

CHAPTER 2

2.1

From Fig. (2.2)

For p-type Si,

$$N_A = 1.5 \times 10^{16} \text{ atoms/cm}^3$$

For n-type Si,

$$N_D = 6 \times 10^{15} \text{ atoms/cm}^3$$

2.2

$$R_D = \frac{\rho}{T} = \frac{1 \Omega\text{-cm}}{5 \mu\text{m}} = \frac{1 \Omega\text{-cm}}{5 \times 10^{-4} \text{cm}} = 2000 \Omega/\square$$

2.3

The doping profile has the form

$$N_D(x) = N_{D0} \exp\left(-\frac{x}{L}\right)$$

The first step is to determine

N_{D0} and L . Since at $x=0$,

$$N_D(x) = 10^{17} \text{ cm}^{-3}, \text{ then}$$

$$N_{D0} = 10^{17} \text{ cm}^{-3}$$

$$\text{At } x = 0.5 \mu\text{m}, N_D(x) = \frac{1}{2} N_{D0}$$

$$\therefore N_{D0} e^{-1} = N_{D0} \exp\left(-\frac{0.5 \mu\text{m}}{L}\right)$$

$$\therefore L = 0.5 \mu\text{m}$$

Thus,

$$N_D(x) = 10^{17} \exp\left(-\frac{x}{0.5 \mu\text{m}}\right)$$

The junction will exist at the

point where the net doping

$$N_D(x) - N_A(x) = 0$$

$$\therefore 10^{17} \exp\left(-\frac{x_j}{0.5 \mu\text{m}}\right) = 10^{15}$$

$$\therefore x_j = 0.5 \mu\text{m} \ln \frac{10^{17}}{10^{15}} = 2.3 \mu\text{m}$$

The sheet resistance is given by:

$$R_D = \left[q \mu_n \int_0^{x_j} [N_D(x) - N_A(x)] dx \right]^{-1}$$

Since the effective doping

$$\text{is } N_D(x) - N_A(x)$$

$$\therefore q \left[\frac{800 \text{ cm}^2}{\text{V}\cdot\text{sec}} \right] \int_0^{2.3 \mu\text{m}} \left[10^{17} e^{-\frac{x}{0.5 \mu\text{m}}} - 10^{15} \right] dx$$

$$= q \left[\frac{800 \text{ cm}^2}{\text{V}\cdot\text{sec}} \right] \left\{ \left[-0.5 \times 10^{11} - 2.3 \times 10^{11} \right] + 0.5 \times 10^{13} \right\}$$

$$= 1.6 \times 10^{-19} \times 800 \times 4.72 \times 10^{12}$$

$$= 1.6 \times 8 \times 4.72 \times 10^{-5}$$

$$\therefore R_D = \left[6.04 \times 10^{-4} \right]^{-1} = 1655 \Omega/\square$$

2.4

$$\text{For the resistor } \frac{L}{W} = \frac{200 \mu\text{m}}{5 \mu\text{m}} = 40$$

Base-diffused

$$R = 40 \times 100 \Omega/\square = 4 \text{ k}\Omega$$

Emitter-diffused

$$R = 40 \times 5 \Omega/\square = 200 \Omega$$

Pinch

$$R = 40 \times 5 \text{ k}\Omega/\square = 200 \text{ k}\Omega$$

2.5

From (2.17), for an npn-transistor,

$$Q_B = \frac{A D_n q n_i^2}{I_c} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\text{For } V_{BE} = 520 \text{ mV},$$

$$Q_B = \frac{(10^{-4} \text{ cm}^2)(13 \frac{\text{cm}^2}{\text{sec}})(1.6 \times 10^{-19})(2 \times 10^{20})}{10^{-5}} e^{(520/26)}$$

$$= 2.02 \times 10^{12} \text{ atoms/cm}^2$$

For $V_{BE} = 580 \text{ mV}$, same expression

$$\text{gives: } Q_B = 2 \times 10^{13} \text{ atoms/cm}^2$$

For $Q_B = 2.02 \times 10^{12}$

$$R_{\square} = [q \mu_p Q_B]^{-1}$$

$$= \left[(1.6 \times 10^{-19}) (150 \frac{\text{cm}^2}{\text{V}\cdot\text{sec}}) (2.02 \times 10^{12}) \right]^{-1}$$

$$= (485 \times 10^{-17})^{-1} = 21 \text{ k}\Omega/\square$$

2.6

(a) Series base R :

Emitter periphery adjacent to a base contact is :

$$p = 4 \times 40 \mu\text{m} = 160 \mu\text{m}$$

Distance from base contact to emitter is : $10 \mu\text{m}$

$$\therefore R_B = R_{\square} \left(\frac{10}{160} \right) = 100 \frac{\Omega}{\square} \left(\frac{1}{16} \right)$$

$$= 6.2 \Omega$$

(b) Series collector resistance :

For each of two emitters the effective buried layer dimensions are :

$$W_{BL} = (W + 2T) = 20 \mu\text{m} + 2(10 \mu\text{m})$$

$$= 40 \mu\text{m}$$

$$L_{BL} = (L + 2T) = 40 \mu\text{m} + 2(10 \mu\text{m})$$

$$= 60 \mu\text{m}$$

Using (2.18),

$$a = \left(\frac{20}{40} \right)^{-1}, \quad b = \left(\frac{40}{60} \right)^{-1}$$

$$2r_{c1} = \frac{(5 \Omega\text{-cm})(10 \mu\text{m})}{(20 \mu\text{m})(40 \mu\text{m})} \ln \frac{2/1.5}{0.5}$$

$$= 360 \Omega$$

$$\therefore r_{c1} = 180 \Omega$$

$$r_{c2} = R_{BL\square} \left(\frac{L}{W} \right) = 20 \frac{\Omega}{\square} \left(\frac{40}{140} \right) = 5.7 \Omega$$

r_{c3} : for buried layer

$$L_{\text{eff}} = 140 \mu\text{m}$$

$$W_{\text{eff}} = 14 \mu\text{m} + 2T = 34 \mu\text{m}$$

For top N^+ : $L = 140 \mu\text{m}$
 $W = 14 \mu\text{m}$

$$\therefore a = \frac{34}{14} = 2.43 ; b = 1$$

$$\therefore r_{c3} = \frac{(5 \Omega\text{-cm})(10 \mu\text{m})}{(140 \mu\text{m})(14 \mu\text{m})} \ln \frac{2.43}{2.43-1}$$

$$= 158 \Omega$$

$$\therefore R_c = 158 + 180 + 5.7 = 344 \Omega$$

(c) $C_{jc} = (A_{\text{bottom}} + A_{\text{sidewall}}) C_{jc/\text{area}}$

$$= \left[(140 \mu\text{m})(60 \mu\text{m}) + \frac{\pi \times 3 \mu\text{m}(280 \mu\text{m} + 120 \mu\text{m})}{2} \right] \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2}$$

$$= (8400 + 1885) \times 10^{-4}$$

$$= 1.03 \text{ pF}$$

(d) $C_{je} \approx (C_{je\text{bottom}} + C_{je\text{sidewall}})$

$$= 2 \left[A_{\text{bottom}} + A_{\text{sidewall}} \right] C_{je/\text{area}}$$

$$= 2 \left[(20 \mu\text{m})(40 \mu\text{m}) + \frac{\pi \times 2 \mu\text{m}(40 \mu\text{m} + 80 \mu\text{m})}{2} \right] \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2}$$

$$= 2.4 \text{ pF}$$

(e) $C_{\text{substrate}} = C_{\text{sidewall}} + C_{\text{epi-sub}} + C_{\text{BL-sub}}$

$$C_{\text{sidewall}} = \frac{\pi}{2} \times (17 \mu\text{m}) \left[175 \mu\text{m} \times 2 + 140 \mu\text{m} \times 2 \right] \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2}$$

$$= 1.6 \text{ pF}$$

$$C_{\text{epi-sub}} = (175 \times 140 - 85 \times 126) \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2}$$

$$= 1.4 \text{ pF}$$

$$C_{\text{BL-sub}} = (85 \times 126) \times 3.3 \times 10^{-4} = 3.5 \text{ pF}$$

$$\therefore C_{\text{substrate}} = 6.5 \text{ pF}$$

$$(f) I_S = \frac{q n_i^2}{Q_B / \bar{D}_n} A_{EB}$$

From text example,

$$\begin{aligned} \frac{Q_B}{\bar{D}_n} &= 5.7 \times 10^{11} \text{ cm}^4 \text{ sec} \\ \therefore I_S &= \frac{1.6 \times 10^{-19} \times 2 \times 10^{20} \times 1600 \text{ } \mu\text{m}^2 \times (10^{-8} \frac{\text{cm}^2}{\text{mm}^2})}{5.7 \times 10^{11}} \\ &= 0.9 \times 10^{-15} \text{ A} \end{aligned}$$

2.7

From (2.24), the β falloff begins

$$\text{at } I_E = q A N_D \frac{D_p}{W_B}$$

For this structure, the area A is the product of the emitter diffusion periphery $120 \text{ } \mu\text{m}$, and the effective sidewall depth.

Regarding the sidewall as a quarter-cylinder this effective depth is the emitter junction depth multiplied by $\pi/2$. Thus,

$$A_{\text{eff}} = (120 \text{ } \mu\text{m})(3 \text{ } \mu\text{m})(\pi/2) = 5.65 \times 10^{-6} \text{ cm}^2$$

From Fig.(2.2), the donor density corresponding to a resistivity of $0.5 \Omega\text{-cm}$ is $1.2 \times 10^{16} \text{ cm}^{-3}$. Thus,

$$\begin{aligned} I_c &= \frac{(1.6 \times 10^{-19})(5.65 \times 10^{-6})(1.2 \times 10^{16})(10)}{8 \times 10^{-4}} \\ &= 13.56 \times 10^{-5} = 136 \text{ } \mu\text{A} \end{aligned}$$

2.8

From (2.17), for a pnp transistor

$$Q_B = q A \bar{D}_p \frac{n_i^2}{I_c} \exp\left(\frac{V_{BE}}{V_T}\right)$$

For this device,

$$\begin{aligned} A &= 2 \times (30 \text{ } \mu\text{m} \times 75 \text{ } \mu\text{m}) + 2 \times (10 \text{ } \mu\text{m} \times 30 \text{ } \mu\text{m}) \\ &= 5100 \text{ } (\mu\text{m})^2 \end{aligned}$$

For $V_{BE} = 525 \text{ mV}$

$$\begin{aligned} Q_B &= \frac{1.6 \times 10^{-19} \times 5100 \times 10^{-8} \times 10 \times 2 \times 10^{20}}{10^{-5}} \exp\left(\frac{525}{26}\right) \\ &= 9.6 \times 10^{11} \end{aligned}$$

Using Fig.(2.2), the donor density corresponding to $2 \Omega\text{-cm}$ is

$$2.5 \times 10^{15} \text{ cm}^{-3}$$

Since, $Q_B = W_B N_D$, then

$$\begin{aligned} W_B &= \frac{Q_B}{N_D} = \frac{9.6 \times 10^{11}}{2.5 \times 10^{15}} = 3.84 \times 10^{-4} \text{ cm} \\ &= 3.84 \text{ } \mu\text{m} \end{aligned}$$

Since the p-diffusion depth is $3 \text{ } \mu\text{m}$, the total epi thickness is $6.84 \text{ } \mu\text{m}$, for $V_{BE} = 525 \text{ mV}$. By a similar calculation, the total depth for $V_{BE} = 560 \text{ mV}$ is $17.45 \text{ } \mu\text{m}$.

The same resistances are :

For $V_{BE} = 525 \text{ mV}$

$$\begin{aligned} R_D &= [q \bar{\mu}_n Q_B]^{-1} \\ &= [(1.6 \times 10^{-19})(800)(9.6 \times 10^{11})]^{-1} \\ &= [12 \times 10^{-5}]^{-1} = 8 \text{ k}\Omega/\square \end{aligned}$$

For $V_{BE} = 560 \text{ mV}$

$$R_D = 2.12 \text{ k}\Omega/\square$$

2.9

First consider the $6 \text{ } \mu\text{m}$ resistor, For $10 \text{ k}\Omega$ we need 100 squares or $600 \text{ } \mu\text{m}$ of length. The total

resistor area is that of the body plus clubheads, or

$$A_{\text{bottom}} = 600 \mu\text{m} \times 6 \mu\text{m} + 2(26 \mu\text{m})^2 \\ = 4952 (\mu\text{m})^2$$

The sidewall area is equal to the total periphery multiplied by $(3 \mu\text{m}) (\pi/2)$.

The periphery is,

$$P = 600 \mu\text{m} + 600 \mu\text{m} + 6 \times 26 \mu\text{m} + 2(26 - 6) \mu\text{m} \\ = 1396 \mu\text{m}$$

Thus,

$$A_{\text{sidewall}} = 1396 \mu\text{m} \times 3 \mu\text{m} \times \frac{\pi}{2} \\ = 6578 (\mu\text{m})^2$$

The total area is,

$$A_{\text{total}} = 11,530 (\mu\text{m})^2$$

For the 10^{15} cm^{-3} epi concentration the capacitance per unit area for zero bias is from Fig.(2.29) equal to $10^{-4} \text{ pF}/(\mu\text{m})^2$

$$\text{Thus, } C_{\text{total}} = 11,530 (\mu\text{m})^2 \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} \\ = 1.15 \text{ pF}$$

For the $12 \mu\text{m}$ resistor, we need $1200 \mu\text{m}$ of length.

$$\text{Thus, } A_{\text{bottom}} = 12 \mu\text{m} \times 1200 \mu\text{m} + 2(26 \mu\text{m})^2 \\ = 15,750 (\mu\text{m})^2$$

$$A_{\text{sidewall}} = \frac{\pi}{2} [1200 \times 2 + 26 \times 6 + 2(26 - 12)] \times 3 \\ = 12,177 (\mu\text{m})^2$$

$$\therefore A_{\text{total}} = 27,929 (\mu\text{m})^2$$

$$\therefore C_{\text{total}} = 27,929 (\mu\text{m})^2 \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} = 2.8 \text{ pF}$$

2.10

$$(1) I_S = A_{EB} \frac{q n_i^2}{Q_B} \bar{D}_P b \\ = [90 \mu\text{m} \times 75 \mu\text{m} - 30 \mu\text{m} \times 55 \mu\text{m}] \\ \times \frac{10^{-8} \text{ cm}^2 \times (1.6 \times 10^{-19}) (2 \times 10^{20}) (10 \text{ cm}^2/\text{sec})}{4^2 \left(\frac{10^{15} \text{ atoms}}{\text{cm}^3} \right) \times 14 \mu\text{m} \times 10^{-4} \frac{\text{cm}}{\mu\text{m}}} \\ = 1.17 \times 10^{-14} \text{ A}$$

$$(2) C_{je} = A_{\text{bottom}} \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} \\ + A_{\text{sidewall}} \times 10^{-3} \frac{\text{pF}}{(\mu\text{m})^2} \\ = (90 \times 75 - 30 \times 55) (\mu\text{m})^2 \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} \\ + \frac{\pi}{2} (60 + 110) (3) (\mu\text{m})^2 \times 10^{-3} \frac{\text{pF}}{(\mu\text{m})^2} \\ = 0.51 + 0.8 = 1.31 \text{ pF}$$

$$(3) C_M = C_{\text{epi-sidewall}} + C_{\text{bottom}} \\ = \frac{\pi}{2} (40 \times 2 + 125 \times 2) \times (17) (\mu\text{m})^2 \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} \\ + (140 \times 125) (\mu\text{m})^2 \times 10^{-4} \frac{\text{pF}}{(\mu\text{m})^2} \\ = 1.4 + 1.75 = 3.15 \text{ pF}$$

$$(4) \tau_F = \frac{W_B^2}{2 D_P} = \frac{[(14 \mu\text{m}) (10^{-4} \text{ cm}/\mu\text{m})]^2}{2 \times 10 \text{ cm}^2/\text{sec}} \\ = 98 \text{ nsec}$$

2.11

For an npn transistor

$$Q_B = \frac{q A \bar{D}_n n_i^2}{I_C} \exp\left(\frac{V_{BE}}{V_T}\right) \\ = \frac{(1.6 \times 10^{-19}) (10^{-4} \text{ cm}^2) (13) (2 \times 10^{20}) e^{\frac{480}{26}}}{10^{-5}} \\ = 4.3 \times 10^{11} \text{ atoms/cm}^2$$

$$R_{\square} = [q \mu_p Q_B]^{-1} \\ = \left[1.6 \times 10^{-19} \times 150 \frac{\text{cm}^2}{\text{V-sec}} \times 4.3 \times 10^{11} \right]^{-1} \\ = 97 \text{ k}\Omega/\square$$

In order to contain $4.3 \times 10^{11} \frac{\text{atoms}}{\text{cm}^2}$ of the n-type epi-impurity, the width of the collector depletion layer at punch-thru is,

$$W_D = \frac{4.3 \times 10^{11} \text{ atoms/cm}^2}{10^{15} \text{ cm}^{-3}} = 4.3 \times 10^{-4} \text{ cm} = 4.3 \mu\text{m}$$

Using (1.15) and assuming that $N_A \gg N_D$,

$$W_D = \sqrt{\frac{2\epsilon(\psi_0 + \psi_R)}{qN_D}} = 4.3 \mu\text{m}$$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12} \times (0.55 + V_R)}{1.6 \times 10^{-19} \times 10^{15}}}$$

$$\therefore V_R = 13.7 \text{ V}$$

For $V_{BE} = 560 \text{ mV}$, same expressions

give, $Q_B = 9.4 \times 10^{12} \text{ atoms/cm}^2$

$$R_D = 4.43 \text{ k}\Omega/\square$$

$$W_D = 94 \mu\text{m}$$

$$V_R = 6,796 \text{ V}$$

2.12

From (1.157),

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$100 \mu\text{A} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (1.5 - V_t)^2 \rightarrow (1)$$

$$10 \mu\text{A} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (0.8 - V_t)^2 \rightarrow (2)$$

Divide (1) by (2) and solve for $V_t \rightarrow$ gives $V_t = 0.48 \text{ V}$.

Substituting for V_t in (1) gives,

$$\mu_n C_{ox} \frac{W}{L} = 191 \mu\text{A/V}$$

$$2.13 \quad V_t = \phi_{Ms} + 2\phi_f + \frac{Q_b}{C_{ox}} - \frac{Q_{ss}}{C_{ox}}, \quad (2.27)$$

(i) For unimplanted transistors,

From Table 2.1, $\phi_{Ms} = -0.1 \text{ V}$

From (2.28),

$$\phi_f = \frac{KT}{q} \ln \frac{N_A}{n_i} = 0.026 \ln \frac{10^{16}}{1.45 \times 10^{10}} = 0.35 \text{ V}$$

From (2.30),

$$C_{ox} = 0.86 \text{ fF}/(\mu\text{m})^2$$

$$\frac{Q_{ss}}{C_{ox}} = \frac{1.6 \times 10^{-19} \times 10^{11}}{8.6 \times 10^{-8}} = 0.19 \text{ V}$$

From (1.137),

$$Q_{bo} = \sqrt{2qN_A \epsilon \cdot 2\phi_f}$$

$$= [2 \times 1.6 \times 10^{-19} \times 10^{16} \times 11.6 \times 8.86 \times 10^{-14} \times 2 \times 0.35]^{1/2} = 4.8 \times 10^{-8} \frac{\text{Coulombs}}{\text{cm}^2}$$

$$\therefore V_t = -0.1 - 2 \times 0.35 - \frac{4.8 \times 10^{-8}}{8.6 \times 10^{-8}} - 0.19 = -1.55$$

(ii) For implanted transistors,

$$N_A = 1 \times 10^{16} - 0.9 \times 10^{16} = 10^{15} \text{ cm}^{-3}$$

From Fig.(2.29), the depletion layer width corresponding to a doping level of 10^{15} cm^{-3} is $\sim 1 \mu\text{m}$, while the implant depth is only $0.3 \mu\text{m}$

$$\therefore V_{t(\text{implant})} = -1.55$$

$$+ \frac{0.9 \times 10^{16} \times 0.3 \times 10^{-4} \times 1.6 \times 10^{-19}}{8.6 \times 10^{-8}}$$

$$= -1.05$$

2.14

The metallurgical channel length is

$$L = L_{\text{drawn}} - 2L_d = 7 \mu\text{m} - 2(0.3) = 6.4 \mu\text{m}$$

The effective channel length is L minus the width of the depletion region at the drain. In the active region, the voltage at the drain end of the channel is $= V_{GS} - V_t = V_{OV}$

To calculate V_{OV} , assume at first that $L_{\text{eff}} \approx L$. Then from (1.166)

$$V_{OV} = \sqrt{\frac{2I_D}{k' \frac{W}{L}}} = \sqrt{\frac{2(10 \mu\text{A})}{700 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} \cdot 0.86 \frac{\text{fF}}{\mu^2} \left(\frac{100}{6.4}\right)}} = 0.15 \text{ V}$$

Thus, the voltage across the drain depletion region $= 5 - 0.15 = 4.85 \text{ V}$

To estimate the depletion-region width, assume it is a one-sided step junction that mainly exists in the lightly doped side. Since the channel and the drain are both n-type regions, the built-in potential of the junction is near zero. Using (1.14) and assuming $N_D \gg N_A$,

$$X_d = \sqrt{\frac{2\epsilon(V_{DS} - V_{OV})}{qN_A}} = \sqrt{\frac{2(1.04 \times 10^{-12})(5 - 0.15)}{1.6 \times 10^{-19}(2 \times 10^{16} + 10^{15})}} = 0.55 \mu\text{m}$$

$$\text{So, } L_{\text{eff}} = 7 \mu\text{m} - 2(0.3 \mu\text{m}) - 0.55 \mu\text{m} = 5.85 \mu\text{m}$$

Therefore, since $r_o = \frac{1}{\lambda I_D} = \frac{L_{\text{eff}}}{I_D} \frac{dX_d}{dV_{DS}}$

$$\Rightarrow r_o = 5 \text{ M}\Omega = \frac{5.85 \mu\text{m}}{10 \mu\text{A}} \frac{dX_d}{dV_{DS}}$$

So, $\frac{dX_d}{dV_{DS}} = 0.12 \frac{\mu\text{m}}{\text{V}}$. Since, the other device uses the same technology, $\frac{dX_d}{dV_{DS}}$ is unchanged. But, should calculate V_{OV} for 2nd transistor,

$$V_{OV} = \sqrt{\frac{2I}{k' \frac{W}{L}}} = \sqrt{\frac{2(30)}{(60.2) \frac{50}{12 - 2(0.3)}}} = 0.48 \text{ V}$$

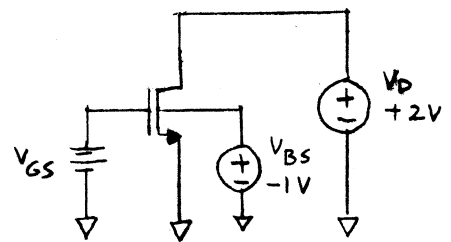
$$\text{So, } X_d = \sqrt{\frac{2(1.04 \times 10^{-12})(5 - 0.48)}{1.6 \times 10^{-19}(2.1 \times 10^{16})}} = 0.53 \mu\text{m}$$

$$L_{\text{eff}2} = 12 - 2(0.3) - 0.53 = 10.87 \mu\text{m}$$

$$r_o = \frac{L_{\text{eff}}}{I_D} \frac{dX_d}{dV_{DS}} = \frac{10.87 \mu\text{m}}{30 \mu\text{A}} \frac{1}{0.12} = 3.02 \text{ M}\Omega$$

Note that using the same V_{OV} as for the 1st transistor (0.15V) gives an answer of 3.1 M Ω . (The change hardly matters)

2.15



First to estimate X_d and L_{eff}

$$L_{\text{eff}} = L_{\text{drawn}} - 2L_d - X_d \rightarrow (1)$$

$$X_d = \sqrt{\frac{2\epsilon_s(V_{DS} - V_{OV})}{q(N_A + N_i)}} \rightarrow (2)$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L_{\text{eff}}} (V_{GS} - V_t)^2 \rightarrow (3)$$

L_{eff} , X_d and $(V_{GS} - V_t)$ can be

found by solving (1), (2), (3).

To avoid solving non-linear equations, the following procedure is used: find x_d by substituting $L_{\text{drawn}} - 2L_D$ for L_{eff} in (3) and use the result in (2).

From (3),

$$V_{GS} - V_t = \sqrt{\frac{2I_D}{\mu_n C_{ox} W / L_{\text{eff}}}}$$

$$= \sqrt{\frac{2 \times 20 \mu A}{450 \frac{\text{cm}^2}{\text{V} \cdot \text{sec}} \cdot 4.32 \frac{\text{fF}}{(\mu\text{m})^2} \left(\frac{10 \mu\text{m}}{1 \mu\text{m} - 2(0.09 \mu\text{m})} \right)}}$$

$$= 0.130 \text{ V}$$

From (2),

$$x_d = \sqrt{\frac{2 \times 11.6 \times 8.86 \times 10^{-14} (2 - 0.13)}{1.6 \times 10^{-19} (5 \times 10^{15} + 4 \times 10^{16})}}$$

$$= 0.231 \mu\text{m}$$

From (1),

$$L_{\text{eff}} = 1 \mu\text{m} - 2(0.09) \mu\text{m} - 0.231 \mu\text{m}$$

$$= 0.589 \mu\text{m}$$

$$(a) \quad g_m = \sqrt{\frac{2 K' W}{L_{\text{eff}}} I_D}$$

$$= \sqrt{\frac{2 \times 194 \frac{\mu A}{V^2} \times \frac{10}{0.589} \times 20 \mu A}{}}$$

$$= 363 \mu A/V$$

$$(b) \quad \text{From (1.200), } g_{mb} = \frac{\gamma g_m}{2 \sqrt{2\phi_f + V_{SB}}}$$

$$\text{where, } \gamma = \frac{\sqrt{2q\epsilon_{si} N_T}}{C_{ox}} \quad \text{and}$$

N_T is the effective dopant density at the bottom of the channel-substrate depletion layer, it can be

either N_A (substrate doping) or $N_A + N_i$. To determine which one applies, we have to find $x'_{d\text{max}}$, i.e., the depletion depth under the channel,

$$x'_{d\text{max}} = \sqrt{\frac{2\epsilon_{si}(2|\phi_p| + V_{SB})}{q(N_A + N_i)}}$$

But,

$$\phi_p = \frac{kT}{q} \ln \frac{N_A + N_i}{n_i}$$

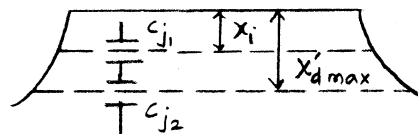
$$= 0.026 \ln \frac{4 \times 10^{16} + 5 \times 10^{15}}{1.45 \times 10^{10}} = 0.389$$

$$\therefore x'_{d\text{max}} = \sqrt{\frac{2 \times 11.6 \times 8.86 \times 10^{-14} (2 \times 0.389 + 1)}{1.6 \times 10^{-19} (4 \times 10^{16} + 5 \times 10^{15})}}$$

$$= 0.225 \mu\text{m}$$

$> 0.16 \mu\text{m}$ (effective implant depth)

Since $x'_{d\text{max}} > x_i$, the depletion capacitor is the series combination of the capacitors as shown,



Now χ can be calculated from the following relationships,

$$\chi = \frac{C_{js}}{C_{ox}}, \quad C_{js} = \frac{C_{j1} C_{j2}}{C_{j1} + C_{j2}}$$

$$C_{j1} = \frac{\epsilon_{si}}{x_i}, \quad C_{j2} = \frac{\epsilon_{si}}{x'_{d\text{max}} - x_i}$$

The actual depletion depth is

$$X_{d\max} = \left[\frac{2 \epsilon_{si}}{q N_A} (\phi_f + |\phi_p| + V_{SB} - x_i^2 \frac{N_i}{N_A}) \right]^{1/2}$$

(cf. Muller and Kamins : Device Electronics for Integrated Circuits, 2nd ed., 1986; equation (10.6.2))

$$\phi_f = \frac{KT}{q} \ln \frac{N_A}{N_i} = 0.026 \ln \frac{5 \times 10^5}{1.45 \times 10^{10}} = 0.33 \text{ V}$$

$$X_{d\max} = \left[\frac{2 \times 11.6 \times 8.86 \times 10^{-14} (0.33 + 0.389 + 1)}{1.6 \times 10^{-19} \times 5 \times 10^{15}} - \frac{(0.16 \times 10^{-4} \text{ cm})^2 (4 \times 10^{16})}{5 \times 10^{15}} \right]^{1/2}$$

$$= 0.453 \text{ } \mu\text{m}$$

$$C_{j1} = \frac{11.6 \times 8.86 \times 10^{-14}}{0.16 \times 10^{-4}} = 6.42 \times 10^{-8} \text{ F/cm}^2$$

$$C_{j2} = \frac{11.6 \times 8.86 \times 10^{-14}}{(0.453 - 0.16) \times 10^{-4}} = 3.51 \times 10^{-8} \text{ F/cm}^2$$

$$C_{js} = 2.27 \times 10^{-8} \text{ F/cm}^2$$

$$\therefore \chi = \frac{2.27 \times 10^{-8}}{4.32 \times 10^{-7}} = 0.0526$$

$$g_{mb} = \chi g_m = 0.0526 \times 363 \times 10^{-6} = 19.1 \frac{\mu\text{A}}{\text{V}}$$

(c) From (2.37),

$$V_{ov} = \sqrt{\frac{2 I_D}{\chi' (W/L_{eff})}} = \sqrt{\frac{2 \times 20 \mu\text{A}}{194 \mu\text{A/V}^2 (10 \mu\text{m}/0.589 \mu\text{m})}}$$

$$= 0.110 \text{ V}$$

$$X_d = \sqrt{\frac{2 \epsilon_{si} (V_{DS} - V_{ov})}{q (N_A + N_i)}}$$

$$\frac{dX_d}{dV_{DS}} = \frac{X_d}{2(V_{DS} - V_{ov})} = \frac{0.231 \mu\text{m}}{2(2 - 0.11)} = 0.061 \mu\text{m/V}$$

$$\therefore r_o = \left(\frac{I_D}{L_{eff}} \frac{dX_d}{dV_{DS}} \right)^{-1} = \left(\frac{20 \mu\text{A}}{0.589 \mu\text{m}} \times 0.061 \mu\text{m/V} \right)^{-1}$$

$$= 482.78 \text{ k}\Omega$$

$$(d) C_{gs} = \frac{2}{3} W L_{eff} C_{ox} + C_{OL}$$

$$= \frac{2}{3} \times 10 \times 0.589 \times 4.32 \times 10^{-15} + 0.35 \times 10^{-15} \times 10$$

$$= 20.5 \text{ fF}$$

$$(e) C_{gd} = C_{OL} = 0.35 \times 10^{-15} \times 10 = 3.5 \text{ fF}$$

$$(f) C_{db} = \frac{C_{j0} \times (\text{drain area}) + C_{jsw0} \times \text{periphery}}{\sqrt{1 + V_{DB}/\phi_0}}$$

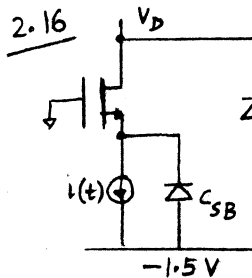
$$= \frac{0.2 \times 10 \times 1 + 1.2 (10 + 1 \times 2)}{\sqrt{1 + (3/0.7)}} \text{ fF}$$

$$= 7.1 \text{ fF}$$

$$(g) C_{sb} = \frac{C_{j0} \times (\text{source area}) + C_{jsw0} \times \text{periphery}}{\sqrt{1 + V_{SB}/\phi_0}}$$

$$= \frac{0.2 \times 10 \times 1 + 1.2 (10 + 1 \times 2)}{\sqrt{1 + (1/0.7)}} \text{ fF}$$

$$= 10.5 \text{ fF}$$



$$V_D(t=0) = V_S(t=0) = 1.5 \text{ V}$$

(i) Initially transistor is off, current source discharges $C_{OL} + C_{SB}$. The rate of voltage change is,

$$\frac{V}{t} = \frac{I}{C}$$

$$\approx \frac{10 \mu\text{A}}{0.35 \frac{\text{fF}}{\mu\text{m}} \times 10 \mu\text{m} + 0.2 \frac{\text{fF}}{\mu\text{m}^2} \times 10 \mu\text{m} \times (1 \mu\text{m}) + 0.09 \mu\text{m} + 1.2 \frac{\text{fF}}{\mu\text{m}} (12 \mu\text{m})}$$

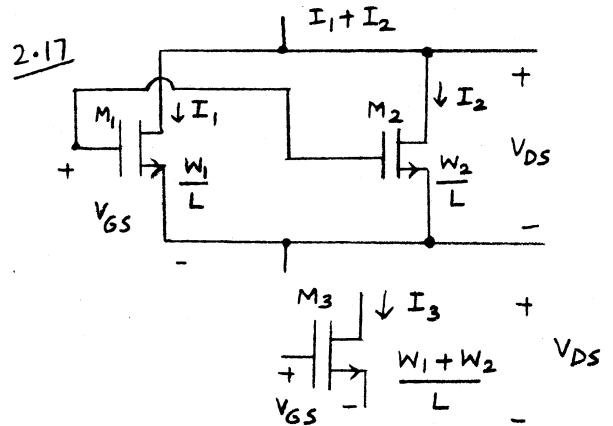
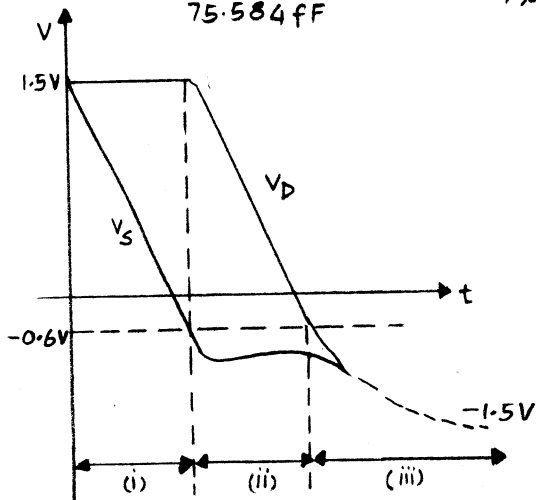
$$= 500 \text{ V}/\mu\text{sec}$$

(ii) Transistor enters saturation region ; drain current starts to flow, discharges $C_{OL} + C_{DB}$ and $\frac{2}{3} C_{ox} WL + C_{OL} + C_{SB}$. The discharge rate of drain voltage is

$$\begin{aligned} \frac{V}{t} &\approx \frac{10 \mu A}{C_{OL} + C_{DB}} \\ &= \frac{10 \mu A}{0.35 \frac{fF}{\mu m} \times 10 \mu m + 0.2 \frac{fF}{\mu m^2} \times 10 \mu m \times (1 \mu m + 0.09 \mu m) + 1.2 \frac{fF}{\mu m} (12 \mu m)} \\ &= 500 V/\mu s \end{aligned}$$

(iii) Transistor enters triode region, current discharges $C_{ox} WL + C_{DB} + C_{SB} + 2C_{OL}$ at a rate of,

$$\begin{aligned} \frac{V}{t} &\approx \frac{10 \mu A}{\{4.32(10)(1-2(0.09)) + 0.2(10)(1+0.09)2 + 1.2(12)2 + 0.35(2)(10)\}} \\ &= \frac{10 \mu A}{75.584 fF} = 132.30 V/\mu s \end{aligned}$$



Note that all the transistors have equal terminal voltages. So,

$$V_{GS1} = V_{GS2} = V_{GS3} = V_{GS}$$

$$V_{DS1} = V_{DS2} = V_{DS3} = V_{DS}$$

$$V_{SB1} = V_{SB2} = V_{SB3} = V_{SB}$$

If $V_{SB} \neq 0$, there is a body effect

$$\text{but } V_{t1} = V_{t2} = V_{t3} = V_t$$

Case 1 : All active

$$I_1 = \frac{K'}{2} \frac{W_1}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_2 = \frac{K'}{2} \frac{W_2}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_1 + I_2 = \frac{K'}{2} \left(\frac{W_1}{L} + \frac{W_2}{L} \right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$\begin{aligned} I_3 &= \frac{K'}{2} \left(\frac{W_1 + W_2}{L} \right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS}) \\ &= I_1 + I_2 \end{aligned}$$

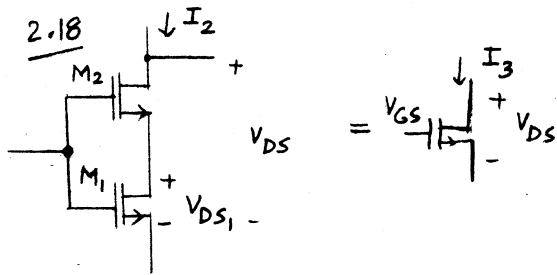
Case 2 : All triode

$$I_1 = \frac{K'}{2} \frac{W_1}{L} [2(V_{GS} - V_t) V_{DS} - V_{DS}^2]$$

$$I_2 = \frac{K'}{2} \frac{W_2}{L} [2(V_{GS} - V_t) V_{DS} - V_{DS}^2]$$

$$I_1 + I_2 = \frac{K'}{2} \left(\frac{W_1}{L} + \frac{W_2}{L} \right) [2(V_{GS} - V_t) V_{DS} - V_{DS}^2]$$

$$\begin{aligned} I_3 &= \frac{K'}{2} \left(\frac{W_1 + W_2}{L} \right) [2(V_{GS} - V_t) V_{DS} - V_{DS}^2] \\ &= I_1 + I_2 \end{aligned}$$



M_2 can operate in active or triode region. M_1 always operates in triode region.

$$I_1 = \frac{k'}{2} \left(\frac{W}{L} \right)_1 [2(V_{GS} - V_t)V_{DS1} - V_{DS1}^2] = I_2$$

Solve for V_{DS1} ,

$$V_{DS1}^2 - 2(V_{GS} - V_t)V_{DS1} + \frac{2I_2}{k'(\frac{W}{L})_1} = 0$$

$$V_{DS1} = \frac{2(V_{GS} - V_t) \pm \sqrt{4(V_{GS} - V_t)^2 - 4 \frac{2I_2}{k'(\frac{W}{L})_1}}}{2}$$

V_{DS1} must be $< V_{GS} - V_t$ or M_1 would be in active region.

$$V_{DS1} = V_{GS} - V_t - \sqrt{(V_{GS} - V_t)^2 - \frac{2I_2}{k'(\frac{W}{L})_1}}$$

Assume M_2 is active,

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 (V_{GS} - V_{DS1} - V_t)^2$$

$$= \frac{k'}{2} \left(\frac{W}{L} \right)_2 \left[V_{GS} - V_t - (V_{GS} - V_t) + \sqrt{(V_{GS} - V_t)^2 - \frac{2I_2}{k'(\frac{W}{L})_1}} \right]^2$$

$$= \frac{k'}{2} \left(\frac{W}{L} \right)_2 (V_{GS} - V_t)^2 - \left(\frac{W}{L} \right)_2 \frac{I_2}{\left(\frac{W}{L} \right)_1}$$

$$I_2 \left(1 + \left(\frac{W}{L} \right)_2 / \left(\frac{W}{L} \right)_1 \right) = \frac{k'}{2} \left(\frac{W}{L} \right)_2 (V_{GS} - V_t)^2$$

$$I_2 = \left[\frac{k'}{2} \left\{ \left(\frac{W}{L} \right)_2 / 1 + \left(\frac{W}{L} \right)_2 / \left(\frac{W}{L} \right)_1 \right\} \right] (V_{GS} - V_t)^2$$

$$= \frac{k'}{2} \left(\frac{W}{L_1 + L_2} \right) (V_{GS} - V_t)^2$$

If M_2 and M_3 are active,

$$I_3 = \frac{k'}{2} \left(\frac{W}{L_1 + L_2} \right) (V_{GS} - V_t)^2 = I_2$$

Assume M_2 is in triode region,

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 [2(V_{GS} - V_{DS1} - V_t)(V_{DS} - V_{DS1}) - (V_{DS} - V_{DS1})^2]$$

$$V_{DS1} = V_{GS} - V_t - \sqrt{(V_{GS} - V_t)^2 - \frac{2I_2}{k'(\frac{W}{L})_1}}$$

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 \left[2 \sqrt{(V_{GS} - V_t)^2 - \frac{I_2}{k'(\frac{W}{L})_1}} \left[V_{DS} - (V_{GS} - V_t) + \sqrt{(V_{GS} - V_t)^2 - \frac{2I_2}{k'(\frac{W}{L})_1}} \right] - \left[V_{DS} - (V_{GS} - V_t) + \sqrt{(V_{GS} - V_t)^2 - \frac{2I_2}{k'(\frac{W}{L})_1}} \right]^2 \right]$$

Let $x = V_{DS} - (V_{GS} - V_t)$ and

$$y = \sqrt{(V_{GS} - V_t)^2 - \frac{I_2}{k'(\frac{W}{L})_1}}$$

Then,

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 [2y(x+y) - (x+y)^2]$$

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 [2xy + 2y^2 - x^2 - y^2 - 2xy]$$

$$I_2 = \frac{k'}{2} \left(\frac{W}{L} \right)_2 [y^2 - x^2]$$

$$= \frac{k'}{2} \left(\frac{W}{L} \right)_2 \left[(V_{GS} - V_t)^2 - \frac{I_2}{k'(\frac{W}{L})_1} - \{V_{DS} - (V_{GS} - V_t)\}^2 \right]$$

$$= \frac{k'}{2} \left(\frac{W}{L} \right)_2 \left[(V_{GS} - V_t)^2 - \frac{I_2}{k'(\frac{W}{L})_1} - V_{DS}^2 + 2V_{DS}(V_{GS} - V_t) - (V_{GS} - V_t)^2 \right]$$

$$I_2 \left(1 + \left(\frac{W}{L} \right)_2 / \left(\frac{W}{L} \right)_1 \right) = \frac{k'}{2} \left(\frac{W}{L} \right)_2 \left\{ -V_{DS}^2 + 2V_{DS}(V_{GS} - V_t) \right\}$$

$$I_2 = \frac{k'}{2} \frac{W}{L_1 + L_2} [2(V_{GS} - V_t)V_{DS} - V_{DS}^2]$$

$$= I_3$$

2.19(a) curve B ;(i) The cost of a 40,000 mil² chip is

$$C = \frac{C_w}{(NY_{ws}) Y_{df} Y_{ft}} = \frac{100}{(47)(10.9)(0.8)} + \frac{0.6}{0.8}$$

$$= 3.71$$

 $NY_{ws} = 47$ is obtained from fig.(2.70)(ii) The cost of two 20,000 mil² chips is,

$$C_{Y_2} = \frac{100}{(200)(0.9)(10.8)} + \frac{0.6}{0.8} = 1.44$$

$$C_{total} = C_{1/2} \times 2 = 2.88$$

∴ Putting the system on two chips is more economical

(b) curve A ;(i) For a 40,000 mil² chip,

$$C = \frac{100}{(115)(0.9)(0.8)} + \frac{0.6}{0.8} = 1.1$$

$$C_{total} = 1.1 \times 2 = 2.2$$

∴ one chip is more economical.

(c) curve C ;(i) For a 40,000 mil² chip,

$$C = \frac{100}{12(0.9)(0.8)} + \frac{0.6}{0.8} = 12.3$$

(ii) For two 20,000 mil² chips,

$$C_{1/2} = \frac{100}{(80)(0.9)(0.8)} + \frac{0.6}{0.8} = 2.49$$

$$C_{total} = 2.49 \times 2 = 4.98$$

∴ using two chips is more economical.

2.20

$$\text{Area} = (150 \text{ mils})^2 = 22,500 \text{ mil}^2$$

(a) curve A ,

$$C = \frac{C_w}{(NY_{ws})(Y_{df})(Y_{ft})} + \frac{C_p}{Y_{ft}}$$

$$= \frac{130}{(360)(0.8)(0.8)} + \frac{0.4}{0.8} = 1.06$$

$NY_{ws} = 360$ is obtained from Fig.(2.70)

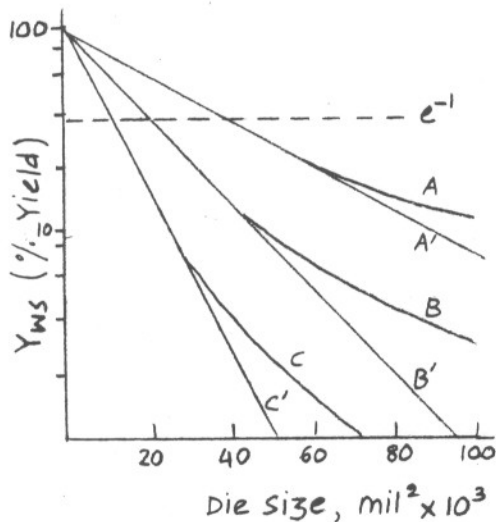
(b) curve B,

$$C = \frac{130}{(140)(0.8)(0.8)} + \frac{0.4}{0.8} = 1.95$$

(c) curve C,

$$C = \frac{130}{(60)(0.8)(0.8)} + \frac{0.4}{0.8} = 3.89$$

2.21

(a) When $Y_{ws} = e^{-1} = 0.37$ From Curve A, $A_0 = 38,000 \text{ mil}^2$ Curve B, $A_0 = 20,000 \text{ mil}^2$ Curve C, $A_0 = 11,000 \text{ mil}^2$ 

Curves A', B' and C' are predicted by the equation.

(b) In fig (2.69), gross die/wafer is inversely proportional to the die size, i.e., $N = KA^{-1}$, K is the proportionality constant related to the wafer size (more specifically, K is the effective or usable area on the wafer). By (2.56), the cost per unit silicon area is,

$$\begin{aligned} \frac{C}{A} &= \frac{C_w}{A N Y_{ws} Y_{df} Y_{ft}} + \frac{C_p}{A Y_{ft}} \\ &= \frac{C_w}{K e^{-1/A_0} Y_{df} Y_{ft}} + \frac{C_p}{A Y_{ft}} \end{aligned}$$

Each K can be obtained from fig (2.69). e.g., $K = 1.15 \times 10^5 \text{ mil}^2$ for 4" wafer.

2.22

First estimate x_d and L_{eff}

$$L_{eff} = L_{drawn} - 2L_d - x_d \quad (1)$$

$$x_d = \sqrt{\frac{2\epsilon_{si}(V_{DS} - V_{ov})}{q(N_A + N_i)}} \quad (2)$$

$$I_D = \frac{k'_n}{2} \frac{W}{L_{eff}} (V_{GS} - V_t)^2 \quad (3)$$

x_d , L_{eff} and $V_{ov} = V_{GS} - V_t$ can be found by solving equations (1), (2) and (3). However, solving these nonlinear equations is difficult. Instead, we approximately solve them. First, estimate x_d by substituting $L_{drawn} - 2L_d$ for L_{eff} in (3) and use that result in (2).

From (3) and using values in Table 2.5:

$$\begin{aligned} V_{ov} &= V_{GS} - V_t = \sqrt{\frac{2I_D}{k'_n W/L_{eff}}} \\ &\approx \sqrt{\frac{2(100\mu A)}{(246\frac{\mu A}{V^2}) \frac{0.9\mu m}{0.2\mu m - 2(0.01\mu m)}}} \\ &= 0.40V \end{aligned}$$

where $k'_n = \mu_n C_{ox} = 300 \frac{cm^2}{V \cdot s} (8.21 \frac{fF}{\mu m^2}) \frac{10^4 \mu m}{1cm}$
 $= 246 \mu A/V^2$ and $C_{ox} = 8.21 \frac{fF}{\mu m^2}$ (based on $t_{ox} = 42$ Angstroms) have been used.

Using (2) and $V_{DS} = 1.0$ V:

$$\begin{aligned} x_d &= \sqrt{\frac{2(11.6)(8.86 \times 10^{-14})(1 - 0.40)}{1.6 \times 10^{-19}(8 \times 10^{16} + 1 \times 10^{17})}} \text{ cm} \\ &= 0.065\mu m \end{aligned}$$

From (1),

$$L_{eff} = 0.2\mu m - 2(0.01\mu m) - 0.065\mu m \approx 0.12\mu m$$

Now we can calculate g_m :

$$\begin{aligned} g_m &= \sqrt{2k'_n(W/L_{eff})I_D} \\ &= \sqrt{2(246\mu A/V^2)(0.9/0.12)100\mu A} \\ &= 607\mu A/V \end{aligned}$$

Next we find r_o :

$$\begin{aligned} r_o &= [(I_D/L_{eff})\partial x_d/\partial V_{DS}]^{-1} \\ &= [(100\mu A/0.12\mu m)0.028\mu m/V]^{-1} \\ &= 43k\Omega \end{aligned}$$

Since the transistor is active (or saturated):

$$\begin{aligned} C_{gs} &= \frac{2}{3}WL_{eff}C_{ox} + C_{ol} \\ &= \frac{2}{3}(0.9\mu m)(0.12\mu m)8.21fF/\mu m^2 \\ &\quad + (0.9\mu m)0.36fF/\mu m \\ &= 0.59fF + 0.32fF = 0.91fF \end{aligned}$$

and

$$\begin{aligned} C_{gd} &= C_{ol} \\ &= (0.9\mu m)0.36fF/\mu m \\ &= 0.32fF \end{aligned}$$

2.23

First estimate x_d and L_{eff}

$$L_{eff} = L_{drawn} - 2L_d - x_d \quad (1)$$

$$x_d = \sqrt{\frac{2\epsilon_{si}(V_{DS} - V_{ov})}{q(N_A + N_i)}} \quad (2)$$

$$I_D = \frac{k'_n}{2} \frac{W}{L_{eff}} (V_{GS} - V_t)^2 \quad (3)$$

x_d , L_{eff} and $V_{ov} = V_{GS} - V_t$ can be found by solving equations (1), (2) and (3). However, solving these nonlinear equations is difficult. Instead, we approximately solve them. First, estimate x_d by substituting $L_{drawn} - 2L_d$ for L_{eff} in (3) and use that result in (2).

From (3) and using values in Table 2.6:

$$\begin{aligned} V_{ov} &= V_{GS} - V_t = \sqrt{\frac{2I_D}{k'_n W/L_{eff}}} \\ &\approx \sqrt{\frac{2(100\mu A)}{(538\frac{\mu A}{V^2}) \frac{0.5\mu m}{0.1\mu m - 2(0.005\mu m)}}} \\ &= 0.27V \end{aligned}$$

where $k'_n = \mu_n C_{ox} = 390 \frac{cm^2}{V \cdot s} (\frac{13.8fF}{\mu m^2}) \frac{10^4 \mu m}{1cm}$
 $= 538\mu A/V^2$ and $C_{ox} = 13.8 \frac{fF}{\mu m^2}$ (based on $t_{ox} = 25$ Angstroms) have been used.

Using (2) and $V_{DS} = 0.6$ V:

$$\begin{aligned} x_d &= \sqrt{\frac{2(11.6)(8.86 \times 10^{-14})(0.6 - 0.27)}{1.6 \times 10^{-19}(1 \times 10^{17} + 5 \times 10^{17})}} \text{ cm} \\ &= 0.027\mu m \end{aligned}$$

From (1),

$$L_{eff} = 0.1\mu m - 2(0.005\mu m) - 0.027\mu m = 0.063\mu m$$

Now we can find g_m :

$$\begin{aligned} g_m &= \sqrt{2k'_n(W/L_{eff})I_D} \\ &= \sqrt{2(538\mu A/V^2)(0.5/0.063)100\mu A} \\ &= 924\mu A/V \end{aligned}$$

Next we calculate r_o :

$$\begin{aligned} r_o &= [(I_D/L_{eff})\partial x_d/\partial V_{DS}]^{-1} \\ &= [(100\mu A/0.063\mu m)0.06\mu m/V]^{-1} \\ &= 10.5k\Omega \end{aligned}$$

Since the transistor is active (or saturated):

$$\begin{aligned} C_{gs} &= \frac{2}{3}WL_{eff}C_{ox} + C_{ol} \\ &= \frac{2}{3}(0.5\mu m)(0.063\mu m)13.8fF/\mu m^2 \\ &\quad + (0.5\mu m)0.10fF/\mu m \\ &= 0.29fF + 0.05fF = 0.34fF \end{aligned}$$

and

$$\begin{aligned} C_{gd} &= C_{ol} \\ &= (0.5\mu m)0.10fF/\mu m \\ &= 0.05fF \end{aligned}$$

The gate-leakage current can be estimated by

$$\begin{aligned} I_G &\approx J_GWL \\ &\approx (1.2 \text{ nA}/\mu m^2)(0.50\mu m)[0.1\mu m - 2(0.005\mu m)] \\ &= 54pA \end{aligned}$$