}\right)\left(\frac{9}{499}\right)\left(\frac{8}{498}\right) = 5.795\times10^{-6}$

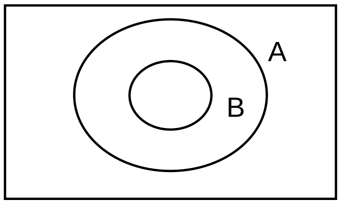
2-97. a) P = (8-1)/(300-1)=0.023
b) P = (8/300)×[(8-1)/(300-1)] = 6.243×10^{-4}
c) P = (292/300) ×[(292-1)/(300-1)]=0.947

2-98. (a) $\frac{1}{36^7}$

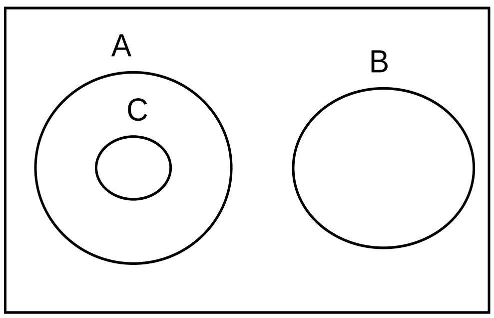
(b) $\frac{1}{5(36^6)}$

(c) $\frac{1}{5(36^5)5}$

2-99. No, if $B \subset A$, then P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$



2-100.



2-101. a) $P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} = \frac{(40+16)/204}{(40+16+36)/204} = \frac{56}{92} = 0.6087$

b) $P(A'\,|\,B) = \frac{P(A' \cap B)}{P(B)} = \frac{36/204}{(40+16+36)/204} = \frac{36}{92} = 0.3913$

c) $P(A\,|\,B') = \frac{P(A \cap B')}{P(B')} = \frac{56/204}{(12+44+56)/204} = \frac{56}{112} = 0.5$

d) $P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} = \frac{(40+16)/204}{(40+12+16+44)/204} = \frac{40+16}{112} = 0.5$

2-102. a) $P(A\,|\,B) = \frac{P(A \cap B)}{P(B)} = \frac{242/22252}{953/22252} = \frac{242}{953} = 0.2539$

b) $P(A'\,|\,B) = \frac{P(A' \cap B)}{P(B)} = \frac{(195+270+246)/22252}{953/22252} = \frac{711}{953} = 0.7461$

c) $P(A\,|\,B') = \frac{P(A \cap B')}{P(B')} = \frac{(984+3103)/22252}{(22252-953)/22252} = \frac{4087}{21299} = 0.1919$

d) $P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} = \frac{242/22252}{4329/22252} = \frac{242}{4329} = 0.0559$

2-103. a) $P(B\,|\,A) = \frac{P(A \cap B)}{P(A)} = \frac{(170+443+60)/8493}{(1685+3733+1403)/8493} = \frac{673}{6821} = 0.0987$

Also the probability of failure for fewer than 1000 wells is

$$P(B \mid A') = \frac{P(B \cap A')}{P(B')} = \frac{(2+14+29+46+3)/8493}{(28+363+309+933+39)/8493} = \frac{92}{1672} = 0.0562$$

Let *C* denote the event that fewer than 500 wells are present. $P(B \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{(2+14+29+46+3)/8493}{(28+363+309+39)/8493} = \frac{48}{739} = 0.0650$

2-104. Let A denote the event that an egg survives to an adult
Let EL denote the event that an egg survives at early larvae stage
Let LL denote the event that an egg survives at late larvae stage
Let PP denote the event that an egg survives at pre-pupae larvae stage
Let LP denote the event that an egg survives at late pupae stage

a) $P(A) = 31/421 = 0.0736$

b) $P(A \mid LL) = \frac{P(A \cap LL)}{P(LL)} = \frac{31/421}{306/421} = 0.1013$

c) $P(EL) = 412/421 = 0.9786$

$$P(LL \mid EL) = \frac{P(LL \cap EL)}{P(EL)} = \frac{306/421}{412/421} = 0.7427$$

$$P(PP \mid LL) = \frac{P(PP \cap LL)}{P(LL)} = \frac{45/421}{306/421} = 0.1471$$

$$P(LP \mid PP) = \frac{P(LP \cap PP)}{P(PP)} = \frac{35/421}{45/421} = 0.7778$$

$$P(A \mid LP) = \frac{P(A \cap LP)}{P(LP)} = \frac{31/421}{35/421} = 0.8857$$

The late larvae stage has the lowest probability of survival to the pre-pupae stage.

<u>Section 2-5</u>

2-105. a) $P(A \cap B) = P(A|B)P(B) = (0.3)(0.6) = 0.18$

b) $P(A' \cap B) = P(A'|B)P(B) = (0.7)(0.6) = 0.42$

2-106.

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= (0.25)(0.8) + (0.35)(0.2) \\ &= 0.2 + 0.07 = 0.27 \end{aligned}$$

2-107. Let F denote the event that a connector fails and let W denote the event that a connector is wet.

$$\begin{aligned} P(F) &= P(F|W)P(W) + P(F|W')P(W') \\ &= (0.06)(0.10) + (0.02)(0.9) = 0.024 \end{aligned}$$

2-108. Let F denote the event that a roll contains a flaw and let C denote the event that a roll is cotton.

$$\begin{aligned} P(F) &= P(F|C)P(C) + P(F|C')P(C') \\ &= (0.01)(0.7) + (0.02)(0.3) = 0.013 \end{aligned}$$

2-109.

Let R denote the event that a product exhibits surface roughness. Let N, A, and W denote the events that the blades are new, average, and worn, respectively. Then,

P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)
= (0.02)(0.25) + (0.03) (0.6) + (0.06)(0.15)
= 0.032

2-110. Let A denote the event that a respondent is a college graduate and let B denote the event that an individual votes for Bush.
P(B) = P(A)P(B|A) + P(A')P(B|A') = (0.38 × 0.52) + (0.62 × 0.5) = 0.0613

2-111. a) (0.88)(0.27) = 0.2376
b) (0.12)(0.13+0.52) = 0.0.078

2-112. a)P = 0.13 × 0.73=0.0949
b)P = 0.87 × (0.27+0.17)=0.3828

2-113. Let A and B denote the event that the first and second part selected has excessive shrinkage, respectively.
a) P(B)= P(B|A)P(A) + P(B|A ')P(A')
= (4/24)(5/25) + (5/24)(20/25) = 0.20
b) Let C denote the event that the third part selected has excessive shrinkage.

$$P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B')P(A\cap B')$$
$$+P(C|A'\cap B)P(A'\cap B) + P(C|A'\cap B')P(A'\cap B')$$
$$= \frac{3}{23}\left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \frac{5}{23}\left(\frac{19}{24}\right)\left(\frac{20}{25}\right)$$
$$= 0.20$$

2-114. Let A and B denote the events that the first and second chips selected are defective, respectively.
a) P(B) = P(B|A)P(A) + P(B|A')P(A') = (19/99)(20/100) + (20/99)(80/100) = 0.2
b) Let C denote the event that the third chip selected is defective.

$$P(A\cap B\cap C) = P(C|A\cap B)P(A\cap B) = P(C|A\cap B)P(B|A)P(A)$$
$$= \frac{18}{98}\left(\frac{19}{99}\right)\left(\frac{20}{100}\right)$$
$$= 0.00705$$

2-115.

Open surgery

	success	failure	sample size	sample percentage	conditional success rate
large stone	192	71	263	75%	73%
small stone	81	6	87	25%	93%
overall summary	273	77	350	100%	78%

PN

	success	failure	sample size	sample percentage	conditional success rate
large stone	55	25	80	23%	69%
small stone	234	36	270	77%	83%
overall summary	289	61	350	100%	83%

The overall success rate depends on the success rates for each stone size group, but also the probability of the groups. It is the weighted average of the group success rate weighted by the group size as follows
P(overall success) = P(success| large stone)P(large stone)) + P(success| small stone)P(small stone).
For open surgery, the dominant group (large stone) has a smaller success rate while for PN, the dominant group (small stone) has a larger success rate.

2-116. P(A) = 112/204 = 0.5490, P(B) = 92/204 = 0.4510

a) P(A ∩ B) = P(A | B)P(B) = (56/92)(92/204) = 0.2745
b) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.5490 + 0.4510 – 0.2745 = 0.7255
c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.2745 = 0.7255
d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (56/92) (92/204) + (56/112)(112/204) = 112/204 = 0.5490

2-117. P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428
a) P(A ∩ B) = P(A | B)P(B) = (242/953)(953/22252) = 0.0109
b) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.1945 + 0.0428 – 0.0109 = 0.2264
c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0109 = 0.9891
d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (242/953)(953/22252) + (4087/21299)(21299/22252) = 0.1945

2-118. a) $P = \dfrac{\binom{270}{3}}{\binom{953}{3}} = 0.0226$

b) $P = \dfrac{\binom{195}{3}+\binom{270}{3}+\binom{246}{3}+\binom{242}{3}}{\binom{953}{3}} = 0.0643$

2-119. P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903
a) P(A ∩ B) = P(B | A)P(A) = (673/6821)(6821/8493) = 0.0792
b) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.8031 + 0.0903 – 0.0792 = 0.8142
c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0792 = 0.9208
d) P(A) = P(A | B)P(B) + P(A | B')P(B') = (673/767)(767/8493) + (6148/7726)(7726/8493) = 0.8031

2-120. a) $P = \dfrac{\binom{170}{2}}{\binom{767}{2}} = 0.0489$

b) $P = \dfrac{\binom{170}{2}+\binom{2}{2}+\binom{443}{2}+\binom{14}{2}+\binom{29}{2}+\binom{60}{2}+\binom{46}{2}+\binom{3}{2}}{\binom{767}{2}} = 0.3934$

2-121. Let R denote red color and F denote that the font size is not the smallest. Then P(R) = 1/4, P(F) = 4/5. Because the Web sites are generated randomly these events are independent. Therefore, P(R ∩ F) = P(R)P(F) = (1/4)(4/5) = 0.2

<u>Section 2-6</u>

2-122. Because P(A | B) ≠ P(A), the events are not independent.

2-123. P(A') = 1 – P(A) = 0.7 and P(A'|B) = 1 – P(A | B) = 0.8. Since P(A' | B) ≠ P(A'), A' and B are not independent events.

2-124. If A and B are mutually exclusive, then P(A∩B) = 0 and P(A)P(B) = 0.09.
As P(A ∩ B) ≠ P(A)P(B), A and B are <u>not</u> independent.

2-125. a) $P(B \mid A)$ = 5/599 = 0.0067
$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/599)(5/600) + (5/599)(595/600) = 0.0083$
As P(B | A) ≠ P(B), *A* and *B* are <u>not</u> independent.
b) *A* and *B* are independent.

2-126. P(A∩B) = 70/100, P(A) = 86/100, P(B) = 80/100.

Then, P($A \cap B$) $\neq$ P(A)P(B), so A and B are <u>not</u> independent.

2-127. a) P($A \cap B$)= 22/100, P(A) = 30/100, P(B) = 77/100, Then P($A \cap B$)$\neq$ P(A)P(B), therefore, A and B are not independent.
b) P(B|A) = P(A ∩ B)/P(A) = (22/100)/(30/100) = 0.733

2-128. (a) $P = (0.001)^2 = 10^{-6}$
(b) $P = 1 - (0.999)^2 = 0.002$

2-129. It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the *i*th sample contains high levels of contamination.

a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is 5(0.0656) = 0.328.

c) Let B denote the event that no sample contains high levels of contamination. The requested probability is P(B') = 1 - P(B). From part (a), P(B') = 1 - 0.59 = 0.41.

2-130. Let A_i denote the event that the ith bit is a one.

a) By independence $P(A_1 \cap A_2 \cap ... \cap A_{10}) = P(A_1)P(A_2)...P(A_{10}) = (\frac{1}{2})^{10} = 0.000976$

b) By independence, $P(A_1' \cap A_2' \cap ... \cap A_{10}') = P(A_1')P(A_2')...P(A_{10}^c) = (\frac{1}{2})^{10} = 0.000976$

c) The probability of the following sequence is

$P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5' \cap A_6 \cap A_7 \cap A_8 \cap A_9 \cap A_{10}) = (\frac{1}{2})^{10}$, by independence. The number of sequences consisting of five "1"'s, and five "0"'s is $\binom{10}{5} = \frac{10!}{5!5!} = 252$. The answer is $252\left(\frac{1}{2}\right)^{10} = 0.246$

2-131.

(a) Let I and G denote an infested and good sample. There are 4 ways to obtain four consecutive samples showing the signs of the infestation: IIIIGGG, GIIIIGG, GGIIIIG, GGGIIII. Therefore, the probability is $4 \times (0.3)^4 (0.7)^3 = 0.0111$.

(b) There are 14 ways to obtain three out of four consecutive samples showing the signs of infestation. The probability is $14 \times (0.3^3 \times 0.7^4) = 0.0908$

2-132. (a) $P = (0.8)^4 = 0.4096$
(b) $P = 1 - 0.2 - 0.8 \times 0.2 = 0.64$
(c) Probability defeats all four in a game = 0.8^4 = 0.4096. Probability defeats all four at least once = 1 – (1 – 0.4096)^3 = 0.7942

2-133. (a) The probability that one technician obtains equivalence at 100 mL is 0.1.
So the probability that both technicians obtain equivalence at 100 mL is $0.1^2 = 0.01$.
(b) The probability that one technician obtains equivalence between 98 and 104 mL is 0.7.

So the probability that both technicians obtain equivalence between 98 and 104 mL is $0.7^2 = 0.49$.

(c) The probability that the average volume at equivalence from the technician is 100 mL is $9(0.1^2) = 0.09$.

2-134. (a) $P = \dfrac{10^6}{10^{16}} = 10^{-10}$

(b) $P = 0.25 \times (\dfrac{1}{12}) = 0.020833$

2-135. Let A denote the event that a sample is produced in cavity one of the mold.

a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup ... \cup B_8) = P(B_1) + P(B_2) + ... + P(B_8)$

From part a., $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4 (\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4 (\frac{7}{8}) = 0.00107$.

2-136. Let A denote the upper devices function. Let B denote the lower devices function.
P(A) = (0.8)(0.7)(0.6) = 0.336
P(B) = (0.95)(0.95)(0.95) = 0.8574
P(A∩B) = (0.336)(0.8574) = 0.288
Therefore, the probability that the circuit operates = P(A∪B) = P(A) + P(B) – P(A∩B) = 0.9054

2-137. *P* = [1 – (0.1)(0.05)][1 – (0.1)(0.05)][1 – (0.2)(0.1)] = 0.9702

2-138. P(A) = 112/204 = 0.5490, P(B) = 92/204 = 0.4510, P(A ∩ B) = 56/204 = 0.2745
Because P(A) P(B) = (0.5490)(0.4510) = 0.2476 ≠ 0.2745 = P(A ∩ B), A and B are not independent.

2-139. P(A) = 4329/22252 = 0.1945, P(B) = 953/22252 = 0.0428, P(A ∩ B) = 242/22252 = 0.0109
Because P(A)*P(B) = (0.1945)(0.0428) = 0.0083 ≠ 0.0109 = P(A ∩ B), A and B are not independent.

2-140. P(A) = (1685+3733+1403)/8493 = 0.8031, P(B) = (170+2+443+14+29+60+46+3)/8493 = 0.0903,
P(A ∩ B) = (170+443+60)/8493 = 0.0792
Because P(A)*P(B) = (0.8031)(0.0903) = 0.0725 ≠ 0.0792 = P(A ∩ B), A and B are not independent.

2-141. P(A) = (3*5*3*5)/(4*3*5*3*5) = 0.25, P(B) = (4*3*4*3*5)/(4*3*5*3*5) = 0.8,
P(A ∩ B) = (3*4*3*5) /(4*3*5*3*5) = 0.2
Because P(A)*P(B) = (0.25)(0.8) = 0.2 = P(A ∩ B), A and B are independent.

Section 2-7

2-142. Because, $P(A|B)\,P(B) = P(A \cap B) = P(B|A)\,P(A)$,

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.6(0.3)}{0.4} = 0.45$$

2-143. $$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

$$= \frac{0.5 \times 0.7}{(0.5 \times 0.7) + (0.1 \times 0.3)} = 0.921$$

2-144. Let F denote a fraudulent user and let T denote a user that originates calls from two or more metropolitan areas in a day. Then,

$$P(F|T)=\frac{P(T|F)P(F)}{P(T|F)P(F)+P(T|F')P(F')}=\frac{0.30(0.0001)}{0.30(0.0001)+0.01(.9999)}=0.003$$

2-145. (a) P=(0.31)(0.978)+(0.27)(0.981)+(0.21)(0.965)+(0.13)(0.992)+(0.08)(0.959)
= 0.97638

(b) $P=\frac{(0.21)(0.965)}{0.97638}=0.207552$

2-146. Let A denote the event that a respondent is a college graduate and let B denote the event that a voter votes for Bush.

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B\mid A)}{P(A)P(B\mid A)+P(A')P(B\mid A')}=\frac{(0.38)(0.54)}{(0.38)(0.54)+(0.62)(0.48)}=0.4081\text{, i.e. }40.8\%$$

2-147. Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.

a)

$$\begin{aligned}P(G) &= P(G|H)P(H)+P(G|M)P(M)+P(G|P)P(P)\\ &= 0.95(0.40)+0.60(0.35)+0.10(0.25)\\ &= 0.615\end{aligned}$$

b) Using the result from part a.,

$$P(H|G)=\frac{P(G|H)P(H)}{P(G)}=\frac{0.95(0.40)}{0.615}=0.618$$

c) $P(H|G')=\frac{P(G'|H)P(H)}{P(G')}=\frac{0.05(0.40)}{1-0.615}=0.052$

2-148. a) P(D)=P(D|G)P(G)+P(D|G')P(G')=(.005)(.991)+(.99)(.009)=0.013865
b) P(G|D')=P(G∩D')/P(D')=P(D'|G)P(G)/P(D')=(.995)(.991)/(1-.013865)=0.9999

2-149. Denote as follows: S = signal, O = organic pollutants, V = volatile solvents, C = chlorinated compounds
a) P(S) = P(S|O)P(O)+P(S|V)P(V)+P(S|C)P(C) = 0.997(0.60) + 0.9995(0.27) + 0.897(0.13) = 0.9847
b) P(C|S) = P(S|C)P(C)/P(S) = (0.897)(0.13)/0.9847 = 0.1184

2-150. Let A denote the event that a reaction final temperature is 271 K or less
Let B denote the event that the heat absorbed is above target

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)P(B\mid A)}{P(A)P(B\mid A)+P(A')P(B\mid A')}$$
$$=\frac{(0.5490)(0.5)}{(0.5490)(0.5)+(0.4510)(0.3913)}=0.6087$$

2-151. Let L denote the event that a person is LWBS
Let A denote the event that a person visits Hospital 1
Let B denote the event that a person visits Hospital 2
Let C denote the event that a person visits Hospital 3
Let D denote the event that a person visits Hospital 4

$$P(D\mid L)=\frac{P(L\mid D)P(D)}{P(L\mid A)P(A)+P(L\mid B)P(B)+P(L\mid C)P(C)+P(L\mid D)P(D)}$$
$$=\frac{(0.0559)(0.1945)}{(0.0368)(0.2378)+(0.0386)(0.3142)+(0.0436)(0.2535)+(0.0559)(0.1945)}$$
$$=0.2540$$

2-152. Let A denote the event that a well is failed
Let B denote the event that a well is in Gneiss
Let C denote the event that a well is in Granite

Let D denote the event that a well is in Loch raven schist
Let E denote the event that a well is in Mafic
Let F denote the event that a well is in Marble
Let G denote the event that a well is in Prettyboy schist
Let H denote the event that a well is in Other schist
Let I denote the event that a well is in Serpentine

$$P(B\mid A)=\frac{P(A\mid B)P(B)}{P(A\mid B)P(B)+P(A\mid C)P(C)+P(A\mid D)P(D)+P(A\mid E)P(E)+P(A\mid F)P(F)+P(A\mid G)P(E)+P(A\mid G)P(G)+P(A\mid H)P(H)}$$

$$=\frac{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right)}{\left(\frac{170}{1685}\right)\left(\frac{1685}{8493}\right)+\left(\frac{2}{28}\right)\left(\frac{28}{8493}\right)+\left(\frac{443}{3733}\right)\left(\frac{3733}{8493}\right)+\left(\frac{14}{363}\right)\left(\frac{363}{8493}\right)+\left(\frac{29}{309}\right)\left(\frac{309}{8493}\right)+\left(\frac{60}{1403}\right)\left(\frac{1403}{8493}\right)+\left(\frac{46}{933}\right)\left(\frac{933}{8493}\right)+\left(\frac{3}{39}\right)\left(\frac{39}{8493}\right)}$$

$$=0.2216$$

2-153. Denote as follows: A = affiliate site, S = search site, B =blue, G =green

$$P(S\mid B)=\frac{P(B\mid S)P(S)}{P(B\mid S)P(S)+P(B\mid A)P(A)}$$

$$=\frac{(0.4)(0.7)}{(0.4)(0.7)+(0.8)(0.3)}$$

$$=0.5$$

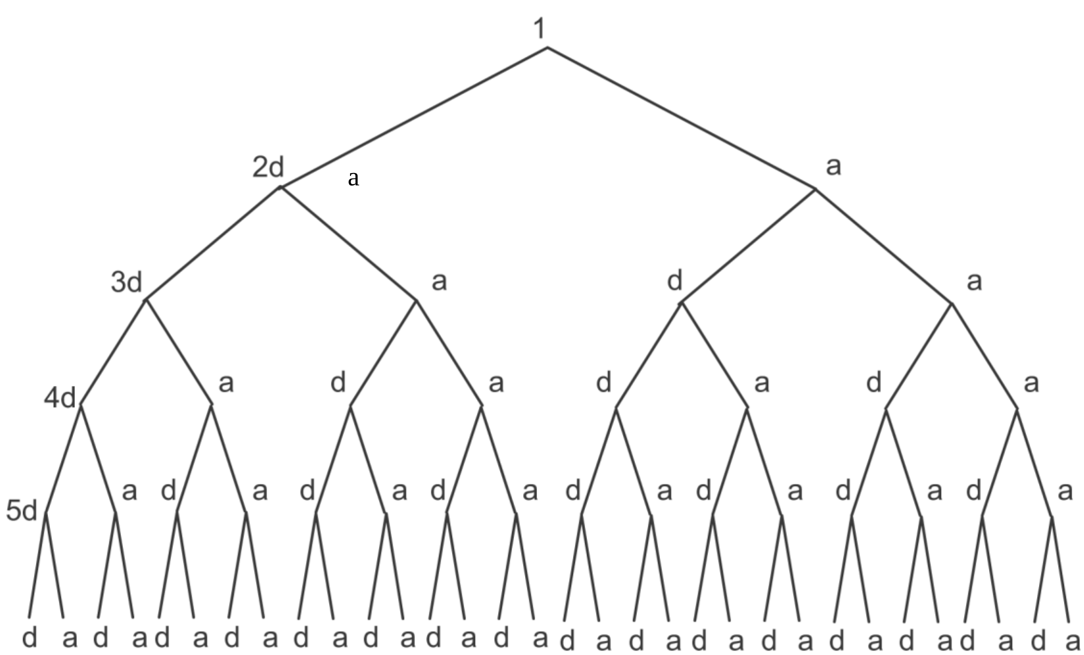
<u>Section 2-8</u>

2-154. Continuous: a, c, d, f, h, i; Discrete: b, e, and g

<u>Supplemental Exercises</u>

2-155. Let B denote the event that a glass breaks.
Let L denote the event that large packaging is used.
P(B)= P(B|L)P(L) + P(B|L')P(L')
= 0.02(0.60) + 0.03(0.40) = 0.024

2-156. Let "d" denote a defective calculator and let "a" denote an acceptable calculator



a) $S = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ daadd, daada, daaad, daaaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa, \\ aaddd, aadda, aadad, aadaa, \\ aaadd, aaada, aaaad, aaaaa \end{Bmatrix}$

b) $A = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ daadd, daada, daaad, daaaa \end{Bmatrix}$

c) $B = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa \end{Bmatrix}$

d) $A \cap B = \begin{Bmatrix} dddda, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa \end{Bmatrix}$

e) To determine $B \cup C$, we need C.

$C = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ daddd, dadda, dadad, dadaa, \\ adddd, addda, addad, addaa, \\ aaddd, aadda, aadad, aadaa \end{Bmatrix}$

$B \cup C = \begin{Bmatrix} ddddd, dddda, dddad, dddaa, \\ ddadd, ddada, ddaad, ddaaa, \\ daddd, dadda, dadad, dadaa, \\ adddd, addda, addad, addaa, \\ adadd, adada, adaad, adaaa, \\ aaddd, aadda, aadad, aadaa \end{Bmatrix}$

2-157. Let A = excellent surface finish; B = excellent length
a) P(A) = 73/100 = 0.73
b) P(B) = 90/100 = 0.90
c) P(A') = 1 – 0.73 = 0.27
d) P(A∩B) = 70/100 = 0.70
e) P(A∪B) = 0.93
f) P(A′∪B) = 0.97

2-158. a) (207 + 350 + 357 - 201 - 204 - 345 + 200)/370 = 0.9838
b) 366/370 = 0.989
c) (200 + 163)/370 = 363/370 = 0.981
d) (201 + 163)/370 = 364/370 = 0.984

2-159. If A,B,C are mutually exclusive, then P($A \cup B \cup C$) = P(A) + P(B) + P(C) = 0.3 + 0.4 + 0.5 = 1.2, which greater than 1. Therefore, P(A), P(B),and P(C) cannot equal the given values.

2-160. a) 350/367 b) 10/23

2-161. (a) P(the first one selected is not ionized) = 20/100 = 0.2
(b) P(the second is not ionized given the first one was ionized) = 20/99 = 0.202
(c) P(both are ionized)
= P(the first one selected is ionized) × P(the second is ionized given the first one was ionized)
= (80/100) × (79/99) = 0.638
(d) If samples selected were replaced prior to the next selection,
P(the second is not ionized given the first one was ionized) = 20/100 = 0.2.
The event of the first selection and the event of the second selection are independent.

2-162. a) P(A) = 20/50 = 0.4
b) $P(B|A)$ = 19/49
c) $P(A\cap B)$ = P(A) P(B/A) = (0.4) (19/49) = 0.155

d) $P(A\cup B)$ = 1 – P(A’ and B’) = $1-\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)=0.645$
A = first is local, B = second is local, C = third is local
e) $P(A \cap B \cap C)$ = (20/50)(19/49)(18/48) = 0.058
f) $P(A \cap B \cap C')$ = (20/50)(19/49)(30/48) = 0.9496

2-163. a) P(A) = 0.02
b) P(A') = 0.98
c) P(B|A) = 0.5
d) P(B|A') = 0.04
e) $P(A\cap B)$ = $P(B|A)$P(A) = (0.5)(0.02) = 0.01
f) $P(A\cap B')$ = $P(B'|A)$P(A) = (0.5)(0.02) = 0.01
g) P(B) = $P(B|A)$P(A) + $P(B|A')$P(A') = (0.5)(0.02) + (0.04)(0.98) = 0.0492

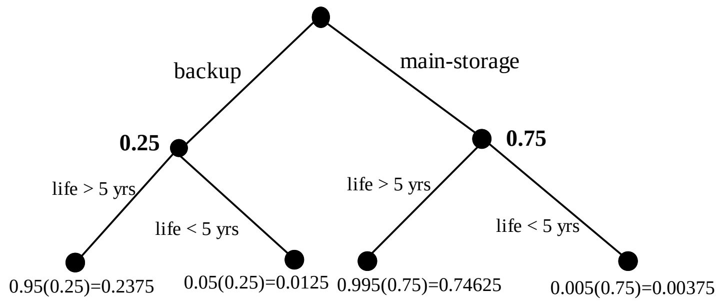
2-164. Let U denote the event that the user has improperly followed installation instructions.
Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.75)(0.03) = 0.0225
b) P(P|R)P(R) = (0.50)(0.25) = 0.125

2-165. (a) $P=1-(1-0.002)^{100}=0.18143$
(b) $P=C_3^1(0.998^2)0.002=0.005976$
(c) $P=1-[(1-0.002)^{100}]^{10}=0.86494$

2-166. $P(A\cap B)$ = 80/100, P(A) = 82/100, P(B) = 90/100.
Then, $P(A\cap B) \neq$ P(A)P(B), so A and B are <u>not</u> independent.

2-167. Let A_i denote the event that the ith readback is successful. By independence,
$P(A_1'\cap A_2'\cap A_3')=P(A_1')P(A_2')P(A_3')=(0.04)^3=0.000064$.

2-168.



a) P(B) = 0.25

b) $P(A|B) = 0.95$

c) $P(A|B') = 0.995$

d) $P(A \cap B) = P(A|B)P(B) = 0.95(0.25) = 0.2375$

e) $P(A \cap B') = P(A|B')P(B') = 0.995(0.75) = 0.74625$

f) $P(A) = P(A \cap B) + P(A \cap B') = 0.95(0.25) + 0.995(0.75) = 0.98375$
g) 0.95(0.25) + 0.995(0.75) = 0.98375.
h)

$$P(B|A') = \frac{P(A'|B)P(B)}{P(A'|B)P(B) + P(A'|B')P(B')} = \frac{0.05(0.25)}{0.05(0.25) + 0.005(0.75)} = 0.769$$

2-169. (a) $A' \cap B = 50$
(b) B'=37
(c) $A \cup B = 93$

2-170. a) 0.25
b) 0.75

2-171. Let D_i denote the event that the primary failure mode is type i and let A denote the event that a board passes the test.
The sample space is S = $\{A, A'D_1, A'D_2, A'D_3, A'D_4, A'D_5\}$.

2-172. a) 20/200 b) 135/200 c) 65/200

2-173. a) P(A) = 29/120 = 0.24
b) $P(A \cap B)$ = 20/120 = 0.1667
c) $P(A \cup B)$ = (29 + 105 – 20)/120 = 0.95
d) $P(A' \cap B)$ = 91/120 = 0.7583
e) $P(A|B) = P(A \cap B)/P(B)$ = (0.1667)(0.875) = 0.146

2-174. Let A_i denote the event that the *i*th order is shipped on time.

a) By independence, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = (0.96)^3 = 0.885$

b) Let

$B_1 = A_1' \cap A_2 \cap A_3$

$B_2 = A_1 \cap A_2' \cap A_3$

$B_3 = A_1 \cap A_2 \cap A_3'$

Then, because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3) = P(B_1) + P(B_2) + P(B_3)$$
$$= 3(0.96)^2(0.04)$$
$$= 0.11$$

c) Let

$B_1 = A_1' \cap A_2' \cap A_3$

$B_2 = A_1' \cap A_2 \cap A_3'$

$B_3 = A_1 \cap A_2' \cap A_3'$

$B_4 = A_1' \cap A_2' \cap A_3'$

Because the B's are mutually exclusive,

$$P(B_1 \cup B_2 \cup B_3 \cup B_4) = P(B_1)+P(B_2)+P(B_3)+P(B_4)$$
$$= 3(0.04)^2(0.96) + (0.04)^3$$
$$= 0.00467$$

2-175. a) No, $P(E_1 \cap E_2 \cap E_3) \neq 0$

b) No, $E_1' \cap E_2'$ is not $\varnothing$

c) $P(E_1' \cup E_2' \cup E_3') = P(E_1') + P(E_2') + P(E_3') – P(E_1' \cap E_2') - P(E_1' \cap E_3') - P(E_2' \cap E_3') + P(E_1' \cap E_2' \cap E_3') = 40/240$

d) $P(E_1 \cap E_2 \cap E_3) = 200/240$

e) $P(E_1 \cup E_3) = P(E_1) + P(E_3) – P(E_1 \cap E_3) = 234/240$

f) $P(E_1 \cup E_2 \cup E_3) = 1 – P(E_1' \cap E_2' \cap E_3') = 1 - 0 = 1$

2-176. a) (0.20)(0.30) +(0.7)(0.9) = 0.69

2-177. Let A_i denote the event that the *i*th bolt selected is not torqued to the proper limit.

a) Then,

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_4 | A_1 \cap A_2 \cap A_3)P(A_1 \cap A_2 \cap A_3)$$
$$= P(A_4 | A_1 \cap A_2 \cap A_3)P(A_3 | A_1 \cap A_2)P(A_2 | A_1)P(A_1)$$
$$= \left(\frac{12}{17}\right)\left(\frac{13}{18}\right)\left(\frac{14}{19}\right)\left(\frac{15}{20}\right) = 0.282$$

b) Let B denote the event that at least one of the selected bolts are not properly torqued. Thus, B' is the event that all bolts are properly torqued. Then,

$$P(B) = 1 - P(B') = 1 - \left(\frac{15}{20}\right)\left(\frac{14}{19}\right)\left(\frac{13}{18}\right)\left(\frac{12}{17}\right) = 0.718$$

2-178. Let A,B denote the event that the first, second portion of the circuit operates. Then, P(A) = (0.99)(0.99)+0.9-(0.99)(0.99)(0.9) = 0.998

P(B) = 0.9+0.9-(0.9)(0.9) = 0.99 and

$P(A \cap B) = P(A) P(B) = (0.998) (0.99) = 0.988$

2-179. A_1 = by telephone, A_2 = website; $P(A_1) = 0.92$, $P(A_2) = 0.95$;

By independence $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.92 + 0.95 - 0.92(0.95) = 0.996$

2-180. P(Possess) = 0.95(0.99) +(0.05)(0.90) = 0.9855

2-181. Let D denote the event that a container is incorrectly filled and let H denote the event that a container is filled under high-speed operation. Then,

a) $P(D) = P(D|H)P(H) + P(D|H')P(H') = 0.01(0.30) + 0.001(0.70) = 0.0037$

b) $$P(H|D) = \frac{P(D|H)P(H)}{P(D)} = \frac{0.01(0.30)}{0.0037} = 0.8108$$

2-182. a) $P(E' \cap T' \cap D') = (0.995)(0.99)(0.999) = 0.984$

b) $P(E \cup D) = P(E) + P(D) – P(E \cap D) = 0.005995$

2-183. D = defective copy

a) P(D = 1) = $\left(\frac{2}{80}\right)\left(\frac{78}{79}\right)\left(\frac{77}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{77}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{1}{78}\right)=0.049$

b) P(D = 2) = $\left(\frac{2}{80}\right)\left(\frac{1}{79}\right)\left(\frac{78}{78}\right)+\left(\frac{2}{80}\right)\left(\frac{78}{79}\right)\left(\frac{1}{78}\right)+\left(\frac{78}{80}\right)\left(\frac{2}{79}\right)\left(\frac{1}{78}\right)=0.00095$

c) Let A represent the event that the two items NOT inspected are not defective. Then, P(A) = (78/80)(77/79) = 0.95.

2-184. The tool fails if any component fails. Let F denote the event that the tool fails. Then, P(F') = $(0.99)^{20}$ by independence and P(F) = 1 − $(0.99)^{20}$ = 0.182

2-185. a) (0.3)(0.99)(0.985) + (0.7)(0.98)(0.997) = 0.9764

b) $P(route1|E)=\frac{P(E|route1)P(route1)}{P(E)}=\frac{0.02485(0.30)}{1-0.9764}=0.3159$

2-186. a) By independence, $0.15^5 = 7.59 \times 10^{-5}$

b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

$$P(A_1 A_2 A_3 A_4 A_5^{'} \ \ or \ \ A_1 A_2 A_3 A_4^{'} A_5 \ \ or \ \ A_1 A_2 A_3^{'} A_4 A_5 \ \ or \ \ A_1 A_2^{'} A_3 A_4 A_5 \ \ or \ \ A_1^{'} A_2 A_3 A_4 A_5)$$
$$= 5(0.15^4)(0.85^1)$$
$$= 0.000000215$$

c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A_4^{'} A_5^{'} \ \ or \ \ A_1 A_2 A_3^{'} A_4 A_5^{'} \ \ or \ \ A_1 A_2 A_3^{'} A_4^{'} A_5 \ \ or \ \ A_1 A_2^{'} A_3 A_4 A_5^{'} \ \ or \ \ A_1 A_2^{'} A_3 A_4^{'} A_5 \ or$$
$$A_1 A_2^{'} A_3^{'} A_4 A_5 \ \ or \ \ A_1^{'} A_2 A_3 A_4 A_5^{'} \ \ or \ \ A_1^{'} A_2 A_3 A_4^{'} A_5 \ \ or \ \ A_1^{'} A_2 A_3^{'} A_4 A_5 \ \ or \ \ A_1^{'} A_2^{'} A_3 A_4 A_5)$$
$$= 10(0.15^3)(0.85^2)$$
$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2-187. Let A_i denote the event that the ith washer selected is thicker than target.

a) $$\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{48}\right) = 0.207$$

b) 30/48 = 0.625

c) The requested probability can be written in terms of whether or not the first and second washer selected are thicker than the target. That is,

$$\left(\frac{30}{50}\right)\left(\frac{29}{49}\right)\left(\frac{28}{48}\right) + \left(\frac{30}{50}\right)\left(\frac{20}{49}\right)\left(\frac{29}{48}\right) + \left(\frac{20}{50}\right)\left(\frac{30}{49}\right)\left(\frac{29}{48}\right) + \left(\frac{20}{50}\right)\left(\frac{19}{49}\right)\left(\frac{30}{48}\right) = 0.60$$

2-188. a) If n washers are selected, then the probability they are all less than the target is $\frac{20}{50} \cdot \frac{19}{49} \cdots \frac{20-n+1}{50-n+1}$.

n	probability all selected washers are less than target
1	20/50 = 0.4
2	(20/50)(19/49) = 0.155
3	(20/50)(19/49)(18/48) = 0.058

Therefore, the answer is n = 3

b) Then event E that one or more washers is thicker than target is the complement of the event that all are less than target. Therefore, P(E) equals one minus the probability in part a. Therefore, n = 3.

2-189.

$$a) \quad P(A \cup B) = \frac{112+60+254}{940} = 0.453$$

$$b) \quad P(A \cap B) = \frac{254}{940} = 0.27$$

$$c) \quad P(A' \cup B) = \frac{514+60+254}{940} = 0.881$$

$$d) \quad P(A' \cap B') = \frac{514}{940} = 0.547$$

. e) $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \left(\frac{254}{940}\right) \Big/ \left(\frac{314}{940}\right) = 0.809$$

f) $$P(B|A) = \frac{P(B \cap A)}{P(A)} = \left(\frac{254}{940}\right) \Big/ \left(\frac{366}{940}\right) = 0.694$$

2-190. Let E denote a read error and let S,O,P denote skewed, off-center, and proper alignments, respectively. Then,

a) P(E) = P(E|S) P(S) + P(E|O) P (O) + P(E|P) P(P)
= 0.01(0.10) + 0.02(0.05) + 0.001(0.85)
= 0.00285

b) P(S|E) = $\frac{P(E|S)P(S)}{P(E)} = \frac{0.01(0.10)}{0.00285} = 0.351$

2-191. Let A_i denote the event that the *i*th row operates. Then,
$P(A_1) = 0.98, P(A_2) = (0.99)(0.99) = 0.9801, P(A_3) = 0.9801, \quad P(A_4) = 0.98.$
The probability the circuit does not operate is
$P(A_1')P(A_2')P(A_3')P(A_4') = (0.02)(0.0199)(0.0199)(0.02) = 1.58 \times 10^{-7}$

2-192. a) (0.4)(0.1) + (0.3)(0.1) +(0.2)(0.2) + (0.4)(0.1) = 0.15
b) P(4 or more | provided info) = (0.4)(0.1)/0.15 = 0.267

2-193. (a) P=(0.93)(0.91)(0.97)(0.90)(0.98)(0.93)=0.67336
(b) P=(1-0.93)(1-0.91)(1-0.97)(1-0.90)(1-0.98)(1-0.93)=2.646×10^{-8}
(c) P=1-(1-0.91)(1-0.97)(1-0.90)=0.99973

2-194. (a) P=(24/36)(23/35)(22/34)(21/33)(20/32)(19/31)=0.069
(b) P=1-0.069=0.931

2-195. (a) 36^7
(b) Number of permutations of six letters is 26^6. Number of ways to select one number = 10. Number of positions among the six letters to place the one number = 7. Number of passwords = $26^6 \times 10 \times 7$
(c) $26^5 10^2$

2-196. (a) $P(A) = \frac{15+35+40+7+40}{1200} = 0.1142$

(b) $P(A \cap B) = \frac{35+7}{1200} = 0.035$

(c) $P(A \cup B) = 1 - \frac{900}{1200} = 0.25$

(d) $P(A' \cap B) = \frac{63+70+30}{1200} = \frac{163}{1200} = 0.136$

(e) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.035}{(35+63+30+7+70)/1200} = 0.1708$

(f) $P = \frac{15}{1200} = 0.0125$

2-197. (a) Let A denote that a part conforms to specifications and let B denote a simple component.
For supplier 1:
P(A) = 1988/2000 = 0.994
For supplier 2:
P(A)= 1990/2000 = 0.995
(b)
For supplier 1: P(A|B') = 990/1000 = 0.99
For supplier 2: P(A|B') = 394/400 = 0.985
(c)
For supplier 1: P(A|B) = 998/1000 = 0.998
For supplier 2: P(A|B) = 1596/1600 = 0.9975

(d) The unusual result is that for both a simple component and for a complex assembly, supplier 1 has a greater probability that a part conforms to specifications. However, supplier 1 has a lower probability of conformance overall. The overall conforming probability depends on both the conforming probability of each part type and also the probability of each part type. Supplier 1 produces more of the complex parts so that overall conformance from supplier 1 is lower.

Mind-Expanding Exercises

2-198. a) Let X denote the number of documents in error in the sample and let n denote the sample size.

$P(X \geq 1) = 1 - P(X = 0)$ and $P(X = 0) = \dfrac{\binom{2}{0}\binom{58}{n}}{\binom{60}{n}}$

Trials for n result in the following results

n	$P(X = 0)$	$1 - P(X = 0)$
30	0.4915	0.5085
40	0.2147	0.7853
45	0.1186	0.8814
46	0.1028	0.8972
47	0.0881	0.9119

Therefore $n = 47$.

b) A large proportion of the set of documents needs to be inspected in order for the probability of a document in error to be detected to exceed 0.9.

2-199. Let n denote the number of washers selected.

a) The probability that none are thicker, that is, all are less than the target is 0.4^n by independence.

The following results are obtained:

n	0.4^n
1	0.4
2	0.16
3	0.064

Therefore, $n = 3$

b) The requested probability is the complement of the probability requested in part a). Therefore, $n = 3$

2-200. Let x denote the number of kits produced.

Revenue at each demand

	0	50	100	200
$0 \leq x \leq 50$	-5x	100x	100x	100x
Mean profit = 100x(0.95)-5x(0.05)-20x				
$50 \leq x \leq 100$	-5x	100(50)-5(x-50)	100x	100x
Mean profit = [100(50)-5(x-50)](0.4) + 100x(0.55)-5x(0.05)-20x				
$100 \leq x \leq 200$	-5x	100(50)-5(x-50)	100(100)-5(x-100)	100x
Mean profit = [100(50)-5(x-50)](0.4) + [100(100)-5(x-100)](0.3) + 100x(0.25) - 5x(0.05) - 20x				

	Mean Profit	Maximum Profit
$0 \leq x \leq 50$	74.75 x	$ 3737.50 at x=50
$50 \leq x \leq 100$	32.75 x + 2100	$ 5375 at x=100
$100 \leq x \leq 200$	1.25 x + 5250	$ 5500 at x=200

Therefore, profit is maximized at 200 kits. However, the difference in profit over 100 kits is small.

2-201. Let E denote the event that none of the bolts are identified as incorrectly torqued.

Let X denote the number of bolts in the sample that are incorrect. The requested probability is P(E').

Then,

P(E) = P(E|X=0)P(X=0) + P(E|X=1) P(X=1) + P(E|X=2) P(X=2) + P(E|X=3) P(X=3) + P(E|X=4)P(X=4)

and P(X=0) = (15/20)(14/19)(13/18)(12/17) = 0.2817.

The remaining probability for X can be determined from the counting methods. Then

$$P(X=1)=\frac{\binom{5}{1}\binom{15}{3}}{\binom{20}{4}}=\frac{\left(\frac{5!}{4!1!}\right)\left(\frac{15!}{3!12!}\right)}{\left(\frac{20!}{4!16!}\right)}=\frac{5!15!4!16!}{4!3!12!20!}=0.4696$$

$$P(X=2)=\frac{\binom{5}{2}\binom{15}{2}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{2!13!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.2167$$

$$P(X=3)=\frac{\binom{5}{3}\binom{15}{1}}{\binom{20}{4}}=\frac{\left(\frac{5!}{3!2!}\right)\left(\frac{15!}{1!14!}\right)}{\left(\frac{20!}{4!16!}\right)}=0.0309$$

P(X = 4) = (5/20)(4/19)(3/18)(2/17) = 0.0010, P(E | X = 0) = 1, P(E | X = 1) = 0.05,
P(E | X = 2) = 0.05^2 = 0.0025, P(E|X=3) = 0.05^3 = $1.25\text{x}10^{-4}$, P(E | X=4) = 0.05^4 = $6.25\text{x}10^{-6}$. Then,

$$P(E)=1(0.2817)+0.05(0.4696)+0.0025(0.2167)+1.25\times10^{-4}(0.0309)+6.25\times10^{-6}(0.0010)=0.306$$

and P(E') = 0.694

2-202.

$$\begin{aligned}P(A'\cap B')&=1-P([A'\cap B']')=1-P(A\cup B)\\&=1-[P(A)+P(B)-P(A\cap B)]\\&=1-P(A)-P(B)+P(A)P(B)\\&=[1-P(A)][1-P(B)]\\&=P(A')P(B')\end{aligned}$$

2-203. The total sample size is $ka + a + kb + b = (k + 1)a + (k + 1)b$. Therefore

$$P(A)=\frac{k(a+b)}{(k+1)a+(k+1)b}\quad ,P(B)=\frac{ka+a}{(k+1)a+(k+1)b}$$

and

$$P(A\cap B)=\frac{ka}{(k+1)a+(k+1)b}=\frac{ka}{(k+1)(a+b)}$$

Then,

$$P(A)P(B)=\frac{k(a+b)(ka+a)}{[(k+1)a+(k+1)b]^2}=\frac{k(a+b)(k+1)a}{(k+1)^2(a+b)^2}=\frac{ka}{(k+1)(a+b)}=P(A\cap B)$$