Applied Mathematics for the Managerial Life 6th Edition Tan Solutions Manual
Full Download: https://alibabadownload.com/product/applied-mathematics-for-the-managerial-life-6th-edition-tan-solutions-manu

1 FUNDAMENTALS OF ALGEBRA

1.1 Real Numbers

Concept Questions page 6

1. The set of natural numbers is $N = \{1, 2, 3, \ldots\}$; the set of whole numbers is $W = \{0, 1, 2, 3, \ldots\}$; the set of integers is $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$; the set of rational numbers is $Q = \{a/b \mid a$ and b are integers and $b \neq 0\}$ (example: $1/2$), and the set of irrational numbers contains all real numbers that cannot be expressed in the form a/b, where a and b are integers and $b \neq 0$ (example: π). The set of real numbers contains all irrational and rational numbers.

2. **a.** The associative law of addition states that $a + (b + c) = (a + b) + c$.
   b. The distributive law states that $ab + ac = a(b + c)$.

3. If $ab \neq 0$, then neither a nor b is equal to zero. If $abc \neq 0$, then none of a, b, and c is equal to zero.

Exercises page 6

1. The number -3 is an integer, a rational number, and a real number.

2. The number -420 is an integer, a rational number, and a real number.

3. The number $\frac{3}{8}$ is a rational real number.

4. The number $-\frac{4}{125}$ is a rational real number.

5. The number $\sqrt{11}$ is an irrational real number.

6. The number $-\sqrt{5}$ is an irrational real number.

7. The number $\frac{\pi}{2}$ is an irrational real number.

8. The number $\frac{2}{\pi}$ is an irrational real number.

9. The number $2.\overline{421}$ is a rational real number.

10. The number $2.71828\ldots$ is an irrational real number.

11. False. -2 is not a whole number.

12. True.

13. True.

14. True.

15. False. No natural number is irrational.

16. True.

17. $(2x + y) + z = z + (2x + y)$: The Commutative Law of Addition.

18. $3x + (2y + z) = (3x + 2y) + z$: The Associative Law of Addition.

19. $u(3v + w) = (3v + w)u$: The Commutative Law of Multiplication.

20. $a^2(b^2c) = (a^2b^2)c$: The Associative Law of Multiplication.

This is sample only, Download all chapters at: AlibabaDownload.com

21. $u(2v + w) = 2uv + uw$: The Distributive Law.

22. $(2u + v)w = 2uw + vw$: The Distributive Law.

23. $(2x + 3y) + (x + 4y) = 2x + [3y + (x + 4y)]$: The Associative Law of Addition.

24. $(a + 2b)(a - 3b) = a(a - 3b) + 2b(a - 3b)$: The Distributive Law.

25. $a - [-(c + d)] = a + (c + d)$: Property 1 of negatives.

26. $-(2x + y)[-(3x + 2y)] = (2x + y)(3x + 2y)$: Property 3 of negatives.

27. $0(2a + 3b) = 0$: Property 1 involving zero.

28. If $(x - y)(x + y) = 0$, then $x = y$ or $x = -y$. Property 2 involving zero.

29. If $(x - 2)(2x + 5) = 0$, then $x = 2$, or $x = -\frac{5}{2}$. Property 2 involving zero.

30. If $x(2x - 9) = 0$, then $x = 0$ or $x = \frac{9}{2}$. Property 2 involving zero.

31. $\dfrac{(x + 1)(x - 3)}{(2x + 1)(x - 3)} = \dfrac{x + 1}{2x + 1}$. Property 2 of quotients.

32. $\dfrac{(2x + 1)(x + 3)}{(2x - 1)(x + 3)} = \dfrac{2x + 1}{2x - 1}$. Property 2 of quotients.

33. $\dfrac{a + b}{b} \div \dfrac{a - b}{ab} = \dfrac{a(a + b)}{a - b}$. Properties 2 and 5 of quotients.

34. $\dfrac{x + 2y}{3x + y} \div \dfrac{x}{6x + 2y} = \dfrac{x + 2y}{3x + y} \cdot \dfrac{2(3x + y)}{x} = \dfrac{2(x + 2y)}{x}$. Properties 2 and 5 of quotients and the Distributive Law.

35. $\dfrac{a}{b + c} + \dfrac{c}{b} = \dfrac{ab + bc + c^2}{b(b + c)}$. Property 6 of quotients and the Distributive Law.

36. $\dfrac{x + y}{x + 1} - \dfrac{y}{x} = \dfrac{x^2 - y}{x(x + 1)}$. Property 7 of quotients and the Distributive Law.

37. False. Consider $a = 2$ and $b = \frac{1}{2}$. Then $ab = 1$, but $a \neq 1$ and $b \neq 1$.

38. True. Multiplying both sides of the equation by $\dfrac{1}{a}$ (which exists because $a \neq 0$), we have $\dfrac{1}{a}(ab) = \dfrac{1}{a}(0)$, or $b = 0$.

39. False. Consider $a = 3$ and $b = 2$. Then $a - b = 3 - 2 \neq b - a = 2 - 3 = -1$.

40. False. Consider $a = 3$ and $b = 2$. Then $\dfrac{a}{b} = \dfrac{3}{2} \neq \dfrac{b}{a} = \dfrac{2}{3}$.

41. False. Consider $a = 1$, $b = 2$, and $c = 3$. Then $(a - b) - c = (1 - 2) - 3 = -4 \neq a - (b - c) = 1 - (2 - 3) = 2$.

42. False. Consider $a = 1$, $b = 2$, and $c = 3$. Then $\dfrac{a}{b/c} = \dfrac{1}{2/3} = \dfrac{3}{2} \neq \dfrac{a/b}{c} = \dfrac{1/2}{3} = \dfrac{1}{6}$.

1.2 Polynomials

Concept Questions

page 13

1. A polynomial of degree n in x is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$. One polynomial of degree 4 in x is $x^4 + 2x^3 - 2x^2 - 5x - 7$.

2. (a) $1 + 2b + b^2$ **b.** $a^2 - 2ab + b^2$ **c.** $a^2 - b^2$

Exercises

page 13

1. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81.$

2. $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32.$

3. $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$

4. $\left(-\frac{3}{4}\right)^2 = \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) = \frac{9}{16}.$

5. $-3^4 = -3 \cdot 3 \cdot 3 \cdot 3 = -81.$

6. $-\left(-\frac{4}{5}\right)^3 = -\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right) = \frac{64}{125}.$

7. $-3\left(\frac{3}{5}\right)^3 = (-3)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = -\frac{81}{125}.$

8. $\left(-\frac{2}{3}\right)^2\left(-\frac{3}{4}\right)^3 = \left(\frac{4}{9}\right)\left(-\frac{27}{64}\right) = -\frac{3}{16}.$

9. $2^3 \cdot 2^5 = 2^8 = 256.$

10. $(-3)^2 \cdot (-3)^3 = (-3)^5 = -243.$

11. $(3y)^2(3y)^3 = (3y)^5 = 243y^5.$

12. $(-2x)^3(-2x)^2 = (-2x)^5 = -32x^5.$

13. $(2x + 3) + (4x - 6) = 2x + 3 + 4x - 6 = 6x - 3.$

14. $(-3x + 2) - (4x - 3) = -3x + 2 - 4x + 3 = -7x + 5.$

15. $\left(7x^2 - 2x + 5\right) + \left(2x^2 + 5x - 4\right) = 7x^2 - 2x + 5 + 2x^2 + 5x - 4 = 7x^2 + 2x^2 - 2x + 5x + 5 - 4 = 9x^2 + 3x + 1.$

16. $\left(3x^2 + 5xy + 2y\right) + \left(4 - 3xy - 2x^2\right) = x^2 + 2xy + 2y + 4.$

17. $\left(5y^2 - 2y + 1\right) - \left(y^2 - 4y - 8\right) = 5y^2 - 2y + 1 - y^2 + 4y + 8 = 5y^2 - y^2 - 2y + 4y + 1 + 8 = 4y^2 + 2y + 9.$

18. $\left(2x^2 - 3x + 4\right) - \left(-x^2 + 2x - 6\right) = 2x^2 - 3x + 4 + x^2 - 2x + 6 = 3x^2 - 5x + 10.$

19.
$$\begin{aligned}\left(2.4x^3 - 3x^2 + 1.7x - 6.2\right) - \left(1.2x^3 + 1.2x^2 - 0.8x + 2\right) &= 2.4x^3 - 3x^2 + 1.7x - 6.2 - 1.2x^3 - 1.2x^2 + 0.8x - 2 \\ &= 1.2x^3 - 4.2x^2 + 2.5x - 8.2.\end{aligned}$$

20.
$$\begin{aligned}\left(1.4x^3 - 1.2x^2 + 3.2\right) - \left(-0.8x^3 - 2.1x - 1.8\right) &= 1.4x^3 - 1.2x^2 + 3.2 + 0.8x^3 + 2.1x + 1.8 \\ &= 2.2x^3 - 1.2x^2 + 2.1x + 5.\end{aligned}$$

21. $\left(3x^2\right)\left(2x^3\right) = 6x^5.$

22. $\left(-2rs^2\right)\left(4r^2s^2\right)(2s) = -16r^3s^5.$

23. $-2x\left(x^2 - 2\right) + 4x^3 = -2x^3 + 4x + 4x^3 = 2x^3 + 4x.$

24. $xy\,(2y-3x)=2xy^2-3x^2y.$

25. $2m\,(3m-4)+m\,(m-1)=6m^2-8m+m^2-m=7m^2-9m.$

26. $-3x\left(2x^2+3x-5\right)+2x\left(x^2-3\right)=-6x^3-9x^2+15x+2x^3-6x=-4x^3-9x^2+9x.$

27. $3\,(2a-b)-4\,(b-2a)=6a-3b-4b+8a=6a+8a-3b-4b=14a-7b.$

28. $2\,(3m-1)-3\,(-4m+2n)=6m-2+12m-6n=18m-6n-2.$

29. $(2x+3)\,(3x-2)=2x\,(3x-2)+3\,(3x-2)=6x^2-4x+9x-6=6x^2+5x-6.$

30. $(3r-1)\,(2r+5)=3r\,(2r+5)-(2r+5)=6r^2+15r-2r-5=6r^2+13r-5.$

31. $(2x-3y)\,(3x+2y)=2x\,(3x+2y)-3y\,(3x+2y)=6x^2+4xy-9xy-6y^2=6x^2-5xy-6y^2.$

32. $(5m-2n)\,(5m+3n)=5m\,(5m+3n)-2n\,(5m+3n)=25m^2+15mn-10mn-6n^2=25m^2+5mn-6n^2.$

33. $(3r+2s)\,(4r-3s)=3r\,(4r-3s)+2s\,(4r-3s)=12r^2-9rs+8rs-6s^2=12r^2-rs-6s^2.$

34. $(2m+3n)\,(3m-2n)=2m\,(3m-2n)+3n\,(3m-2n)=6m^2-4mn+9mn-6n^2=6m^2+5mn-6n^2.$

35.
$$\begin{aligned}(0.2x+1.2y)\,(0.3x-2.1y)&=0.2x\,(0.3x-2.1y)+1.2y\,(0.3x-2.1y)=0.06x^2-0.42xy+0.36xy-2.52y^2\\&=0.06x^2-0.06xy-2.52y^2.\end{aligned}$$

36.
$$\begin{aligned}(3.2m-1.7n)\,(4.2m+1.3n)&=3.2m\,(4.2m+1.3n)-1.7n\,(4.2m+1.3n)\\&=13.44m^2+4.16mn-7.14mn-2.21n^2=13.44m^2-2.98mn-2.21n^2.\end{aligned}$$

37. $(2x-y)\left(3x^2+2y\right)=2x\left(3x^2+2y\right)-y\left(3x^2+2y\right)=6x^3-3x^2y+4xy-2y^2.$

38. $\left(3m-2n^2\right)\left(2m^2+3n\right)=3m\left(2m^2+3n\right)-2n^2\left(2m^2+3n\right)=6m^3+9mn-4m^2n^2-6n^3.$

39. $(2x+3y)^2=(2x)^2+2\,(2x)\,(3y)+(3y)^2=4x^2+12xy+9y^2.$

40. $(3m-2n)^2=(3m)^2-2\,(3m)\,(2n)+(2n)^2=9m^2-12mn+4n^2.$

41. $(2u-v)\,(2u+v)=(2u)^2-v^2=4u^2-v^2.$

42. $(3r+4s)\,(3r-4s)=(3r)^2-(4s)^2=9r^2-16s^2.$

43. $(2x-1)^2+3x-2\left(x^2+1\right)+3=4x^2-4x+1+3x-2x^2-2+3=2x^2-x+2.$

44. $(3m+2)^2-2m\,(1-m)-4=9m^2+12m+4-2m+2m^2-4=11m^2+10m.$

45.
$$\begin{aligned}(2x+3y)^2-(2y+1)\,(3x-2)+2\,(x-y)&=4x^2+12xy+9y^2-6xy-3x+4y+2+2x-2y\\&=4x^2+6xy+9y^2-x+2y+2.\end{aligned}$$

46. $(x-2y)\,(y+3x)-2xy+3\,(x+y-1)=xy+3x^2-2y^2-6xy-2xy+3x+3y-3=3x^2-7xy-2y^2+3x+3y-3.$

47. $\left(t^2-2t+4\right)\left(2t^2+1\right)=\left(t^2-2t+4\right)\left(2t^2\right)+\left(t^2-2t+4\right)(1)=2t^4-4t^3+8t^2+t^2-2t+4$
$=2t^4-4t^3+9t^2-2t+4.$

48. $\left(3m^2-1\right)\left(2m^2+3m-4\right)=3m^2\left(2m^2+3m-4\right)-\left(2m^2+3m-4\right)=6m^4+9m^3-12m^2-2m^2-3m+4$
$=6m^4+9m^3-14m^2-3m+4.$

49. $2x-\{3x-[x-(2x-1)]\}=2x-\{3x-[x-2x+1]\}=2x-[3x-(-x+1)]=2x-(3x+x-1)$
$=2x-(4x-1)=2x-4x+1=-2x+1.$

50. $3m-2\{m-3[2m-(m-5)]+4\}=3m-2[m-3(2m-m+5)+4]=3m-2[m-3(m+5)+4]$
$=3m-2(m-3m-15+4)=3m-2(-2m-11)=3m+4m+22=7m+22.$

51. $x-\{2x-[-x-(1+x)]\}=x-[2x-(-x-1-x)]=x-[2x-(-2x-1)]=x-(2x+2x+1)$
$=x-4x-1=-3x-1.$

52. $3x^2-\left\{x^2+1-x[x-(2x-1)]\right\}+2=3x^2-\left[x^2+1-x(x-2x+1)\right]+2$
$=3x^2-\left[x^2+1-x(-x+1)\right]+2=3x^2-\left(x^2+1+x^2-x\right)+2=3x^2-2x^2-1+x+2=x^2+x+1.$

53. $(2x-3)^2-3(x+4)(x-4)+2(x-4)+1=(2x)^2-2(2x)(3)+3^2-3\left(x^2-16\right)+2x-8+1$
$=4x^2-12x+9-3x^2+48+2x-7=x^2-10x+50.$

54. $(x-2y)^2+2(x+y)(x-3y)+x(2x+3y+2)$
$=x^2-2x(2y)+(2y)^2+2\left(x^2-3xy+xy-3y^2\right)+2x^2+3xy+2x$
$=x^2-4xy+4y^2+2x^2-4xy-6y^2+2x^2+3xy+2x=5x^2-5xy-2y^2+2x.$

55. $2x\{3x[2x-(3-x)]+(x+1)(2x-3)\}=2x\left[3x(2x-3+x)+2x^2-3x+2x-3\right]$
$=2x\left[3x(3x-3)+2x^2-x-3\right]=2x\left(9x^2-9x+2x^2-x-3\right)=2x\left(11x^2-10x-3\right)=22x^3-20x^2-6x.$

56. $-3\left[(x+2y)^2-(3x-2y)^2+(2x-y)(2x+y)\right]=-3\left[x^2+4xy+4y^2-\left(9x^2-12xy+4y^2\right)+\left(4x^2-y^2\right)\right]$
$=-3\left(x^2+4xy+4y^2-9x^2+12xy-4y^2+4x^2-y^2\right)$
$=-3\left(-4x^2+16xy-y^2\right)=12x^2-48xy+3y^2.$

57. The total weekly profit is given by the revenue minus the cost:
$\left(-0.04x^2+2000x\right)-\left(0.000002x^3-0.02x^2+1000x+120{,}000\right)$
$=-0.04x^2+2000x-0.000002x^3+0.02x^2-1000x-120{,}000$
$=-0.000002x^3-0.02x^2+1000x-120{,}000.$

58. The total revenue is given by $xp=x(-0.0004x+10)=-0.0004x^2+10x$. Therefore, the total profit is given by the revenue minus the cost: $-0.0004x^2+10x-\left(0.0001x^2+4x+400\right)=-0.0005x^2+6x-400.$

59. The total revenue is given by $\left(0.2t^2+150t\right)+\left(0.5t^2+200t\right)=0.7t^2+350t$ thousand dollars t months from now, where $0\le t\le 12$.

60. In month t, the revenue of the second gas station will exceed that of the first gas station by $\left(0.5t^2+200t\right)-\left(0.2t^2+150t\right)=0.3t^2+50t$ thousand dollars, where $0<t\leq 12$.

61. The gap is given by $\left(3.5t^2+26.7t+436.2\right)-(24.3t+365)=3.5t^2+2.4t+71.2$.

62. The difference is given by $\left(2.5t^2+18.5t+509\right)-\left(-1.1t^2+29.1t+429\right)=3.6t^2-10.6t+80$ dollars. The difference at the beginning of 1998 is obtained by replacing t with 4, giving $3.6\,(4)^2-10.6\,(4)+80=95.2$, or \$95.20. The difference at the beginning of 2000 is given by $3.6\,(6)^2-10.6\,(6)+80=146$, or \$146.

63. False. Let $a=2$, $b=3$, $m=3$, and $n=2$. Then $2^3\cdot 3^2=8\cdot 9=72\neq(2\cdot 3)^{3+2}=6^5$.

64. True.

65. False. For example, x^2+1 is a polynomial of degree 2 and x is a polynomial of degree 1, but $\left(x^2+1\right)x=x^3+x$ is a polynomial of degree 3, not 2.

66. False. For example, $p=x^3+x+1$ is a polynomial of degree 3 and $q=-x^3+2$ is a polynomial of degree 3, but $p+q=x^3+x+1+\left(-x^3+2\right)=x+3$ is a polynomial of degree 1.

67. The degree of $p-q$ is m. To see this, suppose that $p=a_mx^m+\cdots+a_nx^n+\cdots+a_0$ and $q=b_nx^n+\cdots+b_0$. Because $m>n$, $p-q=a_mx^m+\cdots+(a_n-b_n)\,x^n+\cdots+(a_0-b_0)$ has degree m.

1.3 Factoring Polynomials

Concept Questions page 19

1. A polynomial is completely factored over the set of integers if it is expressed as a product of prime polynomials with integral coefficients. An example is $4x^2-9y^2=(2x-3y)\,(2x+3y)$.

2. a. $(a+b)\left(a^2-ab+b^2\right)$ **b.** $(a-b)\left(a^2+ab+b^2\right)$

Exercises page 19

1. $6m^2-4m=2m\,(3m-2)$.

2. $4t^4-12t^3=4t^3\,(t-3)$.

3. $9ab^2-6a^2b=3ab\,(3b-2a)$.

4. $12x^3y^5+16x^2y^3=4x^2y^3\left(3xy^2+4\right)$.

5. $10m^2n-15mn^2+20mn=5mn\,(2m-3n+4)$.

6. $6x^4y-4x^2y^2+2x^2y^3=2x^2y\left(3x^2-2y+y^2\right)$.

7. $3x\,(2x+1)-5\,(2x+1)=(2x+1)\,(3x-5)$.

8. $2u\left(3v^2+w\right)+5v\left(3v^2+w\right)=\left(3v^2+w\right)(2u+5v)$.

9. $(3a+b)\,(2c-d)+2a\,(2c-d)^2=(2c-d)\,[3a+b+2a\,(2c-d)]=(2c-d)\,(3a+b+4ac-2ad)$.

10. $4uv^2\,(2u-v)+6u^2v\,(v-2u)=\left(4uv^2-6u^2v\right)(2u-v)=2uv\,(2u-v)\,(2v-3u)$.

11. $2m^2-11m-6=(2m+1)\,(m-6)$.

12. $6x^2-x-1=(3x+1)\,(2x-1)$.

13. $x^2-xy-6y^2=(x-3y)\,(x+2y)$.

14. $2u^2+5uv-12v^2=(2u-3v)\,(u+4v)$.

15. $x^2 - 3x - 1$ is prime.

16. $m^2 + 2m + 3$ is prime.

17. $4a^2 - b^2 = (2a - b)(2a + b)$.

18. $12x^2 - 3y^2 = 3\left(4x^2 - y^2\right) = 3(2x - y)(2x + y)$.

19. $u^2v^2 - w^2 = (uv)^2 - w^2 = (uv - w)(uv + w)$.

20. $4a^2b^2 - 25c^2 = (2ab)^2 - (5c)^2 = (2ab - 5c)(2ab + 5c)$.

21. $z^2 + 4$ is prime.

22. $u^2 + 25v^2$ is prime.

23. $x^2 + 6xy + y^2$ is prime.

24. $4u^2 - 12uv + 9v^2 = (2u - 3v)^2$.

25. $x^2 + 3x - 4 = (x + 4)(x - 1)$.

26. $3m^3 + 3m^2 - 18m = 3m\left(m^2 + m - 6\right) = 3m(m + 3)(m - 2)$.

27. $12x^2y - 10xy - 12y = 2y\left(6x^2 - 5x - 6\right) = 2y(3x + 2)(2x - 3)$.

28. $12x^2y - 2xy - 24y = 2y\left(6x^2 - x - 12\right) = 2y(3x + 4)(2x - 3)$.

29. $35r^2 + r - 12 = (7r - 4)(5r + 3)$.

30. $6uv^2 + 9uv - 6v = 3v(2uv + 3u - 2)$.

31. $9x^3y - 4xy^3 = xy\left(9x^2 - 4y^2\right) = xy\left[(3x)^2 - (2y)^2\right] = xy(3x - 2y)(3x + 2y)$.

32. $4u^4v - 9u^2v^3 = u^2v\left(4u^2 - 9v^2\right) = u^2v\left[(2u)^2 - (3v)^2\right] = u^2v(2u - 3v)(2u + 3v)$.

33. $x^4 - 16y^2 = \left(x^2\right)^2 - (4y)^2 = \left(x^2 - 4y\right)\left(x^2 + 4y\right)$.

34. $16u^4v - 9v^3 = v\left(16u^4 - 9v^2\right) = v\left[\left(4u^2\right)^2 - (3v)^2\right] = v\left(4u^2 - 3v\right)\left(4u^2 + 3v\right)$.

35. $(a - 2b)^2 - (a + 2b)^2 = [(a - 2b) - (a + 2b)][(a - 2b) + (a + 2b)] = (-4b)(2a) = -8ab$.

36. $2x(x + y)^2 - 8x\left(x + y^2\right)^2 = 2x\left[(x + y)^2 - 4\left(x + y^2\right)^2\right] = 2x\left[(x + y) - 2\left(x + y^2\right)\right]\left[(x + y) + 2\left(x + y^2\right)\right]$
$= 2x\left(y - x - 2y^2\right)\left(3x + y + 2y^2\right)$.

37. $8m^3 + 1 = (2m)^3 + 1 = (2m + 1)\left(4m^2 - 2m + 1\right)$.

38. $27m^3 - 8 = (3m)^3 - 2^3 = (3m - 2)\left(9m^2 + 6m + 4\right)$.

39. $8r^3 - 27s^3 = (2r)^3 - (3s)^3 = (2r - 3s)\left(4r^2 + 6rs + 9s^2\right)$.

40. $x^3 + 64y^3 = x^3 + (4y)^3 = (x + 4y)\left(x^2 - 4xy + 16y^2\right)$.

41. $u^2v^6 - 8u^2 = u^2\left(v^6 - 8\right) = u^2\left(v^2 - 2\right)\left(v^4 + 2v^2 + 4\right)$.

42. $r^6s^6 + 8s^3 = s^3\left(r^6s^3 + 8\right) = s^3\left[\left(r^2s\right)^3 + 2^3\right] = s^3\left(r^2s + 2\right)\left(r^4s^2 - 2r^2s + 4\right)$.

43. $2x^3 + 6x + x^2 + 3 = 2x\left(x^2 + 3\right) + \left(x^2 + 3\right) = \left(x^2 + 3\right)(2x + 1)$.

44. $2u^4 - 4u^2 + 2u^2 - 4 = 2u^4 - 2u^2 - 4 = 2\left(u^4 - u^2 - 2\right) = 2\left(u^2 + 1\right)\left(u^2 - 2\right)$.

45. $3ax + 6ay + bx + 2by = 3a(x + 2y) + b(x + 2y) = (x + 2y)(3a + b)$.

46. $6ux - 4uy + 3vx - 2vy = 2u(3x - 2y) + v(3x - 2y) = (3x - 2y)(2u + v)$.

47. $u^4 - v^4 = \left(u^2\right)^2 - \left(v^2\right)^2 = \left(u^2 - v^2\right)\left(u^2 + v^2\right) = (u - v)(u + v)\left(u^2 + v^2\right)$.

48. $u^4 - u^2v^2 - 6v^4 = \left(u^2 - 3v^2\right)\left(u^2 + 2v^2\right)$.

49. $4x^3 - 9xy^2 + 4x^2y - 9y^3 = x\left(4x^2 - 9y^2\right) + y\left(4x^2 - 9y^2\right) = \left[(2x)^2 - (3y)^2\right](x + y)$

$= (2x - 3y)(2x + 3y)(x + y)$.

50. $4u^4 + 11u^2v^2 - 3v^4 = \left(4u^2 - v^2\right)\left(u^2 + 3v^2\right) = (2u - v)(2u + v)\left(u^2 + 3v^2\right)$.

51. $x^4 + 3x^3 - 2x - 6 = x^3(x + 3) - 2(x + 3) = (x + 3)\left(x^3 - 2\right)$.

52. $a^2 - b^2 + a + b = (a - b)(a + b) + (a + b) = (a + b)(a - b + 1)$.

53. $au^2 + (a + c)u + c = au^2 + au + cu + c = au(u + 1) + c(u + 1) = (u + 1)(au + c)$.

54. $ax^2 - (1 + ab)xy + by^2 = ax^2 - xy - abxy + by^2 = ax(x - by) - y(x - by) = (x - by)(ax - y)$.

55. $P + Prt = P(1 + rt)$.

56. $-t^3 + 6t^2 + 15t = -t\left(t^2 - 6t - 15\right)$.

57. $8000x - 100x^2 = 100x(80 - x)$.

58. $R = kQx - kx^2 = kx(Q - x)$.

59. $kMx - kx^2 = kx(M - x)$.

60. $-0.1x^2 + 500x = -0.1x(x - 5000)$.

61. $V = V_0 + \frac{V_0}{273}T = \frac{V_0}{273}(273 + T)$.

62. $\frac{kD^2}{2} - \frac{D^3}{3} = D^2\left(\frac{k}{2} - \frac{D}{3}\right)$.

1.4 Rational Expressions

Concept Questions

page 25

1. a. Quotients of polynomials are rational expressions; $\frac{2x^2 + 1}{3x^2 - 3x + 4}$.

b. Any polynomial P can be written in the form $\frac{P}{1}$, but not all rational expressions can be written as a polynomial.

2. a. $\frac{PR}{QS}$; $\frac{PS}{RQ}$.

b. $\frac{P + Q}{R}$; $\frac{P - Q}{R}$.

Exercises

page 25

1. $\dfrac{28x^2}{7x^3} = \dfrac{4}{x}.$

2. $\dfrac{3y^4}{18y^2} = \frac{1}{6}y^2.$

3. $\dfrac{4x+12}{5x+15} = \dfrac{4(x+3)}{5(x+3)} = \dfrac{4}{5}.$

4. $\dfrac{12m-6}{18m-9} = \dfrac{6(2m-1)}{9(2m-1)} = \dfrac{2}{3}.$

5. $\dfrac{6x^2-3x}{6x^2} = \dfrac{3x(2x-1)}{6x^2} = \dfrac{2x-1}{2x}.$

6. $\dfrac{8y^2}{4y^3-4y^2+8y} = \dfrac{8y^2}{4y\left(y^2-y+2\right)} = \dfrac{2y}{y^2-y+2}.$

7. $\dfrac{x^2+x-2}{x^2+3x+2} = \dfrac{(x+2)(x-1)}{(x+2)(x+1)} = \dfrac{x-1}{x+1}.$

8. $\dfrac{2y^2-y-3}{2y^2+y-1} = \dfrac{(2y-3)(y+1)}{(2y-1)(y+1)} = \dfrac{2y-3}{2y-1}.$

9. $\dfrac{x^2-9}{2x^2-5x-3} = \dfrac{(x-3)(x+3)}{(2x+1)(x-3)} = \dfrac{x+3}{2x+1}.$

10. $\dfrac{6y^2+11y+3}{4y^2-9} = \dfrac{(3y+1)(2y+3)}{(2y-3)(2y+3)} = \dfrac{3y+1}{2y-3}.$

11. $\dfrac{x^3+y^3}{x^2-xy+y^2} = \dfrac{(x+y)\left(x^2-xy+y^2\right)}{x^2-xy+y^2} = x+y.$

12. $\dfrac{8r^3-s^3}{2r^2+rs-s^2} = \dfrac{(2r-s)\left(4r^2+2rs+s^2\right)}{(2r-s)(r+s)} = \dfrac{4r^2+2rs+s^2}{r+s}.$

13. $\dfrac{6x^3}{32} \cdot \dfrac{8}{3x^2} = \dfrac{1}{2}x.$

14. $\dfrac{25y^4}{12y} \cdot \dfrac{3y^2}{5y^3} = \frac{5}{4}y^2.$

15. $\dfrac{3x^3}{8x^2} \div \dfrac{15x^4}{16x^5} = \dfrac{3x^3}{8x^2} \cdot \dfrac{16x^5}{15x^4} = \dfrac{2x^8}{5x^6} = \frac{2}{5}x^2$

16. $\dfrac{6x^5}{21x^2} \div \dfrac{4x}{7x^3} = \dfrac{6x^5}{21x^2} \cdot \dfrac{7x^3}{4x} = \frac{1}{2}x^5.$

17. $\dfrac{3x}{x+2y} \cdot \dfrac{5x+10y}{6} = \dfrac{(3x)\,5(x+2y)}{6(x+2y)} = \dfrac{5x}{2}.$

18. $\dfrac{4y+12}{y+2} \cdot \dfrac{3y+6}{2y-1} = \dfrac{4(y+3)\,3(y+2)}{(y+2)(2y-1)} = \dfrac{12(y+3)}{2y-1}.$

19. $\dfrac{2m+6}{3} \div \dfrac{3m+9}{6} = \dfrac{2(m+3)}{3} \cdot \dfrac{6}{3(m+3)} = \dfrac{4}{3}.$

20. $\dfrac{3y-6}{4y+6} \div \dfrac{6y+24}{8y+12} = \dfrac{3(y-2)}{2(2y+3)} \cdot \dfrac{4(2y+3)}{6(y+4)} = \dfrac{y-2}{y+4}.$

21. $\dfrac{6r^2-r-2}{2r+4} \cdot \dfrac{6r+12}{4r+2} = \dfrac{(3r-2)(2r+1)\,6(r+2)}{2(r+2)\,2(2r+1)} = \dfrac{3(3r-2)}{2}.$

22. $\dfrac{x^2-x-6}{2x^2+7x+6} \cdot \dfrac{2x^2-x-6}{x^2+x-6} = \dfrac{(x-3)(x+2)(2x+3)(x-2)}{(2x+3)(x+2)(x+3)(x-2)} = \dfrac{x-3}{x+3}.$

23. $\dfrac{k^2-2k-3}{k^2-k-6} \div \dfrac{k^2-6k+8}{k^2-2k-8} = \dfrac{(k-3)(k+1)}{(k-3)(k+2)} \cdot \dfrac{(k-4)(k+2)}{(k-4)(k-2)} = \dfrac{k+1}{k-2}.$

24. $\dfrac{6y^2-5y-6}{6y^2+13y+6} \div \dfrac{6y^2-13y+6}{9y^2-12y+4} = \dfrac{(3y+2)(2y-3)}{(3y+2)(2y+3)} \cdot \dfrac{(3y-2)(3y-2)}{(3y-2)(2y-3)} = \dfrac{3y-2}{2y+3}.$

25. $\dfrac{2}{2x+3} + \dfrac{3}{2x-1} = \dfrac{2(2x-1)+3(2x+3)}{(2x+3)(2x-1)} = \dfrac{4x-2+6x+9}{(2x+3)(2x-1)} = \dfrac{10x+7}{(2x+3)(2x-1)}.$

26. $\dfrac{2x-1}{x+2} - \dfrac{x+3}{x-1} = \dfrac{(2x-1)(x-1)-(x+3)(x+2)}{(x+2)(x-1)} = \dfrac{2x^2-3x+1-x^2-5x-6}{(x+2)(x-1)} = \dfrac{x^2-8x-5}{(x+2)(x-1)}.$

27. $\dfrac{3}{x^2-x-6}+\dfrac{2}{x^2+x-2}=\dfrac{3}{(x-3)(x+2)}+\dfrac{2}{(x+2)(x-1)}=\dfrac{3(x-1)+2(x-3)}{(x-3)(x+2)(x-1)}$
$=\dfrac{3x-3+2x-6}{(x-3)(x+2)(x-1)}=\dfrac{5x-9}{(x-3)(x+2)(x-1)}.$

28. $\dfrac{4}{x^2-9}-\dfrac{5}{x^2-6x+9}=\dfrac{4}{(x-3)(x+3)}-\dfrac{5}{(x-3)^2}=\dfrac{4(x-3)-5(x+3)}{(x+3)(x-3)^2}=\dfrac{4x-12-5x-15}{(x+3)(x-3)^2}$
$=-\dfrac{x+27}{(x+3)(x-3)^2}.$

29. $\dfrac{2m}{2m^2-2m-1}+\dfrac{3}{2m^2-3m+3}=\dfrac{2m\left(2m^2-3m+3\right)+3\left(2m^2-2m-1\right)}{\left(2m^2-2m-1\right)\left(2m^2-3m+3\right)}$
$=\dfrac{4m^3-6m^2+6m+6m^2-6m-3}{\left(2m^2-2m-1\right)\left(2m^2-3m+3\right)}=\dfrac{4m^3-3}{\left(2m^2-2m-1\right)\left(2m^2-3m+3\right)}.$

30. $\dfrac{t}{t^2+t-2}-\dfrac{2t-1}{2t^2+3t-2}=\dfrac{t}{(t+2)(t-1)}-\dfrac{2t-1}{(t+2)(2t-1)}=\dfrac{t}{(t+2)(t-1)}-\dfrac{1}{t+2}$
$=\dfrac{t-1(t-1)}{(t+2)(t-1)}=\dfrac{t-t+1}{(t+2)(t-1)}=\dfrac{1}{(t+2)(t-1)}.$

31. $\dfrac{x}{1-x}+\dfrac{2x+3}{x^2-1}=-\dfrac{x}{x-1}+\dfrac{2x+3}{(x+1)(x-1)}=\dfrac{-x(x+1)+2x+3}{(x+1)(x-1)}=\dfrac{-x^2-x+2x+3}{(x+1)(x-1)}=-\dfrac{x^2-x-3}{(x+1)(x-1)}.$

32. $2+\dfrac{1}{a+2}-\dfrac{2a}{a-2}=\dfrac{2(a+2)(a-2)+a-2-2a(a+2)}{(a+2)(a-2)}=\dfrac{2a^2-8+a-2-2a^2-4a}{(a+2)(a-2)}=-\dfrac{3a+10}{(a+2)(a-2)}.$

33. $x-\dfrac{x^2}{x+2}+\dfrac{2}{x-2}=\dfrac{x(x+2)(x-2)-x^2(x-2)+2(x+2)}{(x+2)(x-2)}=\dfrac{x^3-4x-x^3+2x^2+2x+4}{(x+2)(x-2)}$
$=\dfrac{2x^2-2x+4}{(x+2)(x-2)}=\dfrac{2\left(x^2-x+2\right)}{(x+2)(x-2)}.$

34. $\dfrac{y}{y^2-1}+\dfrac{y-1}{y+1}-\dfrac{2y}{1-y}=\dfrac{y}{(y+1)(y-1)}+\dfrac{y-1}{y+1}+\dfrac{2y}{y-1}=\dfrac{y+(y-1)(y-1)+2y(y+1)}{(y+1)(y-1)}$
$=\dfrac{y+y^2-2y+1+2y^2+2y}{(y+1)(y-1)}=\dfrac{3y^2+y+1}{(y+1)(y-1)}.$

35. $\dfrac{x}{x^2+5x+6}+\dfrac{2}{x^2-4}-\dfrac{3}{x^2+3x+2}=\dfrac{x}{(x+3)(x+2)}+\dfrac{2}{(x-2)(x+2)}-\dfrac{3}{(x+1)(x+2)}$
$=\dfrac{x(x-2)(x+1)+2(x+3)(x+1)-3(x+3)(x-2)}{(x+3)(x+2)(x-2)(x+1)}=\dfrac{x^3-x^2-2x+2x^2+8x+6-3x^2-3x+18}{(x+3)(x+2)(x-2)(x+1)}$
$=\dfrac{x^3-2x^2+3x+24}{(x+3)(x+2)(x-2)(x+1)}.$

36. $\dfrac{2x+1}{2x^2-x-1}-\dfrac{x+1}{2x^2+3x+1}+\dfrac{4}{x^2+2x-3}=\dfrac{2x+1}{(2x+1)(x-1)}-\dfrac{x+1}{(2x+1)(x+1)}+\dfrac{4}{(x+3)(x-1)}$
$=\dfrac{1}{x-1}-\dfrac{1}{2x+1}+\dfrac{4}{(x+3)(x-1)}=\dfrac{(2x+1)(x+3)-(x-1)(x+3)+4(2x+1)}{(x-1)(2x+1)(x+3)}$
$=\dfrac{2x^2+7x+3-x^2-2x+3+8x+4}{(x-1)(2x+1)(x+3)}=\dfrac{x^2+13x+10}{(x-1)(2x+1)(x+3)}$

37. $\dfrac{x}{ax-ay}+\dfrac{y}{by-bx}=\dfrac{x}{a(x-y)}-\dfrac{y}{b(x-y)}=\dfrac{bx-ay}{ab(x-y)}$.

38. $\dfrac{ax+by}{ax-bx}+\dfrac{ay-bx}{by-ay}=\dfrac{ax+by}{x(a-b)}+\dfrac{ay-bx}{-y(a-b)}=\dfrac{-y(ax+by)+x(ay-bx)}{-(a-b)xy}=\dfrac{-by^2-bx^2}{-(a-b)xy}$
$=\dfrac{-b\left(x^2+y^2\right)}{-xy(a-b)}=\dfrac{b\left(x^2+y^2\right)}{(a-b)xy}$.

39. $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\dfrac{x+1}{x}}{\dfrac{x-1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}$.

40. $\dfrac{2+\dfrac{2}{x}}{x-\dfrac{2}{x}}=\dfrac{\dfrac{2x+2}{x}}{\dfrac{x^2-2}{x}}=\dfrac{2(x+1)}{x}\cdot\dfrac{x}{x^2-2}=\dfrac{2(x+1)}{x^2-2}$.

41. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{1-\dfrac{1}{xy}}=\dfrac{\dfrac{y+x}{xy}}{\dfrac{xy-1}{xy}}=\dfrac{y+x}{xy}\cdot\dfrac{xy}{xy-1}=\dfrac{y+x}{xy-1}$.

42. $\dfrac{1+\dfrac{x}{y}}{1-\dfrac{x^2}{y^2}}=\dfrac{\dfrac{y+x}{y}}{\dfrac{y^2-x^2}{y^2}}=\dfrac{x+y}{y}\cdot\dfrac{y^2}{(y-x)(y+x)}=\dfrac{y}{y-x}$.

43. $\dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{x+y}=\dfrac{\dfrac{y^2-x^2}{x^2y^2}}{x+y}=\dfrac{(y+x)(y-x)}{x^2y^2}\cdot\dfrac{1}{x+y}=\dfrac{y-x}{x^2y^2}$.

44. $\dfrac{\dfrac{1}{x^3}-\dfrac{1}{y^3}}{\dfrac{1}{x}-\dfrac{1}{y}}=\dfrac{\dfrac{y^3-x^3}{x^3y^3}}{\dfrac{y-x}{xy}}=\dfrac{y^3-x^3}{x^3y^3}\cdot\dfrac{xy}{y-x}=\dfrac{(y-x)\left(y^2+xy+x^2\right)}{x^2y^2(y-x)}=\dfrac{y^2+xy+x^2}{x^2y^2}$.

45. $\dfrac{\dfrac{1}{2(x+h)}-\dfrac{1}{2x}}{h}=\dfrac{\dfrac{x-(x+h)}{2x(x+h)}}{h}=-\dfrac{h}{2x(x+h)}\cdot\dfrac{1}{h}=-\dfrac{1}{2x(x+h)}$.

46. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\dfrac{\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}=\dfrac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\cdot\dfrac{1}{h}=-\dfrac{2x+h}{x^2(x+h)^2}$.

47. **a.** $2.2+\dfrac{2500}{x}=\dfrac{2.2x+2500}{x}$.

b. The total cost is $x\left(\dfrac{2.2x+2500}{x}\right)=2.2x+2500$.

48. $A=\dfrac{km}{q}+cm+\dfrac{hq}{2}=\dfrac{2km+2cmq+hq^2}{2q}$.

49. $P=\dfrac{R}{i}-\dfrac{R}{i(1+i)^n}=\dfrac{R(1+i)^n-R}{i(1+i)^n}=\dfrac{R\left[(1+i)^n-1\right]}{i(1+i)^n}$.

50. $P=\dfrac{kT}{V-b}+\dfrac{ab}{V^2(V-b)}-\dfrac{a}{V(V-b)}=\dfrac{kTV^2+ab-aV}{V^2(V-b)}$.

51. $A = \frac{136}{1+0.25(t-4.5)^2} + 28 = \frac{136 + 28\left[1+0.25(t-4.5)^2\right]}{1+0.25(t-4.5)^2} = \frac{164+7(t-4.5)^2}{1+0.25(t-4.5)^2}$.

1.5 Integral Exponents

Concept Questions

page 30

1. If a is any real number and n is a natural number, then the expression a^n is defined as the number $a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$, where the number a is the base and the superscript n is the exponent, or power, to which the base is raised. For any real number a, $a^0 = 1$. If n is a negative number and $a \neq 0$, then $a^n = \frac{1}{a^{-n}}$.

2. a. $a^m \cdot a^n = a^{m+n}$. For example, $2x^2 \cdot x^7 = 2x^{2+7} = 2x^9$.

b. $\frac{a^m}{a^n} = a^{m-n}$. For example, $\frac{y^6}{2y^3} = \frac{1}{2}y^{6-3} = \frac{1}{2}y^3$.

c. $(a^m)^n = a^{mn}$. For example, $\left(2^4\right)^3 = 2^{4\cdot 3} = 2^{12}$.

d. $(ab)^n = a^n \cdot b^n$. For example, $(3 \cdot 2)^4 = 3^4 \cdot 2^4 = 81 \cdot 16 = 1296$.

e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$. For example, $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$.

Exercises

page 30

1. $(-2)^3 = -8$.

2. $\left(-\frac{2}{3}\right)^4 = \frac{16}{81}$.

3. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$.

4. $\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \frac{16}{9}$.

5. $-\left(-\frac{1}{4}\right)^{-2} = -\frac{1}{\left(-\frac{1}{4}\right)^2} = -\frac{1}{\frac{1}{16}} = -16$.

6. $-4^2 = -16$.

7. $2^{-2} + 3^{-1} = \frac{1}{2^2} + \frac{1}{3} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$.

8. $-3^{-2} - \left(-\frac{2}{3}\right)^2 = -\frac{1}{9} - \frac{4}{9} = -\frac{5}{9}$.

9. $(0.03)^2 = 0.0009$.

10. $(-0.3)^{-2} = 11.111\overline{1}$.

11. $1996^0 = 1$.

12. $(18+25)^0 = 1$.

13. $\left(ab^2\right)^0 = 1$.

14. $\left(3x^2y^3\right)^0 = 1$.

15. $\frac{2^3 \cdot 2^5}{2^4 \cdot 2^9} = 2^{3+5-4-9} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$.

16. $\frac{6 \cdot 10^4}{3 \cdot 10^2} = 2 \cdot 10^2 = 200$.

17. $\frac{2^{-3} \cdot 2^{-4}}{2^{-5} \cdot 2^{-2}} = 2^{-3-4+5+2} = 2^0 = 1$.

18. $\frac{4 \cdot 2^{-3}}{2 \cdot 4^{-2}} = \frac{2^2 \cdot 2^{-3}}{2 \cdot \left(2^2\right)^{-2}} = \frac{2^2 2^{-3}}{2 \cdot 2^{-4}} = 2^{2-3-1+4} = 2^2 = 4$.

19. $\left(\dfrac{3^4\cdot 3^{-3}}{3^{-2}}\right)^{-1} = \left(3^{4-3+2}\right)^{-1} = \left(3^3\right)^{-1} = \dfrac{1}{3^3} = \dfrac{1}{27}.$

20. $\left(\dfrac{5^{-2}\cdot 5^{-2}}{5^{-5}}\right)^{-2} = \left(5^{-2-2+5}\right)^{-2} = (5)^{-2} = \dfrac{1}{25}.$

21. $\left(2x^3\right)\left(\frac{1}{8}x^2\right) = \frac{1}{4}x^5.$

22. $\left(-2x^2\right)\left(3x^{-4}\right) = -6x^{-2} = -\dfrac{6}{x^2}.$

23. $\dfrac{3x^3}{2x^4} = \dfrac{3}{2x}.$

24. $\dfrac{\left(3x^2\right)\left(4x^3\right)}{2x^4} = 6x^{2+3-4} = 6x.$

25. $\left(a^{-2}\right)^3 = a^{-6} = \dfrac{1}{a^6}.$

26. $\left(-a^2\right)^{-3} = (-1)^{-3}\left(a^2\right)^{-3} = -a^{-6} = -\dfrac{1}{a^6}.$

27. $\left(2x^{-2}y^2\right)^3 = 8x^{-6}y^6 = \dfrac{8y^6}{x^6}.$

28. $\left(3u^{-1}v^{-2}\right)^{-3} = 3^{-3}u^3v^6 = \dfrac{u^3v^6}{27}.$

29. $\left(4x^2y^{-3}\right)\left(2x^{-3}y^2\right) = 8x^{-1}y^{-1} = \dfrac{8}{xy}.$

30. $\left(\frac{1}{2}u^{-2}v^3\right)\left(4v^3\right) = 2u^{-2}v^6 = \dfrac{2v^6}{u^2}.$

31. $\left(-x^2y\right)^3\left(\dfrac{2y^2}{x^4}\right) = -\dfrac{2x^6y^3y^2}{x^4} = -2x^2y^5.$

32. $\left(-\frac{1}{2}x^2y\right)^{-2} = \dfrac{(-1)^{-2}}{2^{-2}}x^{-4}y^{-2} = \dfrac{4}{x^4y^2}.$

33. $\left(\dfrac{2u^2v^3}{3uv}\right)^{-1} = \left(\dfrac{2uv^2}{3}\right)^{-1} = \dfrac{3}{2uv^2}.$

34. $\left(\dfrac{a^{-2}}{2b^2}\right)^{-3} = \dfrac{a^6}{2^{-3}b^{-6}} = 8a^6b^6.$

35. $\left(3x^{-2}\right)^3\left(2x^2\right)^5 = \left(27x^{-6}\right)\left(32x^{10}\right) = 864x^4.$

36. $\left(2^{-1}r^3\right)^{-2}\left(3s^{-1}\right)^2 = 2^2r^{-6}3^2s^{-2} = \dfrac{36}{r^6s^2}.$

37. $\dfrac{3^0\cdot 4x^{-2}}{16\cdot\left(x^2\right)^3} = \dfrac{4x^{-2}}{16x^6} = \dfrac{1}{4x^8}.$

38. $\dfrac{5x^2\left(3x^{-2}\right)}{\left(4x^{-1}\right)\left(x^3\right)^{-2}} = \dfrac{15x^0}{4x^{-1}x^{-6}} = \dfrac{15x^7}{4}.$

39. $\dfrac{2^2u^{-2}\left(v^{-1}\right)^3}{3^2\left(u^{-3}v\right)^2} = \dfrac{4u^{-2}v^{-3}}{9u^{-6}v^2} = \dfrac{4u^4}{9v^5}.$

40. $\dfrac{\left(3a^{-1}b^2\right)^{-2}}{\left(2a^2b^{-1}\right)^{-3}} = \dfrac{3^{-2}a^2b^{-4}}{2^{-3}a^{-6}b^3} = \dfrac{8a^8}{9b^7}.$

41. $(-2x)^{-2}(3y)^{-3}(4z)^{-2} = (-2)^{-2}x^{-2}3^{-3}y^{-3}4^{-2}z^{-2} = \dfrac{1}{4\cdot 27\cdot 16x^2y^3z^2} = \dfrac{1}{1728x^2y^3z^2}.$

42. $\left(3x^{-1}\right)^2\left(4y^{-1}\right)^3(2z)^{-2} = 3^2x^{-2}4^3y^{-3}2^{-2}z^{-2} = \dfrac{9\cdot 64}{4x^2y^3z^2} = \dfrac{144}{x^2y^3z^2}.$

43. $\left(a^2b^{-3}\right)^2\left(a^{-2}b^2\right)^{-3} = a^4b^{-6}a^6b^{-6} = \dfrac{a^{10}}{b^{12}}.$

44. $\left(5u^2v^{-3}\right)^{-1}\cdot 3\left(2u^2v^2\right)^{-2} = 5^{-1}u^{-2}v^3 3\cdot 2^{-2}u^{-4}v^{-4} = \dfrac{3}{20u^6v}.$

45. $\left[\left(\dfrac{a^{-2}b^{-2}}{3a^{-1}b^2}\right)^2\right]^{-1} = \left[\left(\dfrac{1}{3ab^4}\right)^2\right]^{-1} = \left(\dfrac{1}{9a^2b^8}\right)^{-1} = 9a^2b^8.$

46. $\left[\left(\dfrac{x^2y^{-3}z^{-4}}{x^{-2}y^{-1}z^2}\right)^{-2}\right]^3 = \left(x^4y^{-2}z^{-6}\right)^{-6} = x^{-24}y^{12}z^{36} = \dfrac{y^{12}z^{36}}{x^{24}}.$

47. $\left(\dfrac{3^2u^{-2}v^2}{2^2u^3v^{-3}}\right)^{-2}\left(\dfrac{3^2v^5}{4^2u}\right)^2=\left(3^22^{-2}u^{-5}v^5\right)^{-2}\left(3^24^{-2}v^5u^{-1}\right)^2=3^{-4}2^4u^{10}v^{-10}3^44^{-4}v^{10}u^{-2}=2^{-4}u^8v^0=\dfrac{u^8}{16}.$

48. $\left[\left(-\dfrac{2^2x^{-2}y^0}{3^2x^3y^{-2}}\right)^{-2}\right]^{-2}=\left(-2^2\cdot 3^{-2}x^{-5}y^2\right)^4=2^8\cdot 3^{-8}x^{-20}y^8=\dfrac{256y^8}{6561x^{20}}.$

49. $\dfrac{x^{-1}-1}{x^{-1}+1}=\dfrac{\frac{1}{x}-1}{\frac{1}{x}+1}=\dfrac{\frac{1-x}{x}}{\frac{1+x}{x}}=\dfrac{1-x}{1+x}.$

50. $\dfrac{x^{-1}-y^{-1}}{x^{-1}+y^{-1}}=\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}=\dfrac{\frac{y-x}{xy}}{\frac{y+x}{xy}}=\dfrac{y-x}{y+x}.$

51. $\dfrac{u^{-1}-v^{-1}}{v-u}=\dfrac{\frac{1}{u}-\frac{1}{v}}{v-u}=\dfrac{\frac{v-u}{uv}}{v-u}=\dfrac{v-u}{uv}\cdot\dfrac{1}{v-u}=\dfrac{1}{uv}.$

52. $\dfrac{(uv)^{-1}}{u^{-1}+v^{-1}}=\dfrac{\frac{1}{uv}}{\frac{1}{u}+\frac{1}{v}}=\dfrac{\frac{1}{uv}}{\frac{v+u}{uv}}=\dfrac{1}{uv}\cdot\dfrac{uv}{v+u}=\dfrac{1}{v+u}.$

53. $\left(\dfrac{a^{-1}-b^{-1}}{a^{-1}+b^{-1}}\right)^{-1}=\dfrac{a^{-1}+b^{-1}}{a^{-1}-b^{-1}}=\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}=\dfrac{\frac{b+a}{ab}}{\frac{b-a}{ab}}=\dfrac{b+a}{b-a}.$

54. $\left[\left(a^{-1}+b^{-1}\right)\left(a^{-1}-b^{-1}\right)\right]^{-2}=\left(a^{-2}-b^{-2}\right)^{-2}=\left(\dfrac{1}{a^2}-\dfrac{1}{b^2}\right)^{-2}=\left(\dfrac{b^2-a^2}{a^2b^2}\right)^{-2}=\dfrac{a^4b^4}{\left(b^2-a^2\right)^2}.$

55. False. For example, if $a=2$, $b=3$, $m=2$, and $n=3$, then $a^mb^n=2^2\cdot 3^3=108$, and this is not equal to $(ab)^{mn}=6^6=46{,}656$.

56. False. For example, if $a=1$, $b=2$, $m=3$, and $n=2$, then we have $\dfrac{a^m}{b^n}=\dfrac{1^3}{2^2}=\dfrac{1}{4}$, whereas $\left(\dfrac{a}{b}\right)^{m-n}=\left(\dfrac{1}{2}\right)^{3-2}=\dfrac{1}{2}$.

57. False. For example, if $a=1$, $b=2$, and $n=2$, then $(a+b)^n=(1+2)^2=3^2=9$, whereas $a^n+b^n=1^2+2^2=5$.

1.6 Solving Equations

Concept Questions page 35

1. An equation is a statement that two mathematical expressions are equal. A solution of an equation involving one variable is a number that renders the equation a true statement when it is substituted for the variable. The solution set of an equation is the set of all solutions to the equation.

One example: $2x=3$ is an equation. Its solution is $x=\frac{3}{2}$ because $2\left(\frac{3}{2}\right)=3$.

Another example: $\dfrac{5x}{2}=10$ is an equation. Its solution is $x=4$ because $\dfrac{5\cdot 4}{2}=\dfrac{20}{2}=10$.

2. **a.** If $a = b$, then $a + c = b + c$ and $a - c = b - c$. Example: If $a = 2$, $b = 2$, and $c = 3$, then $a + c = 2 + 3 = 5 = b + c$ and $a - c = 2 - 3 = -1 = b - c$.

b. If $a = b$ and $c \neq 0$, then $ca = cb$ and $\dfrac{a}{c} = \dfrac{b}{c}$. Example: If $a = 2$, $b = 2$, and $c = 4$, then $ca = 2 \cdot 4 = cb$ and $\dfrac{a}{c} = \dfrac{2}{4} = \dfrac{b}{c}$.

3. A linear equation in the variable x is an equation that can be written in the form $ax + b = 0$, where a and b are constants with $a \neq 0$. Example: $3x + 4 = 5$. Solving for x, we have $3x = 1$, so $x = \frac{1}{3}$.

Exercises

page 35

1.
$$\begin{aligned} 3x &= 12 \\ \tfrac{1}{3}(3x) &= \tfrac{1}{3}(12) \\ x &= 4. \end{aligned}$$

2.
$$\begin{aligned} 2x &= 0 \\ \tfrac{1}{2}(2x) &= \tfrac{1}{2}(0) \\ x &= 0. \end{aligned}$$

3.
$$\begin{aligned} 0.3y &= 2 \\ \tfrac{1}{0.3}(0.3y) &= \tfrac{1}{0.3}(2) \\ y &= \tfrac{2}{0.3} \\ &= \tfrac{20}{3}. \end{aligned}$$

4.
$$\begin{aligned} 2x + 5 &= 11 \\ 2x &= 6 \\ x &= 3. \end{aligned}$$

5.
$$\begin{aligned} 3x + 4 &= 2 \\ 3x + 4 - 4 &= 2 - 4 \\ 3x &= -2 \\ \tfrac{1}{3}(3x) &= \tfrac{1}{3}(-2) \\ x &= -\tfrac{2}{3}. \end{aligned}$$

6.
$$\begin{aligned} 2 - 3y &= 8 \\ 2 - 3y - 2 &= 8 - 2 \\ -3y &= 6 \\ \left(-\tfrac{1}{3}\right)(-3y) &= \left(-\tfrac{1}{3}\right)6 \\ y &= -2. \end{aligned}$$

7.
$$\begin{aligned} -2y + 3 &= -7 \\ -2y + 3 - 3 &= -7 - 3 \\ -2y &= -10 \\ -\tfrac{1}{2}(-2y) &= -\tfrac{1}{2}(-10) \\ y &= 5. \end{aligned}$$

8.
$$\begin{aligned} \tfrac{1}{3}k + 1 &= \tfrac{1}{4}k - 2 \\ 12\left(\tfrac{1}{3}k + 1\right) &= 12\left(\tfrac{1}{4}k - 2\right) \\ 4k + 12 &= 3k - 24 \\ 4k + 12 - 12 &= 3k - 24 - 12 \\ 4k &= 3k - 36 \\ 4k - 3k &= 3k - 36 - 3k \\ k &= -36. \end{aligned}$$

9.
$$\begin{aligned}
\tfrac{1}{5}p - 3 &= -\tfrac{1}{3}p + 5 \\
15\left(\tfrac{1}{5}p - 3\right) &= 15\left(-\tfrac{1}{3}p + 5\right) \\
3p - 45 &= -5p + 75 \\
3p - 45 + 45 &= -5p + 75 + 45 \\
3p &= -5p + 120 \\
8p &= 120 \\
p &= 15.
\end{aligned}$$

10.
$$\begin{aligned}
3.1m + 2 &= 3 - 0.2m \\
3.1m + 2 - 2 &= 3 - 0.2m - 2 \\
3.1m &= 1 - 0.2m \\
3.1m + 0.2m &= 1 - 0.2m + 0.2m \\
3.3m &= 1 \\
\tfrac{1}{3.3}(3.3m) &= \tfrac{1}{3.3}(1) \\
m &= \tfrac{1}{3.3} = \tfrac{1}{3.3} \cdot \tfrac{10}{10} = \tfrac{10}{33}.
\end{aligned}$$

11.
$$\begin{aligned}
0.4 - 0.3p &= 0.1(p + 4) \\
0.4 - 0.3p &= 0.1p + 0.4 \\
0.4 - 0.3p - 0.4 &= 0.1p + 0.4 - 0.4 \\
-0.3p &= 0.1p \\
-0.3p - 0.1p &= 0.1p - 0.1p \\
-0.4p &= 0 \\
p &= 0.
\end{aligned}$$

12.
$$\begin{aligned}
\tfrac{1}{3}k + 4 &= -2\left(k + \tfrac{1}{3}\right) \\
\tfrac{1}{3}k + 4 &= -2k - \tfrac{2}{3} \\
3\left(\tfrac{1}{3}k + 4\right) &= 3\left(-2k - \tfrac{2}{3}\right) \\
k + 12 &= -6k - 2 \\
7k &= -14 \\
k &= -2.
\end{aligned}$$

13.
$$\begin{aligned}
\tfrac{3}{5}(k + 1) &= \tfrac{1}{4}(2k + 4) \\
12(k + 1) &= 5(2k + 4) \\
12k + 12 &= 10k + 20 \\
2k &= 8 \\
k &= 4.
\end{aligned}$$

14.
$$\begin{aligned}
3\left(\tfrac{3m}{4} - 1\right) + \tfrac{m}{5} &= \tfrac{42-m}{4} \\
\tfrac{9m}{4} - 3 + \tfrac{m}{5} &= \tfrac{42-m}{4} \\
20\left(\tfrac{9m}{4} - 3 + \tfrac{m}{5}\right) &= 20\left(\tfrac{42-m}{4}\right) \\
45m - 60 + 4m &= 210 - 5m \\
54m &= 270 \\
m &= \tfrac{270}{54} = 5.
\end{aligned}$$

15.
$$\begin{aligned}
\tfrac{2x-1}{3} + \tfrac{3x+4}{4} &= \tfrac{7(x+3)}{10} \\
60\left(\tfrac{2x-1}{3} + \tfrac{3x+4}{4}\right) &= 60\left[\tfrac{7(x+3)}{10}\right] \\
20(2x - 1) + 15(3x + 4) &= 42(x + 3) \\
40x - 20 + 45x + 60 &= 42x + 126 \\
85x + 40 &= 42x + 126 \\
85x &= 42x + 86 \\
43x &= 86 \\
x &= 2.
\end{aligned}$$

16.
$$\begin{aligned}
\tfrac{w-1}{3} + \tfrac{w+1}{4} &= -\tfrac{w+1}{6} \\
12\left(\tfrac{w-1}{3} + \tfrac{w+1}{4}\right) &= -12\left(\tfrac{w+1}{6}\right) \\
4(w - 1) + 3(w + 1) &= -2(w + 1) \\
4w - 4 + 3w + 3 &= -2w - 2 \\
9w &= -1 \\
w &= -\tfrac{1}{9}.
\end{aligned}$$

17.
$$\begin{aligned}
\tfrac{1}{2}\left[2x-3(x-4)\right] &= \tfrac{2}{3}(x-5)\\
6\left\{\tfrac{1}{2}\left[2x-3(x-4)\right]\right\} &= 6\left[\tfrac{2}{3}(x-5)\right]\\
3(2x-3x+12) &= 4(x-5)\\
3(-x+12) &= 4x-20\\
-3x+36 &= 4x-20\\
-7x+36 &= -20\\
-7x &= -56\\
x &= 8.
\end{aligned}$$

18.
$$\begin{aligned}
\tfrac{1}{3}\left[2-3(x+2)\right] &= \tfrac{1}{4}\left[(-3x+1)+\tfrac{1}{2}x\right]\\
4(2-3x-6) &= 3\left(-3x+1+\tfrac{1}{2}x\right)\\
4(-3x-4) &= 3\left(-\tfrac{5}{2}x+1\right)\\
-12x-16 &= -\tfrac{15}{2}x+3\\
-\tfrac{9}{2}x &= 19\\
x &= -\tfrac{38}{9}.
\end{aligned}$$

19.
$$\begin{aligned}
(2x+1)^2-(3x-2)^2 &= 5x(2-x)\\
\left(4x^2+4x+1\right)-\left(9x^2-12x+4\right) &= 10x-5x^2\\
4x^2+4x+1-9x^2+12x-4 &= 10x-5x^2\\
-5x^2+16x-3 &= 10x-5x^2\\
16x-3 &= 10x\\
6x-3 &= 0\\
6x &= 3\\
x &= \frac{1}{2}.
\end{aligned}$$

20.
$$\begin{aligned}
x\left[(2x-3)^2+5x^2\right] &= 3x^2(3x-4)+18\\
x\left(4x^2-12x+9+5x^2\right) &= 9x^3-12x^2+18\\
x\left(9x^2-12x+9\right) &= 9x^3-12x^2+18\\
9x^3-12x^2+9x &= 9x^3-12x^2+18\\
-12x^2+9x &= -12x^2+18\\
9x &= 18\\
x &= 2.
\end{aligned}$$

21.
$$\begin{aligned}
\frac{8}{x} &= 24\\
8 &= 24x\\
\frac{1}{3} &= x.
\end{aligned}$$

22.
$$\begin{aligned}
\frac{1}{x}+\frac{2}{x} &= 6\\
\frac{3}{x} &= 6\\
3 &= 6x\\
\frac{1}{2} &= x.
\end{aligned}$$

23.
$$\begin{aligned}
\frac{2}{y-1} &= 4\\
2 &= 4(y-1)\\
2 &= 4y-4\\
6 &= 4y\\
\frac{3}{2} &= y.
\end{aligned}$$

24.
$$\begin{aligned}
\frac{1}{x+3} &= 0\\
1 &= 0.
\end{aligned}$$

But this is impossible, and so there is no solution.

25.

$$\begin{aligned}\tfrac{2x-3}{x+1} &= \tfrac{2}{5}\\ 5(x+1)\left(\tfrac{2x-3}{x+1}\right) &= 5(x+1)\left(\tfrac{2}{5}\right)\\ 5(2x-3) &= 2(x+1)\\ 10x-15 &= 2x+2\\ 10x &= 2x+17\\ 8x &= 17\\ x &= \tfrac{17}{8}.\end{aligned}$$

26.

$$\begin{aligned}\tfrac{r}{3r-1} &= 4\\ (3r-1)\left(\tfrac{r}{3r-1}\right) &= 4(3r-1)\\ r &= 12r-4\\ 4 &= 11r\\ \tfrac{4}{11} &= r.\end{aligned}$$

27.

$$\begin{aligned}\frac{2}{q-1} &= \frac{3}{q-2}\\ (q-1)(q-2)\left(\frac{2}{q-1}\right) &= (q-1)(q-2)\left(\frac{3}{q-2}\right)\\ (q-2)\,2 &= (q-1)\,3\\ 2q-4 &= 3q-3\\ -4 &= q-3\\ -1 &= q.\end{aligned}$$

28.

$$\begin{aligned}\frac{y}{3}-\frac{2}{y+1} &= \tfrac{1}{3}(y-3)\\ 3(y+1)\left(\frac{y}{3}-\frac{2}{y+1}\right) &= 3(y+1)\left[\tfrac{1}{3}(y-3)\right]\\ (y+1)\,y-6 &= (y+1)(y-3)\\ y^2+y-6 &= y^2-2y-3\\ y-6 &= -2y-3\\ y &= -2y+3\\ 3y &= 3\\ y &= 1.\end{aligned}$$

29.

$$\begin{aligned}\frac{3k-2}{4}-\frac{3k}{4} &= \frac{k+3}{k}\\ -\frac{1}{2} &= \frac{k+3}{k}\\ -k &= 2k+6\\ -3k &= 6\\ k &= -2\end{aligned}$$

30.

$$\begin{aligned}\frac{2x-1}{3x+2} &= \frac{2x+1}{3x+1}\\ (3x+2)(3x+1)\left(\frac{2x-1}{3x+2}\right) &= (3x+2)(3x+1)\left(\frac{2x+1}{3x+1}\right)\\ (3x+1)(2x-1) &= (3x+2)(2x+1)\\ 6x^2-x-1 &= 6x^2+7x+2\\ -x-1 &= 7x+2\\ -x &= 7x+3\\ -8x &= 3\\ x &= -\tfrac{3}{8}\end{aligned}$$

31.
$$\begin{aligned} \frac{m-2}{m}+\frac{2}{m} &= \frac{m+3}{m-3} \\ 1-\frac{2}{m}+\frac{2}{m} &= \frac{m+3}{m-3} \\ 1 &= \frac{m+3}{m-3} \\ m-3 &= m+3 \\ -3 &= 3 \end{aligned}$$

which is impossible. Thus, there is no solution.

32.
$$\begin{aligned} \frac{4}{x\,(x-2)} &= \frac{2}{x-2} \\ x\,(x-2)\left[\frac{4}{x\,(x-2)}\right] &= x\,(x-2)\left(\frac{2}{x-2}\right) \\ 4 &= 2x \\ 2 &= x. \end{aligned}$$

But the original equation is not defined for $x = 2$, so there is no solution.

33. $I = Prt$, so $r = \dfrac{I}{Pt}$.

34. $ax + by + c = 0$, so $by = -ax - c$. Thus, $y = \dfrac{-ax-c}{b} = -\dfrac{a}{b}x - \dfrac{c}{b}$.

35. $p = -3q + 1$, so $-3q = p - 1$. Thus, $q = \dfrac{p-1}{-3} = -\frac{1}{3}p + \frac{1}{3}$.

36. $w = \dfrac{kuv}{s^2}$, so $\dfrac{ws^2}{kv} = u$.

37. $iS = R\left[(1+i)^n - 1\right]$, so $R = \dfrac{iS}{(1+i)^n - 1}$.

38. $iS = R\,(1+i)\left[(1+i)^n - 1\right]$, so $R = \dfrac{iS}{(1+i)\left[(1+i)^n - 1\right]}$.

39.
$$\begin{aligned} V &= \frac{ax}{x+b} \\ V\,(x+b) &= ax \\ Vx + Vb &= ax \\ Vx - ax &= -Vb \\ x\,(V-a) &= -Vb \\ x &= -\frac{Vb}{V-a} \\ &= \frac{Vb}{a-V}. \end{aligned}$$

40.
$$\begin{aligned} V &= C\left(1-\frac{n}{N}\right) \\ &= C - \frac{Cn}{N} \\ V - C &= -\frac{Cn}{N} \\ n &= -\frac{N}{C}\,(V-C) \\ &= \frac{N}{C}\,(C-V). \end{aligned}$$

41.
$$\begin{aligned} r &= \frac{2mI}{B\,(n+1)} \\ rB\,(n+1) &= 2mI \\ m &= \frac{rB\,(n+1)}{2I}. \end{aligned}$$

42.
$$\begin{aligned} p &= \frac{x+10}{x+4} \\ p\,(x+4) &= x+10 \\ px + 4p &= x + 10 \\ px - x &= 10 - 4p \\ x\,(p-1) &= 10 - 4p \\ x &= \frac{2\,(5-2p)}{p-1}. \end{aligned}$$

43.
$$\begin{aligned} r &= \frac{2mI}{B(n+1)} \\ rBn + rB &= 2mI \\ rBn &= 2mI - rB \\ n &= \frac{2mI - rB}{rB}. \end{aligned}$$

44.
$$\begin{aligned} y &= 10\left(1 - \frac{1}{1+2x}\right) \\ &= \frac{10(1+2x-1)}{1+2x} \\ &= \frac{20x}{1+2x} \\ y(1+2x) &= 20x \\ y + 2yx &= 20x \\ y &= 20x - 2yx \\ &= 2x(10-y) \\ x &= \frac{y}{2(10-y)}. \end{aligned}$$

45.
$$\begin{aligned} \frac{1}{f} &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{p} &= \frac{1}{f} - \frac{1}{q} \\ &= \frac{q-f}{fq} \\ p &= \frac{fq}{q-f}. \end{aligned}$$

46.
$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ \frac{1}{R_3} &= \frac{1}{R} - \frac{1}{R_1} - \frac{1}{R_2} \\ &= \frac{R_1R_2 - RR_2 - RR_1}{RR_1R_2} \\ R_3 &= \frac{RR_1R_2}{R_1R_2 - RR_2 - RR_1}. \end{aligned}$$

47. $I = Prt$, so $t = \frac{I}{Pr}$. If $I = 90$, $P = 1000$, and $r = 6\% = 0.06$, then $t = \frac{90}{(0.06)(1000)} = 1.5$, or 1.5 years.

48. $F = \frac{9}{5}C + 32$, so $\frac{9}{5}C = F - 32$ and $C = \frac{5}{9}(F - 32)$. If $F = 70$, then $C = \frac{5}{9}(70 - 32) = \frac{190}{9} \approx 21.11$, or about 21.11°C.

49. $S = \frac{a}{t} + b = \frac{a+bt}{t}$, so $tS = a + bt$, $tS - bt = a$, $(S - b)t = a$, and $t = \frac{a}{S-b}$.

50. $V = \frac{ax}{x+b}$, so $V(x+b) = ax$, $Vx + Vb = ax$, $Vx - ax = -Vb$, $(V - a)x = -Vb$, and $x = -\frac{Vb}{V-a} = \frac{Vb}{a-V}$.

51. **a.** $V = C - \left(\frac{C-S}{N}\right)t$, so $V - \frac{St}{N} = C\left(1 - \frac{t}{N}\right)$ and $C = \dfrac{\frac{NV-St}{N}}{1-\frac{t}{N}} = \frac{NV-St}{N-t}$.

b. If $N = 5$, $t = 3$, $S = 40{,}000$, and $V = 70{,}000$, we have $C = \frac{70{,}000(5) - 40{,}000(3)}{5-3} = \frac{230{,}000}{2} = 115{,}000$, or \$115, 000.

52. **a.** $v^2 = u^2 + 2as$, so $2as = v^2 - u^2$ and $a = \frac{v^2 - u^2}{2s}$.

b. If $v = 88$, $u = 0$, and $s = 1320$, we have $a = \frac{(88)^2 - 0}{2(1320)} = \frac{44}{15}$, or approximately 2.93 ft/sec^2.

53. a. $c = \left(\frac{t+1}{24}\right)a$, so $\frac{t+1}{24} = \frac{c}{a}$, $t + 1 = \frac{24c}{a}$, and $t = \frac{24c}{a} - 1 = \frac{24c - a}{a}$.

b. Here $a = 500$ and $c = 125$, so the child's age is $t = \frac{24(125)-500}{500} = 5$, or 5 years.

54. a. $T = \frac{0.8t}{t+4.1}$, so $(t + 4.1)\,T = 0.8t$, $tT + 4.1T = 0.8t$, $0.8t - tT = 4.1T$, $(0.8 - T)\,t = 4.1T$, and $t = \frac{4.1T}{0.8 - T}$.

b. If $T = 0.4$, then the time taken is $t = \frac{4.1 \cdot 0.4}{0.8 - 0.4} = 4.1$, or 4.1 hours.

1.7 Rational Exponents and Radicals

Concept Questions page 44

1. If n is a natural number and a and b are real numbers such that $a^n = b$, then a is the nth root of b. For example, 3 is the 4th root of 81; that is $\sqrt[4]{81} = 3$.

2. The principal nth root of a positive real number b, when n is even, is the positive root of b. If n is odd, it is the unique nth root of b. The principal 4th root of 16 is 2, and the principal (and only) 3rd root of 8 is 2.

3. The process of eliminating a radical from the denominator of an algebraic expression is referred to as rationalizing the denominator. For example, $\frac{1}{1-\sqrt{6}} = \frac{1}{1-\sqrt{6}} \cdot \frac{1+\sqrt{6}}{1+\sqrt{6}} = \frac{1+\sqrt{6}}{1-6} = -\frac{1}{5}\left(1+\sqrt{6}\right)$.

Exercises page 44

1. $\sqrt{81} = 9$.

2. $\sqrt[3]{-27} = -3$.

3. $\sqrt[4]{256} = 4$.

4. $\sqrt[5]{-32} = -2$.

5. $16^{1/2} = 4$.

6. $625^{1/4} = 5$.

7. $8^{2/3} = 2^2 = 4$.

8. $32^{2/5} = 2^2 = 4$.

9. $-25^{1/2} = -5$.

10. $-16^{3/2} = -4^3 = -64$.

11. $(-8)^{2/3} = (-2)^2 = 4$.

12. $(-32)^{3/5} = (-2)^3 = -8$.

13. $\left(\frac{4}{9}\right)^{1/2} = \frac{2}{3}$.

14. $\left(\frac{9}{25}\right)^{3/2} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$.

15. $\left(\frac{27}{8}\right)^{2/3} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

16. $\left(-\frac{8}{125}\right)^{1/3} = -\frac{2}{5}$.

17. $8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{2^2} = \frac{1}{4}$.

18. $81^{-1/4} = \frac{1}{81^{1/4}} = \frac{1}{3}$.

19. $-\left(\frac{27}{8}\right)^{-1/3}=-\left(\frac{8}{27}\right)^{1/3}=-\frac{2}{3}.$

20. $-\left(-\frac{8}{27}\right)^{-2/3}=-\frac{1}{\left(-\frac{8}{27}\right)^{2/3}}=-\frac{1}{\left(-\frac{2}{3}\right)^{2}}=-\frac{1}{\frac{4}{9}}=-\frac{9}{4}.$

21. $3^{1/3}\cdot 3^{5/3}=3^{(1/3)+(5/3)}=3^{2}=9.$

22. $2^{6/5}\cdot 2^{-1/5}=2^{(6/5)-(1/5)}=2^{1}=2.$

23. $\frac{3^{1/2}}{3^{5/2}}=\frac{1}{3^{2}}=\frac{1}{9}.$

24. $\frac{3^{-5/4}}{3^{-1/4}}=\frac{1}{3^{-1/4+5/4}}=\frac{1}{3^{1}}=\frac{1}{3}.$

25. $\frac{2^{-1/2}\cdot 3^{2/3}}{2^{3/2}\cdot 3^{-1/3}}=\frac{3^{(2/3)+(1/3)}}{2^{(3/2)+(1/2)}}=\frac{3^{1}}{2^{2}}=\frac{3}{4}.$

26. $\frac{4^{1/3}\cdot 4^{-2/5}}{4^{2/3}}=4^{(1/3)-(2/5)-(2/3)}=4^{(5-6-10)/15}=4^{-11/15}=\frac{1}{4^{11/15}}.$

27. $\left(2^{3/2}\right)^{4}=2^{(3/2)4}=2^{6}=64.$

28. $\left[(-3)^{1/3}\right]^{2}=(-3)^{2/3}=3^{2/3}.$

29. $x^{2/5}\cdot x^{-1/5}=x^{1/5}.$

30. $y^{-3/8}\cdot y^{1/4}=y^{(-3/8)+(1/4)}=y^{-1/8}=\frac{1}{y^{1/8}}.$

31. $\frac{x^{3/4}}{x^{-1/4}}=x^{(3/4)+(1/4)}=x.$

32. $\frac{x^{7/3}}{x^{-2}}=x^{(7/3)+2}=x^{13/3}.$

33. $\left(\frac{x^{3}}{-27x^{-6}}\right)^{-2/3}=\left(\frac{x^{9}}{-27}\right)^{-2/3}=\frac{x^{-18/3}}{\frac{1}{9}}=9x^{-6}=\frac{9}{x^{6}}.$

34. $\left(\frac{27x^{-3}y^{2}}{8x^{-2}y^{-5}}\right)^{1/3}=\frac{3x^{-1}y^{2/3}}{2x^{-2/3}y^{-5/3}}=\frac{3y^{7/3}}{2x^{1/3}}.$

35. $\left(\frac{x^{-3}}{y^{-2}}\right)^{1/2}\left(\frac{y}{x}\right)^{3/2}=\frac{x^{-3/2}y^{3/2}}{y^{-1}x^{3/2}}=\frac{y^{5/2}}{x^{3}}.$

36. $\left(\frac{r^{n}}{r^{5-2n}}\right)^{4}=\frac{r^{4n}}{r^{20-8n}}=r^{12n-20}.$

37. $x^{2/5}\left(x^{2}-2x^{3}\right)=x^{12/5}-2x^{17/5}.$

38. $s^{1/3}\left(2s-s^{1/4}\right)=2s^{4/3}-s^{7/12}.$

39. $2p^{3/2}\left(2p^{1/2}-p^{-1/2}\right)=4p^{2}-2p.$

40. $3y^{1/3}\left(y^{2/3}-1\right)^{2}=3y^{1/3}\left(y^{4/3}-2y^{2/3}+1\right)=3y^{5/3}-6y+3y^{1/3}.$

41. $\sqrt{32}=\sqrt{4^{2}\cdot 2}=4\sqrt{2}.$

42. $\sqrt{45}=\sqrt{3^{2}\cdot 5}=3\sqrt{5}.$

43. $\sqrt[3]{-54}=\sqrt[3]{(-1)\left(3^{3}\right)(2)}=-3\sqrt[3]{2}.$

44. $-\sqrt[4]{48}=-\sqrt[4]{2^{4}\cdot 3}=-2\sqrt[4]{3}.$

45. $\sqrt{16x^{2}y^{3}}=\sqrt{4^{2}x^{2}y^{2}y}=4xy\sqrt{y}.$

46. $\sqrt{40a^{3}b^{4}}=\sqrt{4\cdot 10\cdot a^{2}\cdot a\cdot b^{4}}=2ab^{2}\sqrt{10a}.$

47. $\sqrt[3]{m^{6}n^{3}p^{12}}=\sqrt[3]{\left(m^{2}\right)^{3}n^{3}\left(p^{4}\right)^{3}}=m^{2}np^{4}.$

48. $\sqrt[3]{-27p^{2}q^{3}r^{4}}=\sqrt[3]{(-1)\left(3^{3}\right)p^{2}q^{3}r^{3}r}=-3qr\sqrt[3]{p^{2}r}.$

49. $\sqrt[3]{\sqrt{9}}=\sqrt[3]{3}.$

50. $\sqrt[5]{\sqrt[3]{9}}=\sqrt[15]{9}.$

51. $\sqrt[3]{\sqrt{x}} = \sqrt[6]{x}$.

52. $\sqrt[3]{-\sqrt[4]{x^3}} = \sqrt[12]{-x^3} = -\sqrt[4]{x}$.

53. $\dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$.

54. $\dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$.

55. $\dfrac{3}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{3\sqrt{x}}{2x}$.

56. $\dfrac{3}{\sqrt{xy}} \cdot \dfrac{\sqrt{xy}}{\sqrt{xy}} = \dfrac{3\sqrt{xy}}{xy}$.

57. $\dfrac{2y}{\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{2y\sqrt{3y}}{3y} = \frac{2}{3}\sqrt{3y}$.

58. $\dfrac{5x^2}{\sqrt{3x}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{5x^2\sqrt{3x}}{3x} = \dfrac{5x}{3}\sqrt{3x}$.

59. $\dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$.

60. $\sqrt{\dfrac{2x}{y}} = \dfrac{\sqrt{2x}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{2xy}}{y}$.

61. $\dfrac{2}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{2\left(1-\sqrt{3}\right)}{1-3} = \dfrac{2\left(1-\sqrt{3}\right)}{-2} = \sqrt{3}-1$.

62. $\dfrac{3}{1-\sqrt{2}} \cdot \dfrac{1+\sqrt{2}}{1+\sqrt{2}} = \dfrac{3\left(1+\sqrt{2}\right)}{1-2} = -3\left(1+\sqrt{2}\right)$.

63. $\dfrac{1+\sqrt{2}}{1-\sqrt{2}} \cdot \dfrac{1+\sqrt{2}}{1+\sqrt{2}} = \dfrac{\left(1+\sqrt{2}\right)^2}{1-2} = -\left(1+\sqrt{2}\right)^2$.

64. $\dfrac{9+\sqrt{2}}{3-\sqrt{2}} \cdot \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = \dfrac{\left(9+\sqrt{2}\right)\left(3+\sqrt{2}\right)}{9-2} = \frac{1}{7}\left(9+\sqrt{2}\right)\left(3+\sqrt{2}\right) = \frac{1}{7}\left(27+3\sqrt{2}+9\sqrt{2}+2\right)$
$= \frac{1}{7}\left(29+12\sqrt{2}\right)$.

65. $\dfrac{q}{\sqrt{q}-1} \cdot \dfrac{\sqrt{q}+1}{\sqrt{q}+1} = \dfrac{q\left(\sqrt{q}+1\right)}{q-1}$.

66. $\dfrac{xy}{\sqrt{x}+\sqrt{y}} \cdot \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \dfrac{xy\left(\sqrt{x}-\sqrt{y}\right)}{x-y}$.

67. $\dfrac{y}{\sqrt[3]{x^2z}} \cdot \dfrac{\sqrt[3]{xz^2}}{\sqrt[3]{xz^2}} = \dfrac{y\sqrt[3]{xz^2}}{\sqrt[3]{x^3z^3}} = \dfrac{y\sqrt[3]{xz^2}}{xz}$.

68. $\dfrac{2x}{\sqrt[3]{xy^2}} \cdot \dfrac{\sqrt[3]{x^2y}}{\sqrt[3]{x^2y}} = \dfrac{2x\sqrt[3]{x^2y}}{\sqrt[3]{x^3y^3}} = \dfrac{2x\sqrt[3]{x^2y}}{xy} = \dfrac{2\sqrt[3]{x^2y}}{y}$.

69. $\sqrt{\dfrac{16}{3}} = \dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$.

70. $-\sqrt{\dfrac{8}{3}} = -\dfrac{2\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = -\dfrac{2\sqrt{6}}{3}$.

71. $\sqrt[3]{\dfrac{2}{3}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{3}} \cdot \dfrac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \dfrac{\sqrt[3]{18}}{3}$.

72. $\sqrt[3]{\dfrac{81}{4}} = \dfrac{\sqrt[3]{81}}{\sqrt[3]{4}} = \dfrac{3\sqrt[3]{3}}{\sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{3\sqrt[3]{3}\sqrt[3]{2}}{2} = \dfrac{3\sqrt[3]{6}}{2}$.

73. $\sqrt{\dfrac{3}{2x^2}} = \dfrac{\sqrt{3}}{x\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}}{2x}$.

74. $\sqrt{\dfrac{x^3y^5}{4}} = \dfrac{xy^2\sqrt{xy}}{2}$.

75. $\sqrt[3]{\dfrac{2y^2}{3}} = \dfrac{\sqrt[3]{2y^2}}{\sqrt[3]{3}} \cdot \dfrac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \dfrac{\sqrt[3]{18y^2}}{3}$.

76. $\sqrt[3]{\dfrac{3a^3}{b^2}} = \dfrac{\sqrt[3]{3a^3}}{\sqrt[3]{b^2}} \cdot \dfrac{\sqrt[3]{b}}{\sqrt[3]{b}} = \dfrac{a\sqrt[3]{3}}{\sqrt[3]{b^2}} \cdot \dfrac{\sqrt[3]{b}}{\sqrt[3]{b}} = \dfrac{a\sqrt[3]{3b}}{b}$.

77. $\dfrac{1}{\sqrt{a}}+\sqrt{a}=\dfrac{1+a}{\sqrt{a}}=\dfrac{1+a}{\sqrt{a}}\cdot\dfrac{\sqrt{a}}{\sqrt{a}}=\dfrac{\sqrt{a}\,(1+a)}{a}.$

78. $\dfrac{x}{\sqrt{x-y}}-\sqrt{x-y}=\dfrac{x-(x-y)}{\sqrt{x-y}}=\dfrac{y}{\sqrt{x-y}}=\dfrac{y}{\sqrt{x-y}}\cdot\dfrac{\sqrt{x-y}}{\sqrt{x-y}}=\dfrac{y\sqrt{x-y}}{x-y}.$

79. $\dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\dfrac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}=\dfrac{\sqrt{x}\left(\sqrt{x}-\sqrt{y}\right)+\sqrt{y}\left(\sqrt{x}+\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)}=\dfrac{x-\sqrt{xy}+\sqrt{xy}+y}{x-y}=\dfrac{x+y}{x-y}.$

80. $\dfrac{a}{\sqrt{a^2-b^2}}-\dfrac{\sqrt{a^2-b^2}}{a}=\dfrac{a^2-\left(a^2-b^2\right)}{a\sqrt{a^2-b^2}}=\dfrac{b^2}{a\sqrt{a^2-b^2}}\cdot\dfrac{\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}}=\dfrac{b^2\sqrt{a^2-b^2}}{a\left(a^2-b^2\right)}.$

81. $(x+1)^{1/2}+\frac{1}{2}x\,(x+1)^{-1/2}=\frac{1}{2}\,(x+1)^{-1/2}\,[2\,(x+1)+x]=\frac{1}{2}\,(x+1)^{-1/2}\,(3x+2)=\dfrac{\sqrt{x+1}\,(3x+2)}{2\,(x+1)}.$

82.
$$\begin{aligned}\tfrac{1}{2}x^{-1/2}\,(x+y)^{1/3}+\tfrac{1}{3}x^{1/2}\,(x+y)^{-2/3}&=\tfrac{1}{6}x^{-1/2}\,(x+y)^{-2/3}\left[3\,(x+y)+2x\right]\\&=\frac{5x+3y}{6x^{1/2}\,(x+y)^{2/3}}=\frac{x^{1/2}\,(x+y)^{1/3}\,(5x+3y)}{6x\,(x+y)}.\end{aligned}$$

83.
$$\begin{aligned}\frac{\frac{1}{2}\left(1+x^{1/3}\right)x^{-1/2}-\frac{1}{3}x^{1/2}\cdot x^{-2/3}}{\left(1+x^{1/3}\right)^2}&=\frac{\frac{1}{2}x^{-1/2}+\frac{1}{2}x^{-1/6}-\frac{1}{3}x^{-1/6}}{\left(1+x^{1/3}\right)^2}=\frac{\frac{1}{2}x^{-1/2}+\frac{1}{6}x^{-1/6}}{\left(1+x^{1/3}\right)^2}=\frac{\frac{1}{6}x^{-1/2}\left(3+x^{1/3}\right)}{\left(1+x^{1/3}\right)^2}\\&=\frac{3+x^{1/3}}{6x^{1/2}\left(1+x^{1/3}\right)^2}.\end{aligned}$$

84. $\dfrac{\frac{1}{2}x^{-1/2}\,(x+y)^{1/2}-\frac{1}{2}x^{1/2}\,(x+y)^{-1/2}}{x+y}=\dfrac{\frac{1}{2}x^{-1/2}\,(x+y)^{-1/2}\left[(x+y)-x\right]}{x+y}=\dfrac{y}{2x^{1/2}\,(x+y)^{3/2}}.$

85.
$$\begin{aligned}\sqrt{3x+1}&=2\\3x+1&=4\\3x&=3\\x&=1.\end{aligned}$$

Check: $\sqrt{3\,(1)+1}\stackrel{?}{=}2.$
Yes, $x=1$ is a solution.

86.
$$\begin{aligned}\sqrt{2x-3}&=3\\2x-3&=9\\2x&=12\\x&=6.\end{aligned}$$

Check: $\sqrt{2\,(6)-3}\stackrel{?}{=}3.$
Yes, $x=6$ is a solution.

87. $\sqrt{k^2-4} = 4-k$

$$k^2-4 = 16-8k+k^2$$
$$-4 = 16-8k$$
$$8k = 20$$
$$k = \frac{20}{8} = \frac{5}{2}.$$

Check: $\sqrt{\left(\frac{5}{2}\right)^2-4} \stackrel{?}{=} 4-\frac{5}{2}$

$$\frac{3}{2} \stackrel{?}{=} \frac{3}{2}.$$

Yes, $k=\frac{5}{2}$ is a solution.

88. $\sqrt{4k^2-3} = 2k+1$

$$4k^2-3 = 4k^2+4k+1$$
$$4k = -4$$
$$k = -1.$$

Check: $\sqrt{4(-1)^2-3} = 1 \neq 2(-1)+1 = -1.$
Therefore there is no solution.

89. $\sqrt{k+1}+\sqrt{k} = 3\sqrt{k}$

$$\sqrt{k+1} = 2\sqrt{k}$$
$$k+1 = 4k$$
$$3k = 1$$
$$k = \frac{1}{3}.$$

Check: $\sqrt{\frac{1}{3}+1}+\sqrt{\frac{1}{3}} \stackrel{?}{=} 3\sqrt{\frac{1}{3}}$

$$\sqrt{\frac{4}{3}}+\sqrt{\frac{1}{3}} \stackrel{?}{=} 2\sqrt{\frac{1}{3}}+\sqrt{\frac{1}{3}}$$
$$3\sqrt{\frac{1}{3}} \stackrel{?}{=} 3\sqrt{\frac{1}{3}}.$$

Yes, $k=\frac{1}{3}$ is a solution.

90. $\sqrt{x+1}-\sqrt{x} = \sqrt{4x-3}$

$$x+1-2\sqrt{x^2+x}+x = 4x-3$$
$$-2\sqrt{x^2+x} = 2x-4 = 2(x-2)$$
$$\sqrt{x^2+x} = -x+2$$
$$x^2+x = x^2-4x+4$$
$$5x = 4$$
$$x = \frac{4}{5}.$$

Check: $\sqrt{\frac{4}{5}+1}-\sqrt{\frac{4}{5}} \stackrel{?}{=} \sqrt{4\cdot\frac{4}{5}-3}$

$$\sqrt{\frac{9}{5}}-\sqrt{\frac{4}{5}} \stackrel{?}{=} 3\sqrt{\frac{1}{5}}-2\sqrt{\frac{1}{5}}$$
$$\sqrt{\frac{1}{5}} \stackrel{?}{=} \sqrt{\frac{1}{5}}.$$

Yes, $x=\frac{4}{5}$ is a solution.

91. $x=\sqrt{144-p}$, so $x^2 = 144-p$ and $p = 144-x^2$.

92. $x = 10\sqrt{\dfrac{50-p}{p}}$, so $\dfrac{x}{10} = \sqrt{\dfrac{50-p}{p}}$, $\dfrac{x^2}{100} = \dfrac{50-p}{p}$, $x^2p = 100(50-p) = 5000-100p$, $x^2p+100p = 5000$, $(x^2+100)p = 5000$, and $p = \dfrac{5000}{x^2+100}$.

93. True

94. False

95. True

96. False

1.8 Quadratic Equations

Concept Questions

page 51

1. A quadratic equation in the variable x is any equation that can be written in the form $ax^2+bx+c=0$. For example, $4x^2+3x-4=0$ is a quadratic equation.

2. Step 1 Write the equation in the form $x^2 + \frac{b}{a}x = -\frac{c}{a}$ where the coefficient of x^2 is 1 and the constant term is on the right side of the equation. For example, $3x^2 + 2x - 3 = 0$ can be written as $x^2 + \frac{2}{3}x = 1$.

Step 2 Square half of the coefficient of x. Continuing our example, $\left(\frac{\frac{2}{3}}{2}\right)^2 = \frac{4}{9} \cdot \frac{1}{4} = \frac{1}{9}$.

Step 3 Add the number obtained in step 2 to both sides of the equation, factor, and solve for x. Continuing our example, $x^2 + \frac{2}{3}x + \frac{1}{9} = 1 + \frac{1}{9}$, so $\left(x + \frac{1}{3}\right)^2 = \sqrt{\frac{10}{9}}$, and therefore $x = -\frac{1}{3} \pm \frac{1}{3}\sqrt{10} = \frac{1}{3}\left(-1 \pm \sqrt{10}\right)$.

3. The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Using it to solve $2x^2 - 3x - 5 = 0$ for x, we substitute $a = 2$, $b = -3$, and $c = -5$, obtaining $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} = \frac{3 \pm \sqrt{49}}{4}$. Simplifying, the solutions are $x = \frac{5}{2}$ and $x = -1$.

Exercises page 51

1. $(x+2)(x-3) = 0$. So $x + 2 = 0$ or $x - 3 = 0$; that is, $x = -2$ or $x = 3$.

2. Here $y - 3 = 0$ or $y - 4 = 0$, and so $y = 3$ or $y = 4$.

3. $x^2 - 4 = (x-2)(x+2) = 0$, so $x = 2$ or $x = -2$.

4. $2m^2 - 32 = 2\left(m^2 - 16\right) = 2(m+4)(m-4) = 0$, so $m = -4$ or $m = 4$.

5. $x^2 + x - 12 = (x+4)(x-3) = 0$, so $x = -4$ or $x = 3$.

6. $3x^2 - x - 4 = (3x-4)(x+1) = 0$, so $x = -1$ or $x = \frac{4}{3}$.

7. $4t^2 + 2t - 2 = 2(t+1)(2t-1) = 0$, so $t = -1$ or $t = \frac{1}{2}$.

8. $-6x^2 + x + 12 = 0$ is equivalent to $6x^2 - x - 12 = 0$. Factoring, we have $(3x+4)(2x-3) = 0$, and so $x = -\frac{4}{3}$ or $x = \frac{3}{2}$.

9. $\frac{1}{4}x^2 - x + 1 = 0$ is equivalent to $x^2 - 4x + 4 = 0$, or $(x-2)^2 = 0$. So $x = 2$ is a double root.

10. $\frac{1}{2}a^2 + a - 12 = 0$ is equivalent to $a^2 + 2a - 24 = 0$, or $(a+6)(a-4) = 0$, and so $a = -6$ or $a = 4$.

11. Rewrite the given equation in the form $2m^2 - 7m + 6 = 0$. Then $(2m-3)(m-2) = 0$ and $m = \frac{3}{2}$ or $m = 2$.

12. Rewrite the given equation in the form $6x^2 + 5x - 6 = 0$. Factoring, we have $(3x-2)(2x+3) = 0$, and so $x = \frac{2}{3}$ or $x = -\frac{3}{2}$.

13. $4x^2 - 9 = (2x)^2 - 3^2 = (2x+3)(2x-3) = 0$, and so $x = -\frac{3}{2}$ or $x = \frac{3}{2}$.

14. $8m^2 + 64m = 8m(m+8) = 0$, and so $m = -8$ or $m = 0$.

15. $z(2z+1) = 6$ is equivalent to $2z^2 + z - 6 = 0$, so $(2z-3)(z+2) = 0$. Thus, $z = -2$ or $z = \frac{3}{2}$.

16. Rewrite the given equation in the form $6m^2 + 13m + 5 = 0$. Then $(2m+1)(3m+5) = 0$, and so $m = -\frac{1}{2}$ or $m = -\frac{5}{3}$.

17. $x^2 + 2x + (1)^2 = 8 + 1$, so $(x+1)^2 = 9$, $x + 1 = \pm 3$, and the solutions are $x = -4$ and $x = 2$.

18. $x^2 - x + \left(-\frac{1}{2}\right)^2 = 6 + \left(-\frac{1}{2}\right)^2$, so $\left(x - \frac{1}{2}\right)^2 = \frac{25}{4}$ and $x - \frac{1}{2} = \pm\frac{5}{2}$. Thus, $x = \frac{1}{2} - \frac{5}{2} = -2$ or $x = \frac{1}{2} + \frac{5}{2} = 3$.

19. Rewrite the given equation in the form $6\left[x^2 - 2x + (-1)^2\right] = 3 + 6(-1)^2$. Then $6(x-1)^2 = 9$, $(x-1)^2 = \frac{3}{2}$, and $x - 1 = \pm\sqrt{\frac{3}{2}} = \pm\frac{1}{2}\sqrt{6}$. Therefore, $x = 1 - \frac{\sqrt{6}}{2}$ or $x = 1 + \frac{\sqrt{6}}{2}$.

20. Rewrite the given equation as $2\left[x^2 - 3x + \left(-\frac{3}{2}\right)^2\right] = 20 + 2\left(-\frac{3}{2}\right)^2$, so $2\left(x - \frac{3}{2}\right)^2 = 20 + \frac{9}{2} = \frac{49}{2}$, $\left(x - \frac{3}{2}\right)^2 = \frac{49}{4}$, and $x - \frac{3}{2} = \pm\frac{7}{2}$. Therefore, $x = -2$ or $x = 5$.

21. $m^2 + m = 3$, so $m^2 + m + \left(\frac{1}{2}\right)^2 = 3 + \left(\frac{1}{2}\right)^2$, $\left(m + \frac{1}{2}\right)^2 = \frac{13}{4}$, and $m + \frac{1}{2} = \pm\frac{1}{2}\sqrt{13}$. Therefore, $m = -\frac{1}{2} - \frac{1}{2}\sqrt{13}$ or $m = -\frac{1}{2} + \frac{1}{2}\sqrt{13}$.

22. $p^2 + 2p = 4$, so $p^2 + 2p + (1)^2 = 4 + 1$, $(p+1)^2 = 5$, and $p + 1 = \pm\sqrt{5}$. Therefore, $p = -1 - \sqrt{5}$ or $p = -1 + \sqrt{5}$.

23. $2x^2 + 3x = 4$, so $2\left[x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right] = 4 + 2\left(\frac{3}{4}\right)^2$, $2\left(x + \frac{3}{4}\right)^2 = 4 + \frac{9}{8} = \frac{41}{8}$, $\left(x + \frac{3}{4}\right)^2 = \frac{41}{16}$, and $x + \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$. Therefore, $x = -\frac{3}{4} - \frac{\sqrt{41}}{4}$ or $x = -\frac{3}{4} + \frac{\sqrt{41}}{4}$.

24. $4x^2 - 10x = -5$, $4\left[x^2 - \frac{5}{2}x + \left(-\frac{5}{4}\right)^2\right] = -5 + 4\left(-\frac{5}{4}\right)^2 = -5 + \frac{25}{4} = \frac{5}{4}$. Thus, $4\left(x - \frac{5}{4}\right)^2 = \frac{5}{4}$, $\left(x - \frac{5}{4}\right)^2 = \frac{5}{16}$, and $x - \frac{5}{4} = \pm\frac{1}{4}\sqrt{5}$. Therefore, $x = \frac{5}{4} - \frac{\sqrt{5}}{4}$ or $x = \frac{5}{4} + \frac{\sqrt{5}}{4}$.

25. $4x^2 = 13$, so $x^2 = \frac{13}{4}$ and $x = \pm\frac{\sqrt{13}}{2}$.

26. $7p^2 = 20$, so $p^2 = \frac{20}{7}$ and $p = \pm\sqrt{\frac{20}{7}} = \pm 2\sqrt{\frac{5}{7}}$.

27. Using the quadratic formula with $a = 2$, $b = -1$, and $c = -6$, we obtain $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)} = \dfrac{1 \pm \sqrt{1+48}}{4} = \dfrac{1 \pm 7}{4} = -\frac{3}{2}$ or 2.

28. Using the quadratic formula with $a = 6$, $b = -7$, and $c = -3$, we obtain $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)} = \dfrac{7 \pm \sqrt{49+72}}{12} = \dfrac{7 \pm \sqrt{121}}{12} = \dfrac{7 \pm 11}{12} = -\frac{1}{3}$ or $\frac{3}{2}$.

29. Rewrite the given equation in the form $m^2 - 4m + 1 = 0$. Then using the quadratic formula with $a = 1$, $b = -4$, and $c = 1$, we obtain $m = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2} = \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$.

30. Rewrite the given equation in the form $2x^2-8x+3=0$. Then using the quadratic formula with $a=2$, $b=-8$, and $c=3$, we obtain $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(3)}}{2(2)}=\frac{8\pm\sqrt{64-24}}{4}=\frac{8\pm\sqrt{40}}{4}=\frac{8\pm2\sqrt{10}}{4}=2\pm\frac{1}{2}\sqrt{10}$.

31. Rewrite the given equation in the form $8x^2-8x-3=0$. Then using the quadratic formula with $a=8$, $b=-8$, and $c=-3$, we obtain $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(8)(-3)}}{2(8)}=\frac{8\pm\sqrt{64+96}}{16}=\frac{8\pm\sqrt{160}}{16}=\frac{8\pm4\sqrt{10}}{16}=\frac{1}{2}\pm\frac{1}{4}\sqrt{10}$.

32. Rewrite the given equation in the form $p^2-6p+6=0$. Then using the quadratic formula with $a=1$, $b=-6$, and $c=6$, we obtain $p=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(6)}}{2(1)}=\frac{6\pm\sqrt{36-24}}{2}=\frac{6\pm\sqrt{12}}{2}=\frac{6\pm2\sqrt{3}}{2}=3\pm\sqrt{3}$.

33. Rewrite the given equation in the form $2x^2+4x-3=0$. Then using the quadratic formula with $a=2$, $b=4$, and $c=-3$, we obtain $x=\frac{-4\pm\sqrt{4^2-4(2)(-3)}}{2(2)}=\frac{-4\pm\sqrt{16+24}}{4}=\frac{-4\pm\sqrt{40}}{4}=\frac{-4\pm2\sqrt{10}}{4}=-1\pm\frac{1}{2}\sqrt{10}$.

34. Rewrite the given equation in the form $2y^2+7y-15=0$. Then using the quadratic formula with $a=2$, $b=7$, and $c=-15$, we obtain $y=\frac{-7\pm\sqrt{49-4(2)(-15)}}{4}=\frac{-7\pm\sqrt{169}}{4}=\frac{-7\pm13}{4}=-5$ or $\frac{3}{2}$.

35. Using the quadratic formula with $a=2.1$, $b=-4.7$, and $c=-6.2$, we obtain $x=\frac{4.7\pm\sqrt{(-4.7)^2-4(2.1)(-6.2)}}{2(2.1)}=\frac{4.7\pm\sqrt{74.17}}{4.2}\approx\frac{4.7\pm8.6122}{4.2}\approx-0.93$ or 3.17.

36. Using the quadratic formula with $a=0.2$, $b=1.6$, and $c=1.2$, we obtain $x=\frac{-1.6\pm\sqrt{1.6^2-4(0.2)(1.2)}}{2(0.2)}=\frac{-1.6\pm\sqrt{1.6}}{0.4}\approx\frac{-1.6\pm1.2649}{0.4}\approx-7.16$ or -0.84.

37. $x^4-5x^2+6=0$. Let $m=x^2$. Then the equation reads $m^2-5m+6=0$. Now, factoring, we obtain $(m-3)(m-2)=0$, and so $m=2$ or $m=3$. Therefore, $x=\pm\sqrt{2}$ or $\pm\sqrt{3}$.

38. $m^4-13m^2+36=0$. Let $x=m^2$. Then, we have $x^2-13x+36=0$. Now, factoring, we obtain $(x-9)(x-4)=0$, and so $x=4$ or 9. Therefore, $m=\pm2$ or $m=\pm3$.

39. $y^4-7y^2+10=0$. Let $x=y^2$. Then we have $x^2-7x+10=0$. Factoring, we obtain $(x-2)(x-5)=0$, and so $x=2$ or 5. Therefore, $y=\pm\sqrt{2}$ or $y=\pm\sqrt{5}$.

40. $4x^4-21x^2+5=0$. Let $y=x^2$. Then we have $4y^2-21y+5=0$. Factoring, we obtain $(4y-1)(y-5)=0$, and so $y=\frac{1}{4}$ or 5. Therefore, $x=\pm\frac{1}{2}$, or $\pm\sqrt{5}$.

41. $6(x+2)^2+7(x+2)-3=0$. Let $y=x+2$. Then we have $6y^2+7y-3=0$. Factoring, we obtain $(2y+3)(3y-1)=0$, and so $y=-\frac{3}{2}$ or $\frac{1}{3}$. Therefore, $x+2=-\frac{3}{2}$ or $\frac{1}{3}$, and so $x=-\frac{7}{2}$ or $-\frac{5}{3}$.

42. $8(2m+3)^2+14(2m+3)-15=0$. Let $x=2m+3$. Then we have $8x^2+14x-15=0$. Factoring, we obtain $(4x-3)(2x+5)=0$, and so $x=-\frac{5}{2}$ or $\frac{3}{4}$. Therefore, $2m+3=-\frac{5}{2}$ or $\frac{3}{4}$, from which we obtain $m=-\frac{11}{4}$ and $m=-\frac{9}{8}$.

43. $6w-13\sqrt{w}+6=0$. Let $x=\sqrt{w}$. Then $6x^2-13x+6=0$, $(2x-3)(3x-2)=0$, and so $x=\frac{3}{2}$ or $x=\frac{2}{3}$. Then the solutions are $w=x^2=\frac{9}{4}$ or $\frac{4}{9}$.

Check $w=\frac{4}{9}$: $6\left(\frac{4}{9}\right)-13\sqrt{\frac{4}{9}}+6=\frac{24}{9}-13\cdot\frac{2}{3}+6\stackrel{?}{=}0$. Yes, $\frac{4}{9}$ is a solution.

Check $w=\frac{9}{4}$: $6\left(\frac{9}{4}\right)-13\sqrt{\frac{9}{4}}+6=\frac{54}{4}-13\cdot\frac{3}{2}+6\stackrel{?}{=}0$. Yes, $\frac{9}{4}$ is also a solution.

44. $\left(\frac{t}{t-1}\right)^2-\frac{2t}{t-1}-3=0$. Let $x=\frac{t}{t-1}$. Then $x^2-2x-3=0$, $(x-3)(x+1)=0$, and $x=3$ or $x=-1$. Next, either $\frac{t}{t-1}=3$, in which case $3t-3=t$, $2t=3$, and $t=\frac{3}{2}$; or $\frac{t}{t-1}=-1$, in which case $-t+1=t$, $-2t=-1$, and $t=\frac{1}{2}$. The solutions are $t=\frac{3}{2}$ and $t=\frac{1}{2}$.

45.
$$\frac{2}{x+3}-\frac{4}{x}=4$$
$$2(x)-4(x+3)=4(x)(x+3)$$
$$2x-4x-12=4x^2+12x$$
$$-2x-12=4x^2+12x$$
$$4x^2+14x+12=0$$
$$2x^2+7x+6=0$$
$$(2x+3)(x+2)=0.$$
Thus, the solutions are $x=-\frac{3}{2}$ and $x=-2$.

46.
$$\frac{3y-1}{4}+\frac{4}{y+1}=\frac{5}{2}$$
$$(3y-1)(y+1)+16=\tfrac{5}{2}(4)(y+1)$$
$$3y^2+2y-1+16=\tfrac{5}{2}(4)(y+1)$$
$$3y^2+2y-1+16=10y+10$$
$$3y^2-8y+5=0$$
$$(3y-5)(y-1)=0.$$
Thus, $y=\frac{5}{3}$ or $y=1$.

47.
$$x+2-\frac{3}{2x-1}=0$$
$$x(2x-1)+2(2x-1)-3=0$$
$$2x^2-x+4x-2-3=0$$
$$2x^2+3x-5=0$$
$$(2x+5)(x-1)=0.$$
Thus, the solutions are $x=-\frac{5}{2}$ and $x=1$.

48.
$$\frac{x^2}{x-1}=\frac{3-2x}{x-1}$$
Because the fractions on both sides of the equation have the same denominator, we can write
$$x^2=3-2x \text{ (for } x\neq 1)$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0.$$
But because $x=1$ results in division by zero in the original equation, we discard it. Thus, the only solution is $x=-3$.

49.
$$2-\frac{7}{2y}-\frac{15}{y^2}=0$$
$$4y^2-7y-30=0$$
$$(y+2)(4y-15)=0.$$
Thus, $y=-2$ or $y=\frac{15}{4}$.

50.
$$6+\frac{1}{k}-\frac{2}{k^2}=0$$
$$6k^2+k-2=0$$
$$(3k+2)(2k-1)=0.$$
Thus, $k=-\frac{2}{3}$ or $k=\frac{1}{2}$.

51. $\dfrac{3}{x^2-1}+\dfrac{2x}{x+1}=\dfrac{7}{3}$

$$9+6x(x-1)=7\left(x^2-1\right)$$
$$9+6x^2-6x=7x^2-7$$
$$x^2+6x-16=0$$
$$(x+8)(x-2)=0.$$

Thus, $x=-8$ or $x=2$.

52. $\dfrac{m}{m-2}-\dfrac{27}{7}=\dfrac{2}{m^2-m-2}=\dfrac{2}{(m-2)(m+1)}$

$$7m(m+1)-27\left(m^2-m-2\right)=2(7)$$
$$7m^2+7m-27m^2+27m+54=14$$
$$-20m^2+34m+54=14$$
$$-20m^2+34m+40=0$$
$$10m^2-17m-20=0$$
$$(5m+4)(2m-5)=0.$$

Thus, $m=-\frac{4}{5}$ or $m=\frac{5}{2}$.

53. $\dfrac{3x}{x-2}+\dfrac{4}{x+2}=\dfrac{24}{x^2-4}$

$$3x(x+2)+4(x-2)=24$$
$$3x^2+6x+4x-8=24$$
$$3x^2+10x-32=0$$
$$(3x+16)(x-2)=0.$$

Thus, $x=-\frac{16}{3}$ or $x=2$. But because $x=2$ results in division by zero in the original equation, we discard it. The only solution is $x=-\frac{16}{3}$.

54. $\dfrac{3x}{x+1}+\dfrac{2}{x}+5=\dfrac{3}{x^2+x}$

$$3x^2+2x+2+5\left(x^2+x\right)=3$$
$$3x^2+2x+2+5x^2+5x=3$$
$$8x^2+7x-1=0$$
$$(8x-1)(x+1)=0.$$

Thus, $x=\frac{1}{8}$ or $x=-1$. But because division by zero is not allowed in the original equation, we discard $x=-1$. The only solution is $x=\frac{1}{8}$.

55. $\dfrac{2t+1}{t-2}-\dfrac{t}{t+1}=-1$

$$(2t+1)(t+1)-t(t-2)=-1(t-2)(t+1)$$
$$2t^2+3t+1-t^2+2t=-t^2+t+2$$
$$2t^2+4t-1=0.$$

Using the quadratic formula with $a=2$, $b=4$, and $c=-1$, we obtain

$$t=\frac{-4\pm\sqrt{16-4(2)(-1)}}{4}=-1\pm\frac{\sqrt{24}}{4}$$
$$=-1\pm\tfrac{1}{2}\sqrt{6}.$$

56. $\dfrac{x}{x+1}-\dfrac{3}{x-2}+\dfrac{2}{x^2-x-2}=0$

$$x(x-2)-3(x+1)+2=0$$
$$x^2-2x-3x-3+2=0$$
$$x^2-5x-1=0.$$

Using the quadratic formula with $a=1$, $b=-5$, and $c=-1$, we obtain

$$x=\frac{+5\pm\sqrt{25-4(1)(-1)}}{2}=\frac{+5\pm\sqrt{29}}{2}$$
$$\approx 5.19 \text{ or } -0.19.$$

57.
$$\sqrt{u^2+u-5}=1$$
$$u^2+u-5=1$$
$$u^2+u-6=0$$
$$(u+3)(u-2)=0.$$
Thus, $u=-3$ or $u=2$.

Check $u=-3$: $\sqrt{(-3)^2-3-5}=\sqrt{1}\stackrel{?}{=}1$. Yes, so $u=-3$ is a solution.

Check $u=2$: $\sqrt{2^2+2-5}=\sqrt{1}\stackrel{?}{=}1$. Yes, so $u=2$ is also a solution.

58.
$$\sqrt{6x^2-5x}-2=0$$
$$\sqrt{6x^2-5x}=2$$
$$6x^2-5x-4=0$$
$$(3x-4)(2x+1)=0.$$
Thus, $x=\frac{4}{3}$ or $x=-\frac{1}{2}$.

Check $x=\frac{4}{3}$: $\sqrt{6\left(\frac{4}{3}\right)^2-5\left(\frac{4}{3}\right)}-2\stackrel{?}{=}0$. Yes, so $x=\frac{4}{3}$ is a solution.

Check $x=-\frac{1}{2}$: $\sqrt{6\left(-\frac{1}{2}\right)^2-5\left(-\frac{1}{2}\right)}-2\stackrel{?}{=}0$. Yes, so $x=-\frac{1}{2}$ is also a solution.

59.
$$\sqrt{2r+3}=r$$
$$2r+3=r^2$$
$$r^2-2r-3=0$$
$$(r-3)(r+1)=0.$$
Thus, $r=3$ or $r=-1$.

Check $r=3$: $\sqrt{2(3)+3}\stackrel{?}{=}3$. Yes, so $r=3$ is a solution.

Check $r=-1$: $\sqrt{2(-1)+3}\stackrel{?}{=}-1$. No, so $r=-1$ is not a solution.

60.
$$\sqrt{3-4x}=-2x$$
$$3-4x=4x^2$$
$$4x^2+4x-3=0$$
$$(2x+3)(2x-1)=0.$$
Thus, $x=-\frac{3}{2}$ or $x=\frac{1}{2}$.

Check $x=-\frac{3}{2}$: $\sqrt{3-4\left(-\frac{3}{2}\right)}\stackrel{?}{=}-2\left(-\frac{3}{2}\right)=3$. Yes, so $x=-\frac{3}{2}$ is a solution.

Check $x=\frac{1}{2}$: $\sqrt{3-4\left(\frac{1}{2}\right)}\stackrel{?}{=}-2\left(\frac{1}{2}\right)=-1$. No, so $x=\frac{1}{2}$ is not a solution.

61.
$$\sqrt{s-2}-\sqrt{s+3}+1=0$$
$$\sqrt{s-2}=\sqrt{s+3}-1$$
$$s-2=s+3-2\sqrt{s+3}+1$$
$$2\sqrt{s+3}=6$$
$$s+3=3^2=9$$
$$s=6.$$
Check: $\sqrt{6-2}-\sqrt{6+3}+1\stackrel{?}{=}0$. Yes, so $s=6$ is the solution.

62.
$$\sqrt{x+1}-\sqrt{2x-5}+1=0$$
$$\sqrt{x+1}=\sqrt{2x-5}-1$$
$$x+1=2x-5-2\sqrt{2x-5}+1$$
$$x+1-2x+5-1=-2\sqrt{2x-5}$$
$$-x+5=-2\sqrt{2x-5}$$
$$x^2-10x+25=4(2x-5)$$
$$x^2-10x-8x+25+20=0$$
$$x^2-18x+45=0$$
$$(x-15)(x-3)=0.$$
Thus, $x=15$ or $x=3$.

Check $x=15$: $\sqrt{15+1}-\sqrt{2(15)-5}+1\stackrel{?}{=}0$. Yes, so $x=15$ is a solution.

Check $x=3$: $\sqrt{3+1}-\sqrt{2(3)-5}+1\stackrel{?}{=}0$. No, so $x=3$ is not a solution.

63. $$\frac{1}{(x-3)^2} - \frac{10}{x-3} + 21 = 0$$
$$1 - 10(x-3) + 21(x-3)^2 = 0$$
$$31 - 10x + 21x^2 - 126x + 189 = 0$$
$$21x^2 - 136x + 220 = 0$$
$$(7x-22)(3x-10) = 0.$$
Thus, $x = \frac{22}{7}$ or $x = \frac{10}{3}$.

64. $$\frac{2}{(2x-1)^2} - \frac{5}{2x-1} + 3 = 0$$
$$2 - 5(2x-1) + 3(2x-1)^2 = 0$$
$$7 - 10x + 12x^2 - 12x + 3 = 0$$
$$12x^2 - 22x + 10 = 0$$
$$2(6x-5)(x-1) = 0.$$
Thus, $x = \frac{5}{6}$ or $x = 1$.

65. $x^2 - 6x + 5 = 0$. Here $a = 1$, $b = -6$, and $c = 5$. $b^2 - 4ac = (-6)^2 - 4(1)(5) = 16 > 0$, and so the equation has two real solutions.

66. $2m^2 + 5m + 3 = 0$. Here $a = 2$, $b = 5$, and $c = 3$. $b^2 - 4ac = 5^2 - 4(2)(3) = 1 > 0$, and so the equation has two real solutions.

67. $3y^2 - 4y + 5 = 0$. Here $a = 3$, $b = -4$, and $c = 5$. $b^2 - 4ac = (-4)^2 - 4(3)(5) = -44 < 0$, and so the equation has no real solution.

68. $2p^2 + 5p + 6 = 0$Here $a = 2$, $b = 5$, and $c = 6$. $b^2 - 4ac = 5^2 - 4(2)(6) = -23 < 0$, and so the equation has no real solution.

69. $4x^2 + 12x + 9 = 0$. Here $a = 4$, $b = 12$, and $c = 9$. $b^2 - 4ac = 12^2 - 4(4)(9) = 0$, and so the equation has one real solution.

70. $25x^2 - 80x + 64 = 0$. Here $a = 25$, $b = -80$, and $c = 64$. $b^2 - 4ac = (-80)^2 - 4(25)(64) = 0$, and so the equation has one real solution.

71. $\frac{6}{k^2} + \frac{1}{k} - 2 = 0$. Multiplying by k^2, we have $6 + k - 2k^2 = 0$ or $2k^2 - k - 6 = 0$. Here $a = 2$, $b = -1$, and $c = -6$, so the discriminant is $b^2 - 4ac = (-1)^2 - 4(2)(-6) = 49 > 0$, and the equation has two real solutions.

72. $(2p+1)^2 - 3(2p+1) + 4 = 0$. Let $x = 2p + 1$. Then the equation becomes $x^2 - 3x + 4 = 0$. Here $a = 1$, $b = -3$, and $c = 4$, so because $b^2 - 4ac = (-3)^2 - 4(1)4 = -7 < 0$, the new equation has no real solution, and therefore the given equation also has no real solution.

73. The ball reaches the ground when $h = 0$; that is, when $16t^2 - 64t - 768 = 0$, and $t^2 - 4t - 48 = 0$. Using the quadratic formula with $a = 1$, $b = -4$, and $c = -48$, we find $t = \frac{-(-4) \pm \sqrt{16 - 4(1)(-48)}}{2} = \frac{4 \pm \sqrt{208}}{2} \approx 9.21$, or approximately 9.2 seconds. (We discard the negative root.)

74. a. The rocket is at a height of 1284 ft when $h(t) = 1284$.

$$-16t^2 + 384t - 1280 = 0$$
$$16t^2 - 384t + 1280 = 0$$
$$t^2 - 24t + 80 = 0$$
$$(t - 20)(t - 4) = 0.$$

Thus, $t = 20$ seconds or 4 seconds.

b. The rocket reaches the ground when $h(t) = 0$.

$$-16t^2 + 384t + 4 = 0$$
$$4t^2 - 96t - 1 = 0$$

Using the quadratic formula with $a = 4$, $b = -96$, and $c = -1$, we obtain

$$t = \frac{96 \pm \sqrt{(96)^2 - 4(4)(-1)}}{8} \approx -0.01 \text{ or } 24.01.$$

Discarding the negative root, we see that the time of the flight is approximately 24.01 seconds.

75. Substituting $u = 10$, $a = 4$, and $v = 22$ into the equation $v = ut + at^2$, we have $22 = 10t + 4t^2$. Then $4t^2 + 10t - 22 = 0$, or $2t^2 + 5t - 11 = 0$. Using the quadratic formula with $a = 2$, $b = 5$, and $c = -11$, we have $t = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(-11)}}{2(2)} \approx \dfrac{-5 \pm \sqrt{113}}{4} \approx 1.41$ or -3.91. We reject the negative root, so the time taken is approximately 1.41 seconds after passing the tree.

76. We solve the equation $-0.0002x^2 + 3x + 50{,}000 = 60{,}800$, rewriting it as $0.0002x^2 - 3x + 10{,}800 = 0$. Using the quadratic formula with $a = 0.0002$, $b = -3$, and $c = 10{,}800$, we have $x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(0.0002)(10{,}800)}}{2(0.0002)} = \dfrac{3 \pm \sqrt{0.36}}{0.0004} = \dfrac{3 \pm 0.6}{0.0004} = 6000$ or 9000. Thus, a production level of either $6000 + 10{,}000 = 16{,}000$ or $9000 + 10{,}000 = 19{,}000$ will yield a profit of \$60,800.

77. Substituting $p = 10$ into $p = \dfrac{30}{0.02x^2 + 1}$, we have $10\left(0.02x^2 + 1\right) = 30$. Solving this equation for x, we have $0.2x^2 + 10 = 30$, $0.2x^2 = 20$, $x^2 = 100$, and $x = \pm 10$. Rejecting the negative root, we see that the quantity demanded is 10,000. (Remember that x is measured in units of one thousand.)

78. Substituting $p = 6$ into $p = \sqrt{-x^2 + 100}$, we have $6 = \sqrt{-x^2 + 100}$. Solving this equation, we have $36 = -x^2 + 100$, $x^2 = 64$, and $x = \pm 8$. We reject the negative root, and see that the quantity demanded is 8000.

79. Substituting $p = 30$ into the equation $p = \frac{1}{10}\sqrt{x} + 10$, we have $300 = \sqrt{x} + 100$, so $\sqrt{x} = 200$ and $x = 200^2 = 40{,}000$. Thus, 40,000 satellite radios will be made available at the unit price of \$30.

80. Substituting $p = 20$ into the equation $p = 0.1x^2 + 0.5x + 15$, we have $20 = 0.1x^2 + 0.5x + 15$. Solving this equation, we have $x^2 + 5x - 50 = 0$, so $(x + 10)(x - 5) = 0$ and $x = -10$ or $x = 5$. We reject the negative root, and see that at a unit price of \$20, 5000 lamps will be made available.

81. We solve the equation $100\left(\dfrac{t^2 + 10t + 100}{t^2 + 20t + 100}\right) = 80$, obtaining $5\left(t^2 + 10t + 100\right) = 4\left(t^2 + 20t + 100\right)$, $5t^2 + 50t + 500 = 4t^2 + 80t + 400$, and $t^2 - 30t + 100 = 0$. Using the quadratic formula with $a = 1$, $b = -30$, and $c = 100$, we get $t = \dfrac{30 \pm \sqrt{30^2 - 4(1)(100)}}{2} = \dfrac{30 \pm \sqrt{500}}{2} \approx 3.82$ or 26.18. So the oxygen content first drops to 80% of its natural level approximately 4 days after the waste was dumped into the pond and is restored to that level approximately 26 days after the waste was dumped.

82. We solve the equation $\frac{x}{y} = \frac{x+y}{x}$. Let $\frac{x}{y} = r$. Then $r = 1 + \frac{1}{r}$, $r^2 = r + 1$, and $r^2 - r - 1 = 0$. Using the quadratic formula with $a = 1$, $b = -1$, and $c = -1$, we obtain $r = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2} \approx 1.62$. (We discard the negative root.)

83. The total surface area is given by

$$S = (10 - 2x)(16 - 2x) + 2x(10 - 2x) + 2x(16 - 2x) = 160 - 20x - 32x + 4x^2 + 20x - 4x^2 + 32x - 4x^2$$
$$= -4x^2 + 160.$$

Since the total surface area is to be 144 square inches, we have $-4x^2 + 160 = 144$, $4x^2 = 16$, and $x^2 = 4$. Thus, $x = 2$ because x must be positive. The dimensions are therefore $12'' \times 6'' \times 2''$.

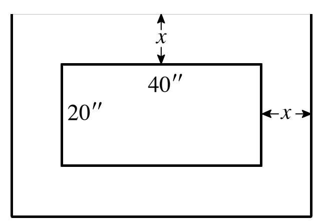
84. Let x be the width of the garden, so that its length is $2x$. Then $2x^2 = 200$, $x^2 = 100$, and $x = \pm 10$. Discarding the negative root, we see that $x = 10$, so the amount of fencing Carmen needs is $2(2x + x) = 6x$, or 60 feet.

85. Let x denote the length of one piece of fencing so that the second piece has length $(120 - x)$ ft. The squares' side lengths are $\frac{x}{4}$ and $\frac{120 - x}{4}$, and so the sum of the areas is

$$A = \left(\frac{x}{4}\right)^2 + \left(\frac{120 - x}{4}\right)^2 = \frac{1}{16}\left[x^2 + (120 - x)^2\right] = \frac{1}{16}\left(x^2 + 14{,}400 - 240x + x^2\right)$$
$$= \frac{1}{16}\left(2x^2 - 240x + 14{,}400\right).$$

Since the sum of the areas of the two rectangles is to be 562.5 ft^2, we have $\frac{1}{16}\left(2x^2 - 240x + 14{,}400\right) = 562.5$, $2x^2 - 240x + 14{,}400 = 9000$, $2x^2 - 240x + 5400 = 0$, $x^2 - 120x + 2700 = 0$, and $(x - 30)(x - 90) = 0$. Therefore $x = 30$ or $x = 90$, and the lengths of the pieces of fencing are 30 ft and 90 ft.

86. Let x denote the width of the walkway. Then the area of the walkway is given by $4x^2 + 80x + 40x = 325$, so $4x^2 + 120x - 325 = 0$, $(2x - 5)(2x + 65) = 0$, and $x = \frac{5}{2}$ or $x = -\frac{65}{2}$. We discard the negative root.



87. Let x denote the width and y the length. Then $2x + y = 3000$. The area is given by $A = xy = x(3000 - 2x) = -2x^2 + 3000x$. The quadratic function $A = -2x^2 + 3000x$ has a maximum at $x = -\frac{b}{2a} = -\frac{3000}{2(-2)} = 750$. Therefore, $y = 3000 - 2(750) = 1500$. The dimensions are 750 yards by 1500 yards.

88. $S = 2\pi r^2 + 2\pi rh$. Substituting $S = 100$ and $h = 3$, we have $100 = 2\pi r^2 + 6\pi r$, so $\pi r^2 + 3\pi r - 50 = 0$. Using the quadratic formula with $a = \pi$, $b = 3\pi$, and $c = -50$, we find $r = \frac{-3\pi \pm \sqrt{9\pi^2 - 4(\pi)(-50)}}{2\pi} \approx \frac{-3\pi \pm \sqrt{717}}{2\pi} \approx 2.76$. (We discard the negative root.) Thus, the radius is approximately 2.76 inches.

89. We solve the equation $2\pi r\ell + 4\pi r^2 = 28\pi$ with $\ell = 4$, obtaining $28\pi = 8\pi r + 4\pi r^2$, $4\pi r^2 + 8\pi r - 28\pi = 0$, and $r^2 + 2r - 7 = 0$. Using the quadratic formula with $a = 1$, $b = 2$, and $c = -7$, we get $r = \dfrac{-2 \pm \sqrt{4 + 4\,(1)\,(7)}}{2} = \dfrac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$. Since r must be positive, we discard the negative root. Thus, $r = -1 + 2\sqrt{2} \approx 1.83$, and the radius of each hemisphere is approximately 1.83 ft.

90. Let x denote the increase in the radius. Then $10{,}000\pi + 4400\pi = (100 + x)^2\,\pi$. Rewriting, we have $14400 = 10000 + 200x + x^2$, $x^2 + 200x - 4400 = 0$, and so $(x + 220)\,(x - 20) = 0$. Because x cannot be negative, we discard the negative root and conclude that $x = 20$, so the radius had increased by 20 ft.

91. False. In fact both a and b must be nonzero.

92. True

93. True.

94. True.

1.9 Inequalities and Absolute Value

Concept Questions page 62

1. Let a, b, and c be any real numbers.

Property 1 If $a < b$ and $b < c$, then $a < c$. Example: $3 < 4$ and $5 < 9$, so $3 < 9$.

Property 2 If $a < b$, then $a + c < b + c$. Example: $-6 < -2$, so $-6 + 3 < -2 + 3$; that is, $-3 < 1$.

Property 3 If $a < b$ and $c > 0$, then $ac < bc$. Example: $-7 < -2$ and $3 > 0$, so $(-7)\,(3) < (-2)\,(3)$; that is, $-21 < -6$.

Property 4 If $a < b$ and $c < 0$, then $ac > bc$. Example: $-7 < -2$ and $-3 < 0$, so $(-7)\,(-3) > (-2)\,(-3)$; that is, $21 > 6$.

2. The absolute value of a number a is defined as $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$ The absolute value of a number cannot be negative.

3. Let a, b, and c be any real numbers.

Property 1 $|-a| = |a|$. Example: $|-5| = |5| = 5$.

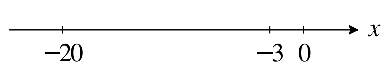
Property 2 $|ab| = |a|\,|b|$. Example: $|(3)\,(-4)| = |(3)|\,|(-4)| = 12$.

Property 3 $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$. Example: $\left|\dfrac{-4}{3}\right| = \dfrac{|-4|}{|3|} = \dfrac{4}{3}$.

Property 4 $|a + b| \leq |a| + |b|$. Example: $|9 + (-4)| = |5| = 5 \leq |9| + |-4| = 13$.

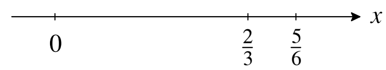
Exercises page 62

1. The statement is false because -3 is greater than -20. See the number line below.



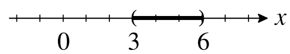
2. The statement is true because -5 is equal to -5.

3. The statement is false because $\frac{2}{3} = \frac{4}{6}$ is less than $\frac{5}{6}$.

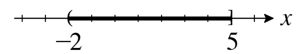


4. The statement is false because $-\frac{5}{6} = -\frac{10}{12}$ is greater than $-\frac{11}{12}$.

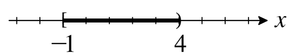
5. The interval $(3, 6)$ is shown on the number line below. Note that this is an open interval indicated by "(" and ")".



6. The interval $(-2, 5]$ is shown on the number line below.

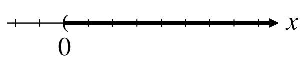


7. The interval $[-1, 4)$ is shown on the number line below. Note that this is a half-open interval indicated by "**[**'' (closed) and "**)**"(open).

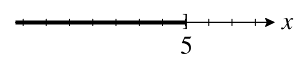


8. The closed interval $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ is shown on the number line below.

9. The infinite interval $(0, \infty)$ is shown on the number line below.



10. The infinite interval $(-\infty, 5]$ is shown on the number line below.



11. We are given $2x + 2 < 8$. Add -2 to each side of the inequality to obtain $2x < 6$, then multiply each side of the inequality by $\frac{1}{2}$ to obtain $x < 3$. We write this in interval notation as $(-\infty, 3)$.

12. We are given $-6 > 4 + 5x$. Add -4 to each side of the inequality to obtain $-6 - 4 > 5x$, so $-10 > 5x$. Dividing by 2, we obtain $-2 > x$, so $x < -2$. We write this in interval notation as $(-\infty, -2)$.

13. We are given the inequality $-4x \geq 20$. Multiply both sides of the inequality by $-\frac{1}{4}$ and reverse the sign of the inequality to obtain $x \leq -5$. We write this in interval notation as $(-\infty, -5]$.

14. $-12 \leq -3x \Rightarrow 4 \geq x$, or $x \leq 4$. We write this in interval notation as $(-\infty, 4]$.

15. We are given the inequality $-6 < x - 2 < 4$. First add 2 to each member of the inequality to obtain $-6 + 2 < x < 4 + 2$ and $-4 < x < 6$, so the solution set is the open interval $(-4, 6)$.

16. We add -1 to each member of the given double inequality $0 \leq x + 1 \leq 4$ to obtain $-1 \leq x \leq 3$, and the solution set is $[-1, 3]$.

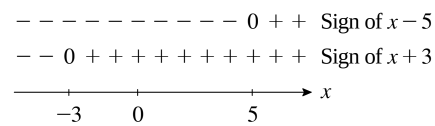
17. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 4$ and $x + 2 < -1$. Adding -1 to both sides of the first inequality, we obtain $x + 1 - 1 > 4 - 1$, so $x > 3$. Similarly, adding -2 to both sides of the second inequality, we obtain $x + 2 - 2 < -1 - 2$, so $x < -3$. Therefore, the solution set is $(-\infty, -3) \cup (3, \infty)$.

18. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 2$ and $x - 1 < -2$. Solving these inequalities, we find that $x > 1$ or $x < -1$, and the solution set is $(-\infty, -1) \cup (1, \infty)$.

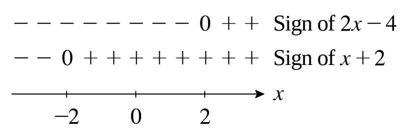
19. We want to find the values of x that satisfy the inequalities $x+3>1$ and $x-2<1$. Adding -3 to both sides of the first inequality, we obtain $x+3-3>1-3$, or $x>-2$. Similarly, adding 2 to each side of the second inequality, we obtain $x-2+2<1+2$, so $x<3$. Because both inequalities must be satisfied, the solution set is $(-2,3)$.

20. We want to find the values of x that satisfy the inequalities $x-4\le 1$ and $x+3>2$. Solving these inequalities, we find that $x\le 5$ and $x>-1$, and the solution set is $(-1,5]$.

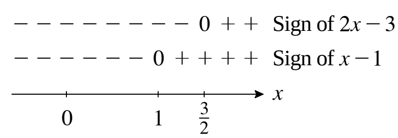
21. We want to find the values of x that satisfy the inequality $(x+3)(x-5)\le 0$. From the sign diagram, we see that the given inequality is satisfied when $-3\le x\le 5$, that is, when the signs of the two factors are different or when one of the factors is equal to zero. The solution set is $[-3,5]$.



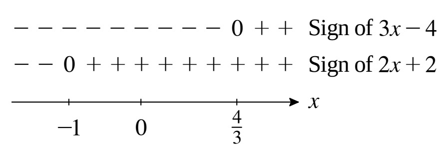
22. We want to find the values of x that satisfy the inequality $(2x-4)(x+2)\ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x\le -2$ or $x\ge 2$; that is, when the signs of both factors are the same or one of the factors is equal to zero. The solution set is $(-\infty,-2]\cup[2,\infty)$.



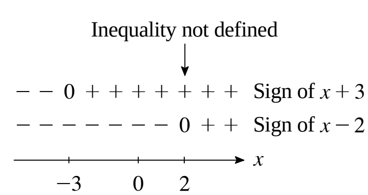
23. We want to find the values of x that satisfy the inequality $(2x-3)(x-1)\le 0$. From the sign diagram, we see that the given inequality is satisfied when $x\ge 1$ and $x\le\frac{3}{2}$; that is, when the signs of the two factors differ or one of the two factors is 0. The solution set is $\left[1,\frac{3}{2}\right]$.



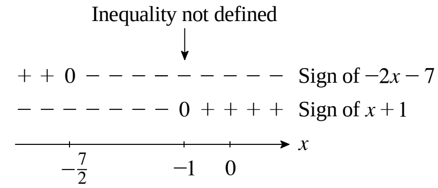
24. We want to find the values of x that satisfy the inequality $(3x-4)(2x+2)\le 0$.
From the sign diagram, we see that the given inequality is satisfied when $-1\le x\le\frac{4}{3}$, that is, when the signs of the two factors differ or when one of the factors is equal to zero. The solution set is $\left[-1,\frac{4}{3}\right]$.



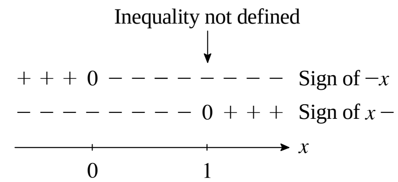
25. We want to find the values of x that satisfy the inequality $\dfrac{x+3}{x-2}\ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x\le -3$ or $x>2$, that is, when the signs of the two factors are the same. The solution set is $(-\infty,-3]\cup(2,\infty)$. Notice that $x=2$ is not included because the inequality is not defined at that value of x.



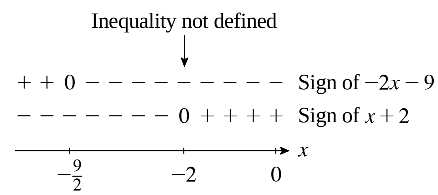
26. We want to find the values of x that satisfy the inequality $\dfrac{2x-3}{x+1} \geq 4$. We rewrite the inequality as $\dfrac{2x-3}{x+1} - 4 \geq 0$, $\dfrac{2x-3-4x-4}{x+1} \geq 0$, and $\dfrac{-2x-7}{x+1} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $-\frac{7}{2} \leq x < -1$; that is, when the signs of the two factors are the same. The solution set is $\left[-\frac{7}{2}, -1\right)$. Notice that $x = -1$ is not included because the inequality is not defined at that value of x.



27. We want to find the values of x that satisfy the inequality $\dfrac{x-2}{x-1} \leq 2$. Subtracting 2 from each side of the given inequality and simplifying gives $\dfrac{x-2}{x-1} - 2 \leq 0$, $\dfrac{x-2-2(x-1)}{x-1} \leq 0$, and $-\dfrac{x}{x-1} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq 0$ or $x > 1$; that is, when the signs of the two factors differ. The solution set is $(-\infty, 0] \cup (1, \infty)$. Notice that $x = 1$ is not included because the inequality is undefined at that value of x.



28. We want to find the values of x that satisfy the inequality $\dfrac{2x-1}{x+2} \leq 4$. Subtracting 4 from each side of the given inequality and simplifying gives $\dfrac{2x-1}{x+2} - 4 \leq 0$, $\dfrac{2x-1-4(x+2)}{x+2} \leq 0$, $\dfrac{2x-1-4x-8}{x+2} \leq 0$, and finally $\dfrac{-2x-9}{x+2} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -\frac{9}{2}$ or $x > -2$. The solution set is $\left(-\infty, -\frac{9}{2}\right] \cup (-2, \infty)$.



29. $|-6+2| = 4$.

30. $4 + |-4| = 4 + 4 = 8$.

31. $\dfrac{|-12+4|}{|16-12|} = \dfrac{|-8|}{|4|} = 2$.

32. $\left|\dfrac{0.2-1.4}{1.6-2.4}\right| = \left|\dfrac{-1.2}{-0.8}\right| = 1.5$.

33. $\sqrt{3}\,|-2| + 3\left|-\sqrt{3}\right| = \sqrt{3}\,(2) + 3\sqrt{3} = 5\sqrt{3}$.

34. $|-1| + \sqrt{2}\,|-2| = 1 + 2\sqrt{2}$.

35. $|\pi - 1| + 2 = \pi - 1 + 2 = \pi + 1$.

36. $|\pi - 6| - 3 = 6 - \pi - 3 = 3 - \pi$.

37. $\left|\sqrt{2} - 1\right| + \left|3 - \sqrt{2}\right| = \sqrt{2} - 1 + 3 - \sqrt{2} = 2$.

38. $\left|2\sqrt{3} - 3\right| - \left|\sqrt{3} - 4\right| = 2\sqrt{3} - 3 - \left(4 - \sqrt{3}\right) = 3\sqrt{3} - 7$.

39. False. If $a > b$, then $-a < -b$, $-a + b < -b + b$, and $b - a < 0$.

40. False. Let $a = -2$ and $b = -3$. Then $a/b = \frac{-2}{-3} = \frac{2}{3} < 1$.

41. False. Let $a = -2$ and $b = -3$. Then $a^2 = 4$ and $b^2 = 9$, and $4 < 9$. (Note that we need only provide a counterexample to show that the statement is not always true.)

42. False. Let $a = -2$ and $b = -3$. Then $\dfrac{1}{a} = -\dfrac{1}{2}$ and $\dfrac{1}{b} = -\dfrac{1}{3}$, and $-\dfrac{1}{2} < -\dfrac{1}{3}$.

43. True. There are three possible cases.
Case 1: If $a > 0$ and $b > 0$, then $a^3 > b^3$, since $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right) > 0$.
Case 2: If $a > 0$ and $b < 0$, then $a^3 > 0$ and $b^3 < 0$, and it follows that $a^3 > b^3$.
Case 3: If $a < 0$ and $b < 0$, then $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right) > 0$, and we see that $a^3 > b^3$. (Note that $a - b > 0$ and $ab > 0$.)

44. True. If $a > b$, then it follows that $-a < -b$ because an inequality symbol is reversed when both sides of the inequality are multiplied by a negative number.

45. $|x - a| < b$ is equivalent to $-b < x - a < b$ or $a - b < x < a + b$.

46. $|x - a| \geq b$ is equivalent to $x - a \geq b$ or $a - x \geq b$; that is, $x \geq a + b$ or $-x \geq b - a$; or $x \geq a + b$ or $x \leq a - b$.

47. False. If we take $a = -2$, then $|-a| = |-(-2)| = |2| = 2 \neq a$.

48. True. If $b < 0$, then $b^2 > 0$, and $\left|b^2\right| = b^2$.

49. True. If $a - 4 < 0$, then $|a - 4| = 4 - a = |4 - a|$. If $a - 4 > 0$, then $|4 - a| = a - 4 = |a - 4|$.

50. False. If we let $a = -2$, then $|a + 1| = |-2 + 1| = |-1| = 1 \neq |-2| + 1 = 3$.

51. False. If we take $a = 3$ and $b = -1$, then $|a + b| = |3 - 1| = 2 \neq |a| + |b| = 3 + 1 = 4$.

52. False. If we take $a = 3$ and $b = -1$, then $|a - b| = 4 \neq |a| - |b| = 3 - 1 = 2$.

53. Simplifying $5(C - 25) \geq 1.75 + 2.5C$, we obtain $5C - 125 \geq 1.75 + 2.5C$, $5C - 2.5C \geq 1.75 + 125$, $2.5C \geq 126.75$, and finally $C \geq 50.7$. Therefore, the minimum cost is \$50.70.

54. $6(P - 2500) \leq 4(P + 2400)$ can be rewritten as $6P - 15{,}000 \leq 4P + 9600$, $2P \leq 24{,}600$, or $P \leq 12{,}300$. Therefore, the maximum profit is \$12,300.

55. If the car is driven in the city, then it can be expected to cover (18.1 gallons) $\left(20\ \frac{\text{miles}}{\text{gallon}}\right) = 362$ miles on a full tank. If the car is driven on the highway, then it can be expected to cover (18.1 gallons) $\left(27\ \frac{\text{miles}}{\text{gallon}}\right) = 488.7$ miles on a full tank. Thus, the driving range of the car may be described by the interval $[362, 488.7]$.

56. **a.** We want to find a formula for converting Centigrade temperatures to Fahrenheit temperatures. Thus, $C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$. Therefore, $\frac{5}{9}F = C + \frac{160}{9}$, $5F = 9C + 160$, and $F = \frac{9}{5}C + 32$. Calculating the lower temperature range, we have $F = \frac{9}{5}(-15) + 32 = 5$, or 5 degrees. Calculating the upper temperature range, $F = \frac{9}{5}(-5) + 32 = 23$, or 23 degrees. Therefore, the temperature range is $5^\circ < F < 23^\circ$.

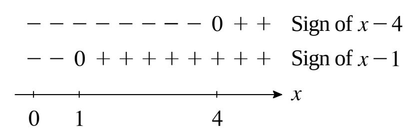
b. For the lower temperature range, $C = \frac{5}{9}(63 - 32) = \frac{155}{9} \approx 17.2$, or 17.2 degrees. For the upper temperature range, $C = \frac{5}{9}(80 - 32) = \frac{5}{9}(48) \approx 26.7$, or 26.7 degrees. Therefore, the temperature range is $17.2^\circ < C < 26.7^\circ$.

57. Let x represent the salesman's monthly sales in dollars. Then $0.15(x - 12{,}000) \geq 6000$, $15(x - 12{,}000) \geq 600{,}000$, $15x - 180{,}000 \geq 600{,}000$, $15x \geq 780{,}000$, and $x \geq 52{,}000$. We conclude that the salesman must have sales of at least \$52,000 to reach his goal.

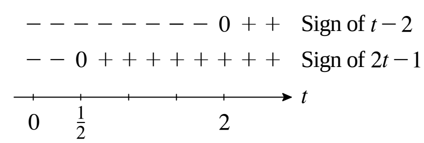
58. Let x represent the wholesale price of the car. Then $\dfrac{\text{Selling price}}{\text{Wholesale price}} - 1 \geq \text{Markup}$; that is, $\dfrac{11{,}200}{x} - 1 \geq 0.30$, whence $\dfrac{11{,}200}{x} \geq 1.30$, $1.3x \leq 11{,}200$, and $x \leq 8615.38$. We conclude that the maximum wholesale price is \$8615.38.

59. We want to solve the inequality $-6x^2 + 30x - 10 \geq 14$. (Remember that x is expressed in thousands.) Adding -14 to both sides of this inequality, we have $-6x^2 + 30x - 10 - 14 \geq 14 - 14$, or $-6x^2 + 30x - 24 \geq 0$. Dividing both sides of the inequality by -6 (which reverses the sign of the inequality), we have $x^2 - 5x + 4 \leq 0$. Factoring this last expression, we have $(x - 4)(x - 1) \leq 0$.

From the sign diagram, we see that x must lie between 1 and 4. (The inequality is satisfied only when the two factors have different signs.) Because x is expressed in thousands of units, we see that the manufacturer must produce between 1000 and 4000 units of the commodity.



60. We solve the inequality $\dfrac{0.2t}{t^2 + 1} \geq 0.08$, obtaining $0.08t^2 + 0.08 \leq 0.2t$, $0.08t^2 - 0.2t + 0.08 \leq 0$, $2t^2 - 5t + 2 \leq 0$, and $(2t - 1)(t - 2) \leq 0$. From the sign diagram, we see that the required solution is $\left[\frac{1}{2}, 2\right]$, so the concentration of the drug is greater than or equal to 0.08 mg/cc between $\frac{1}{2}$ hr and 2 hr after injection.



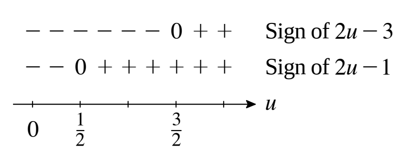
61. We solve the inequalities $25 \leq \dfrac{0.5x}{100 - x} \leq 30$, obtaining $2500 - 25x \leq 0.5x \leq 3000 - 30x$, which is equivalent to $2500 - 25x \leq 0.5x$ and $0.5x \leq 3000 - 30x$. Simplifying further, $25.5x \geq 2500$ and $30.5x \leq 3000$, so $x \geq \dfrac{2500}{25.5} \approx 98.04$ and $x \leq \dfrac{3000}{30.5} \approx 98.36$. Thus, the city could expect to remove between 98.04% and 98.36% of the toxic pollutant.

62. We simplify the inequality $20t - 40\sqrt{t} + 50 \leq 35$ to $20t - 40\sqrt{t} + 15 \leq 0$ (1). Let $u = \sqrt{t}$. Then $u^2 = t$, so we have $20u^2 - 40u + 15 \leq 0$, $4u^2 - 8u + 3 \leq 0$, and $(2u - 3)(2u - 1) \leq 0$.

From the sign diagram, we see that we must have u in $\left[\frac{1}{2}, \frac{3}{2}\right]$.

Because $t = u^2$, we see that the solution to Equation (1) is $\left[\frac{1}{4}, \frac{9}{4}\right]$.

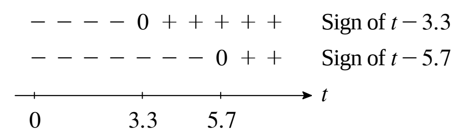
Thus, the average speed of a vehicle is less than or equal to 35 miles per hour between 6:15 a.m. and 8:15 a.m.



63. We solve $\dfrac{10{,}000}{t^2+1} + 2000 < 4000$, obtaining $\dfrac{10{,}000}{t^2+1} < 2000$, $10{,}000 < 2000\left(t^2+1\right)$, and $t^2+1 > 5$. Rewriting, we have $t^2 - 4 > 0$, or $(t-2)(t+2) > 0$. The solution of this inequality is $t < -2$ or $t > 2$. Because t must be positive, we conclude that the number of bacteria will have dropped below 4000 after 2 minutes.

64. We solve the inequality $\dfrac{136}{1+0.25(t-4.5)^2} + 28 \geq 128$ or $\dfrac{136}{1+0.25(t-4.5)^2} \geq 100$. Next, $136 \geq 100\left[1+0.25(t-4.5)^2\right]$, so $136 \geq 100 + 25(t-4.5)^2$, $36 \geq 25(t-4.5)^2$, $(t-4.5)^2 \leq \frac{36}{25}$, $\left(t-\frac{9}{2}\right)^2 - \left(\frac{6}{5}\right)^2 \leq 0$, $\left[\left(t-\frac{9}{2}\right)+\frac{6}{5}\right]\left[\left(t-\frac{9}{2}\right)-\frac{6}{5}\right] \leq 0$, or $(t-3.3)(t-5.7) \leq 0$.

From the sign diagram, we see that the required solution is $[3.3, 5.7]$. Thus, the amount of nitrogen dioxide is greater than or equal to 128 PSI between 10:18 a.m. and 12:42 p.m.



65. The ball's height is 196 ft or greater when $128t - 16t^2 + 4 \geq 196$, that is, $16t^2 - 128t + 192 \leq 0$. Simplifying and factoring, this is equivalent to the inequality $t^2 - 8t + 12 = (t-6)(t-2) \leq 0$. The solution of this inequality is $2 \leq t \leq 6$. We conclude that the ball's height is greater than or equal to 196 ft for 4 seconds.

66. **a.** $\left(5.6 \times 10^{11}\right)(30{,}000)^{-1.5} \approx 107{,}772$, or 107,772 families.

b. $\left(5.6 \times 10^{11}\right)(60{,}000)^{-1.5} \approx 38{,}103$, or 38,103 families.

c. $\left(5.6 \times 10^{11}\right)(150{,}000)^{-1.5} \approx 9639$, or 9639 families.

67. The rod is acceptable if $0.49 \leq x \leq 0.51$ or $-0.01 \leq x - 0.5 \leq 0.01$. This gives the required inequality, $|x - 0.5| \leq 0.01$.

68. $|x - 0.1| \leq 0.01$ is equivalent to $-0.01 \leq x - 0.1 \leq 0.01$ or $0.09 \leq x \leq 0.11$. Therefore, the smallest diameter a ball bearing in the batch can have is 0.09 inch, and the largest diameter is 0.11 inch.

CHAPTER 1 Concept Review Questions page 65

1. **a.** rational; repeating; terminating

b. irrational, terminates, repeats

2. **a.** $b + a$; $(a + b) + c$; a; 0

b. ba; $(ab)\,c$; $1 \cdot a = a$; 1

c. $ab + ac$

3. **a.** a; $-(ab) = a(-b)$; ab

b. 0; 0

4. **a.** polynomial; x; degree; term; polynomial; coefficient

b. like

5. product; prime; $x(x+2)(x-1)$

6. **a.** polynomials

b. numerator; denominator; factors; 1; -1

c. denominator; fractions

7. complex; $\dfrac{1+\frac{1}{x}}{1-\frac{1}{y}}$

8. a. $\underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$; base; exponent; power

b. 1; not defined

c. $\dfrac{1}{a^n}$

9. a. equation

b. number

c. $ax + b = 0$; 1

10. a. $a^n = b$

b. pairs

c. no

d. real root

11. a. radical; $b^{1/n}$

b. radical

12. a. $ax^2 + bx + c = 0$

b. factoring; completing the square;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CHAPTER 1 Review Exercises page 65

1. The number $\frac{7}{8}$ is a rational number and a real number.

2. The number $\sqrt{13}$ is an irrational number and a real number.

3. The number -2π is an irrational number and a real number.

4. The number 0 is a whole number, an integer, a rational number, and a real number.

5. The number $2.\overline{71}$ is a rational number and a real number.

6. The number 3.14159 . . .is an irrational number and a real number.

7. $\left(\dfrac{9}{4}\right)^{3/2} = \dfrac{9^{3/2}}{4^{3/2}} = \dfrac{27}{8}.$

8. $\dfrac{5^6}{5^4} = 5^{6-4} = 5^2 = 25.$

9. $(3 \cdot 4)^{-2} = 12^{-2} = \dfrac{1}{12^2} = \dfrac{1}{144}.$

10. $(-8)^{5/3} = \left[\left(-8^{1/3}\right)^5\right] = (-2)^5 = -32.$

11. $\left(\dfrac{16}{9}\right)^{3/2} = \left(\dfrac{4}{3}\right)^3 = \dfrac{64}{27}.$

12. $\dfrac{\left(3 \cdot 2^{-3}\right)\left(4 \cdot 3^5\right)}{2 \cdot 9^3} = \dfrac{3 \cdot 2^{-3} \cdot 2^2 \cdot 3^5}{2 \cdot \left(3^2\right)^3} = \dfrac{2^{-1} \cdot 3^6}{2 \cdot 3^6} = \dfrac{1}{4}.$

13. $\sqrt[3]{\dfrac{27}{125}} = \dfrac{3}{5}.$

14. $\dfrac{3\sqrt[3]{54}}{\sqrt[3]{18}} = \dfrac{3 \cdot \left(2 \cdot 3^3\right)^{1/3}}{\left(2 \cdot 3^2\right)^{1/3}} = \dfrac{3^2 \cdot 2^{1/3}}{2^{1/3} \cdot 3^{2/3}} = 3^{4/3} = 3\sqrt[3]{3}.$

15. $\dfrac{4\left(x^2+y\right)^3}{x^2+y} = 4\left(x^2+y\right)^2.$

16. $\dfrac{a^6 b^{-5}}{\left(a^3 b^{-2}\right)^{-3}} = \dfrac{a^6 b^{-5}}{a^{-9} b^6} = \dfrac{a^{15}}{b^{11}}.$

17. $\dfrac{\sqrt[4]{16x^5yz}}{\sqrt[4]{81xyz^5}} = \dfrac{\left(2^4x^5yz\right)^{1/4}}{\left(3^4xyz^5\right)^{1/4}} = \dfrac{2x^{5/4}y^{1/4}z^{1/4}}{3x^{1/4}y^{1/4}z^{5/4}} = \dfrac{2x}{3z}$. **18.** $\left(2x^3\right)\left(-3x^{-2}\right)\left(\frac{1}{6}x^{-1/2}\right) = -x^{1/2}$.

19. $\left(\dfrac{3xy^2}{4x^3y}\right)^{-2}\left(\dfrac{3xy^3}{2x^2}\right)^3 = \left(\dfrac{3y}{4x^2}\right)^{-2}\left(\dfrac{3y^3}{2x}\right)^3 = \left(\dfrac{4x^2}{3y}\right)^2\left(\dfrac{3y^3}{2x}\right)^3 = \dfrac{\left(16x^4\right)\left(27y^9\right)}{\left(9y^2\right)\left(8x^3\right)} = 6xy^7$.

20. $\left(-3a^2b^3\right)^2\left(2a^{-1}b^{-2}\right)^{-1} = 9a^4b^6 \cdot \frac{1}{2}ab^2 = \frac{9}{2}a^5b^8$.

21. $\sqrt[3]{81x^5y^{10}}\sqrt[3]{9xy^2} = 3^{4/3}x^{5/3}y^{10/3} \cdot 3^{2/3}x^{1/3}y^{2/3} = 3^2x^2y^4 = 9x^2y^4$.

22. $\left(\dfrac{-x^{1/2}y^{2/3}}{x^{1/3}y^{3/4}}\right)^6 = \dfrac{x^3y^4}{x^2y^{9/2}} = \dfrac{x}{y^{1/2}}$.

23. $\left(3x^4 + 10x^3 + 6x^2 + 10x + 3\right) + \left(2x^4 + 10x^3 + 6x^2 + 4x\right)$
$= 3x^4 + 2x^4 + 10x^3 + 10x^3 + 6x^2 + 6x^2 + 10x + 4x + 3 = 5x^4 + 20x^3 + 12x^2 + 14x + 3$.

24. $(3x - 4)\left(3x^2 - 2x + 3\right) = 3x\left(3x^2 - 2x + 3\right) - 4\left(3x^2 - 2x + 3\right)$
$= 9x^3 - 6x^2 + 9x - 12x^2 + 8x - 12 = 9x^3 - 18x^2 + 17x - 12$

25. $(2x + 3y)^2 - (3x + 1)(2x - 3) = 4x^2 + 12xy + 9y^2 - 6x^2 + 7x + 3 = -2x^2 + 9y^2 + 12xy + 7x + 3$.

26. $2(3a + b) - 3[(2a + 3b) - (a + 2b)] = 6a + 2b - 3(2a + 3b - a - 2b) = 6a + 2b - 3a - 3b = 3a - b$.

27. $\dfrac{(t + 6)(60) - (60t + 180)}{(t + 6)^2} = \dfrac{60t + 360 - 60t - 180}{(t + 6)^2} = \dfrac{180}{(t + 6)^2}$.

28. $\dfrac{6x}{2\left(3x^2 + 2\right)} + \dfrac{1}{4(x + 2)} = \dfrac{(6x)\,2(x + 2) + \left(3x^2 + 2\right)}{4\left(3x^2 + 2\right)(x + 2)} = \dfrac{12x^2 + 24x + 3x^2 + 2}{4\left(3x^2 + 2\right)(x + 2)} = \dfrac{15x^2 + 24x + 2}{4\left(3x^2 + 2\right)(x + 2)}$.

29. $\dfrac{2}{3}\left(\dfrac{4x}{2x^2 - 1}\right) + 3\left(\dfrac{3}{3x - 1}\right) = \dfrac{8x}{3\left(2x^2 - 1\right)} + \dfrac{9}{3x - 1} = \dfrac{8x(3x - 1) + 27\left(2x^2 - 1\right)}{3\left(2x^2 - 1\right)(3x - 1)} = \dfrac{78x^2 - 8x - 27}{3\left(2x^2 - 1\right)(3x - 1)}$.

30. $-\dfrac{2x}{\sqrt{x + 1}} + 4\sqrt{x + 1} = \dfrac{-2x + 4(x + 1)}{\sqrt{x + 1}} = \dfrac{2(x + 2)}{\sqrt{x + 1}}\dfrac{\sqrt{x + 1}}{\sqrt{x + 1}} = \dfrac{2(x + 2)\sqrt{x + 1}}{x + 1}$.

31. $-2\pi^2r^3 + 100\pi r^2 = -2\pi r^2(\pi r - 50)$.

32. $2v^3w + 2vw^3 + 2u^2vw = 2vw\left(v^2 + w^2 + u^2\right)$.

33. $16 - x^2 = 4^2 - x^2 = (4 - x)(4 + x)$.

34. $12t^3 - 6t^2 - 18t = 6t\left(2t^2 - t - 3\right) = 6t(2t - 3)(t + 1)$.

35. $-2x^2 - 4x + 6 = -2\left(x^2 + 2x - 3\right) = -2(x + 3)(x - 1)$.

36. $12x^2 - 92x + 120 = 4\left(3x^2 - 23x + 30\right) = 4(3x - 5)(x - 6)$.

37. $9a^2 - 25b^2 = (3a)^2 - (5b)^2 = (3a - 5b)(3a + 5b)$.

38. $8u^6v^3 + 27u^3 = u^3\left(8u^3v^3 + 27\right) = u^3\left[(2uv)^3 + 3^3\right] = u^3(2uv + 3)\left(4u^2v^2 - 6uv + 9\right)$.

39. $6a^4b^4c - 3a^3b^2c - 9a^2b^2 = 3a^2b^2\left(2a^2b^2c - ac - 3\right)$.

40. $6x^2 - xy - y^2 = (3x + y)(2x - y)$.

41. $\dfrac{2x^2 + 3x - 2}{2x^2 + 5x - 3} = \dfrac{(2x - 1)(x + 2)}{(2x - 1)(x + 3)} = \dfrac{x + 2}{x + 3}$.

42. $\dfrac{\left[\left(t^2 + 4\right)(2t - 4)\right] - \left(t^2 - 4t + 4\right)(2t)}{\left(t^2 + 4\right)^2} = \dfrac{2t^3 + 8t - 4t^2 - 16 - 2t^3 + 8t^2 - 8t}{\left(t^2 + 4\right)^2} = \dfrac{4t^2 - 16}{\left(t^2 + 4\right)^2} = \dfrac{4\left(t^2 - 4\right)}{\left(t^2 + 4\right)^2}$.

43. $\dfrac{2x - 6}{x + 3} \cdot \dfrac{x^2 + 6x + 9}{x^2 - 9} = \dfrac{2(x - 3)}{x + 3} \cdot \dfrac{(x + 3)(x + 3)}{(x + 3)(x - 3)} = \dfrac{2(x + 3)}{x + 3} = 2$.

44.
$$\frac{3x}{x^2 + 2} + \frac{3x^2}{x^3 + 1} = \frac{3x\left(x^3 + 1\right) + 3x^2\left(x^2 + 2\right)}{\left(x^2 + 2\right)\left(x^3 + 1\right)} = \frac{3x^4 + 3x + 3x^4 + 6x^2}{\left(x^2 + 2\right)\left(x^3 + 1\right)} = \frac{6x^4 + 6x^2 + 3x}{\left(x^2 + 2\right)\left(x^3 + 1\right)}$$
$$= \frac{3x\left(2x^3 + 2x + 1\right)}{\left(x^2 + 2\right)\left(x^3 + 1\right)}.$$

45. $\dfrac{1 + \dfrac{1}{x + 2}}{x - \dfrac{9}{x}} = \dfrac{x + 2 + 1}{x + 2} \cdot \dfrac{x}{x^2 - 9} = \dfrac{x + 3}{x + 2} \cdot \dfrac{x}{(x + 3)(x - 3)} = \dfrac{x}{(x + 2)(x - 3)}$.

46. $\dfrac{x\left(3x^2 + 1\right)}{x - 1} \cdot \dfrac{x\left(3x^2 - 5x + 1\right)}{x(x - 1)\left(3x^2 + 1\right)^{1/2}} = \dfrac{x\sqrt{3x^2 + 1}\left(3x^2 - 5x + 1\right)}{(x - 1)^2}$

47. $8x^2 + 2x - 3 = (4x + 3)(2x - 1) = 0$, so the solutions are $x = -\frac{3}{4}$ and $x = \frac{1}{2}$.

48. $-6x^2 - 10x + 4 = 0$, $3x^2 + 5x - 2 = (3x - 1)(x + 2) = 0$, and so $x = -2$ or $\frac{1}{3}$.

49. $2x^2 - 3x - 4 = 0$. Using the quadratic formula with $a = 2$, $b = -3$, and $c = -4$, we have $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} = \dfrac{3 \pm \sqrt{9 + 32}}{4} = \dfrac{3 \pm \sqrt{41}}{4}$.

50. $x^2 + 5x + 3 = 0$. Using the quadratic formula with $a = 1$, $b = 5$, and $c = 3$, we have $x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2} = \dfrac{-5 \pm \sqrt{25 - 12}}{2} = \dfrac{-5 \pm \sqrt{13}}{2}$.

51. $2y^2 - 3y + 1 = (2y - 1)(y - 1) = 0$, and so $y = \frac{1}{2}$ or 1.

52. $0.3m^2 - 2.1m - 3.2 = 0$. Using the quadratic formula with $a = 0.3$, $b = -2.1$, and $c = -3.2$, we have $m = \dfrac{-(-2.1) \pm \sqrt{(-2.1)^2 - 4(0.3)(-3.2)}}{2(0.3)} = \dfrac{2.1 \pm \sqrt{4.41 + 3.84}}{0.6} = \dfrac{2.1 \pm \sqrt{8.25}}{0.6} \approx -1.2871$ or 8.2871.

53. $-x^3 - 2x^2 + 3x = -x\left(x^2 + 2x - 3\right) = -x(x + 3)(x - 1) = 0$, and so the roots of the equation are $x = 0$, $x = -3$, and $x = 1$.

54. $2x^4 + x^2 = 1$. Let $y = x^2$ and we can write the equation as $2y^2 + y - 1 = (2y - 1)(y + 1) = 0$, giving $y = \frac{1}{2}$ or $y = -1$. We reject the second root because $y = x^2$ must be nonnegative. Therefore, $x^2 = \frac{1}{2}$ or $x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$.

55. $\frac{1}{4}x + 2 = \frac{3}{4}x - 5$, so $-\frac{1}{2}x = -7$ and $x = 14$.

56. $\dfrac{3p+1}{2} - \dfrac{2p-1}{3} = \dfrac{5p}{12}$, so $6(3p + 1) - 4(2p - 1) = 5p$, $18p + 6 - 8p + 4 = 5p$, $5p + 10 = 0$, $5p = -10$, and $p = -2$.

57. $(x + 2)^2 - 3x(1 - x) = (x - 2)^2$. Thus, $x^2 + 4x + 4 - 3x + 3x^2 = x^2 - 4x + 4$, $3x^2 + 5x = 0$, and $x(3x + 5) = 0$, and so $x = 0$ or $x = -\frac{5}{3}$.

58. $\dfrac{3(2q+1)}{4q-3} = \dfrac{3q+1}{2q+1}$, so $3(2q + 1)(2q + 1) = (3q + 1)(4q - 3)$, $3\left(4q^2 + 4q + 1\right) = 12q^2 - 5q - 3$, $12q^2 + 12q + 3 = 12q^2 - 5q - 3$, $17q = -6$, and $q = -\frac{6}{17}$.

Check: $\dfrac{3\left[2\left(-\frac{6}{17}\right)+1\right]}{4\left(-\frac{6}{17}\right)-3} = -\dfrac{1}{5}$ and $\dfrac{3\left(-\frac{6}{17}\right)+1}{2\left(-\frac{6}{17}\right)+1} = -\dfrac{1}{5}$, so $q = -\frac{6}{17}$ is the solution.

59. $\sqrt{k-1} = \sqrt{2k-3}$, so $k - 1 = 2k - 3$ and $2 = k$. Check: $\sqrt{2-1} = 1$ and $\sqrt{2(2)-3} = 1$, so $k = 2$ is the solution.

60. $\sqrt{x} - \sqrt{x-1} = \sqrt{4x-3}$, so $x - 2\sqrt{x}\sqrt{x-1} + x - 1 = 4x - 3$, $-2\sqrt{x(x-1)} = 4x - 2x + 1 - 3 = 2x - 2 = 2(x - 1)$, $-\sqrt{x(x-1)} = x - 1$, $x^2 - x = x^2 - 2x + 1$, and thus $x = 1$.

Check: $\sqrt{4(1)-3} = \sqrt{1}$, so $x = 1$ is the solution.

61. Solve $C = \dfrac{20x}{100-x}$. $C(100 - x) = 20x$, $100C - Cx = 20x$, $-Cx - 20x = -100C$, $x(20 + C) = 100C$, and so $x = \dfrac{100C}{20+C}$.

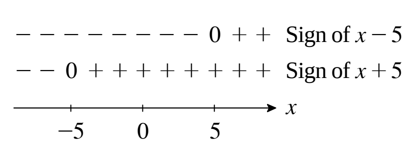
62. $r = \dfrac{2mI}{B(n+1)}$, so $rB(n + 1) = 2mI$ and $\dfrac{rB(n+1)}{2m} = I$.

63. $-x + 3 \le 2x + 9$. Adding x to both sides yields $3 \le 3x + 9$, so $3x \ge -6$ and thus $x \ge -2$. We conclude that the solution set is $[-2, \infty)$.

64. $-2 \le 3x + 1 \le 7$ implies $-3 \le 3x \le 6$, or $-1 \le x \le 2$, and so the solution set is $[-1, 2]$.

65. The inequalities $x - 3 > 2$ and $x + 1 < -1$ imply $x > 5$ or $x < -4$, so the solution set is $(-\infty, -4) \cup (5, \infty)$.

66. $2x^2 > 50$ is equivalent to $x^2 - 25 > 0$, or $(x + 5)(x - 5) > 0$. From the sign diagram, we see that the solution set is $(-\infty, -5) \cup (5, \infty)$.



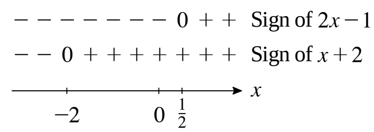
67. $|-5 + 7| + |-2| = |2| + |-2| = 2 + 2 = 4$.

68. $\left|\dfrac{5-12}{-4-3}\right| = \dfrac{|5-12|}{|-7|} = \dfrac{|-7|}{7} = \dfrac{7}{7} = 1$.

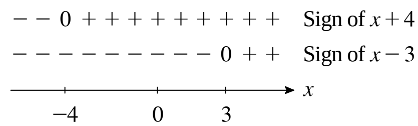
69. $|2\pi - 6| - \pi = 2\pi - 6 - \pi = \pi - 6.$

70. $\left|\sqrt{3} - 4\right| + \left|4 - 2\sqrt{3}\right| = \left(4 - \sqrt{3}\right) + \left(4 - 2\sqrt{3}\right) = 8 - 3\sqrt{3}.$

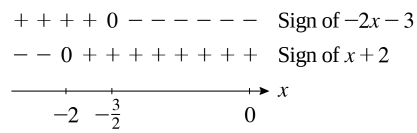
71. Factoring the left-hand side of $2x^2 + 3x - 2 \le 0$, we have $(2x - 1)(x + 2) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-2 \le x \le \frac{1}{2}$. The solution set is $\left[-2, \frac{1}{2}\right]$.



72. Factoring the left-hand side of $x^2 + x - 12 \le 0$, we have $(x + 4)(x - 3) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-4 \le x \le 3$. The solution set is $[-4, 3]$.



73. $\dfrac{1}{x+2} > 2$ gives $\dfrac{1}{x+2} - 2 > 0$, $\dfrac{1 - 2x - 4}{x+2} > 0$, and finally $\dfrac{-2x-3}{x+2} > 0$. From the sign diagram, we see that the given inequality is satisfied when $-2 < x < -\frac{3}{2}$.The solution set is $\left(-2, -\frac{3}{2}\right)$.



74. The given inequality $|2x - 3| < 5$ is equivalent to $-5 < 2x - 3 < 5$. Thus, $-2 < 2x < 8$, or $-1 < x < 4$. The solution set is $(-1, 4)$.

75. The given inequality $|3x - 4| \le 2$ is equivalent to $3x - 4 \le 2$ or $3x - 4 \ge -2$. Solving the first inequality, we have $3x \le 6$, so $x \le 2$. Similarly, we solve the second inequality and obtain $3x \ge 2$, so $x \ge \frac{2}{3}$. We conclude that $\frac{2}{3} \le x \le 2$. The solution set is $\left[\frac{2}{3}, 2\right]$.

76. The given equation $\left|\dfrac{x+1}{x-1}\right| = 5$ implies that either $\dfrac{x+1}{x-1} = 5$ or $\dfrac{x+1}{x-1} = -5$. Solving the first equation, we have $x + 1 = 5(x - 1) = 5x - 5$, $-4x = -6$, and $x = \frac{3}{2}$. Similarly, we solve the second equation and obtain $x + 1 = -5(x - 1) = -5x + 5$, $6x = 4$, and $x = \frac{2}{3}$. Thus, the two values of x that satisfy the equation are $x = \frac{3}{2}$ and $x = \frac{2}{3}$.

77. $\dfrac{\sqrt{x}-1}{x-1} = \dfrac{\sqrt{x}-1}{x-1} \cdot \dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \dfrac{\left(\sqrt{x}\right)^2 - 1}{(x-1)\left(\sqrt{x}+1\right)} = \dfrac{x-1}{(x-1)\left(\sqrt{x}+1\right)} = \dfrac{1}{\sqrt{x}+1}.$

78. $\dfrac{\sqrt[3]{x^2}}{\sqrt[3]{yz^3}} \cdot \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}} = \dfrac{x}{\sqrt[3]{xyz^3}} = \dfrac{x}{z\sqrt[3]{xy}}.$

79. $\dfrac{\sqrt{x}-1}{2\sqrt{x}} = \dfrac{\sqrt{x}-1}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{x - \sqrt{x}}{2x}.$

80. $\dfrac{3}{1+2\sqrt{x}} \cdot \dfrac{1-2\sqrt{x}}{1-2\sqrt{x}} = \dfrac{3\left(1-2\sqrt{x}\right)}{1-4x}.$

81. $x^2 - 2x - 5 = 0$. Using the quadratic formula with $a = 1$, $b = -2$, and $c = -5$, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}.$$

82. $2x^2 + 8x + 7 = 0$. Using the quadratic formula with $a = 2$, $b = 8$, and $c = 7$, we have $x = \dfrac{-8 \pm \sqrt{64-56}}{4} = \dfrac{-8 \pm \sqrt{8}}{4} = -2 \pm \frac{1}{2}\sqrt{2}$.

83. $2(1.5C + 80) \le 2(2.5C - 20)$. Simplifying, we obtain $1.5C + 80 \le 2.5C - 20$, so $C \ge 100$ and the minimum cost is \$100.

84. $12(2R - 320) \le 4(3R + 240)$. Dividing by 4 and simplifying, we obtain $3(2R - 320) \le 3R + 240$, $6R - 960 \le 3R + 240$, $3R \le 1200$, and finally $R \le 400$. We conclude that the maximum revenue is \$400.

CHAPTER 1 Before Moving On... page 67

1. $2(3x-2)^2 - 3x(x+1) + 4 = 2\left(9x^2 - 12x + 4\right) - 3x^2 - 3x + 4 = 18x^2 - 24x + 8 - 3x^2 - 3x + 4$

$$= 15x^2 - 27x + 12 = 3\left(5x^2 - 9x + 4\right).$$

2. a. $x^4 - x^3 - 6x^2 = x^2\left(x^2 - x - 6\right) = x^2(x-3)(x+2)$.

b. $(a-b)^2 - \left(a^2+b\right)^2 = \left[(a-b) - \left(a^2+b\right)\right]\left[(a-b) + \left(a^2+b\right)\right] = \left(a - b - a^2 - b\right)\left(a + a^2 - b + b\right)$

$$= \left(-a^2 - 2b + a\right)(a)(a+1).$$

3. $\dfrac{2x}{3x^2 - 5x - 2} + \dfrac{x-1}{x^2 - x - 2} = \dfrac{2x}{(3x+1)(x-2)} + \dfrac{x-1}{(x-2)(x+1)} = \dfrac{2x(x+1) + (x-1)(3x+1)}{(3x+1)(x-2)(x+1)}$

$$= \frac{2x^2 + 2x + 3x^2 - 2x - 1}{(3x+1)(x-2)(x+1)} = \frac{5x^2 - 1}{(3x+1)(x-2)(x+1)}.$$

4. $\left(\dfrac{8x^2y^{-3}}{9x^{-3}y^2}\right)^{-1}\left(\dfrac{2x^2}{2y^3}\right)^2 = \dfrac{8^{-1}x^{-2}y^3}{9^{-1}x^3y^{-2}} \cdot \dfrac{2^2x^4}{3^2y^6} = \dfrac{1}{2} \cdot \dfrac{1}{xy} = \dfrac{1}{2xy}$.

5. $2s = \dfrac{r}{s+r}$, so $2s(s+r) = r$, $2s^2 + 2sr = r$, $r(1-2s) = 2s^2$, and $r = \dfrac{2s^2}{1-2s}$.

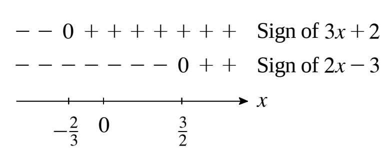
6. $\dfrac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{4 - 4\sqrt{3} + 3}{2^2 - \left(\sqrt{3}\right)^2} = \dfrac{7 - 4\sqrt{3}}{4-3} = 7 - 4\sqrt{3}$.

7. a. $2x^2 + 5x - 12 = 0$, so $(2x-3)(x+4) = 0$. Thus, $x = \frac{3}{2}$ or $x = -4$.

b. $m^2 - 3m - 2 = 0$. Using the quadratic formula with $a = 1$, $b = -3$, and $c = -2$, we obtain $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2} = \dfrac{3 \pm \sqrt{9+8}}{2} = \dfrac{3 \pm \sqrt{17}}{2}$.

8. $\sqrt{x+4} - \sqrt{x-5} - 1 = 0$, so $\sqrt{x+4} - \sqrt{x-5} = 1$, $\sqrt{x+4} = 1 + \sqrt{x-5}$, $x + 4 = 1 + 2\sqrt{x-5} + x - 5$, $8 = 2\sqrt{x-5}$, $4 = \sqrt{x-5}$, $x - 5 = 16$, and $x = 21$.

9. We want to find the values of x for which $(3x+2)(2x-3) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-\frac{2}{3} \le x \le \frac{3}{2}$. The solution set is $\left[-\frac{2}{3}, \frac{3}{2}\right]$.



Applied Mathematics for the Managerial Life 6th Edition Tan Solutions Manual
Full Download: https://alibabadownload.com/product/applied-mathematics-for-the-managerial-life-6th-edition-tan-solutions-manu

10. $|2x + 3| \le 1$ is equivalent to $-1 \le 2x + 3 \le 1$. Thus, $-1 - 3 \le 2x \le 1 - 3$, or $-4 \le 2x \le -2$. We conclude that $-2 \le x \le -1$. The solution set is $[-2, -1]$.

This is sample only, Download all chapters at: AlibabaDownload.com